Rectangular-Waveguide Vector-Network-Analyzer Calibrations With Imperfect Test Ports

Dylan F. Williams, *Fellow, IEEE*

***Abstract*—We present a strategy for correcting for imperfect interfaces between the test ports of a vector network analyzer, the calibration standards and the devices under test. This corrects for the inconsistencies in calibrations introduced by use of flush thrus and flat shorts as calibration standards. The approach is based on equivalent standard definitions that are easy to implement in conventional network analyzers. We present analytic formulas for these definitions and demonstrate them in WR-90 rectangular waveguide.**

***Index Terms*—Calibration, submillimeter wave, vector network analyzer.**

I. Introduction

We develop expressions for "equivalent" vector-network-analyzer (VNA) calibration-standard definitions that correct for imperfections in the calibration standards, the VNA test ports and the device under test. While we have developed methods in the past that correct for imperfections in the calibration standards and test ports of coaxial calibrations [1;2], and some VNA manufacturers have developed proprietary software to accomplish this, the formulation we present here treats this problem in a more explicit way and can be easily used in any table-based VNA calibration software or firmware package. We will demonstrate the approach in WR-90 rectangular waveguide.

The approach is motivated by the difficulty in accurately fabricating rectangular-waveguide calibration standards and test ports for use at submillimeter-wave frequencies. This leads to several problems. First, as mechanical dimensions become more difficult to control, the importance of imperfections in the calibration standards, test ports, and interfaces to the devices under test grows.

Second, it is easier to accurately fabricate flush thrus and flat shorts at these frequencies than it is to fabricate short transmission lines and offset shorts. However, it has long been recognized that the use of flush thrus and flat shorts in VNA calibrations makes the calibration sensitive not only to imperfections in the calibration standards, but also to imperfections in the test ports [3].

This work was supported by the Defense Advanced Research Projects Agency's Terahertz Electronics Program.

D. F. Williams is with the National Institute of Standards and Technology, Boulder, CO 80305 USA (phone: 303-497-3138; e-mail: dylan@ boulder.nist.gov).

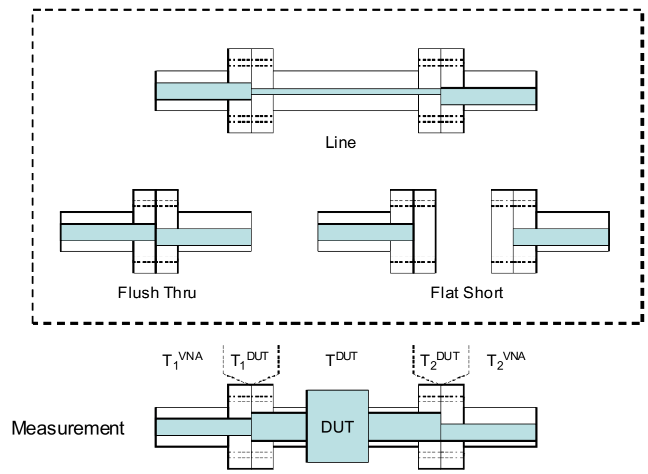

Fig. 1. The calibration and measurement scenario addressed here.

The conventional strategy for use of flush thrus and flat shorts in calibrations is to make the test ports as ideal as possible in order to to minimize their impact on calibration errors. Of course, this becomes difficult at submillimeter-wave frequencies.

The strategy we propose uses simple mechanical measurements to account for the first-order imperfections in the test ports, as well as the calibration standards and interfaces to the device under test. Measurement accuracy is thus improved even when we use flush thrus and flat shorts in the calibrations. The approach is ideally suited to situations in which flush thrus and flat shorts are easier to realize than short transmission lines and offset shorts, and in which aperture sizes, displacements, corner rounding and other mechanical parameters can be more accurately measured than they can be controlled. Although we demonstrate the approach only in rectangular waveguide, it is applicable to other guides as well.

II. Problem Statement

Figure 1 illustrates the problem we address, with rectangular waveguide as an example. The calibration standards consist of a line, a flush thru, and a flat short. The right test port is displaced downward in the flange from its alignment holes and pins. The line standard, while centered in the flange, has a

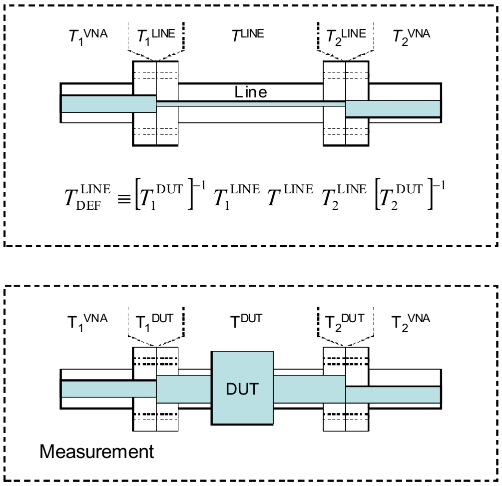

Fig. 2. A transmission-line calibration standard and its equivalent definition.

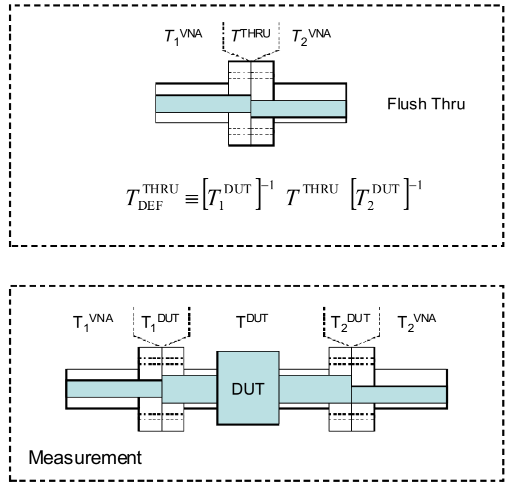

Fig. 3. Flush thru calibration standard and its equivalent definition.

smaller height than the test ports. Finally, the device under test has larger-height access lines than do the test ports. Our goal is to develop a set of equivalent standard definitions that correct for these and other imperfections in the test ports, calibration standards, and interfaces to the device under test, and, after the correction is applied, determine the actual scattering parameters S^{DUT} of the device under test.

After correcting for switch terms and isolation, we measure the scattering parameters S_M^{DUT} of the device under test. The calibration is designed to determine the relationship between the measured scattering parameters S_M^{DUT} and the actual scattering parameters S^{DUT} of the device under test. We can relate T^{DUT} and T_M^{DUT}, the transmission parameters corresponding to S^{DUT} and S_M^{DUT}, by means of

$$T_M^{DUT} = T_1^{VNA} \ T_1^{DUT} \ T^{DUT} \ T_2^{DUT} \ T_2^{VNA} \ . \tag{1}$$

As illustrated in Fig. 1, T_1^{VNA} corresponds to the port 1 correction coefficients of the vector network analyzer to the reference plane just to the left of the left test-port/device-under-test interface, and T_2^{VNA} corresponds to the port 2 correction coefficients of the vector network analyzer to the reference plane just to the right of the right test-port/device-under-test interface. T_1^{DUT} corresponds to the discontinuity at the interface between the left test port and the device under test and T_2^{DUT} corresponds to the discontinuity at the interface between the right test-port and the device under test.

As (1) holds for any device, including the calibration standards, we must define the equivalent definition T_{DEF}^{CS} of a calibration standard so as to satisfy

$$T_M^{CS} = T_1^{VNA} \ T_1^{DUT} \ T_{DEF}^{CS} \ T_2^{DUT} \ T_2^{VNA} \ , \tag{2}$$

where T_M^{CS} correspond to the uncorrected measurements of the calibration standard. This implies that we must define the equivalent definition T_{DEF}^{CS} as

$$T_{DEF}^{CS} \equiv [T_1^{DUT}]^{-1} [T_1^{VNA}]^{-1} \ T_M^{CS} \ [T_2^{VNA}]^{-1} [T_2^{DUT}]^{-1}. \tag{3}$$

We can now easily find the equivalent definition T_{DEF}^{CS} of any calibration standard of interest by writing down the expression for T_M^{CS} and substituting it into (3).

A. Transmission line standard

Figure 2 shows a transmission line standard connected to the two test ports of a VNA. The locations of the various reference planes in the figure can be chosen to simplify the application of (3).

Referring again to Fig. 2, the uncorrected measurement T_M^{LINE} of the transmission line standard can now be written as

$$T_M^{LINE} = T_1^{VNA} \ T_1^{LINE} \ T^{LINE} \ T_2^{LINE} \ T_2^{VNA} \ , \tag{4}$$

where T_1^{LINE} and T_2^{LINE} are the transmission parameters describing the interface between the test ports and the line, and T^{LINE} is the cascade matrix of the line itself. For convenience, we included the impedance transformation between the line in the test port and the calibration standard in T_1^{LINE} and T_2^{LINE}, rather than including them in T^{LINE}. With this definition, T^{LINE} corresponds only to the delay and loss of the transmission line.

We can now substitute (4) into (3) to obtain the equivalent definition

$$T_{DEF}^{LINE} \equiv [T_1^{DUT}]^{-1} T_1^{LINE} \ T^{LINE} \ T_2^{LINE} \ [T_2^{DUT}]^{-1} \tag{5}$$

for the transmission-line standard. As we anticipated above, T_1^{VNA} and T_2^{VNA} cancel out of (5). This leaves a straightforward expression for the equivalent definition of the line standard that we can use directly in table-based VNA calibration models.

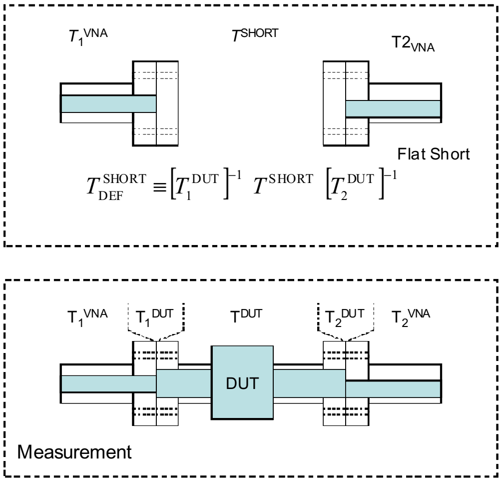

Fig. 4. A flat-short calibration standard and its equivalent definition.

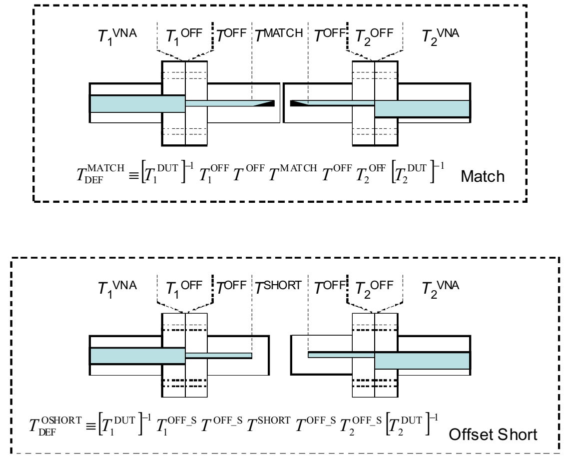

Fig. 5. Offset short and match calibration standards and their equivalent definitions.

B. Flush thru

Figure 3 shows a direct connection of the two test ports of the vector network analyzer, which we call a flush thru. The uncorrected measurement T_M^{THRU} of the flush-thru calibration standard can be written as

$$T_M^{THRU} = T_1^{VNA} \; T^{THRU} \; T_2^{VNA} \;, \tag{6}$$

where T^{THRU} are the transmission parameters describing the interface between the two test ports. T^{THRU} describes any discontinuities between the two test ports when they are connected directly together, including any changes in impedance level.

Substituting (6) into (3), we obtain the equivalent definition

$$T_{DEF}^{THRU} \equiv \left[T_1^{DUT}\right]^{-1} \; T^{THRU} \; \left[T_2^{DUT}\right]^{-1} \tag{7}$$

for the flush thru. Note that the equivalent definition of even a simple flush thru no longer corresponds to a perfect connection (identity matrix) between two lines. The equivalent definition (7) not only takes into account any discontinuities at the interface between the test ports themselves, but also any discontinuities between the imperfect test ports and the interface to the device under test.

C. Flat short and radiating open

Figure 4 shows flat shorts connected to the two test ports of the vector network analyzer. As before, the uncorrected measurement T_M^{SHORT} of the flat-short calibration standard can be written as

$$T_M^{SHORT} = T_1^{VNA} \; T^{SHORT} \; T_2^{VNA} \;, \tag{8}$$

where T^{SHORT} are the transmission parameters of the short-circuited test ports. As there is no transmission between the two ports through T^{SHORT}, some care must be taken to formulate the problem to avoid singularities. An elegant solution based on forming wave vectors based on the reflection coefficients is provided in [4]. Alternatively, Appendix I provides formulas for directly cascading the scattering parameters without the use of transmission parameters.

Substituting (8) into (3) we obtain the equivalent definition

$$T_{DEF}^{SHORT} \equiv \left[T_1^{DUT}\right]^{-1} \; T^{SHORT} \; \left[T_2^{DUT}\right]^{-1} \tag{9}$$

for the transmission-line standard.

Most discontinuities in rectangular waveguide can be described as shunt admittances or impedance transformations. As a short has a zero impedance, applying an impedance transformation to a short or adding a shunt admittance in parallel with a short does not change the impedance of the combination, which remains zero. Thus, in rectangular waveguide, T^{SHORT} corresponds closely to the scattering parameters of a perfect short. Of course, this is a special case, and would not happen if, for example, the test port had discontinuities that could be modeled as a series inductance.

In some instances, a radiating open can also be used as a calibration standard. Like the flush short, the radiating open calibration standard does not have its own interface dimensions. Rather, its definition is completely dependent on the geometry and properties of the test port. Thus (9) applies to radiating open standards as well, with T^{SHORT} replaced by T^{ROPEN}, the reflection coefficient of the radiating open at the test-port interface, and T_{DEF}^{SHORT} replaced by T_{DEF}^{ROPEN}, the equivalent definition of the radiating open.

D. *Offset match and short standards*

Figure 5 shows offset match and short standards connected to the two test ports of the vector network analyzer. Following the approach outlined above, we easily obtain the equivalent definitions

$$T_{\mathrm{DEF}}^{\mathrm{MATCH}} \equiv \left[T_1^{\mathrm{DUT}}\right]^{-1} T_1^{\mathrm{OFF}}\, T^{\mathrm{OFF}}\, T^{\mathrm{MATCH}}\, T^{\mathrm{OFF}}\, T_2^{\mathrm{OFF}} \left[T_2^{\mathrm{DUT}}\right]^{-1} \tag{10}$$

and

$$T_{\mathrm{DEF}}^{\mathrm{OSHORT}} \equiv \left[T_1^{\mathrm{DUT}}\right]^{-1} T_1^{\mathrm{OFF\_S}}\, T^{\mathrm{OFF\_S}}\, T^{\mathrm{SHORT}}\, T^{\mathrm{OFF\_S}}\, T_2^{\mathrm{OFF\_S}} \left[T_2^{\mathrm{DUT}}\right]^{-1}. \tag{11}$$

With this model, T^{OFF} and $T^{\mathrm{OFF\_S}}$ represent the offset transmission line between the interface and the match or short; T^{MATCH} represents the reflection coefficient of the absorbing element embedded in the match standard itself.

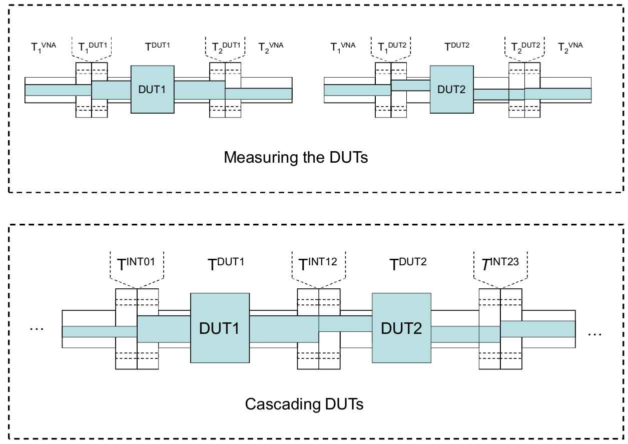

Fig. 6. Illustration of the rigorous calculation of the scattering parameters of the cascade of two or more measured devices.

III. Calibration Reference Planes

The calibration reference plane in our formulation is determined by the manner in which T_1^{DUT} and T_2^{DUT} are defined. There are a number of possible senarios. We examine reference planes set in an ideal transmission line or in the device under test's access line.

A. *Reference plane in an ideal transmission line*

The actual scattering parameters S^{DUT} of the device under test are usually defined with respect to an ideal transmission line centered in the flange and its alignment pins and holes. Of course, this is an approximation, as the actual device under test is never embedded in an ideal transmission line. However, it is often impractical to measure the actual aperture size and displacement of every interface on every device tested, and ideal interfaces on the device under test are probably the best estimates available. This corresponds to the conventional choice of calibration reference planes, and is the goal of most thru-reflect-line (TRL) calibrations, for example.

In this case, the discontinuities T_1^{DUT} and T_2^{DUT} correspond to the transitions between the test ports and an ideal transmission line perfectly centered in the flange. Note that if the test ports were perfect, T_1^{DUT}, T_2^{DUT} and T^{THRU} would reduce to the identity matrix. Thus, T_1^{DUT}, T_2^{DUT}, and T^{THRU} drop out of (1-9), simplifying all of the expressions. *This explains the common wisdom that test ports should be made as ideal as possible when conventional algorithms are used with flush thrus and flat shorts as calibration standards*.

B. *Rigorously setting the calibration reference plane in the access line of the device under test*

The approximations inherent in the conventional choice of a reference plane on an ideal transmission line can be circumvented by placing the reference planes in the actual access lines of the device under test. Of course, this requires measurements of the aperture size and displacement of the access lines with respect to the test port. In this case, the scattering parameters S^{DUT} are defined as the actual scattering parameters of the device under test in its access lines, and the discontinuities T_1^{DUT} and T_2^{DUT} correspond to the transitions between the test ports and the actual access lines on the device under test. This allows the reference plane for the measurements to be moved into test fixtures and placed nearer to the device under test, as was done in [2].

To maintain the accuracy of this approach, some care must be taken to properly include the interfaces between the various access lines when calculating the scattering parameters of a cascade of measured devices. This is because the calibration reference planes are set in the access line of the devices under test, and the interface between the test port and the devices under test are lumped into the VNA error model.

Thus, properly calculating the scattering parameters of a cascade of two or more measured devices together requires inserting the scattering parameters of the appropriate interfaces between the devices access lines between the devices as shown in Fig. 6. The upper box in Fig. 6 shows the measurement configuration. The calibration determines the transmission matrices T^{DUT1} and T^{DUT2}, and these transmission matrices do not include the interface between the test ports and the devices under test. Therefore, to accurately cascade the devices together, the transmission parameters T^{INT12} of the interface between the two devices must be added into the chain to correctly account for any mismatch at their flanges, as shown in the figure.

C. *Placing the reference plane in the middle of the interface*

The reference planes in our formulation are always placed on the device-under-test side of the interface between the test port and the device under test. That is, the scattering-parameters of the interface between the test port and device under test are removed from the measurement by the calibration algorithm, even if the reference plane in the transmission line on the device-under-test side of the interface is mathematically translated through the device-under-test's access line back to a position just on the device-under-test side of the interface. This results in a consistent formalism in which the measured scattering parameters are always defined in a uniform section of transmission line. This is a requirement for the unambiguous definition of equivalent voltages and currents, wave amplitudes, and scattering and other circuit parameters in microwave circuit theory [5].

However, we saw above that rigorously cascading measurements performed in this way requires not only cascading together the measured scattering parameters of the devices, but also cascading the scattering parameters of the interfaces between their access lines. It is logical to ask whether this rigorous process could be simplified by cleverly placing the reference plane in the middle of the interface between the test port and the access line of the device under test. The goal would be to include only the portion of the interface relevant to the device under test in the measured scattering parameters of the device.

In fact, it is generally *not* possible to split the test-port/device-under-test interface in half without introducing some approximation. The difficulty with placing the calibration reference plane in the center of an interface is rooted in the difficulty of consistently defining scattering parameters at a discontinuity that changes when the device is under test is cascaded with another measured device. A simple example based on the quadratic nature of the electrical elements describing E-plane, H-plane and angular displacements at an interface between two rectangular waveguides illustrates some of the difficulties with attempting to do this.

Consider the interface between the two rectangular-waveguide flanges shown in Fig. 7(a). The two flanges and waveguides are perfect, but the waveguide apertures are displaced in the E-plane from the center of their respective flanges and pin and hole patterns by the same amount d. These E-plane displacements are the dominant source of uncertainty in rectangular waveguide calibrations at sub-millimeter wavelengths.

Now recall that an E-plane step between two rectangular waveguides can be described with a shunt capacitance at the interface that is roughly proportional to the *square* of the total displacement between the two waveguide apertures [6]. Thus we can approximate the total capacitance C of an E-plane step as $C \approx C_0\,(d_1\text{-}d_2)^2/d^2$, where C_0 is the capacitance due to an E-plane step of height d. Thus, the total capacitance at the interface between two flanges can only be calculated if the parameters d_1 and d_2 of both flanges are known.

Figure 7 illustrates this difficulty with two flanges whose apertures are offset by equal amounts from the center of the flange. There are two ways of connecting the two flanges in Fig. 7(a) together. If they are connected as shown in Fig. 7(b), there is no net displacement between the two waveguide apertures, $d_1 = d_2 = d$, $C = 0$, and the interface is transparent.

On the other hand, if the second flange was turned upside down, and the two flanges were connected together as shown in Fig. 7(c), the total displacement between the apertures of the two guides is $2d$, $d_1 = d$, $d_2 = -d$, and $C \approx 4C_0$. Thus we see that it is not possible to define any two fixed half models that

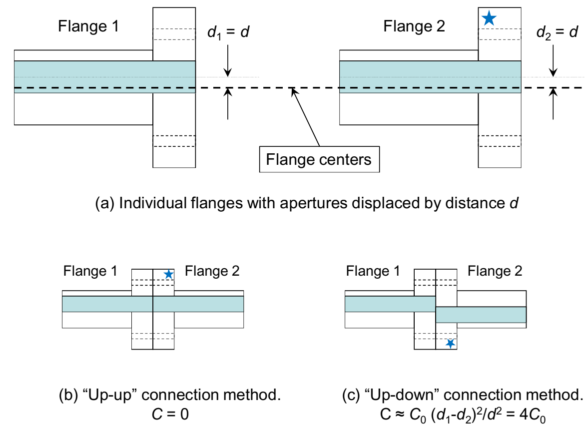

Fig. 7. Connections between two rectangular-waveguide flanges.

will correctly determine the capacitance of the junction, as a calculation of the total capacitance describing the step depends on the offsets of *both* of the flanges.

IV. Closed-Form Expressions for Rectangular Waveguide

To demonstrate our formalism and its advantages, we developed a set of closed-form approximations for the discontinuities at rectangular waveguide interfaces. We first compared analytic approximations from [6-11] to each other, to simulations performed with the Ansoft High-Frequency Structure Simulator (HFSS), and to simulations in the literature to verify their accuracy. After comparing the different approaches, we chose the approximations from Hunter in [7] to approximate the impact of E-plane and H-plane waveguide displacements, the approximations in Marcuvitz [8] to approximate steps in the waveguide width and height, perturbation expressions in Collin [9] to approximate loss in the waveguide, the approximations in Anson [10] to account for impedance changes due to rounded waveguide corners, and the approximations in Brady [11] to determine the cutoff frequency of waveguides with rounded corners. We developed HFSS fits to account for angular displacements, the effective admittance created by rounded waveguide corners, to model radiating opens, and to model the impacts of burrs on radiating open and match standards. We implemented these approximations in the software package [12] for defining equivalent rectangular-waveguide calibration-standard definitions.

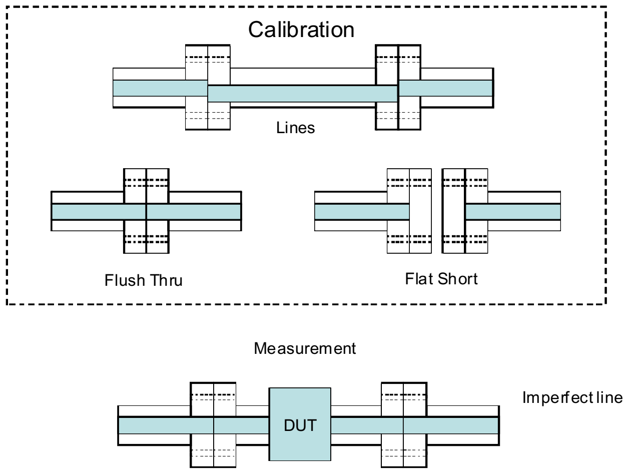

Fig. 8. Calibration with an imperfect transmission-line standard.

V. WR-90 Demonstration

We performed three experiments with WR-90 rectangular waveguide to demonstrate the utility of the equivalent definitions discussed in Section II. In all three cases, we used simple E-plane displacements to test the approach, as these displacements could be easily and accurately introduced into the experiments.

A. *Imperfect line standard*

Figure 8 illustrates the use of a line standard whose aperture is systematically displaced in the E-plane with respect to its flange by 1.53 mm in an otherwise perfect TRL calibration. In this case, only the equivalent definition of the line standard differs from the conventional definition.

We used the calibration comparison method of [13] to investigate the impact of the displaced line on calibration accuracy. The calibration comparison method determines a bound on the worst-case differences of the scattering parameters of two passive devices measured by the two calibrations.

This bound is plotted in the curve shown with triangles in Fig. 9 for a TRL calibration with our imperfect displaced line. Here, the imperfect TRL calibration is compared to a conventional TRL calibration using the same line with no displacement. The figure shows that the 1.53 mm E-plane displacement can introduce errors as high as 0.3 in a conventional TRL calibration.

The bound for a calibration using the equivalent table-based definition for the displaced line is shown with squares in Fig. 9. The figure shows that the worst-case error of the table-based calibration employing the equivalent definition of the displaced line is reduced to about 0.05, approximately one sixth of that of the standard TRL calibration.

Finally, we investigated the magnitude of the error in the table-based calibration due to sources other than the 1.53 mm

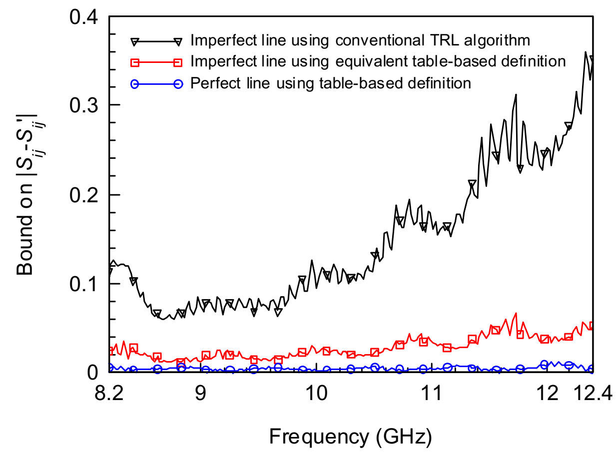

Fig. 9. Calibration with an imperfect transmission-line standard displaced in the E-plane by 1.53 mm.

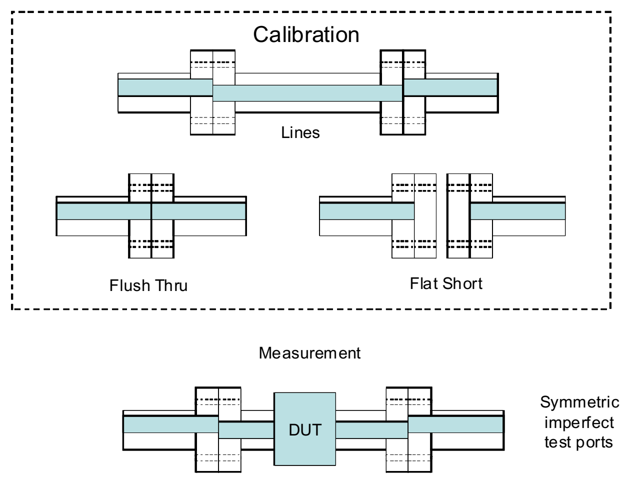

Fig. 10. TRL calibration with symmetric imperfections in the test ports.

E-plane displacement. The curve labeled with circles in Fig. 7 shows the bound for a calibration using the table-based model and the line with no displacement. Here, we see that the table-based model can introduce errors of the order of 0.01 into scattering-parameter measurements of passive devices. This shows that our models are limited in accuracy, and suggests that they are only useful when the systematic errors of the calibration are reasonably large, as was the case in this experiment.

B. *Symmetric test-port displacement*

Figure 10 illustrates the use of test ports with apertures displaced by 1.53 mm in the E-plane from the center of thier flanges in an otherwise perfect TRL calibration. In this case, a conventional calibration would be based on the assumption that there were two steps in the flush thru connection. However, because the two test ports are displaced in the same direction, there is no physical discontinuity at the interface. Thus, the equivalent definition of the flush thru standard differs from its conventional definition, and corresponds to the inverse of the two missing E-plane displacements.

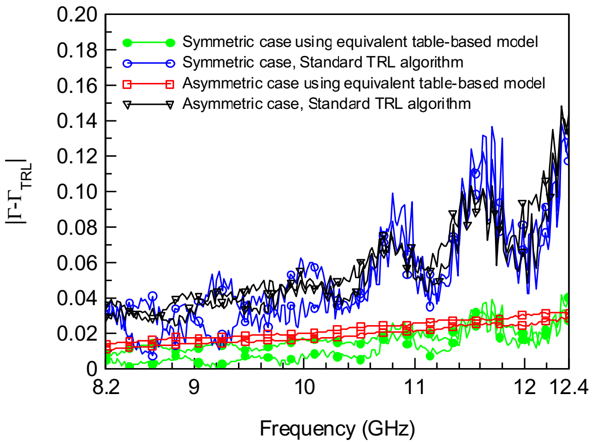

Fig. 11. Measurement of waveguide loads measured with equivalent table-based definitions and the conventional TRL algorithm.

Here, both T_1^{DUT} and T_2^{DUT} correspond to those E-plane displacements of 1.53 mm, while T^{THRU} corresponds to a perfect thru connection. The E-plane displacements are described well by a shunt admittance, and the inverse of these two admittances add together, resulting in an effective definition corresponding to an overall negative admittance of twice the size of the admittance of a single 1.53 mm step in the waveguide.

Figure 11 plots the difference of the reflection coefficient of a load measured by this calibration using the conventional definition and the equivalent definition from (5) to the measurement of the same load with our baseline TRL calibration with no errors. The figure shows clearly that the equivalent definition of the flush thru improves the load measurements significantly. Of course, the formulas we used to evaluate T_1^{DUT} and T_2^{DUT} are approximate, and the corrections are not perfect.

C. *Asymmetric test-port displacement*

Finally, Fig. 12 illustrates the use of test ports with apertures displaced in opposite directions by 1.53 mm in the E-plane from the center of their flanges in an otherwise perfect TRL calibration. Again, only the equivalent definition of the flush thru standard differs from its conventional definition. In this case, the conventional definition would account for two E-plane steps of 1.53 mm. However, the actual discontinuity is a single E-plane step of 3.06 mm.

In the equivalent definition, both T_1^{DUT} and T_2^{DUT} correspond to E-plane displacements of 1.53 mm, while T^{THRU} corresponds to an E-plane displacement of 3.06 mm. Since the admittance due to an E-plane displacement is roughly proportional to the square of the displacement, the admittance captured in T^{THRU} dominates, and the sign of the total admittance required in the equivalent definition of the thru is positive in this case.

Again, Fig. 11 shows that use of the equivalent definition of the flush thru improves the accuracy of the calibration significantly.

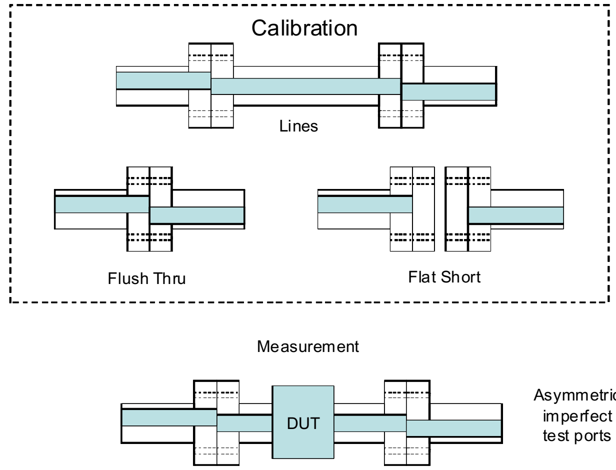

Fig. 12. TRL calibration with asymmetric imperfections in the test ports.

VI. Conclusion

We developed equivalent definitions for calibration standards that correct for imperfections not only in the calibration standards, but also in the test ports. While we demonstrated the approach in WR 90 rectangular waveguide, where we could easily introduce well-controlled imperfections in the waveguide alignment at the interfaces, we expect the approach to be most useful at submillimeter-wave frequencies, where imperfections in the test port can be more easily measured than they can be controlled.

The equivalent definitions we developed are very convenient, and can be easily used with any table-based calibration engine. We also developed a software package [12] that calculates the effective standard definitions described here. The software package is quite flexible, and can be used not only to generate equivalent calibration-standard definitions, but also to generate uncertainties and estimate bias introduced by statistical deviations of the mechanical dimensions of the standards in their equivalent electrical definitions.

VII. Appendix I – Cascading Scattering Parameters

We formulated most of this work in terms of transmission parameters because they cascade easily. That is, if T^{A} and T^{B} are the transmission matrices of circuits A and B, then $T^{\mathrm{AB}} = T^{\mathrm{A}}\, T^{\mathrm{B}}$ is the transmission matrix of the cascade of circuits A and B.

However, the transmission matrix T corresponding to the scattering-parameter matrix S is given by

$$T = \frac{1}{S_{21}} \begin{bmatrix} S_{21}S_{12} - S_{11}S_{22} & S_{11} \\ -S_{22} & 1 \end{bmatrix}, \tag{9}$$

but is not defined when $S_{21} = 0$. Calculating the scattering parameters S^{AB} of the cascade directly with

$$S^{AB} = \begin{bmatrix} S_{11}^{A} + \dfrac{S_{11}^{B} S_{21}^{A} S_{12}^{A}}{1 - S_{11}^{B} S_{22}^{A}} & \dfrac{S_{12}^{A} S_{12}^{B}}{1 - S_{11}^{B} S_{22}^{A}} \\ \dfrac{S_{21}^{A} S_{21}^{B}}{1 - S_{11}^{B} S_{22}^{A}} & S_{22}^{B} + \dfrac{S_{22}^{A} S_{21}^{B} S_{12}^{B}}{1 - S_{11}^{B} S_{22}^{A}} \end{bmatrix} \tag{9}$$

provides a convenient alternative to multiplying transmission matrices. The formulation suggested in [4] provides another alternative.

REFERENCES

[1] A. Lewandowski, D. F. Williams, P. D. Hale, C. M. Wang, and A. Dienstfrey, "Covariance-based VNA Uncertainty Analysis for Time- and Frequency-Domain Measurements," *IEEE Trans. Microwave Theory Tech.*, vol. 58, no. 7, pp. 1877-1886, July 2010.

[2] D. F. Williams and D. K. Walker, "Compensation for Geometrical Variation in Coplanar Waveguide Probe-tip Calibration," *IEEE Microwave and Wireless Comp. Lett.*, vol. 7, no. 4, pp. 97-99, Apr. 1997.

[3] J. R. Juroshek, C. A. Hoer, and R. F. Kaiser, "Calibrating network analyzers with imperfect test ports," *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 4, pp. 898-901, Aug. 1989.

[4] W. Wiatr and D. K. Walker, "Systematic errors of noise parameter determination caused by imperfect source impedance measurement," *IEEE Trans. Inst. Meas.*, vol. 54, no. 2, pp. 696-700, Jan. 2005.

[5] R. B. Marks and D. F. Williams, "A general waveguide circuit theory," *J. Res. Nat. Instit. Standards and Technol.*, vol. 97, no. 5, pp. 533-562, Sept. 1992.

[6] D. Bannister, E. J. Griffin, and T. E. Hodgetts, "On the dimensional tolerances of rectangular waveguide for reflectrometry at millimetric wavelenghts," National Physical Laboratory,1989.

[7] J. D. Hunter, "The displaced rectangular waveguide junction and its use as an adjustable reference reflection," *IEEE Trans. Microwave Theory Tech.*, vol. 32, no. 4, pp. 387-394, Apr. 1984.

[8] N. Marcuvitz, *Waveguide handbook*. New York: McGraw-Hill, 1951.

[9] R. E. Collin, *Foundations for Microwave Engineering* McGraw-Hill, Inc., 1966.

[10] W. J. Anson, R. W. Beatty, D. M. Kerns, and W. T. randy, "Investigations of the reflection from a junction of an ideal rectangular waveguide with one having rounded inside corners," *PGMTT National Symposium Digest*, vol. 62, no. 1, pp. 27-31, 1962.

[11] M. M. Brady, "Cutoff wavelengths and frequencies of standard rectangular waveguides," *Electronics Letters*, pp. 410-412, 1969.

[12] D. F. Williams, "Rectangular-waveguide calculator," National Institute of Standards and Technology, http://www.boulder.nist.gov/dylan, 2010.

[13] D. F. Williams, R. B. Marks, and A. Davidson, "Comparison of On-Wafer Calibrations," *Automatic RF Techniques Group Conference Digest*, vol. 38, pp. 68-81, Dec. 1991.