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LRM Probe-Tip Calibrations using Nonideal Standards

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Abstract—**The line-reflect-match calibration is enhanced to accommodate imperfect match standards and lossy lines typical of monolithic microwave integrated circuits. We characterize the match and line standards using an additional line standard of moderate length. The new method provides a practical means of obtaining accurate, wideband calibrations with compact standard sets. Without the enhancement, calibration errors due to imperfections in typical standards can be severe.**

I. Introduction

This paper, which has been presented in conference [1], shows how line-reflect-match (LRM) calibrations of microwave probe stations can be extended to cases in which the match and line standards are imperfect.

Eul and Schiek [2] introduced LRM as an alternative to the thru-reflect-line (TRL) calibration [3]. They noted that the LRM calibration sets the reference impedance to the impedance of the match standard, which is generally unknown except at dc. This is further discussed in [4].

More recently, Barr and Pervere [5] studied the LRM calibration and noted that a characterization of the lossy line is also necessary in order to translate the reference plane. They did not suggest a means of performing this characterization, however. Davidson, *et al.* [6] applied the LRM technique with the intent of obtaining a probe-tip calibration, that is, a probe-station calibration with reference plane near the probe tips and reference impedance of 50 Ω. As a match standard, these authors used resistors trimmed to a dc resistance of 50 Ω. They attempted to determine the resistor reactance and concluded that it was small. They achieved the reference plane translation by using a very short low-loss line standard, estimating its parameters from lossless approximations. These implementations of the LRM calibration are therefore limited to ideal match standards and to short low-loss line standards.

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In [7], Davidson, *et al.* introduced a procedure which attempts to determine and account for the reactance of the planar resistors they used as match standards. They achieved this by introducing a lossless reflect into the calibration. This method is still limited to match standards with a frequency-independent resistance and with a reactance due only to a frequency-independent inductance, to short low-loss line standards, and to lossless reflects.

The multiline TRL calibration [8] does not suffer from these limitations. Because it is based on the TRL algorithm, it measures the ratios of traveling waves in the transmission lines [4]. The bandwidth and accuracy of the calibration are increased over conventional TRL by the use of multiple lines. The calibration also measures the propagation constant of the line standards so that the calibration reference impedance and the reference plane can be set accurately [9], [10]. The calibration is thus especially well suited to monolithic microwave integrated circuits (MMIC's), in which wide bandwidth is needed and small geometries result in very lossy lines with a complex frequency-dependent characteristic impedance.

The multiline TRL calibration suffers one important drawback, however. To obtain a wide measurement bandwidth, a set of lines, some quite long, is required; this uses expensive space on the wafer. When realized in MMIC form, LRM standards while far more compact than multiline TRL standards, are incompatible with conventional LRM assumptions. Typical imperfections include match standards with process-dependent dc resistance and frequency-dependent resistance and inductance [11], lossy line standards, and lossy reflects, are incompatible with the assumptions of conventional implementations of LRM.

In this paper we show how to modify the LRM calibration to account for the imperfect match and line standards typical of MMIC's. We first study coplanar waveguide (CPW) resistors and lines, evaluating separately their use as match and line standards in LRM probe-tip calibrations. We show that both the real and imaginary parts of the resistor impedance must be known if the LRM reference impedance, which is initially set to the impedance of the match, is to be reset to some standard value (e.g. 50 Ω). We also show that the line loss and characteristic impedance must be considered when setting the reference plane position. Finally, we examine a TRL calibration with a single line moderately longer than the thru line and show that it is accurate enough in practice to characterize the match and line standards. This results in a practical means of obtaining accurate wideband calibrations with a compact standard set consisting of a thru line, a reflect, a match standard, and a second line standard of moderate length.

II. Reference Impedance

For these experiments we constructed a set of CPW calibration artifacts, typical of those found on MMIC's, on a gallium arsenide substrate. The artifacts consisted of a CPW thru line 550 μm long, four longer lines of length 2.685 mm, 3.75 mm, 7.115 mm, and 20.245 mm, and two shorts offset 0.225 mm from the beginning of the line. We also fabricated a match standard by terminating a 275 μm section of the CPW with a single 73 μm by 73 μm nickel-chromium thin-film resistor; the resistor geometry is described in [11]. These artifacts were fabricated with a 0.5 μm evaporated gold film adhered to the 500 μm gallium arsenide substrate with an approximately 50 nm titanium adhesion layer. The lines had a center conductor of width 73 μm separated from two 250 μm ground planes by 49 μm gaps.

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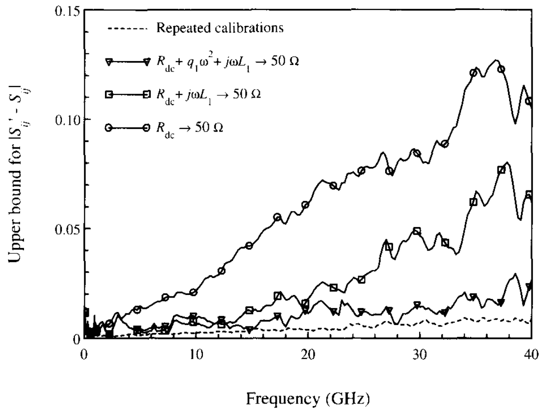

Fig. 1. The maximum possible differences between measurements of passive devices from LRM and our multiline TRL calibration. Each curve corresponds to an LRM calibration to which we applied a different impedance transformation. The dashed curve corresponds to repeated calibrations with identical artifacts.

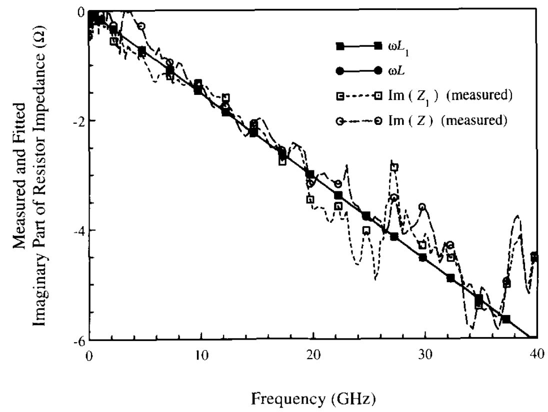

Fig. 2. The imaginary part of the resistor impedance measurements Z and Z_1. The fitted reactances are plotted in solid lines.

We assessed the accuracy of our LRM calibrations by comparing them to a multiline probe-tip TRL calibration [8] using all five lines. The characteristic impedance of the lines was found from the capacitance and propagation constant of the lines, allowing the reference impedance of the TRL calibration to be accurately set to 50 Ω [9]. The capacitance C of the lines was determined from the reflection coefficient and dc resistance of the lumped resistor [10].

We first compared two consecutive multiline TRL calibrations using identical standards in order to assess the limitations on calibration repeatability due to contact error and instrument drift. We used the technique of [12] to determine an upper bound on this repeatability error. The comparison determines the upper bound for $|S_{ij}' - S_{ij}|$ for measurements of any passive device, where S_{ij} is its S-parameter measured with respect to the first calibration and S_{ij}' is its S-parameter measured with respect to the second; the bound is obtained from a linearization which assumes that the two calibrations are similar to first order. The result, plotted as a dashed line in Fig. 1, roughly indicates the minimum deviation between any pair of calibrations.

In order to examine the effect of the imperfect match on the LRM calibration, we compared a simple LRM calibration to the multiline TRL calibration, using the same thru and reflect measurements in both calibrations. We found that the maximum possible difference $|S_{ij}' - S_{ij}|$, where in this case S_{ij}' is the S-parameter measured with respect to the LRM calibration, exceeded 0.8. This large difference is not surprising since the reference impedance of the LRM calibration was equal to the match impedance Z_{match} (with dc resistance Rdc= 91.15 Ω) while the reference impedance of the multiline TRL calibration had been adjusted to 50 Ω. While this difference could have been minimized by fabricating resistors with a dc resistance of 50 Ω, this would have required improved process control and, as will be discussed below, still would not guarantee an accurate calibration at high frequencies, where the resistor impedance may depart significantly from its dc resistance [11].

In a second experiment we applied an impedance transformation that would transform a reference impedance of R_{dc} to one of 50 Ω. This would transform the LRM reference impedance Z_{match} to 50 Ω if and only if $Z_{match} = R_{dc}$. This result is labeled with circles in the figure. A comparison to the dashed line in the figure shows that the maximum possible difference in measurements for this impedance-transformed LRM calibration remains significantly larger than the repeatability of the calibrations. As we will show below, the cause for these significant measurement differences is related to the fact that Z_{match} is not equal to R_{dc}.

In Fig. 2 we plot the imaginary part of Z and Z_1, measurements of Z_{match}; Z was determined by the multiline TRL calibration and Z_1 by a TRL calibration using only the 550 μm thru line and the 2.685 mm line. The figure shows that the resistor has a significant negative reactance not atypical of planar resistors, even those carefully fabricated to have a 50 Ω dc resistance [11]. This reactance varies approximately linearly with frequency. To correct for it, we performed a weighted least-squares fit, using a line through the origin, to the imaginary part of Z and Z_1 to determine the "effective inductances" L and L_1 [11]. In fitting, we used the weighting functions suggested in [8]. Then we tried applying two impedance transformations to the LRM calibration, one that would take an initial reference impedance of $R_{dc} + j\omega L$ to 50 Ω and one that would take an initial reference impedance of $R_{dc} + j\omega L_1$ to 50 Ω. When we compared these two impedance-transformed LRM calibrations to our multiline TRL calibration, we obtained almost exactly the same result in each case. The result for the transformation taking $R_{dc} + j\omega L_1$ to 50 Ω is labeled with squares in Fig. 1. While the maximum possible discrepancy in the LRM measurements was significantly reduced, it was nevertheless still significantly larger than the repeatability of the calibrations.

The lumped-element model of our CPW resistors developed in [11] suggests not only a linear reactance but, at very high frequencies, a quadratically increasing or decreasing resistance. We plot the real parts of Z and Z_1 in Fig. 3, which shows that the real part of the resistor impedance decreases quadratically with frequency. This is also not atypical of planar resistors, even those carefully fabricated to have a 50 Ω dc resistance [11]. We fitted the quadratics $R_{dc} + q\omega^2$ and $R_{dc} + q_1\omega^2$ in the least-squares sense to the real parts of Z and Z_1 using the same weighting as above. Then we tried applying two impedance transformations to the LRM calibration, one that would take an initial reference impedance of $R_{dc} + q\omega^2 + j\omega L$ to 50 Ω and one that would take an initial reference impedance of $R_{dc} + q_1\omega^2 + j\omega L_1$ to 50 Ω. We compared the resulting calibrations to our TRL calibration and again obtained almost exactly the same result for the two cases. The result for the transformation from $R_{dc} + q_1\omega^2 + j\omega L_1$ to 50 Ω is labeled with triangles in Fig. 1. In this case the differences in the LRM measurements are reduced to nearly the level to which we could repeat calibrations. This indicates that any further improvements in setting the reference impedance of the LRM calibration would not significantly improve the accuracy of the calibration.

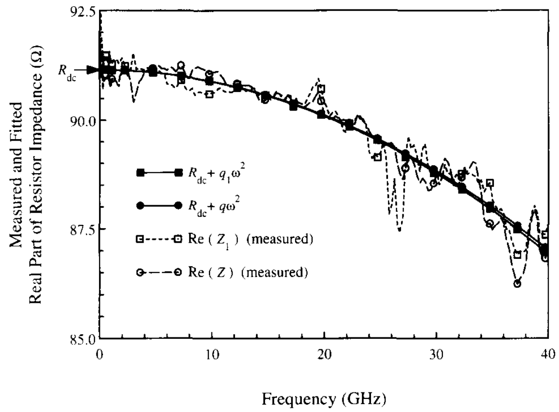

Fig. 3. The real part of the resistor impedance measurements Z and Z_1. The quadratics $R_{dc}+ q\omega^2$ and $R_{dc} + q_1\omega^2$, fitted to Z and Z_1, respectively, are plotted in solid lines.

III. Reference Plane Translation

The reference plane of a probe-tip calibration is located just beyond the probe tips. In our case, we apply a translation of reference plane from the center of the thru line 250 μm toward the probes to bring the reference plane to a position 25 μm in front of the physical beginning of the line. To investigate the effect of line loss on this reference plane translation, we compared our multiline TRL probe-tip calibration to another calibration, identical except that the reference plane translation of the second calibration was accomplished assuming a different effective dielectric constant ϵ_r. In each case, we determined the line characteristic impedance from ϵ_r and the capacitance C of the lines, as described in [9]. C was assumed identical for all cases.

In the first experiment we set ϵ_r to 6.95, the approximate effective dielectric constant obtained from the lossless, thin metal approximation. The maximum possible differences between the LRM and TRL measurements are labeled with circles in Fig. 4 and exceed the repeatability of the calibrations by a significant amount. In the second experiment we set ϵ_r to ϵ_1, the frequency-dependent effective dielectric constant measured by the TRL calibration using only the 550 μm thru line and the 2.685 mm line. The result, labeled by squares in the figure, is less than the repeatability of the calibrations. This indicates that the error introduced into the calibration by determining ϵ_r from a single line is smaller than the repeatability error and is thus of little practical significance.

IV. Probe-Tip Calibrations

Probe-tip calibrations, which have a 50 Ω reference impedance and a reference plane just in front of the physical beginning of the line, require both a reference plane translation and reference impedance transformation. In Fig. 5 we compare several LRM and TRL calibrations to our multiline calibration. The figure shows that differences in measurements using the simple LRM calibration (curve labeled with circles), in which we applied an impedance transformation which would take an initial reference impedance of R_{dc} to 50 Ω and in which ϵ_r was assumed to be 6.95, can be quite large. The maximum possible differences for the single-line TRL calibration (curve labeled with solid squares) are generally small except at low frequencies and near the point where the 2.685 mm line is approximately a half wavelength longer than the thru line, as indicted by the arrow labeled "$\Delta\phi \approx \pi$". By contrast, the measurement differences for the LRM calibration based on the match

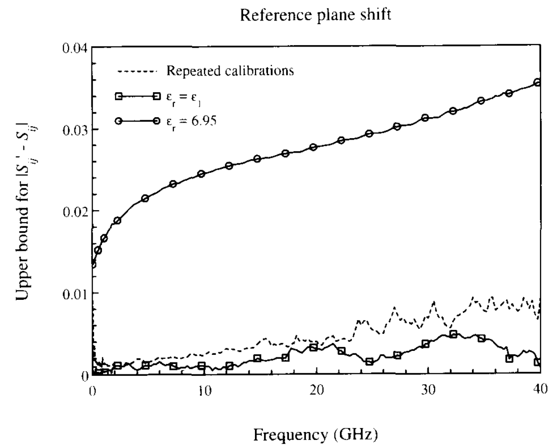

Fig. 4. The maximum possible differences between measurements of passive devices from TRL calibrations in which different effective dielectric constants were used to accomplish the reference plane translations. The dashed curve corresponds to repeated calibrations with identical artifacts.

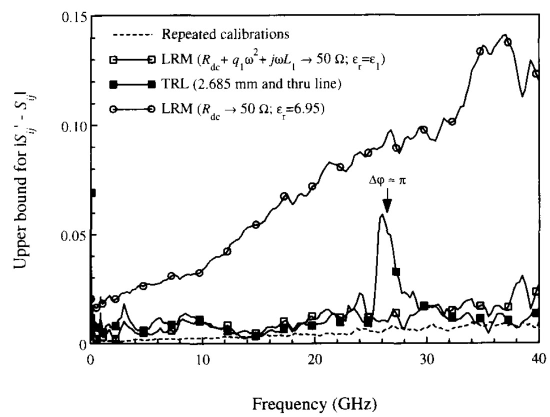

Fig. 5. The maximum possible differences between measurements of passive devices from LRM and TRL calibrations and our multiline TRL calibration. The dashed curve corresponds to repeated calibrations with identical artifacts.

and line standards characterized by the single- line TRL calibration (hollow squares) are never much greater than the repeatability of the calibrations.

V. Conclusions

LRM calibrations can be performed with imperfect CPW artifacts typical of MMIC's with good accuracy. Furthermore, while the imperfections in the match and line standards must be characterized and accounted for, a full multiline TRL calibration is not required for this purpose. In fact, only a line of moderate length need be added to the LRM calibration set. Therefore, accurate broadband LRM calibrations can be achieved using compact sets of calibration artifacts.

The experiments were conducted with well behaved resistors deeply embedded in the CPW line and required only moderate reference plane translations. Thus, the results may be inapplicable to poorly behaved resistors, such as some of those investigated in [11]. The suitability of resistors in microstrip remains to be established. The method may also be inapplicable to resistors placed directly under the probe tips or to calibrations with large reference plane translations.

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