COMPLETE WAVEFORM CHARACTERIZATION AT NIST*

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Abstract

We present a method for calibrating the voltage that a pulse generator produces at a load at every point in the measured waveform epoch. The calibration includes an equivalent circuit model of the generator so that the user can calculate how the generator will behave when it is connected to different instruments. The calibration also includes a covariance-based uncertainty analysis that provides the uncertainty of each sample and the correlations between the uncertainties at the different time points. Given the calibrated waveform and its covariance matrix, various pulse parameters and their uncertainties can also be calculated.

Introduction

Pulse generators and the signals they produce are often used to characterize various electronic devices used in the aerospace, communications, test equipment, and computer industries. In these measurements, it is sometimes adequate to neglect impedance mismatch and to quantify the response of the system under test in terms of pulse parameters, such as transition duration [1]. However, in some applications, more detailed characterization of the signal features is necessary, such as when comparing measurement systems or when checking the fidelity of a high-speed digital system. Furthermore, when the measurement bandwidth increases into the microwave and millimeter wave region, electrical devices are rarely matched to 50 Ω. At these frequencies, impedance mismatch, loss, and dispersion must be accounted for because they can affect the signal measurement in different ways when the generator is connected to different instruments.

We describe a methodology for measuring a repetitive pulsed waveform and calibrating the measurements to obtain the voltage or current that the signal generator produces at an arbitrary load at calibrated times in the measured waveform epoch. The calibration is traceable to fundamental physics through the electro-optic sampling system at the National Institute of Standards and Technology (NIST). Furthermore, the calibration includes a novel covariance-matrix-based uncertainty analysis that describes the uncertainty throughout the measured epoch. By use of the calibrated waveform and its covariance matrix, pulse parameters and their uncertainty can be obtained when linear or nearly linear transformations between the waveform and the pulse parameters are available [2].

Apparatus and calibration

To completely characterize the waveform, we consider errors in both the time and the voltage in our measurements. We measure a step-like waveform with amplitude of about 250 mV and transition duration of about 15 ps to illustrate our methods.

Our apparatus, shown in Fig. 1, and measurement procedures are designed to take advantage of the NIST timebase correction algorithm [3] to correct for timing errors in the measurement system. The signal generator produces sine waves that are measured on channels 1 and 2. The prescaler produces a fast transition that is used to trigger the oscilloscope and the pulse generator, which is measured on channel 3 simultaneously with channels 1 and 2. Because all of the samplers in the oscilloscope are activated by the same trigger pulse, the timing errors in all the channels in the oscilloscope are nearly identical. The timebase correction algorithm fits the sinusoids measured on channels 1 and 2 and estimates the timing error in their measurement. This estimate is then used to correct the timing error in one measurement of the pulse generator. We acquired 100 such waveforms, interpolated them on to a regular time grid, and then averaged them to obtain a

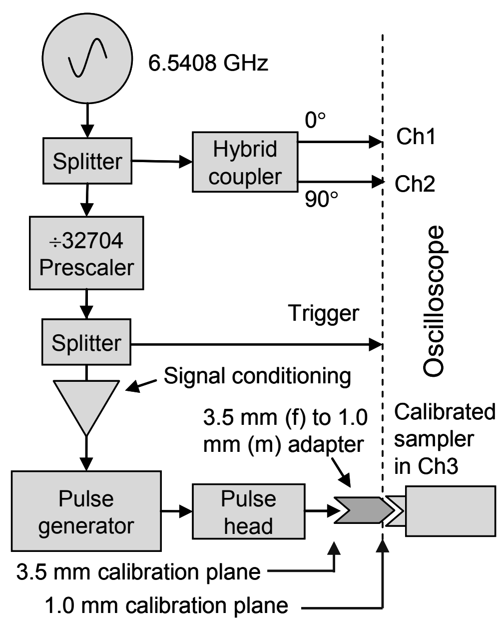

Fig. 1. Schematic diagram of the measurement apparatus.

measurement $\mathbf{y}$.

We corrected for static voltage errors by using the built-in dc calibration of the oscilloscope. We estimated the impulse response of the oscilloscope using the methods described in [4-6]. We also added a 1.0 mm adapter to our oscilloscope to provide an interface that is single-mode to 110 GHz, well above the frequency at which the pulses we are measuring contain significant energy.

We measured the scattering parameters of the 1.0 mm to 3.5 mm adapter in Fig. 1 up to 33 GHz, and extended the measurements to 110 GHz using an empirical model. We also measured the reflection coefficient of the generator directly to 33 GHz and extended this to 110 GHz using measurements of the reflection coefficient of the generator and adapter at the 1.0 mm connector. Based on these measurements, we estimated the (system) response $\mathbf{A}$ of the oscilloscope and adapter at the generator's 3.5 mm connector, accounting for the oscilloscope response and mismatch corrections to 110 GHz.

We deconvolved the system response from the measurement $\mathbf{y}$ using Tikhonov regularization with a second-difference roughness penalty. The regularization parameter $\lambda = \lambda^*$ that balances the roughness and the least-squares error $\|\mathbf{A}\mathbf{x}_\lambda - \mathbf{y}\|^2$ is found at the maximum curvature point in the L-curve [7]. With this parameter fixed we performed the deconvolution to obtain the estimated signal x_{λ^*}.

Uncertainty analysis

We propagated uncertainties using a covariance-matrix based formalism [4]. We refer to the process described above as a procedure $\mathbf{x}_\lambda = D(\mathbf{A}, \mathbf{y}, \lambda)$, and determine the sensitivity of D to its various arguments by calculating the Jacobians

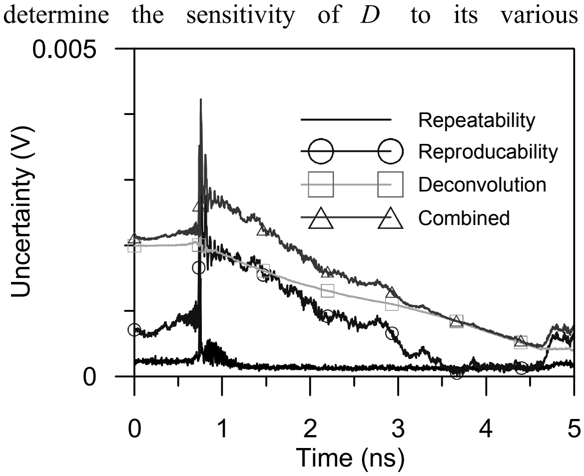

Fig. 2 Components of uncertainty contributing to combined uncertainty in the estimated signal. The pulse transition from about -0.25 V to 0.0 V occurs at about 0.8 ns.

$$\mathbf{J}_\mathbf{A} = \frac{\partial D}{\partial \mathbf{A}}, \quad \mathbf{J}_\mathbf{y} = \frac{\partial D}{\partial \mathbf{y}}, \quad \mathbf{J}_\lambda = \frac{\partial D}{\partial \lambda}. \quad (1)$$

The uncertainty in the estimated signal due to deconvolution is

$$\Sigma_{x_{\lambda^*}} = \mathbf{J}_\mathbf{A}\Sigma_\mathbf{A}\mathbf{J}_\mathbf{A}^\mathrm{T} + \mathbf{J}_\mathbf{y}\Sigma_\mathbf{y}\mathbf{J}_\mathbf{y}^\mathrm{T} + \mathbf{J}_\lambda\sigma_\lambda^2\mathbf{J}_\lambda^\mathrm{T}, \quad (2)$$

where $\Sigma_\mathbf{A}$ and $\Sigma_\mathbf{y}$ are the covariance matrices of the system response function [4] and the mean waveform $\mathbf{y}$. The 1×1 matrix σ_λ^2 is the squared uncertainty of the regularization parameter λ.

We made measurements with three different 1.0 mm to 3.5 mm adapters and oscilloscope samplers to account for the effects of possible high-order modes in the 3.5 mm connectors and discrepancies in the oscilloscope calibrations. We made four repeat measurements with each adapter/sampler combination, giving a total of 36 measurements. We aligned the measurements in time and used the covariance approach to characterize the repeatability within each set of four measurements for a given configuration and the reproducibility of the measurements between different configurations. The combined covariance is the sum of these matrices and $\Sigma_{x_{\lambda^*}}$. The square root of the diagonal elements of the covariance matrices are their uncertainty contribution at a given time, as shown in Fig. 2. Using the estimated signal and combined covariance matrix, we estimated the 10 % to 90 % transition duration as 14.9 ± 0.5 ps [2]. Repeating this measurement for other generators we find the uncertainty in 10 % to 90 % transition duration to be less than 0.7 ps.

References

[1] *IEEE Std 181-2003 IEEE Standard on transitions, pulses, and related waveforms,* IEEE, New York, 2003.

[2] P. D. Hale and C.M. Wang, "Measurement of pulse parameters and propagation of uncertainty," submitted to *IEEE Instrum. Meas.*, Sept. 2007.

[3] P. D. Hale, D. F. Willaims, K. A. Remley, and J. Wepman, "Compensation of random and systematic timing errors in sampling oscilloscopes," *IEEE Trans. Instrum. Meas.*, vol. 55, no. 6, pp. 2146-2154, Dec. 2006.

[4] D. F. Williams, A. Lewandowski, T. S. Clement, C. M. Wang, P. D. Hale, J. M. Morgan, D. A. Keenan, and A. Dienstfrey, "Covariance-based uncertainty analysis of the NIST electro-optic sampling system," *IEEE Trans. Microwave Theory Tech.*, vol. 54, no. 1, pp. 481-491, Jan. 2006.

[5] D. F. Williams, T.S. Clement, P.D. Hale, and A. Dienstfrey, "Terminology for High-Speed Sampling-Oscilloscope Calibration," 68th ARFTG Microwave Measurements Conf. Digest, Boulder, CO, Nov. 30-Dec. 1, 2006.

[6] T. S. Clement, et al., "Calibration of sampling oscilloscopes with high-speed diodes," *IEEE Trans. Microwave Theory and Tech.*, vol. 54, no. 8, pp. 3173-3181, Aug. 2006.

[7] P. C. Hansen, *Rank-deficient and discrete ill-posed problems*, SIAM, Philadelphia, 1997.