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DESIGNS FOR THE CALIBRATION OF STANDARDS OF MASS

by

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This report presents a collection of designs for the intercomparison of sets of weights for use in precision calibration of standards of mass. These include a number of previously unpublished designs which have an additional weight in each set to serve as the check standard for monitoring the performance of the weighing process. Also included are the classical designs of Benoit and Hayford. The complete least squares analysis is presented in integer form (i.e., with a common division) for the most widely used designs; and for the others, the standard deviations are given for various weight combinations when used as an ascending or as a descending series. Designs for sets of nominally equal objects, the 2 2 . . . 1 1 . . . series, the binary sequences, the 5 2 2 1 1 series, and the 5 3 2 1 1 and some miscellaneous series are given.

Key Words: Design of experiments, least squares, mass calibration, statistical design, weighing design

INTRODUCTION

Calibration of a set of weights consists of assigning values for the unknown weights in terms of the known mass of one or more standards. For high precision work, this involves the use of the balance as a comparator which measures the difference between two objects (or two groups of objects) which must have nominally the same mass because of the small "on-scale" range of the comparator. In deriving units which are subdivisions of the basic unit or multiples thereof, a variety of different weighing sets have been used because of convenience or other practical considerations. A typical set is the 5 3 2 1 series which bridges the range from 10 to 1. In this paper, designs are presented for sets of weights of the same nominal size, for the most common subdivisions currently in use, and for a miscellaneous group included for completeness. In most cases, the designs provide for a check standard, treated as an additional unknown weight, to be used for monitoring the performance of the measuring process.

Precision weighing is usually done by some form of transposition weighing on a two-pan balance and by substitution methods on a one-pan balance. Matters relating to weighing procedures are discussed in [8, 9]. For the purposes of this report, it will be assumed that a well behaved comparator is available and that measurements of differences in the mass of two objects or groups of objects are corrected for air buoyancy effects and other environmental or procedural factors. It is further assumed that the measurements are uncorrelated in the statistical sense and all are of equal precision. (These latter two assumptions are non-trivial and special care has to be taken to insure their validity so that the random error component of the uncertainty is properly evaluated.)

NEED FOR A CHECK STANDARD

In a calibration laboratory, it is necessary to have checks on the measurement process to provide assurance that the process measures what it was intended to measure and that it does so with a nearly constant precision [10]. A direct check on the limiting mean of the measurement process is provided if a known weight is calibrated regularly as if it were an unknown test weight. If the value obtained for the weight differs from its accepted value by an amount larger than can be accounted for by the imprecision of measurement, then the process would be regarded as being out of control. One is saying that if he cannot calibrate his own weight correctly, he can have little confidence in the values for the calibration of unknown weights derived from the same data.

There is another equally important reason for routine calibration of the same weight--the results on it provide the true measure of the variability of the process. In the course of a year the weighings would have been done under diverse weighing conditions and, hence, the sequence of values would reflect the actual variability of the process--variability which may not be reflected in the internal agreement of one series of measurements.

If an unknown test weight was repeatedly measured, one would expect variability similar to that shown by the check standard. If one has a single measurement on an unknown, it would be like a random selection from the sequence. From the sequence of values on the check standard, one can establish limits to the variability of the process and, because of the equivalence in the method of measurements for both the standard and the unknown, one can legitimately transfer the properties of this sequence to the unknowns.

To establish that the measurement process is in control requires also that the measurements be internally consistent within the limits of random error. If more weighings are made than there are unknowns, then there will be a "closure" error because the values for the observations calculated from "best" values for the weights will differ from that actually observed. The standard deviation computed from these deviations can be tested against the long-run value of this process parameter.

In summary then, the schedule of measurements for calibration should include provision for a check standard and also for within-run redundancy. The decision as to which one of a number of possible schedules or designs to use for intercomparison of a set of weights depends on items such as the variance associated with individual weights or combinations thereof. The least squares analysis from which the values for the weights and their variances are calculated is presented in the next section.

LEAST SQUARES ANALYSIS

We begin then with a set of n observations, $y_1, y_2, \ldots y_n$ involving k objects whose values, $\beta_1, \beta_2, \ldots \beta_k$ are to be determined. The set of observations can be represented by the equations for their expected values, $E(y_i)$,

$$E(y_1) = x_{11}\beta_1 + x_{12}\beta_2 \ldots x_{1k}\beta_k \tag{1}$$
$$E(y_2) = x_{21}\beta_1 + x_{22}\beta_2 \ldots x_{2k}\beta_k$$
$$\vdots$$
$$E(y_n) = x_{n1}\beta_1 + x_{n2}\beta_2 \ldots x_{nk}\beta_k$$

or in matrix form $E(y) = X\beta$ where the element, x_{ij}, of the X matrix is 0 if the weight is absent, and 1 or -1 depending on the direction of the comparison. In this note we shall adopt the convention of using just the signs so that, for example, all possible comparisons (ignoring direction) of 4 nominally equal objects will have the representation

β_1	β_2	β_3	β_4
+	-		
+		-	
+			-
	+	-	
	+		-
		+	-

$$X = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

In the least squares analysis one forms the normal equations

$$X'X\hat{\beta} = X'y$$

where the entries in X'X are merely the sums of squares and sums of cross products of the columns of X. In the above case, one gets

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \hat{\beta} = \begin{bmatrix} y_1 + y_2 + y_3 \\ -y_1 + y_4 + y_5 \\ -y_2 - y_4 + y_6 \\ -y_3 - y_5 - y_6 \end{bmatrix}$$

where $\hat{\beta}$ is the column vector with elements $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, $\hat{\beta}_4$, the caret being used to denote the fact that the values are functions of the observations, and not the sought-after values, β.

It can easily be verified in this case that the system of equations is not of full rank (e.g., the column totals are zero) and this is a property of all designs where only differences are measured. In mass calibration, one has one or more standards whose value can be taken as known and these provide the restraint on the system needed to give a unique set of answers. Usually these involve a starting kilogram or a unique summation such as 5 + 3 + 2 which has been determined in a previous series or is the initial unit value for an ascending series such as the 1, 2, 3, 5 series. One can write the restraint* in the form

$$r_1\beta_1 + r_2\beta_2 \; . \; . \; . + r_k\beta_k = m \tag{2}$$

and use the method of Lagrangian multipliers (with multipliers 2λ) to minimize the function

*In all cases treated here a single restraint is sufficient. See Zelen [12] and Goldman and Zelen [6] for a discussion of the general case.

$$\Phi = \Sigma(\text{deviations})^2 + 2\lambda(r_1\beta_1 + \ldots r_k\beta_k - m) \qquad (3)$$

The normal equations now contain an additional "unknown," namely λ and written out in full are as follows:

$$\begin{array}{lllll}
\Sigma x_1^2\hat{\beta}_1 & + \Sigma x_1x_2\hat{\beta}_2 & \ldots \Sigma x_1x_k\hat{\beta}_k & + r_1\lambda & = \Sigma x_1y \\
\Sigma x_2x_1\hat{\beta}_1 & + \Sigma x_2^2\hat{\beta}_2 & \ldots \Sigma x_2x_k\hat{\beta}_k & + r_2\lambda & = \Sigma x_2y \\
\cdot & & & & \\
\cdot & & & & \\
\cdot & & & & \\
\Sigma x_kx_1\hat{\beta}_1 & + \Sigma x_kx_2\hat{\beta}_2 & \ldots \Sigma x_k^2\hat{\beta}_k & + r_k\lambda & = \Sigma x_ky \\
r_1\hat{\beta}_1 & + r_2\hat{\beta}_2 & \ldots r_k\hat{\beta}_k & & = m
\end{array} \qquad (4)$$

where

$$\Sigma x_ix_j = \sum_{k=1}^{n} x_{ik}x_{jk}$$

$$\Sigma x_iy = \sum_{k=1}^{n} x_{ik}y_k$$

or in matrix notation

$$\begin{bmatrix} X'X & r \\ r' & o \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \lambda \end{bmatrix} = \begin{bmatrix} X'y \\ m \end{bmatrix} \qquad (5)$$

The solution may be written out formally as follows:

$$\begin{bmatrix} \hat{\beta} \\ \lambda \end{bmatrix} = \begin{bmatrix} C & h \\ h' & o \end{bmatrix} \begin{bmatrix} X'y \\ m \end{bmatrix} = \begin{bmatrix} CX' & h \\ h'X' & o \end{bmatrix} \begin{bmatrix} y \\ m \end{bmatrix} \qquad (6)$$

where $r' = (r_1 r_2 \ldots r_k)$.

To facilitate computation it is convenient to have the values, $\hat{\beta}$, written out as linear functions of the y's and m, i.e., $\hat{\beta} = [CX', h]\begin{bmatrix} y \\ m \end{bmatrix}$. This leads to a set of multipliers of the observations of the form

$$\hat{\beta}_1 = g_{11}y_1 + g_{12}y_2 \cdot \cdot \cdot g_{1n}y_n + h_1 m \tag{7}$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$\hat{\beta}_k = g_{k1}y_1 + g_{k2}y_2 \cdot \cdot \cdot g_{kn}y_n + h_k m$$

These multipliers, g_{ij} and h_i, are given in Appendix B in transposed form for some of the designs. The matrix C is important because the variances and covariances of the estimates are given by

$$\text{Variance } (\hat{\beta}_i) = C_{ii}\sigma^2, \text{ Covariance } (\hat{\beta}_i, \hat{\beta}_j) = C_{ij}\sigma^2 \tag{8}$$

The quantity, σ^2, is the variance (square of the long run value of the standard deviation) associated with the process. In a set of n observations on k items and r = 1 restraints one has $n - k + r = n - k + 1$ degrees of freedom for a standard deviation, s, formed by

$$s^2 = \frac{1}{n - k + 1} \Sigma(\text{deviations})_i^2 \tag{9}$$

$$(\text{deviation})_i = y_i - (x_{i1}\hat{\beta}_1 + x_{i2}\hat{\beta}_2 \cdot \cdot \cdot x_{ik}\hat{\beta}_k)$$

One can write these deviations as a function of the observations by noting that the predicted values are just $X\hat{\beta}$ and the deviations are thus

$$\text{dev} = y - X\hat{\beta} = y - X[CX',h] \begin{bmatrix} y \\ m \end{bmatrix} = y - [XCX',0] \begin{bmatrix} y \\ m \end{bmatrix} \tag{10}$$
$$= [I-XCX']y$$

which can be written as

$$\text{dev}_1 = d_{11}y_1 + d_{12}y_2 \cdot \cdot \cdot d_{1n}y_n \tag{11}$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$\text{dev}_n = d_{n1}y_1 + d_{n2}y_2 \cdot \cdot \cdot d_{nn}y_n$$

The array of coefficients, d_{ij}, is given in Appendix B for some of the designs

Weights are often used in combination and one needs to know the standard deviation for the various sums. For a sum of two items, $\hat{\beta}_i$ and $\hat{\beta}_j$, one has

$$\mathrm{Var}(\hat{\beta}_i + \hat{\beta}_j) = \mathrm{Var}(\hat{\beta}_i) + \mathrm{Var}(\hat{\beta}_j) + 2\mathrm{Cov}(\hat{\beta}_i, \hat{\beta}_j)$$

and for a linear combination

$$L = \ell_1\hat{\beta}_1 + \ell_2\hat{\beta}_2 \; . \; . \; . \; \ell_k\hat{\beta}_k \qquad (12)$$

$$\text{Variance } (L) = \ell' C \ell \sigma^2$$

where $\ell' = (\ell_1, \ell_2 \; . \; . \; . \; \ell_k)$, C comes from the inverse of the matrix of normal equations [see equation (6)]. In Appendix A each design has a list of the factors D_i for computing the standard deviations for all usual weight combinations, L_i where $\mathrm{Variance}(L_i) = D_i^2\sigma^2$.

DESIGNS FOR WEIGHING

The criteria for good weighing designs depend to some extent on the use intended for the resulting values. For example, if the weights are to be used independently of each other, then one would want the standard deviation [$\sigma\sqrt{C_{ii}}$ from formula (8)] for the value for each unknown weight to be the minimum possible. If the weights are to be used in combination, then one wants the variance of all appropriate linear functions to be as small as possible.

Further, the desirability of a design depends somewhat on the restraint being used. In some cases, one's judgment of a design changes depending on whether one starts with a summation as known (e.g., 5 + 3 + 2) and works down, or with a unit as known and works up (e.g., by use of a 1, 2, 3, 5 series). For a given number of measurements only a finite set of possible designs exist for a series and only occasionally is one of these designs uniformly and undeniably "best".

The designs are grouped into categories in Appendix A: A. Designs for Nominally Equal Groups, B. Designs for the 2 2 . . . 1 1 . . . Series, C. The 5, 3, 2, 1 and 5, 2, 2, 1 Series, D. Binary and Miscellaneous Series, and E. Designs for Direct Reading. The two most widely known collections of weighing designs are those of Hayford [7] and Benoit [1] which, although they do not make provision for a check standard, are listed in this appendix. In Appendix B the complete analysis is given for five of the most commonly used designs. For the others, the complete analysis is on file with the authors, and the factors for computing the standard deviations for different weight combinations are given in Appendix A.

A. DESIGNS FOR NOMINALLY EQUAL GROUPS

A.1 All distinct intercomparisons. If k weights are to be intercompared by measuring the difference between weights of the $k(k-1)/2$ distinct pairings, then a general analysis can be written out as a function of the number of weights that are regarded as known and used as the restraint. The inverse of the normal equations with the sum of the first m of the k weights taken as known is as follows:

$$\begin{bmatrix} kI-J & -J & \mathbf{1} \\ -J & kI-J & 0 \\ \mathbf{1}' & 0 & 0 \end{bmatrix}^{-1} = \frac{1}{mk}\begin{bmatrix} mI-J & 0 & k\mathbf{1} \\ 0 & mI+J & k\mathbf{1} \\ k\mathbf{1}' & k\mathbf{1}' & 0 \end{bmatrix} \qquad (13)$$

where $\mathbf{1}' = (1, 1, \ldots 1)$ and J is a matrix of all ones and the matrices on the diagonal are of dimension $m\times m$, $(k-m)\times(k-m)$, and 1×1.

Thus the standard deviation of the value for weights within the restraint is $\sigma\sqrt{\frac{m-1}{mk}}$ and for the unknowns, $\sigma\sqrt{\frac{m+1}{mk}}$. The standard deviation of a sum of h unknowns is $\sigma\sqrt{\frac{h(h+m)}{mk}}$.

The $\hat{\beta}_i$ are given by

$$\hat{\beta}_i = \frac{T_i}{k} - \frac{\sum_{1}^{m} T_i}{mk} + \frac{K}{m} \qquad \text{for weights within restraint}$$

$$\hat{\beta}_i = \frac{T_i}{k} + \frac{\sum_{m+1}^{k} T_i}{mk} + \frac{K}{m} \qquad \text{for unknown weights} \tag{14}$$

where T_i is the sum of the y values invlving β_i in the positive sense minus the sum of y values involving β_i in the negative sense (e.g., if $E(y) = \beta_i - \beta_j$, then y would be added to T_i but subtracted from T_j), and K is the value of the restraint $(\sum_{1}^{m}\beta = K)$.

The standard deviation s is given by

$$s^2 = \frac{2}{(k-1)(k-2)} \{\Sigma \text{dev}^2\}$$

$$= \frac{2}{(k-1)(k-2)} \{\Sigma y_i^2 - \frac{\Sigma T_i^2}{k}\} \tag{15}$$

Designs for which a linear drift with time is balanced out are also included (see [4] for details of the analysis).

A.2 Subsets of all distinct intercomparisons. For large k (say $k \geq 6$) the number of possible pairings becomes large enough that the time involved in completing the measurements leads to a degradation of the precision as environmental changes, operator, fatigue, etc., become important. For that reason, subsets of the $k(k-1)/2$ pairings are used to form the design.

In some of the designs the sum of all weights is taken as the restraint. This is appropriate when the design is used to monitor within group behavior. In others, there is an implied grouping into two classes the sum for one of which is taken as the restraint.

A.3 Designs involving grouping of weights. When differences between groups of two or more weights are measured, a reduction in the variance of the values can be achieved in comparison with an equal number of differences between single weights. However, for large k the problems of identifying and handling the groupings may outweigh the possible gain in efficiency. Bose and Cameron [2, 3] have tabulated all designs up to $k = 13$ and give methods of construction for $k \leq 50$, for the special case of designs balanced so that all weights appear equally often with each other on the same pan and a similar property holds for their occurrence in opposite pans. Partially balanced designs have been developed by Suryanarayana and Chakravarti [5, 11].

B. DESIGNS FOR THE 2 2 . . . 1 1 1 . . . SERIES

When weights of nominal size 1 and 2 are involved, the construction and analysis of the designs does not possess the simplicity and symmetry of the case of all equal weights. The designs given under Section B in Appendix A are a listing of those in common use. It is not known how close they are to an "optimum" design.

C. THE 5,3,2,1 AND 5,2,2,1 SERIES

The most commonly used sequences for weight sets are the 5,3,2,1 and the 5,2,2,1. These series permit one to achieve nominal values from one to ten and are the basic series for calibrating weight sets from 1 kilogram down to one milligram. The designs with the check standard [C2, C10] are the ones used for all the calibrations of such series at the National Bureau of Standards, and the complete analysis for each of these two designs is given in Appendix B. A complete example from mass calibration at NBS is given for the 5,3,2,1,1,1 series on page 14.

D. BINARY AND MISCELLANEOUS SERIES

Binary series used for pound sets and some other miscellaneous series used at NBS are included for completeness.

E. DESIGNS FOR DIRECT READING WITH CONSTANT LOAD

When a single pan balance is used as a direct reading device, the time required to complete a set of measurements can be greatly reduced, often with a significant reduction in the variance associated with the measurements. To eliminate the effects of a linear drift in the measurements, the set of weighings are repeated in reverse order, e.g., A,B,C,D,D,C,B,A for the four nominally equal weights A, B, C, and D. The designs are, of course, applicable for groups of weights which are nominally equal as shown in the tables for the 5,2,2,1 series.

An example is given of the use of these direct reading designs in mass calibration on page 17.

TREND ELIMINATION

When responses are time dependent due to temperature and atmospheric changes, proper ordering can make the values for the weights independent of any drift effect. The designs A.1.3, A.2.5, A.2.2, A.2.4, E.1, and E.2 have this property. If one wishes to use the trend eliminating property of the design (they are valid as given, of course), then one can account for a drift effect of the form . . . -3Δ, -2Δ, $-\Delta$, 0, Δ, 2Δ, 3Δ . . . if n is odd or by . . . -5Δ, -3Δ, $-\Delta$, Δ, 3Δ, 5Δ . . . if n is even. This will not change the computations for the weights or their variances. However, the degrees of freedom are reduced by one and the deviations will be different. An example of this usage is Design E.1 given in Tables 9 and 10. See [4] for details of the analysis.

EXAMPLE OF CALIBRATION DESIGN IN MASS CALIBRATION

As an example of the calibration process employed in the mass laboratory at NBS, consider six weights (50, 30, 20, 10, 10 and 10 grams) which are to be calibrated using Design C.2. The complete analysis for this design is given in Table 5 in Appendix B.

The sum of the 50, 30 and 20 gram weights, estimated from a previous series, forms the restraint, and one of the 10 gram weights is an NBS standard.

If the calibration is done as a double substitution weighing on a one pan balance, the given observations after converting the balance readings to milligrams are:

$$
\begin{aligned}
y_1 &= .370 \\
y_2 &= -.499 \\
y_3 &= -.074 \\
y_4 &= -.079 \\
y_5 &= .395 \\
y_6 &= .395 \\
y_7 &= -.454 \\
y_8 &= .405 \\
y_9 &= .495 \\
y_{10} &= .095 \\
y_{11} &= .490
\end{aligned}
$$

The restraint given is $m = 0.862$

Denote the value of the correction to nominal size of the weights 50, 30, 20, 10, 10 and 10 grams by $\beta_1, \ldots \beta_6$ respectively. Using the multipliers given under "Parameter Values,"

the least squares estimates of the β_i's as functions of the observations are:

$$\hat{\beta}_1 = \frac{1}{920}\{100(y_1+y_2+y_3+y_4) + 60y_5 - 20(y_6+y_7+y_8+ y_9+y_{10}+y_{11}) + 460m\} = .395$$

$$\hat{\beta}_2 = \frac{1}{920}\{-68(y_1+y_2+y_3+y_4) - 4y_5 + 124(y_6+y_7+y_8) - 60(y_9+y_{10}+y_{11}) + 276m\} = .254$$

$$\hat{\beta}_3 = \frac{1}{920}\{-32(y_1+y_2+y_3+y_4) - 56y_5 - 104(y_6+y_7+y_8) + 80(y_9+y_{10}+y_{11}) + 184m\} = .213$$

$$\hat{\beta}_4 = \frac{1}{920}\{119y_1+4y_2-111y_3+4y_4-108y_5+128y_6 - 102(y_7+y_8) - 125(y_9+y_{10}) - 10y_{11}+ 92m\} = .069$$

$$\hat{\beta}_5 = \frac{1}{920}\{-111y_1+119y_2 + 4(y_3+y_4) - 108y_5 - 102y_6 + 128y_7 - 102y_8 - 125y_9 - 10y_{10} - 125y_{11} + 92m\} = -.357$$

$$\hat{\beta}_6 = \frac{1}{920}\{4y_1-111y_2+119y_3+4y_4-108y_5-102(y_6+y_7) + 128y_8 - 10y_9 - 125(y_{10}+y_{11}) + 92m\} = .070$$

Note that $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\beta}_3$ necessarily sum to m.

The deviations or differences between the observed and calculated values can be computed using the multipliers given under "Deviations" in Table 5. For example, the deviation corresponding to y_1 is given by

$$dev_1 = \frac{1}{184}\{98y_1 - 17(y_2+y_3) - 40y_4 - 24y_5 - 38y_6 + 54y_7 + 8(y_8+y_9) + 31y_{10} - 15y_{11}\} = .016$$

Similarly, $dev_2 = -.001$

$dev_3 = -.003$

$dev_4 = -.007$

$dev_5 = -.005$

$dev_6 = -.002$

$dev_7 = .001$

$dev_8 = .007$

$dev_9 = -.006$

$dev_{10} = .021$

$dev_{11} = -.010$

The standard deviation for this calibration is

$$s = \{\frac{1}{6} \sum_{i=1}^{11} dev_i^2\}^{\frac{1}{2}} = .013$$

and would be used as an estimate of the process standard deviation if that were not already established.

From the "Inverse" of normal equations we can compute the variance for each estimate or linear combination in terms of the process standard deviation σ. For example,

$$V(\hat{\beta}_1) = \frac{50}{920}\sigma^2$$

$$V(\hat{\beta}_2) = \frac{82}{920}\sigma^2$$

$$V(\hat{\beta}_3) = \frac{64}{920}\sigma^2$$

$$V(\widehat{\beta_1+\beta_2}) = \frac{1}{920}\{50 + 82 - 34 - 34\}\sigma^2 = \frac{64}{920}\sigma^2$$

The usual correction for buoyancy has not been applied to these data for the purpose of this example since all the weights have the same density with the exception of the check standard.

EXAMPLE OF DIRECT READING DESIGN

In order to eliminate time dependency in a mass series, a schedule was developed as in Table 10 in Appendix B which balances out the drift effect. In this example, the corrections to four kilogram weights are to be computed using $n = 8$ observations. Since n is even, the drift can be represented by -7Δ, -5Δ, -3Δ, $-\Delta$, Δ, 3Δ, 5Δ, 7Δ. In addition to four parameter values, β_1, β_2, β_3 and β_4, for the weights and Δ for drift, a value for the tare weight T is computed. T always appears negatively in the design because a direct reading is actually the difference between the unknown weight and the tare weight in the balance.

The correction to the third weight is taken as known, namely $m = 11.906$, and assuming for the purpose of this example, that any effects due to temperature or air density are negligible, the observations are:

$$y_1 = 39.112$$
$$y_2 = 44.697$$
$$y_3 = 38.655$$
$$y_4 = 44.150$$
$$y_5 = 44.150$$
$$y_6 = 38.685$$
$$y_7 = 44.778$$
$$y_8 = 39.207$$

Using the multipliers given under "Parameter Values" in Table 10, the least squares estimates of the parameters are given by:

$$\hat{\beta}_1 = \frac{1}{2}(y_1 - y_3 - y_6 + y_8) + m = 12.3955$$

$$\hat{\beta}_2 = \frac{1}{2}(y_2 - y_3 - y_6 + y_7) + m = 17.9735$$

$$\hat{\beta}_3 = m \qquad = 11.9060$$

$$\hat{\beta}_4 = \frac{1}{2}(-y_3 + y_4 + y_5 - y_6) + m = 17.3860$$

$$\hat{\tau} = \frac{1}{2}(-y_3 - y_6) + m \qquad = -26.7640$$

$$\hat{\Delta} = \frac{1}{168}(7y_1 \quad 5y_2 \quad 3y_3 - y_4 + y_5 - 3y_6 + 5y_7 + 7y_8) = .0069$$

Associated variances in terms of the process standard deviation σ are as follows:

$$V(\hat{\beta}_1) = \sigma^2$$

$$V(\hat{\beta}_2) = \sigma^2$$

$$V(\hat{\beta}_3) = \sigma$$

$$V(\hat{\beta}_4) = \sigma^2$$

$$V(\hat{\tau}) = \frac{1}{2}\sigma^2$$

$$V(\hat{\Delta}) = \frac{1}{168}\sigma^2$$

The deviations or differences between the observed and calculated values can be computed using the multipliers under "Deviations" in Table 10. For example,

$$dev_1 = \frac{1}{168}\{35(y_1 - y_2 + y_7 - y_8) + 21(y_6 - y_3) + 7(y_5 - y_4)\} = .001$$

Similarly,

$$dev_2 = -.006$$

$$dev_3 = .006$$

$$dev_4 = .007$$

$$\mathrm{dev}_5 = -.007$$

$$\mathrm{dev}_6 = -.006$$

$$\mathrm{dev}_7 = \ \ .006$$

$$\mathrm{dev}_8 = -.001$$

The standard deviation s for this experiment is

$$s = \{\frac{1}{3} \sum_{i=1}^{8} \mathrm{dev}_i{}^2\}^{1/2} = .009$$

and would be used in an F-test to verify that this run had a standard deviation consistent with the process standard deviation σ.

APPENDIX A

LIST OF DESIGNS

A. Designs for Nominally Equal Groups

A.1 All Distinct Intercomparisons

A.1.1.	k = 3	n = 3	
*A.1.2.	k = 4	n = 6	
A.1.3.	k = 4	n = 8	(Cameron-Hailes) Trend Elimination
A.1.4.	k = 5	n = 10	
A.1.5.	k = 5	n = 10	(Cameron-Hailes) Trend Elimination
A.1.6.	k = 6	n = 15	

A.2 Subsets of All Distinct Intercomparisons

A.2.1	k = 6	n = 8	
A.2.2	k = 6	n = 12	Trend Elimination
A.2.3	k = 7	n = 10	
A.2.4	k = 7	n = 14	Trend Elimination
A.2.5	k = 8	n = 8	
A.2.6	k = 8	n = 12	
A.2.7	k = 8	n = 15	
A.2.8	k = 8	n = 16	Trend Elimination
A.2.9	k = 9	n = 12	
A.2.10	k = 10	n = 10	
A.2.11	k = 10	n = 12	
A.2.12	k = 10	n = 15	

*Complete analysis for 1,1,1,1 design given in Tables 1 and 2 in Appendix B.

A.3 Designs Involving Grouping of Weights. v = no. of weights in each group.

A.3.1	k = 6	n = 10	v = 3	(Bose-Cameron)
A.3.2	k = 6	n = 15	v = 2	(Bose-Cameron)
A.3.3	k = 7	n = 7	v = 3	(Bose-Cameron)
A.3.4	k = 8	n = 7	v = 4	(Bose-Cameron)
A.3.5	k = 8	n = 14	v = 2	(Bose-Cameron)
A.3.6	k = 9	n = 9	v = 4	(Bose-Cameron)
A.3.7	k = 9	n = 12	v = 3	(Bose-Cameron)

B. 2,2, ... , 1,1, . . . Designs

B.1	2,1,1,1	k = 4	n = 8
B.2	2,1,1,1,1	k = 5	n = 13
*B.3	2,2,1,1	k = 4	n = 6
B.4	2,2,1,1,1	k = 5	n = 10
B.5	2,2,1,1,1,1	k = 6	n = 15
B.6	2,2,1,1,1,1,1	k = 7	n = 10
B.7	2,2,1,1,1,1,1,1	k = 8	n = 16
B.8	2,2,2,1,1,1	k = 6	n = 12
B.9	2,2,2,1,1,1	k = 6	n = 15
B.10	2,2,2,1,1,1,1	k = 7	n = 12
B.11	2,2,2,2,1,1	k = 6	n = 11

*Complete analysis for 2,2,1,1 design is given in Tables 3 and 4 in Appendix B.

C. <u>5,3,2,1 and 5,2,2,1 Series</u>

C.1	5,3,2,1,1	k = 5	n = 8	(Hayford)
*C.2	5,3,2,1,1,1	k = 6	n = 11	
C.3	5,3,2,2,1,1	k = 6	n = 11	
C.4	5,3,2,2,1,1	k = 6	n = 12	
C.5	5,5,3,2,1,1	k = 6	n = 12	
C.6	5,5,3,2,1,1,1	k = 7	n = 15	
C.7	5,2,1,1,1	k = 5	n = 7	(Hayford)
C.8	5,2,2,1,1	k = 5	n = 7	(Hayford)
C.9	5,2,2,1,1	k = 5	n = 8	(Benoit)
**C.10	5,2,2,1,1,1	k = 6	n = 8	

D. <u>Binary and Miscellaneous Series</u>

D.1	4,2,2,1,1	k = 5	n = 6	
D.2	4,2,2,1,1	k = 5	n = 10	
D.3	4,2,2,1,1	k = 5	n = 11	(Benoit)
D.4	4,2,2,1,1,1	k = 6	n = 10	
D.5	4,4,2,1,1	k = 5	n = 9	(Benoit)
D.6	4,4,2,2,1,1	k = 6	n = 9	
D.7	4,4,2,2,1,1,1	k = 7	n = 14	

*Complete analysis for 5,3,2,1,1,1 design in Tables 5 and 6 in Appendix B.

**Complete analysis for 5,2,2,1,1,1 design in Tables 7 and 8 in Appendix B.

D.8	4,3,2,1,1	k = 5	n = 8	(Hayford)
D.9	4,3,2,1,1	k = 5	n = 9	(Benoit)
D.10	10,4,3,2,1,1	k = 6	n = 10	(Hayford)
D.11	10,5,2,1,1,1	k = 6	n = 8	(Hayford)
D.12	10,5,2,2,1,1	k = 6	n = 8	(Hayford)
D.13	10,5,3,2,1,1	k = 6	n = 10	(Hayford)
D.14	5,4,3,2,1	k = 5	n = 7	(Hayford)
D.15	10,5,4,3,2,1	k = 6	n = 10	(Hayford)
D.16	6,5,4,3,2,1	k = 6	n = 9	(Hayford)
D.17	10,6,5,4,3,2,1	k = 7	n = 12	(Hayford)

E. <u>Direct Readings</u>

* E.1	4 nominally equal weights	Trend elimination
E.2	2,2,1,1	Trend elimination

*Complete analysis for direct reading design 1,1,1,1 is given in Tables 9 and 10 in Appendix B.

USES OF THE DESIGNS

Description of Designs

Each design lists k, the number of weights; n, the number of measurements; and d.f., the degrees of freedom associated with the standard deviation.

The identification or nominal size of each weight β_i is given next to the heading "Observations." Y(1), Y(2), . . . denotes the measurements where + indicates the weight is present positively, and – indicates the weight is present negatively.

For example, Design A.1.1 involves three equal weights. If used as a starting design for a 1 kg to 1 mg set, the three weights would be kilograms, and the first observation Y(1) would be the difference between the first and second kilograms.

Two restraints are listed although others are possible. Usually "Restraint A" is appropriate for descending series involving, for example, 5 + 3 + 2 which would be calibrated as a "Ten" weight in the higher series involving 50, 30, 20, 10. For ascending series, "Restraint B" is usually a single unit weight.

In the case of three equal weights, Restraint A takes the sum of two kilograms as known; Restraint B takes the single third weight as the reference standard.

"Factors for Computing Standard Deviations" give the multipliers needed to calculate the standard deviations of linear combinations of the weights. "Wt" identifies the total weight combination or load L_i where

$$L_i = \sum_{i=1}^{k} \ell_i \beta_i, \ \ell_i = 0 \text{ or } 1$$

The next two columns under Restraints A and B list the factors D_i where D_i is calculated so that

$$\text{Variance}(L_i) = D_i^2\sigma^2$$

The remaining columns, under the nominal sizes of the weights, show the actual weights involved in the combinations, i.e. a + indicates $\ell_i = 1$ and a blank indicates $\ell_i = 0$.

Selection of the Appropriate Design

The selection of the appropriate design is usually dictated by the set of weights to be calibrated and the reference standards one has available. There is occasionally some flexibility as, for example, if one were working up from 1 kg to 100 kg from a single reference kilogram. If either the 1,1,1,1,2 Series (Design B.2) or the 1,1,1,2,2 Series (Design B.4) were usable one would make the decision on the basis of the standard deviation of $\Sigma 5$, the 5 kg summation. In design B.2 the factor for computing the standard deviation is 1.6733, whereas in B.4 it is 2.1909. [Even if one compares the two standard deviations on an equal number of observation basis (n = 13 vs. n = 10) one would have 1.6733 vs. 1.9215 (= 2.1909 $\sqrt{10/13}$)]. The advantage clearly is with design B.2 because in the next series, $\Sigma 5$, 5, 5, 5, 10, the uncertainty associated with the starting standard $\Sigma 5$ is smaller.

When the standard deviation for the weight combinations of interest (computed using the standard deviation associated with the measurement process) exceeds the tolerance given by the requirements, then one has to consider the feasibility of repeating a design to reduce the standard deviation factors by $\frac{1}{\sqrt{2}}$ (or $\frac{1}{\sqrt{n}}$ if n repetitions are made).

DESIGN A.1.1 THREE EQUAL WEIGHTS

K= 3
N= 3
D.F.= 1

OBSERVATIONS	1	1	1
Y(1)	+	-	
Y(2)	+		-
Y(3)		+	-
RESTRAINT A	+	+	
RESTRAINT B			+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1
1	.7071	.0000			+
1	[illegible]	[illegible]		+	
1	.4082	.8165	+		
2	.0000	1.4142	+	+	
3	.7071	1.4142	+	+	+

DESIGN A.1.2 FOUR EQUAL WEIGHTS

K= 4
N= 6
D.F.= 3

OBSERVATIONS	1	1	1	1
Y(1)	+	-		
Y(2)	+		-	
Y(3)	+			-
Y(4)		+	-	
Y(5)		+		-
[illegible]			[illegible]	
RESTRAINT A	+	+		
RESTRAINT B				+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1
1	.6124	.0000				+
1	.6124	.7071			+	
1	.3536	.7071		+		
1	.3536	.7071	+			
2	.0000	1.2247	+	+		
3	.6124	1.7321	+	+	+	
4	1.0000	1.7321	+	+	+	+

DESIGN A.1.3 FOUR EQUAL WEIGHTS TREND ELIMINATION* (CAMERON-HAILES)

K= 4
N= 8
D.F.= 5

OBSERVATIONS	1	1	1	1
Y(1)	+			-
Y(2)	-		+	
Y(3)		+	-	
[illegible]				[illegible]
Y(5)			-	+
Y(6)	-		+	
Y(7)	+	-		
Y(8)		+		-
RESTRAINT A	+	+		
RESTRAINT B				+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1
1	.5204	.0000				+
1	.5204	.6455			+	
1	.3227	.5774		+		
1	.3227	.6455	+			
2	.0000	1.0408	+	+		
3	.5204	1.5275	+	+	+	
4	.8165	1.5275	+	+	+	+

*See page 23.

DESIGN A.1.4 FIVE EQUAL WEIGHTS

K= 5
N=10
D.F.= 6

OBSERVATIONS	1	1	1	1	1
Y(1)	+	-			
Y(2)	+		-		
Y(3)	+			-	
Y(4)	+				-
Y(5)		+	-		
Y(6)		+		-	
Y(7)		+			-
Y(8)			+	-	
Y(9)			+		-
Y(10)				+	-
RESTRAINT A	+	+			
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1
1	.5477	.0000					+
1	.5477	.6325				+	
1	.5477	.6325			+		
1	.3162	.6325		+			
1	.3162	.6325	+				
2	.0000	1.0954	+	+			
3	.5477	1.5492	+	+	+		
4	.8944	2.0000	+	+	+	+	
5	1.2247	2.0000	+	+	+	+	+

DESIGN A.1.5 — FIVE EQUAL WEIGHTS, TREND ELIMINATION * (CAMERON-HAILES) — K= 5, N=10, D.F.= 6

OBSERVATIONS	1	1	1	1	1
Y(1)	+	−			
Y(2)		+	−		
Y(3)			+	−	
Y(4)				+	−
Y(5)	−				+
Y(6)	−			+	
Y(7)		+		−	
Y(8)		−			+
Y(9)			+		−
Y(10)	+		−		
RESTRAINT A	+	+			
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1
1	.5477	.0000					+
1	.5477	.6325				−	
1	.5477	.6325			+		
1	.3162	.6325		−			
1	.3162	.6325	+				
2	.0000	1.0954	−	+			
3	.5477	1.5492	+	+	+		
4	.8944	2.0000	+	+	+	+	
5	1.2247	2.0000	+	+	+	+	+

*See page 13.

DESIGN A.2.1 — SIX EQUAL WEIGHTS — K= 6, N= 8, D.F.= 3

OBSERVATIONS	1	1	1	1	1	1
Y(1)	+		−			
Y(2)	+			−		
Y(3)	+				−	
Y(4)	+					−
Y(5)		+	−			
Y(6)		+		−		
Y(7)		+			−	
Y(8)		+				−
RESTRAINT A	+	+				
RESTRAINT B	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1
1	.7071	.6236						+
1	.7071	.6236					+	
1	.7071	.6236				+		
1	.7071	.6236			+			
1	.3536	.4249		+				
1	.3536	.4249	+					
2	.0000	.4714	+	+				
3	.7071	.7071	+	+	+			
4	1.0000	.7454	+	+	+	+		
5	1.2247	.6236	+	+	+	+	+	
6	1.4142	.0000	+	+	+	+	+	+

DESIGN A.1.6 — SIX EQUAL WEIGHTS — K= 6, N=15, D.F.=10

OBSERVATIONS	1	1	1	1	1	1
Y(1)	+	−				
Y(2)	+		−			
Y(3)	+			−		
Y(4)	+				−	
Y(5)	+					−
Y(6)		+	−			
Y(7)		+		−		
Y(8)		+			−	
Y(9)		+				−
Y(10)			+	−		
Y(11)			+		−	
Y(12)			+			−
Y(13)				+	−	
Y(14)				+		−
Y(15)					+	−
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1
1	.5000	.0000						+
1	.5000	.5774					+	
1	.5000	.5774				+		
1	.5000	.5774			+			
1	.2887	.5774		+				
1	.2887	.5774	+					
2	.0000	1.0000	+	+				
3	.5000	1.4142	+	+	+			
4	.8165	1.8257	+	+	+	+		
5	1.1180	2.2361	+	+	+	+	+	
6	1.4142	2.2361	+	+	+	+	+	+

DESIGN A.2.2 — SIX EQUAL WEIGHTS, TREND ELIMINATION * — K= 6, N=12, D.F.= 7

OBSERVATIONS	1	1	1	1	1	1
Y(1)	+		−			
Y(2)	−			+		
Y(3)			+		−	
Y(4)		+				−
Y(5)		−			+	
Y(6)				−		+
Y(7)		−	+			
Y(8)		+		−		
Y(9)			−			+
Y(10)	−				+	
Y(11)	+					−
Y(12)				+	−	
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1
1	.5401	.0000						+
1	.5401	.7071					+	
1	.5401	.6455				+		
1	.5401	.6455			+			
1	[illegible]	.6455		+				
1	.3536	.6455	+					
2	.0000	1.0801	+	−				
3	.5401	1.5546	+	+	+			
4	.8165	2.0000	+	+	+	+		
5	1.1365	2.5495	+	+	+	+	+	
6	1.4142	2.5495	+	+	+	+	+	+

*See page 13.

DESIGN A.2.3 SEVEN EQUAL WEIGHTS K= 7
N=10
D.F.= 4

OBSERVATIONS	1	1	1	1	1	1	1
Y(1)	+		-				
Y(2)	+			-			
Y(3)	+				-		
Y(4)	+					-	
Y(5)	+						
Y(6)		+	-				
Y(7)		+		-			
Y(8)		+			-		
Y(9)		+				-	
Y(10)		+					-
RESTRAINT A	+	+					
RESTRAINT B	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1
1	.7071	.6389							+
1	.7071	.6389						+	
1	.7071	.6389					+		
1	.7071	.6389				+			
1	.7071	.6389			+				
1	.3162	.3886		+					
1	.3162	.3886	+						
2	.0000	.4518	+	+					
3	.7071	.7284	+	+	+				
4	1.0000	.8267	+	+	+	+			
5	1.2247	.7964	+	+	+	+	+		
6	1.4142	.6385	+	+	+	+	+	+	
7	1.5811	.0000	+	+	+	+	+	+	+

DESIGN A.2.4 SEVEN EQUAL WEIGHTS K= 7
TREND ELIMINATION * N=14
D.F.= 8

OBSERVATIONS	1	1	1	1	1	1	1
Y(1)	+						
Y(2)		+	-				
Y(3)			+	-			
Y(4)				+	-		
Y(5)					+	-	
Y(6)						+	-
Y(7)	-						+
Y(8)	-		+				
Y(9)			-		+		
Y(10)					-		-
Y(11)		+					-
Y(12)		-		+			
Y(13)				-		+	
Y(14)	+					-	
RESTRAINT A	+	+					
RESTRAINT B	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1
1	.5694	.4447							+
1	.6157	.4447						+	
1	.6504	.4447					+		
1	[illegible]	.4447				+			
1	.5694	.4447			+				
1	.3231	.4447		+					
1	.3231	.4447	+						
2	.4030	.6112	+	+					
3	.5694	.7310	+	+	+				
4	.9945	.7313	+	+	+	+			
5	1.4507	.6115	+	+	+	+	+		
6	1.8337	.4447	+	+	+	+	+	+	
7	2.1394	.0000	+	+	+	+	+	+	+

*See page 15.

DESIGN A.2.5 EIGHT EQUAL WEIGHTS K= 8
N= 8
D.F.= 1

OBSERVATIONS	1	1	1	1	1	1	1	1
Y(1)	-				-			
Y(2)	-					-		
Y(3)		+					-	
Y(4)		+						-
Y(5)			+					-
Y(6)			+		-			
Y(7)				+		-		
Y(8)				+			-	
RESTRAINT A	+	+	+	+				
RESTRAINT B	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1	1
1	.8660	.8101								+
1	.8660	.8101							+	
1	.8660	.8101						+		
1	.8660	.8101					+			
1	.7906	.8101				+				
1	.7906	.8101			+					
1	.7906	.8101		+						
1	.7906	.8101	+							
2	.7071	.7906	+	+						
3	.7906	.5520	+	+	+					
4	.9055	.7071	+	+	+	+				
5	.8660	.9520	+	+	+	+	+			
6	1.2247	1.0607	+	+	+	+	+	+		
7	1.3225	.8101	+	+	+	+	+	+	+	
8	1.4142	.0000	+	+	+	+	+	+	+	+

DESIGN A.2.6 EIGHT EQUAL WEIGHTS K= 8
N=12
D.F.= 5

OBSERVATIONS	1	1	1	1	1	1	1	1
Y(1)	+		-					
Y(2)	+			-				
Y(3)	+				-			
Y(4)	+					-		
Y(5)	+						-	
Y(6)	+							-
Y(7)		+	-					
Y(8)		+		-				
Y(9)		+			-			
Y(10)		+				-		
Y(11)		+					-	
Y(12)		+						
RESTRAINT A	+	+						
RESTRAINT B	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1	1
1	.7071	.6495								+
1	.7071	.6495							+	
1	.7071	.6495						+		
1	.7071	.6495					+			
1	.7071	.6495								
1	.7071	.6495			+					
1	.2887	.3600		+						
1	.2887	.3608	+							
2	.0000	.4230	+	+						
3	.7071	.7395	+	+	+					
4	1.0000	.8660	+	+	+	+				
5	1.2247	.0927	+	+	+	+	+			
6	1.4142	.8292	+	+	+	+	+	+		
7	1.5811	.6455	+	+	+	+	+	+	+	
8	1.7321	.0000	+	+	+	+	+	+	+	+

DESIGN A.2.7 EIGHT EQUAL WEIGHTS K= 6 N=15 D.F.= 8

OBSERVATIONS	1	1	1	1	1	1	1	1
Y(1)	+			-				
Y(2)	+				-			
Y(3)	+					-		
Y(4)	+						-	
Y(5)	+							-
Y(6)		+		-				
Y(7)		+			-			
Y(8)		+				-		
Y(9)		+					-	
Y(10)		+						-
Y(11)			+	-				
Y(12)			+		-			
Y(13)			+			-		
Y(14)			+				-	
Y(15)			+					-
RESTRAINT A	+	+	+					
RESTRAINT B	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1	1
1	.5774	.5254								+
1	.5774	.5254							+	
1	.5774	.5254						-		
1	.5774	.5254					+			
1	.5774	.5254				+				
1	.3651	.3992			+					
1	.3651	.3992		+						
1	.3651	.3992	+							
2	.3651	.4873	+	+						
3	.4000	.4841	+	+	+					
4	.5774	.6455	+	+	+	+				
5	.8165	.6960	+	+	+	+	+			
6	1.0000	.6614	+	+	+	+	+	-		
7	1.1547	.5254	+	+	+	+	+	-	+	
8	1.2910	.0000	+	+	+	+	+	-	+	+

DESIGN A.2.8 EIGHT EQUAL WEIGHTS TREND ELIMINATION K= 8 N=16 D.F.= 9

OBSERVATIONS	1	1	1	1	1	1	1	1
Y(1)	+						-	
Y(2)						-	+	
Y(3)			-			+		
Y(4)		-	+					
Y(5)		+						-
Y(6)					-			+
Y(7)				-	+			
Y(8)	-			+				
Y(9)	-		+					
Y(10)			-		+			
Y(11)					-		+	
Y(12)		+					-	
Y(13)		-		+				
Y(14)				-		+		
Y(15)						-		+
Y(16)	+							-
RESTRAINT A	+	+						
RESTRAINT B	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1	1
1	.5590	.4507								+
1	.5590	.4507							+	
1	.6124	.4507						+		
1	.6124	.4507					+			
1	.5590	.4507				+				
1	.5590	.4507			+					
1	.3536	.4507		+						
1	.3536	.4507	+							
2	.5000	.5590	+	+						
3	.5590	.6721	+	+	+					
4	.8660	.7071	+	+	+	+				
5	1.2748	.6721	+	+	+	+	+			
6	1.6583	.5590	+	+	+	+	+	+		
7	1.9526	.4507	+	+	+	+	+	+	+	
8	2.2361	.0000	+	+	+	+	+	+	+	+

DESIGN A.2.9 NINE EQUAL WEIGHTS K=9 N=12 D.F.=4

OBSERVATIONS	1	1	1	1	1	1	1	1	1
Y(1)	+			−					
Y(2)	+						−		
Y(3)	+				−				
Y(4)	+							−	
Y(5)		+			−				
Y(6)		+						−	
Y(7)		+				−			
Y(8)		+							−
Y(9)			+			−			
Y(10)			+						−
Y(11)			+	−					
Y(12)			+				−		
RESTRAINT A	+	+	+						
RESTRAINT B	+	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1	1	1	1
1	.7454	.6939									+
1	.7454	.6939								+	
1	.7454	.6939							+		
1	.7454	.6939						+			
1	.7454	.6939					+				
1	.7454	.6939				+					
1	.4714	.5092			+						
1	.4714	.5092		+							
1	.4714	.5092	+								
2	.4714	.6086	+	+							
3	.5000	.5774	+	+	+						
4	.7454	.8389	+	+	+	+					
5	1.0274	.9329	+	+	+	+	+				
6	1.2247	.9129	+	+	+	+	+	+			
7	1.4337	.8714	+	+	+	+	+	+	+		
8	1.5966	.5939	+	+	+	+	+	+	+	+	
9	1.7321	.0000	+	+	+	+	+	+	+	+	+

DESIGN A.2.10 TEN EQUAL WEIGHTS K=10 N=10 D.F.=1

OBSERVATIONS	1	1	1	1	1	1	1	1	1	1
Y(1)	+					−				
Y(2)	+							−		
Y(3)		+					−			
Y(4)		+							−	
Y(5)			+					−		
Y(6)			+							−
Y(7)				+					−	
Y(8)				+		−				
Y(9)					+					−
Y(10)					+		−			
RESTRAINT A	+	+	+	+	+					
RESTRAINT B	+	+	+	+	+	−	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1	1	1	1	1
1	.9487	.9083										+
1	.9487	.9083									+	
1	.9487	.9083								+		
1	.9487	.9083							+			
1	.9487	.9083						+				
1	.8944	.9083					+					
	.8944	.9083				+						
	.8944	.9083			+							
	.8944	.9083		+								
	.8944	.9083	+									
2	.8944	.9487	+	+								
3	.8944	1.0124	+	+	+							
4	.8944	1.0954	+	+	+	+						
5	[illegible]	.7906	+	+	+	+	+					
6	.9487	1.0954	+	+	+	+	+	+				
7	1.0954	1.0124	+	+	+	+	+	+	+			
8	1.3030	.9487	+	+	+	+	+	+	+	+		
9	1.5492	.9062	+	+	+	+	+	+	+	+	+	
10	1.3911	.0000	+	+	+	+	+	+	+	+	+	+

DESIGN A.2.11 TEN EQUAL WEIGHTS K=10 N=12 D.F.= 3

OBSERVATIONS	1	1	1	1	1	1	1	1	1	1
Y(1)	+				-					
Y(2)	+					-				
Y(3)	+						-			
Y(4)		+			-					
Y(5)		+						-		
Y(6)		+							-	
Y(7)			+			-				
Y(8)			+					-		
Y(9)			+							-
Y(10)				+			-			
Y(11)				+					-	
Y(12)				+						-
RESTRAINT A	+	+	+	+						
RESTRAINT B	+	+	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1	1	1	1	1
1	.7906	.7450										+
1	.7906	.7450									+	
1	.7906	.7450								+		
1	.7906	.7450							+			
1	.7906	.7450						+				
1	.7906	.7450					+					
1	.6124	.6364				+						
1	.6124	.6364			+							
1	.6124	.6364		+								
1	.6124	.6364	+									
2	.7071	.7874	+	+								
3	.6124	.8031	+	+	+							
4	.0000	.6929	+	+	+	+						
5	.7906	.9354	+	+	+	+	+					
6	1.1180	1.0630	+	+	+	+	+	+				
7	1.3693	1.1158	+	+	+	+	+	+	+			
8	1.5000	.9449	+	+	+	+	+	+	+	+		
9	1.6202	.7450	+	+	+	+	+	+	+	+	+	
10	1.7321	.0000	+	+	+	+	+	+	+	+	+	+

DESIGN A.2.12 TEN EQUAL WEIGHTS K=10 N=15 D.F.= 6

OBSERVATIONS	1	1	1	1	1	1	1	1	1	1
Y(1)	+					-				
Y(2)	+						-			
Y(3)	+								-	
Y(4)		+					-			
Y(5)		+						-		
Y(6)		+								-
Y(7)			+					-		
Y(8)			+						-	
Y(9)			+			-				
Y(10)				+					-	
Y(11)				+						-
Y(12)				+			-			
Y(13)					+					-
Y(14)					+	-				
Y(15)					+			-		
RESTRAINT A	+	+	+	+	+					
RESTRAINT B	+	+	+	+	+	+	+	+	+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1	1	1	1	1
[illegible]	.6276	.5865										+
[illegible]	.6276	.5865									+	
[illegible]	.6276	.5865								+		
[illegible]	.6276	.5865							+			
[illegible]	.6276	.5865						+				
[illegible]	.5721	.5865					+					
[illegible]	.5721	.5865				+						
[illegible]	.5721	.5865			+							
[illegible]	.5721	.5865		+								
[illegible]	.5721	.5865	+									
2	.6606	.7052	+	+								
3	.6606	.7657	+	+	+							
4	.5721	.7707	+	+	+	+						
5	.0000	.6455	+	+	+	+	+					
6	.6276	.7707	+	+	+	+	+	+				
7	.8385	.7657	+	+	+	+	+	+	+			
8	1.0180	.7002	+	+	+	+	+	+	+	+		
9	1.1007	.5865	+	+	+	+	+	+	+	+	+	
10	1.2910	.0000	+	+	+	+	+	+	+	+	+	+

DESIGN A.3.1 SIX EQUAL WEIGHTS K= 6
N=10
(BOSE-CAMERON) D.F.= 5

OBSERVATIONS	1	1	1	1	1	1
Y(1)	+	-	-	+	-	+
Y(2)	-	+	+	-	+	+
Y(3)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Y(4)	-	+	-	+	-	+
Y(5)	-	-	+	-	+	+
Y(6)	+	-	-	+	+	-
Y(7)	+	+	-	-	+	-
Y(8)	+	+	+	-	-	-
Y(9)	-	+	+	+	-	-
Y(10)	-	-	+	+	+	-
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1
1	.3536	.0000						+
1	.3536	.4082					+	
1	.3536	.4082				+		
1	.3536	.4082			+			
1	.2041	.4082		+				
1	.2041	.4082	+					
2	.0000	.7071	+	+				
3	.3536	1.0000	+	+	+			
4	.5774	1.2910	+	+	+	+		
5	.7906	1.5811	+	+	+	+	+	
6	1.0000	1.5811	+	+	+	+	+	+

DESIGN A.3.3 SEVEN EQUAL WEIGHTS K= 7
N= 7
(BOSE-CAMERON) D.F.= 1

OBSERVATIONS	1	1	1	1	1	1	1
Y(1)	+	+	-	+	-	-	
Y(2)		+	+	-	+	-	-
Y(3)			[illegible]	-	-	+	-
Y(4)	-	-		+	+	-	+
Y(5)	+	+	-		+	+	-
Y(6)	-	+	-	-		+	+
Y(7)	+	-	+	-	-		+
RESTRAINT A	+	+					
RESTRAINT B							+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1
1	.4629	.0000							+
1	.4629	.5345						+	
1	.4629	.5345					+		
1	.4629	.5345				+			
1	.4629	.5345			+				
1	.2673	.5345		+					
1	.2673	.5345	+						
2	.0000	.9258	+	+					
3	.4085	1.3093	+	+	+				
4	.7559	1.6903	+	+	+	+			
5	1.0351	2.0702	+	+	+	+	+		
6	1.3093	2.4495	+	+	+	+	+	+	
7	1.5811	2.4495	+	+	+	+	+	+	+

DESIGN A.3.2 SIX EQUAL WEIGHTS K= 6
N=15
(BOSE-CAMERON) D.F.=10

OBSERVATIONS	1	1	1	1	1	1
Y(1)	+	-		+		-
Y(2)		+	-		+	-
Y(3)	+		+	-		-
Y(4)		+		+	-	-
Y(5)	-		+		+	-
Y(6)	+	-	-			+
Y(7)		+	-	-		+
Y(8)			+	-	-	+
Y(9)	-			+	-	+
Y(10)	-	-			+	+
Y(11)	-	+	+	-		
Y(12)		-	+	+	-	
Y(13)	-		-	[illegible]	[illegible]	
Y(14)	+	-		-	+	
Y(15)	+	+	-	[illegible]	-	
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1
1	.3536	.0000						+
1	.3536	.4082					+	
1	.3536	.4082				+		
1	.3536	.4082			+			
1	.2041	.4082		+				
1	.2041	.4082	+					
2	.0000	.7071	+	+				
3	.3536	1.0000	+	+	+			
4	.5774	1.2910	+	+	+	+		
5	[illegible]	[illegible]	+	+	+	+	+	
6	1.0000	1.5811	+	+	+	+	+	+

DESIGN A.3.4 EIGHT EQUAL WEIGHTS K= 8
N= 7
(BOSE-CAMERON) D.F.= 0

OBSERVATIONS	1	1	1	1	1	1	1	1
Y(1)	+	+	+	+	-	-	-	-
Y(2)	+	+	-	-	[illegible]	[illegible]		
Y(3)	+	-	+	-	+	-	+	-
Y(4)	+	+	-	-	-	-	+	+
Y(5)	+	-	+	-	-	+	-	+
Y(6)	+	-	-	+	+	-	-	+
Y(7)	+	-	-	+	-	+	+	-
RESTRAINT A	+	+						
RESTRAINT B								+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1	1
1	.4330	.0000								+
1	.4330	.5000							+	
1	.4330	.5000						+		
1	.4330	.5000					+			
1	.4330	.5000				+				
1	.4330	.5000			+					
1	.2500	.5000		+						
1	.2500	.5000	+							
2	.0000	.8660	+	+						
3	.4330	1.2247	+	+	+					
4	.7071	1.5811	+	+	+	+				
5	.9682	1.9365	+	+	+	+	+			
6	1.2247	2.2913	+	+	+	+	+	+		
7	1.4790	2.6456	+	+	+	+	+	+	+	
8	1.7321	2.5452	+	+	+	+	+	+	+	+

DESIGN 8.5.5 EIGHT EQUAL WEIGHTS K= 8
N=14
(ROSE-CAMERON) D.F.= 7

OBSERVATIONS	1	1	1	1	1	1	1	1
Y(1)	+	-		-			+	
Y(2)	+	+	-		-			
Y(3)		+	+	-		-		
Y(4)			+	+	-		-	
Y(5)	-			+	+	-		
Y(6)		-			+	+	-	
Y(7)	-		-			+	+	
Y(8)	+	-		+				-
Y(9)		[illegible]			[illegible]			
Y(10)			+	-		+		-
Y(11)				+	-		+	-
Y(12)	+				+	-		-
Y(13)		+				+	-	-
Y(14)	-		+				+	-
RESTRAINT A	+	+						
RESTRAINT B								+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	1	1	1	1	1	1	1	1
1	.4330	.0000								+
1	.4330	.5000							+	
1	.4330	.5000						+		
1	.4330	.5000					+			
1	.4330	.5000				+				
1	.4330	.5000			+					
1	.2500	.5000		+						
1	.2500	.5000	+							
2	[illegible]	.8660	+	+						
3	.4220	1.2247	+	+	+					
4	.7071	1.5311	+	+	+	+				
5	.9682	1.9365	+	+	+	+	+			
6	1.2247	2.2913	+	+	+	+	+	+		
7	1.4790	2.6458	+	+	+	+	+	+	+	
8	1.7321	2.6458	+	+	+	+	+	+	+	+

DESIGN A.3.4 NINE EQUAL WEIGHTS K= 9

N= 9

(BOSE-CAMERON) D.F.= 1

OBSERVATIONS	1	1	1	1	1	1	1	1	1
Y(1)	+	-	+	-	+	-	+	-	
Y(2)		+	-	+	+	-	-	-	+
Y(3)	+		-	+	-	-	+	+	-
Y(4)	-	-		+	-	+	+	-	+
Y(5)	+	+	+		-	+	-	-	-
Y(6)	+	-	-	-		+	-	+	+
Y(7)	-	-	+	+	+		-	+	-
Y(8)	-	+	+	-	-	-		+	+
Y(9)	-	+	-	-	+	+	+		-
RESTRAINT A	+	+							
RESTRAINT B									+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1	1	1	1
1	.4082	.0000									+
1	.4082	.4714								+	
1	.4082	.4714							+		
1	.4082	.4714						+			
1	.4082	.4714					+				
1	.4082	.4714				+					
1	.4082	.4714			+						
1	.2357	.4714		+							
1	.2357	.4714	+								
2	.0000	.8165	+	+							
3	.4082	1.1547	+	+	+						
4	.6667	1.4907	+	+	+	+					
5	.9129	1.8257	+	+	+	+	+				
6	1.1547	2.1602	+	+	+	+	+	+			
7	1.3944	2.4944	+	+	+	+	+	+	+		
8	1.6330	2.8284	+	+	+	+	+	+	+	+	
9	1.8708	3.8264	+	+	+	+	+	+	+	+	+

DESIGN A.5.7 NINE EQUAL WEIGHTS K= 9

N=12

(BOSE-CAMERON) D.F.= 4

OBSERVATIONS	1	1	1	1	1	1	1	1	1
Y(1)	+	+	+	-	-	-			
Y(2)	+	+	+				-	-	-
Y(3)				+	+	+		-	-
Y(4)	+	-		+	-		+	-	
Y(5)	+		-	+		-	+		-
Y(6)		+	-		+	-		+	-
Y(7)	+	-			+	-	-		+
Y(8)	+		-		+			-	+
Y(9)		+	-	-		+	+	-	
Y(10)	+	-		-		+		+	-
Y(11)	+		-		-	+	-	+	
Y(12)		+	-	+			-		+
RESTRAINT A	+	+							
RESTRAINT B									+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1	1	1	1	1	1
1	.4082	.0000									+
1	.4082	.4714								+	
1	.4082	.4714							+		
1	.4082	.4714						+			
1	.4082	.4714					+				
1	.4082	.4714				+					
1	.4082	.4714			+						
1	.2357	.4714		+							
1	.2357	.4714	+								
2	.0000	.8165	+	+							
3	.4082	1.1547	+	+	+						
4	.6667	1.4907	+	+	+	+					
5	.9129	1.8257	+	+	+	+	+				
6	1.1547	2.1602	+	+	+	+	+	+			
7	1.3944	2.4944	+	+	+	-	+	+	+		
8	1.6330	2.8284	+	+	+	-	+	+	+	+	
9	1.0708	2.8284	+	+	+	-	+	+	+	+	+

DESIGN 8.1 2,1,1,1 K= 4

N= 6

(HAYFORD) D.F.= 3

OBSERVATIONS	2	1	1	1
Y(1)	+	-		-
Y(2)	+		-	-
Y(3)	+	-	-	
Y(4)		+		-
Y(5)		+	-	
Y(6)			+	-
RESTRAINT A	+	+	+	+
RESTRAINT B				+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	1	1	1
1	.4243	.0000				+
1	.4243	.7071			+	
1	.4243	.7071		+		
2	.3464	1.0000	+			
3	.4690	1.5811	+	+		
4	.4243	2.1213	+	+	+	
5	.0000	2.1213	+	+	+	+

DESIGN 8.2 2,1,1,1,1 K= 5

N=13

D.F.= 9

OBSERVATIONS	2	1	1	1	1
Y(1)	+	+	-	-	-
Y(2)	+	-	+	-	-
Y(3)	+	-	-	+	-
Y(4)	+	-	-	-	+
Y(5)	+	-	-		
Y(6)	+	-		-	
Y(7)	+	-			-
Y(8)	+		-	-	
Y(9)	+		-		-
Y(10)	+			-	-
Y(11)		+	+	-	-
Y(12)		+	-	+	-
Y(13)		+	-	-	+
RESTRAINT A	+	+	+	+	
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	1	1	1	1
1	.2347	.0000					+
1	.2683	.4472				+	
1	.2683	.4472			+		
1	.2683	.4472		+			
2	.2191	.6325	+				
3	.2966	1.0000	+	+			
4	.2582	1.3416	+	+	+		
5	.0000	1.6733	+	+	+	+	
6	.2347	1.6733	+	+	+	+	+

DESIGN 8.3 2,2,1,1 K= 4

N= 6

D.F.= 3

OBSERVATIONS	2	2	1	1
Y(1)	+	-	+	-
Y(2)	+	-	-	+
Y(3)	+	-		
Y(4)	+		-	-
Y(5)		+	-	-
Y(6)			+	-
RESTRAINT A	+	+		
RESTRAINT B				+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	1	1
1	.4564	.0000				+
1	.4564	.5774			+	
2	.2673	.9512		+		
2	.2673	.9512	+			
3	.5285	1.3801	+		+	
4	.0000	1.8257	+	+		
5	.4564	2.2361	+	+	+	
6	.7071	2.2361	+	+	+	+

DESIGN 8.4 2,2,1,1,1 K= 5

N=10

D.F.= 6

OBSERVATIONS	2	2	1	1	1
Y(1)	+	-	-	+	
Y(2)	+	-		-	+
Y(3)	+	-	+		-
Y(4)	+	-			
Y(5)	+		-	-	
Y(6)	+		-		-
Y(7)	+			-	-
Y(8)		+	-	+	
Y(9)		+	-		+
Y(10)		+		-	-
RESTRAINT A	+	+	+		
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	1	1	1
1	.4382	.0000					+
1	.4382	.6325				+	
1	.3347	.6325			+		
2	.2710	.8634		+			
2	.2710	.8634	+				
3	.2710	1.3548	+		+		
4	.3347	1.6733	+	+			
5	.0000	2.1909	+	+	+		
6	.4382	2.6833	+	+	+	+	
7	.6066	3.6833	+	+	+	+	+

DESIGN 0.5   2,2,1,1,1,1   K= 6
N=15
D.F.=10

OBSERVATIONS	2	2	1	1	1	1
Y(1)	+	+	-	-	-	-
Y(2)	+	-	+	+	-	-
Y(3)	+	-	+	-	+	-
Y(4)	+	-	+	-	-	+
Y(5)	+	-	-	+	+	-
Y(6)	+	-	-	+	-	+
Y(7)	+	-	-	-	+	+
Y(8)	+		+	-	-	-
Y(9)	+		-	+	-	-
Y(10)		+	-	-	-	-
Y(11)		+	-	-	-	+
Y(12)	+		-	-	+	-
Y(13)	+		-	-	-	+
Y(14)		+	+	-	-	-
Y(15)		+	-	+	-	-
RESTRAINT A	+	+	+			
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	1	1	1	1
1	[illegible]	[illegible]						+
1	.2614	.3536					+	
1	.2614	.3536				+		
1	.2082	.3336			+			
2	.2051	.5456		+				
2	.2051	.5456	+					
3	.2051	.7225	+		+			
4	.2082	1.0468	+	+				
5	.0309	1.3070	+	+	+			
6	.2614	1.5679	+	+	+	+		
7	.3851	1.9287	+	+	+	+	+	
8	.4309	1.8257	+	+	+	+	+	+

DESIGN 0.6   2,2,1,1,1,1,1   K= 7
N=10
D.F.= [illegible]

OBSERVATIONS	2	2	1	1	1	1	1
Y(1)	+	-	+	-	-	+	
Y(2)	+	-		+	-	-	+
Y(3)	-	-	+		+	-	-
Y(4)	+	-	-	+		+	-
Y(5)	+	-	-	-	+		+
Y(6)	+	+	-	-	-	-	
Y(7)	+	+	-	-	-		-
Y(8)	+	+	-	-		-	-
Y(9)	+	+	-		-	-	-
Y(10)	+	+		-	-	-	-
RESTRAINT A	+	+	+				
RESTRAINT B							+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	1	1	1	1	1
1	.4000	.0000							+
1	.4000	.5774						+	
1	.4000	.5774					+		
1	.4000	.5774				+			
1	.3055	.5774			+				
2	.2708	.7559		+					
2	.2708	.7558	+						
3	.2708	1.2780	+		+				
4	.3055	1.5275	+	+					
5	.3000	2.0000	+	+	+				
6	.4000	2.4495	+	+	+	+			
7	.5538	2.8868	+	+	+	+	+		
8	.6633	3.3166	+	+	+	+	+	+	
9	.7483	3.3160	+	+	+	+	+	+	+

DESIGN 8.7   2,2,1,1,1,1,1,1   K= 8
N=15
D.F.= 9

OBSERVATIONS	2	2	1	1	1	1	1	1
Y(1)	+	+	-	-	-	-	-	-
Y(2)	+	+	-	+	-	-	-	-
Y(3)	+	+	-	-	+	-	-	-
Y(4)	+	+	-	-	-	-	-	
Y(5)	+	+	-	-	-	-	+	
Y(6)	+	-	-	-	-	-	-	+
Y(7)	+	-	+	-	-	+	-	+
Y(8)	+	-	-	+	-	-	+	+
Y(9)	+	-	+	-	+	-	-	+
Y(10)	+	-	-	+	-	+	-	+
Y(11)	+	-	-	-	+	-	+	+
Y(12)	+	-	+	-	-	+	+	-
Y(13)	+	-	+	+	-	-	+	-
Y(14)	+	-	+	+	+	-	-	-
Y(15)	+	-	-	+	+	+	-	-
Y(16)	+	-	-	-	+	+	+	-
RESTRAINT A	+	+	+					
RESTRAINT B								+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	1	1	1	1	1	1
1	.2550	.0000								+
1	.2550	.3535							+	
1	.2550	.3536						+		
1	.2550	.3536					+			
1	.2550	.3626				+				
1	.2000	.3536			+					
2	.1871	.5244		+						
2	.1871	.5244	+							
3	.1871	.5462	+		+					
4	.2550	1.0000	+	+						
5	.0606	1.2748	+	+	+					
6	.2550	1.5411	+	+	+	+				
7	.3674	1.8028	+	+	+	+	+			
8	.4253	2.0616	+	+	+	+	+	+		
9	.4330	2.3184	+	+	+	+	+	+	+	
10	.5124	2.3194	+	+	+	+	+	+	+	+

DESIGN 8.8 2,2,2,1,1,1 K= 6 N=12 D.F.= 7

OBSERVATIONS	2	2	2	1	1	1
Y(1)	+	-				
Y(2)	+		-			
Y(3)	+			-	-	
Y(4)	+			-		-
Y(5)	+				-	-
Y(6)		+	-			
Y(7)		+		-	-	
Y(8)		+		-		-
Y(9)		+			-	-
Y(10)			+	-	-	
Y(11)			+	-		-
Y(12)			+		-	-
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	2	1	1	1
1	.5065	.0000						+
1	.5069	.8165					+	
1	.5069	.8165				+		
2	[illegible]	[illegible]			+			
2	.2887	1.0541		+				
2	.2887	1.0541	+					
3	.7583	1.7638			+	+		
4	.5774	2.0276		+	+			
5	.8207	2.7284		+	+	+		
6	.5000	3.0000	+	+	+			
7	.7663	[illegible]	+	+	+	+		
8	.5919	4.3589	+	+	+	+	+	
9	.5014	4.3589	+	+	+	+	+	+

DESIGN 8.9 2,2,2,1,1,1 K= 6 N=15 D.F.=10

OBSERVATIONS	2	2	2	1	1	1
Y(1)	+	-				
Y(2)	+					
Y(3)	+			-	-	
Y(4)	+			-		-
Y(5)	+				-	-
Y(6)		+	-			
Y(7)		+		-	-	
Y(8)		+		-		-
Y(9)		+			-	-
Y(10)			+	-	-	
Y(11)			+	-		-
Y(12)			+		-	-
Y(13)				+	-	
Y(14)				+		-
Y(15)					+	-
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	2	1	1	1
1	.3819	.0000						+
1	.3819	.5774					+	
1	.3819	.5774				+		
2	.5000	.8165			+			
2	.2887	.8165		+				
2	.2887	.8165	+					
3	.6922	1.2910			+	+		
4	.5774	1.8257		+	+			
5	[illegible]	[illegible]		+	+	+		
6	.5000	2.2361	+	+	+			
7	.6922	2.7080	+	+	+	+		
8	.8165	3.1623	+	+	+	+	+	
9	.5014	3.1623	+	+	+	+	+	+

DESIGN 8.10 2,2,2,1,1,1,1 K= 7 N=12 D.F.= 6

OBSERVATIONS	2	2	2	1	1	1	1
Y(1)	+	-					
Y(2)	+		-				
Y(3)	+			-	-		
Y(4)	+					-	-
Y(5)		+	-				
Y(6)		+		-		-	
Y(7)		+			-		-
Y(8)			+	-			-
Y(9)			+		-	-	
Y(10)				+	+	-	-
Y(11)				+	-	+	-
Y(12)				+	-	-	+
RESTRAINT A	+	+					
RESTRAINT B							+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	2	1	1	1	1
1	.4183	.0000							+
1	.4183	.5774						+	
1	.4183	.5774					+		
1	[illegible]	[illegible]				+			
2	.5477	.8944			+				
2	.3162	.8944		+					
2	.3162	.8944	+						
3	.7583	1.3416			+	+			
4	.6325	1.6733		+	+				
5	.8216	2.1134		+	+	+			
6	.5477	2.4495	+	+	+				
7	.7583	2.8868	+	+	+	+			
8	.9305	3.3166	+	+	+	+	+		
9	1.0840	3.7417	+	+	+	+	+	+	
10	1.2247	3.7417	+	+	+	+	+	+	+

DESIGN 8.11 2,2,2,2,1,1 K= 6 N=11 D.F.= 6

OBSERVATIONS	2	2	2	2	1	1
Y(1)	+	-				
Y(2)	+		-			
Y(3)	+			-		
Y(4)	+				-	-
Y(5)		+	-			
Y(6)		+		-		
Y(7)		+			-	-
Y(8)			+	-		
Y(9)			+		-	-
Y(10)				+	-	-
Y(11)					+	-
RESTRAINT A	+	+				
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	2	2	2	2	1	1
1	.5701	.0000						+
1	.5701	1.0000					+	
2	.5477	1.1832				+		
2	.5477	1.1832			+			
2	.3162	1.1832		+				
2	.3162	1.1832	+					
3	.8515	2.0476				+	+	
4	.8944	2.2804			+	+		
5	1.1511	3.1937			+	+	+	
6	.9487	3.3764		+	+	+		
7	1.1937	4.2093		+	+	+	+	
8	.8944	4.4721	+	+	+	+		
9	[illegible]	[illegible]	+	+	+	+	+	
10	1.2247	5.3052	+	+	+	+	+	+

DESIGN C.1 5,3,2,1,1 K= 5
N= 8
D.F.= 4

(HAYFORD)

OBSERVATIONS	5	3	2	1	1
Y(1)	+	-	-	+	-
Y(2)	+	+	-	-	+
Y(3)	+	-	-		
Y(4)	+	-		-	-
Y(5)		+	-	-	
Y(6)		+	-		-
Y(7)			+	-	-
Y(8)				+	-
RESTRAINT A	+	+	+		
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	3	2	1	1
1	.4175	.0000					+
1	.4175	.5345				+	
2	.3846	.9258			+		
3	.3338	1.3093		+			
4	.6237	1.6903		+		+	
5	.2673	2.0702	+				
6	.5237	2.4495	+			+	
7	[illegible]	[illegible]	+		+		
8	.3546	3.3381	+	+			
9	.5632	3.7033	+	+		+	
10	.0000	4.1748	+	+	+		

DESIGN C.3 5,3,2,2,1,1 K= 6
N=11
D.F.= 6

OBSERVATIONS	5	3	2	2	1	1
Y(1)	+	-	+	-	-	-
Y(2)	+	-	-	+	-	-
Y(3)	+	-	-	-	+	+
Y(4)	+	-	-		+	-
Y(5)	+	-		-	+	-
Y(6)	+	-	-		-	+
Y(7)	+	-		-	-	+
Y(8)	+	-			-	-
Y(9)	+		-	-	-	
Y(10)	+		-	-		-
Y(11)			+	-		
RESTRAINT A	+	+	+			
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	3	2	2	1	1
1	.3592	.0000						+
1	.3592	.4714					+	
2	.4259	.7553				+		
2	.3625	.7553			+			
3	[illegible]	[illegible]		+				
2	.6116	1.1123				+	+	
4	.6130	1.4254			+	+		
5	.2266	1.7412	+					
6	.4790	2.0893	+				+	
7	.3836	2.4199	+		+			
8	.3625	2.9493	+	+				
9	.4711	3.2708	+	+			+	
10	.0000	3.5982	+	+	+			

DESIGN C.2 5,3,2,1,1,1 K= 6
N=11
D.F.= 6

OBSERVATIONS	5	3	2	1	1	1
Y(1)	+	-	-	+	-	
Y(2)	+	-	-		+	-
Y(3)	+	-	-	-		+
Y(4)	+	-	-			
Y(5)	+		-	-	-	-
Y(6)		+	-	+	-	-
Y(7)		+	-	-	+	-
Y(8)		+	-	-	-	+
Y(9)			+	-	-	
Y(10)			+	-		-
Y(11)			+		-	-
RESTRAINT A	+	+	+			
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	3	2	1	1	1
1	.3551	.0000						+
1	.3551	.0000					+	
1	.3551	.5900				+		
2	.2638	.7802			+			
3	.2989	1.0425		+				
4	.4772	1.4781		+		+		
5	.2231	1.7846	+					
6	.4299	2.1644	+			+		
7	.2965	2.5216	+		+			
8	.2638	2.9284	+	+				
9	.4616	3.2916	+	+		+		
10	[illegible]	[illegible]	+	+	+			

DESIGN C.4 5,3,2,2,1,1 K= 6
N=12
D.F.= 7

OBSERVATIONS	5	3	2	2	1	1
Y(1)	+	-	-			
Y(2)	+	-		-		
Y(3)	+	-			-	-
Y(4)	+		-	-	-	
Y(5)	+		-	-		-
Y(6)		+	-		-	
Y(7)		+	-			-
Y(8)		+		-	-	
Y(9)		+		-		-
Y(10)			+	-		
Y(11)			+		-	-
Y(12)				+	-	-
RESTRAINT A	+	+	+			
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	3	2	2	1	1
1	.4796	.0000						+
1	.4756	.5165					+	
2	.4355	1.0341				+		
2	.3127	1.0541			+			
3	.2848	1.4530		+				
3	.6173	1.7538				+	+	
4	.4372	2.0276			+	+		
5	.2287	2.3805	+					
6	.5028	3.1091	+				+	
7	.5608	3.3834	+			+		
8	[illegible]	[illegible]	+	+				
9	.4119	4.3216	+	+			+	
10	.0000	4.7930	+	+	+			

DESIGN C.5 5,5,3,2,1,1 K= 6
N=12
D.F.= 7

OBSERVATIONS	5	5	3	2	1	1
Y(1)	+	-		+	-	-
Y(2)	+	-		-	+	+
Y(3)	+	-			+	-
Y(4)	+	-			-	+
Y(5)	+					
Y(6)	+		-		-	-
Y(7)		+	-	-		
Y(8)		+	-		-	-
Y(9)			+	-	-	
Y(10)			+	-		-
Y(11)				+	-	-
Y(12)					+	-
RESTRAINT A		+	+	+		
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	5	5	3	2	1	1
1	.3536	.0000						+
1	.3536	.5345					+	
2	.3384	.7319				+		
3	.3261	1.1802			+			
4	.4236	1.5924			+		+	
5	.4366	1.7743	+					
5	.3010	1.7743		+				
6	.4790	2.2051		+			+	
7	.3201	2.4213		+		+		
8	.3384	2.9041		+	+			
9	.4473	3.3174		+	+		+	
10	.0000	3.5363		+	+	+		

DESIGN C.6 5,5,3,2,1,1,1 K= 7
N=15
D.F.= 9

OBSERVATIONS	5	5	3	2	1	1	1
Y(1)	+	-					
Y(2)	+		-	-			
Y(3)		+	-	-			
Y(4)	+		-		-	-	
Y(5)	+				-		-
Y(6)	+		-			-	-
Y(7)		+	-		-	-	
Y(8)		+	-		-		-
Y(9)		+	-			-	-
Y(10)	+			-	-	-	-
Y(11)		+		-	+	-	-
Y(12)			+		-	-	-
Y(13)				+	-	-	
Y(14)				+	-		-
Y(15)				+		-	-
RESTRAINT A		+	+	+			
RESTRAINT B							+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	5	5	3	2	1	1	1
1	.4967	.0000							+
1	.4967	.8165						+	
1	.4967	.8165					+		
2	.3464	1.0465				+			
3	.3251	1.5884			+				
4	.6040	2.2573			+		+		
5	.2673	2.4689		+					
5	.4629	2.4689	+						
6	.7198	3.1773	+				+		
7	.5529	3.4572	+			+			
8	.5904	3.9946	+		+				
9	.7048	4.6466	+		+		+		
10	.0000	4.9666		+	+	+			

DESIGN C.7 — 5,2,1,1,1 (HAYFORD) — K= 5, N= 7, D.F.= 3

OBSERVATIONS	5	2	1	1	1
Y(1)	+	-	-	-	-
Y(2)		+	-	-	
Y(3)		+	-		-
Y(4)		+		-	-
Y(5)			+	-	
Y(6)			+		-
Y(7)				+	-
RESTRAINT A	+				
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	2	1	1	1
1	.4650	.0000					+
1	.4600	.7071				+	
1	.4650	.7071			+		
2	.5292	1.0000		+			
5	.0030	2.3452	+				
3	.7616	1.5811		+	+		
4	.9055	2.1213		+	+	+	
6	.4690	2.9155	+		+		
7	.5292	3.2404	+	+			
8	.7616	3.8079	+	+	+		
9	.9055	4.3589	+	+	+	+	
10	1.0000	4.3309	+	+	+	+	+

DESIGN C.9 — 5,2,2,1,1 (BENOIT) — K= 5, N= 8, D.F.= 4

OBSERVATIONS	5	2	2	1	1
Y(1)	+	-	-	-	
Y(2)	+	-	-		-
Y(3)		+	-	+	-
Y(4)		+	-	-	+
Y(5)		+	-		
Y(6)		+		-	-
Y(7)			+	-	-
Y(8)				+	-
RESTRAINT A	+	+	+	+	
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	2	2	1	1
1	.4140	.0000					+
1	.2780	.5345				+	
2	.3401	.9258			+		
2	.3381	.9258		+			
3	.4140	1.3093			+	+	
4	.4140	1.7728		+	+		
5	.3780	2.0702	+				
6	.4140	2.4495	+			+	
7	.4140	2.9277	+		+		
8	.3381	3.2950	+		+	+	
9	.3780	3.7790	+	+	+		
10	.3330	4.1404	+	+	+	+	

DESIGN C.8 — 5,2,2,1,1 (HAYFORD) — K= 5, N= 7, D.F.= 3

OBSERVATIONS	5	2	2	1	1
Y(1)	+	-	-	-	
Y(2)		+	-	+	-
Y(3)		+	-	-	+
Y(4)		+	-		
Y(5)		+		-	-
Y(6)			+	-	-
Y(7)				+	-
RESTRAINT A	+	+	+	+	
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	2	2	1	1
1	.4523	.0000					+
1	.2780	.5774				+	
2	.3805	.9512			+		
2	.3805	.9512		+			
3	.4413	1.3801			+	+	
4	.5416	1.8257		+	+		
5	.5000	2.4495	+				
6	.5416	2.8868	+			+	
7	.4413	3.3022	+		+		
8	.3805	3.7289	+		+	+	
9	.3786	4.1633	+	+	+		
10	.0000	4.5826	+	+	+	+	

DESIGN C.10 — 5,2,2,1,1,1 — K= 6, N= 8, D.F.= 3

OBSERVATIONS	5	2	2	1	1	1
Y(1)	+	-	-	-	-	+
Y(2)	+	-	-	-	+	-
Y(3)	+	-	-	+	-	-
Y(4)	+	-		-		-
Y(5)	+		-	+	-	-
Y(6)		+	-	+	-	
Y(7)		+	-	-		+
Y(8)		-	-		+	
RESTRAINT A	+	+	+	-		
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	5	2	2	1	1	1
1	.4649	.0000						+
1	.4545	.5345					-	
1	.4326	.5345				+		
2	.3834	1.1339			+			
2	.3654	1.1339		+				
3	.3761	1.4549			+	+		
4	.3535	2.2039		+	+			
5	.3273	2.1712	+					
6	.5573	2.5355	+			+		
7	.3751	3.2514	+		+			
8	.3834	3.5957	+		+	+		
9	.4326	4.3760	+	+	+			
10	.0000	4.6448	+	+	+	+		

DESIGN D.1 4,2,2,1,1 K= 5 N= 6 D.F.= 2

OBSERVATIONS	4	2	2	1	1
Y(1)	+	-	-	+	-
Y(2)	+	-	-	-	+
Y(3)	+	-		-	-
Y(4)	+		-	-	-
Y(5)		+	-	+	-
Y(6)		+	-	-	+
RESTRAINT A	+				
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	4	2	2	1	1
1	.4677	.0000					+
1	.4677	.5000				+	
2	.4743	1.1619			+		
2	.4743	1.1619		+			
3	.5646	1.4491			+	+	
4	.0000	1.8708	+				
5	.4677	2.1794	+			+	
6	.4743	2.4749	+		+		
7	.5646	3.2558	+		+	+	
8	.7071	[illegible]	+	+	+		
9	.5847	4.3301	+	+	+	+	
10	.7906	4.3301	+	+	+	+	+

DESIGN D.3 4,2,2,1,1 (SENDIT) K= 5 N=11 D.F.= 7

OBSERVATIONS	4	2	2	1	1
Y(1)	+	-	-	+	-
Y(2)	+	-	-	-	+
Y(3)	+	-	-		
Y(4)	+				
Y(5)	+		-	-	-
Y(6)		+	-	+	-
Y(7)		+	-	-	+
Y(8)		+	-		
Y(9)		+		-	-
Y(10)			+	-	-
Y(11)				+	-
RESTRAINT A	+				
RESTRAINT C					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	4	2	2	1	1
1	.3354	.0000					+
1	.3354	.4472				+	
2	.3536	.7583			+		
2	.3536	.7583		+			
3	.4872	1.0840		+		+	
4	.0000	1.3416	+				
5	.3354	1.6733	+			+	
7	.4873	2.3611	+	+		+	
8	.5000	2.7298	+	+	+		
9	.6021	3.0414	+	+	+	+	
10	.7071	3.0414	+	+	+	+	+

DESIGN D.2 4,2,2,1,1 K= 5 N=10 D.F.= 6

OBSERVATIONS	4	2	2	1	1
Y(1)	+	-	-	-	+
Y(2)	+	-	-	+	-
Y(3)	+	-	+		
Y(4)	+	-		-	-
Y(5)	+		-	+	-
Y(6)		+	-	-	+
Y(7)		+	-	+	-
Y(8)		+	-		
Y(9)		+		-	-
Y(10)			+	-	-
RESTRAINT A	+				
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	4	2	2	1	1
1	.3536	.0000					+
1	.3536	.5000				+	
2	.3536	.7906			+		
2	.3536	.7906		+			
3	.5000	1.1726		+		+	
4	.0000	1.4142	+				
5	.3536	1.8028	+			+	
6	.3536	2.1506	+	+			
7	.5000	2.5249	+	+		+	
8	.5000	2.8723	+	+	+		
9	.6124	3.2404	+	+	+	+	
10	.7071	3.2404	+	+	+	+	+

DESIGN D.4 4,2,2,1,1,1 K=6 N=10 D.F.= 5

OBSERVATIONS	4	2	2	1	1	1
Y(1)	+	-	-			
Y(2)	+	-		-	-	
Y(3)	+	-		-		-
Y(4)	+	-			-	-
Y(5)	+		-	-	-	
Y(6)	+		-	-		-
Y(7)	+		-		-	-
Y(8)		+	-	+	-	
Y(9)		+	-	-		+
Y(10)		+	-		+	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	4	2	2	1	1	1
1	.4873	.0000						+
1	.4873	.6325					+	
1	.4873	.6325				+		
2	.3528	1.3229			+			
2	.3528	1.3229		+				
3	.5413	1.7192			+	+		
4	.0000	1.9494	+					
5	.4873	2.4083	+			+		
6	.3528	3.2180	+		+			
7	.5413	3.6271	+		+	+		
8	[illegible]	4.4944	+	+	+			
9	.6588	4.8785	+	+	+	+		
10	.7416	5.2726	+	+	+	+	+	

DESIGN 0.5   4,4,2,1,1   K= 5
N= 9
D.F.= 5

(HENOIT)

OBSERVATIONS	4	4	2	1	1
Y(1)	+	-	+	-	-
Y(2)	+	-	-	+	+
Y(3)	+	-		+	-
Y(4)	+	-		-	+
Y(5)	+	-			
Y(6)	+		-	-	-
Y(7)		+	-	-	-
Y(8)			+	-	-
Y(9)				+	-
RESTRAINT A	+	+	+		
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	4	4	2	1	1
1	.3651	.0000					+
1	.3651	.5774				+	
2	.3651	.8165			+		
3	.5164	1.2910			+	+	
4	.2807	1.4673		+			
4	.2807	1.4673	+				
5	.4606	1.9695	+			+	
6	.2807	2.2088	+		+		
7	.4606	2.6855	+		+	+	
8	.3651	2.9439	+	+			
9	.5164	3.4157	+	+		+	
10	.0000	3.6515	+	+	+		

DESIGN 0.7   4,4,2,2,1,1,1   K= 7
N=14
D.F.= 8

OBSERVATIONS	4	4	2	2	1	1	1
Y(1)	+		-	-			
Y(2)	+		-		-	-	
Y(3)	+		-		-		-
Y(4)	+		-			-	-
Y(5)	+			-	-	-	
Y(6)	+			-	-		-
Y(7)	+			-		-	-
Y(8)		+	-	-			
Y(9)		+	-		-	-	
Y(10)		+	-		-		-
Y(11)		+	-			-	-
Y(12)		+		-	-	-	
Y(13)		+		-	-		-
Y(14)		+		-		-	-
RESTRAINT A	+	+	+				
RESTRAINT B							+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	4	4	2	2	1	1	1
1	.4030	.0000							+
1	.4030	.7071						+	
1	.4030	.7071					+		
2	.4472	1.1547				+			
2	.3651	1.1547			+				
3	.5774	1.6833				+	+		
4	.3237	1.6498		+					
4	.3237	1.6498	+						
5	.6362	2.4640	+				+		
6	.5521	2.9841	+			+			
7	.7105	3.5220	+			+	+		
8	.3651	3.7417	+	+					
9	.7071	4.3012	+	+			+		
10	.0000	4.8305	+	+	+				

DESIGN 0.6   4,4,2,2,1,1   K= 6
N= 9
D.F.= 4

OBSERVATIONS	4	4	2	2	1	1
Y(1)	+	-	+	-	+	-
Y(2)	+	-	+	-	-	+
Y(3)	+	-	-	+		
Y(4)	+	-	+		-	-
Y(5)	+	-		-	+	+
Y(6)	+		-		-	-
Y(7)	+		-	+	-	+
Y(8)		+		-	-	-
Y(9)		+	-	-	+	-
RESTRAINT A	+	+				
RESTRAINT B					+	+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	4	4	2	2	1	1
1	.3578	.2560						+
1	.3578	.2560					+	
2	[illegible]	[illegible]				+		
2	.3907	.6346			+			
3	.5211	.6776				+		+
3	.5211	.6776			+		+	
4	.2195	1.0238		+				
4	.2195	1.0238	+					
5	.4476	1.0667		+				+
5	.4476	1.0667	+				+	
6	.4058	1.5539		+		+		
6	.4058	1.5539	+		+			
7	.5547	1.5796		+		+		+
7	.5547	1.5796	+		+		+	
8	.0000	2.3000	+	+				

DESIGN 0.8   4,3,2,1,1   K= 5
N= 8
D.F.= 4

(HAYFORD)

OBSERVATIONS	4	3	2	1	1
Y(1)	+	-	-		+
Y(2)	+		-	-	-
Y(3)	+	-		-	
Y(4)	+	-			-
Y(5)		+	-	-	
Y(6)		+	-		-
Y(7)			+	-	-
Y(8)				+	-
RESTRAINT A	+	+	+	+	
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	4	3	2	1	1
1	.4275	.0000					+
1	.4350	.6892				+	
2	.3950	.5354			+		
3	.3532	1.2645		+			
4	.3146	1.0956	+				
5	.4677	2.2383	+			+	
5	.4677	2.1154		+	+		
6	.4221	2.5455	+		+		
7	.4180	2.9112	+	+			
8	.3550	3.4456	+	+		+	
9	.4190	3.7850	+	+	+		
10	.0000	4.2749	+	+	+	+	

DESIGN D.9 4,3,2,1,1 K= 5
N= 9
(BENDIT) D.F.= 5

OBSERVATIONS	4	3	2	1	1
Y(1)	+	-	-	+	
Y(2)	+	-	-		+
Y(3)	+	-		-	
Y(4)	+				
Y(5)	+		-	-	-
Y(6)		+	-	-	
Y(7)		+	-		-
Y(8)			+	-	-
Y(9)				+	-
RESTRAINT A	+	+	+	+	
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	4	3	2	1	1
1	.3751	.6325				+	
2	.3507	.9309			+		
3	.3256	1.2049		+			
4	.4011	1.6733	+				
5	.3416	2.1909	+			+	
6	.4131	2.5420	+		+		
7	.4155	2.8963	+	+			
8	.3502	3.4059	+	+		+	
9	.3751	3.7506	+	+	+		
10	.0000	4.2595	+	+	+	+	

DESIGN D.11 10,5,2,1,1,1 K= 6
N= 8
(HAYFORD) D.F.= 3

OBSERVATIONS	10	5	2	1	1	1
Y(1)	+	-	-	-	-	-
Y(2)		+	-	-	-	-
Y(3)			+	-	-	
Y(4)			+			
Y(5)			+	-		-
Y(6)				+	-	
Y(7)				+		-
Y(8)					+	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	10	5	2	1	1	1
1	.4472	.0000						+
1	.4472	.7071					+	
1	.4472	.7071				+		
2	.4472	1.0000			+			
3	.6325	1.5811			+	+		
4	.7071	2.1213			+	+	+	
5	.7071	2.3452		+				
6	[illegible]	[illegible]		+		+		
7	.6367	3.2404		+	+			
8	.9487	3.8079		+	+	+		
9	1.0000	4.3589		+	+	+	+	
10	.0000	4.4721	-					

DESIGN D.10 10,4,3,2,1,1 K= 6
N=10
(HAYFORD) D.F.= 5

OBSERVATIONS	10	4	3	2	1	1
Y(1)	+	-	-	-	-	
Y(2)	+	-	-	-		-
Y(3)		+	-	-		+
Y(4)		+		-	-	-
Y(5)		+	-		-	
Y(6)		+	-			-
Y(7)			+	-	-	
Y(8)			+	-		-
Y(9)				+	-	-
Y(10)					+	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	10	4	3	2	1	1
1	.5045	.0000						+
1	.4411	.6195					+	
2	.4023	.9045				+		
3	.3885	1.2060			+			
4	[illegible]	[illegible]		+				
5	.6552	2.0889		+			+	
5	.6170	2.0220			+	+		
6	.3610	2.4225		+		+		
7	.6092	2.7561		+	+			
8	.7656	3.2193		+	+		+	
9	.6295	3.5061		+	+	+		
10	.7720	4.0113		+	+	+	+	

DESIGN D.12 10,5,2,2,1,1 K= 6
N= 8
(HAYFORD) D.F.= 3

OBSERVATIONS	10	5	2	2	1	1
Y(1)	+	-	-	-	-	
Y(2)		+	-	-	-	
Y(3)			+	-	+	-
Y(4)			+	-	-	+
Y(5)			+	-		
Y(6)			+		-	-
Y(7)				+	-	-
Y(8)					+	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	10	5	2	2	1	1
1	.4650	.0000						+
1	.3916	.5774					+	
2	.4298	.9512				+		
2	.4298	.9512			+			
3	.5336	1.3861			+		+	
4	.6733	1.8257			+	+		
5	.7071	[illegible]		+				
6	.8083	2.8060		+			+	
7	.8275	3.3022		+	+			
8	.8555	3.7289		+	+		+	
9	.9764	4.1533		+	+	+		
10	.0000	4.6904	+					

DESIGN D.13 10,5,3,2,1,1 K= 6

N=10

(HAYFORD) D.F.= 5

OBSERVATIONS	10	5	3	2	1	1
Y(1)	+	-	-	-		
Y(2)	+	-	-		-	-
Y(3)		+	-	-	-	+
Y(4)		+	-	-	+	-
Y(5)		+	-	-		
Y(6)		+	-		-	-
Y(7)			+	-	-	
Y(8)			+	-		-
Y(9)				+	-	-
Y(10)					+	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	10	5	3	2	1	1
1	.3519	.0000						+
1	.3810	.5345					+	
2	.4900	.6545				+		
3	.3844	1.2599			+			
4	.5014	1.6523			+		+	
5	[illegible]	[illegible]		+				
6	.5738	2.3666		+			+	
7	.6692	2.7598		+		+		
8	.6481	3.1910		+	+			
9	.7159	3.5713		+	+		+	
10	.7817	3.5551		+	+	+		

DESIGN D.15 10,5,4,3,2,1 K= 6

N=10

(HAYFORD) D.F.= 5

OBSERVATIONS	10	5	4	3	2	1
Y(1)	+	-	-			.
Y(2)	+	-		-	-	
Y(3)	+		-	-	-	-
Y(4)		+	-	-	+	
Y(5)		+	-		-	+
Y(6)		+	-			-
Y(7)		+		-	-	
Y(8)			+	-	+	+
Y(9)			+	-		-
Y(10)				+	-	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	10	5	4	3	2	1
1	.4096	.0000						+
2	.3545	.5045					+	
3	.4006	1.3104				+		
4	.4299	1.8340			+			
5	.4523	2.2144		+				
6	[illegible]	[illegible]		+				+
7	.6038	3.0928			+	+		
8	.6547	3.4772		+		+		
9	.7294	4.0176		+	+			
10	.0000	6.0561	+					

DESIGN D.14 5,4,3,2,1 K= 5

N= 7

(HAYFORD) D.F.= 3

OBSERVATIONS	5	4	3	2	1
Y(1)	+	-	-	+	
Y(2)	+	-		-	+
Y(3)	+	-			-
Y(4)	+		-	-	
Y(5)		-	-	-	+
Y(6)		-	-		-
Y(7)			+	-	-
RESTRAINT A	+				
RESTRAINT B					+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	5	4	3	2	1
1	.4775	.0000					+
2	.4775	.4663				+	
3	.5912	.5683			+		
4	.5912	.6825		+			
5	.6000	.6825	+				
6	.4775	.6825	+				+
7	.4775	.8776	+			+	
8	.5912	.9292	+		+		
9	.5912	1.1952	+	+			
10	.6782	1.1952	+	+			+

DESIGN D.16 6,5,4,3,2,1 K= 6

N= 9

(HAYFORD) D.F.= 4

OBSERVATIONS	6	5	4	3	2	1
Y(1)	+	-	-	-		
Y(2)	+		-	-	+	-
Y(3)	+	-			-	+
Y(4)	+	-				-
Y(5)	+		-		-	
Y(6)		+	-			-
Y(7)		+		-	-	
Y(8)			-	-		-
Y(9)				+	-	-
RESTRAINT A	+					
RESTRAINT B						+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	B	6	5	4	3	2	1
1	.4342	.0000						+
2	.4646	.9181					+	
3	.5175	1.4630				+		
4	[illegible]	[illegible]			+			
5	.4956	2.1079		+				
6	.0000	2.6049	+					
7	.4342	2.8049	+					+
8	.4646	3.4310	+				+	
9	.5175	4.0409	+			+		
10	.4697	4.4657	+		+			

DESIGN C.17 10,6,5,4,3,2,1 K= 7

N=12

(HAYFORD) D.F.= 6

OBSERVATIONS	10	6	5	4	3	2	1
Y(1)	+	-		-			
Y(2)	+		-	-			-
Y(3)	+		-		-	-	
Y(4)		+			-		
Y(5)		+		-	-	+	-
Y(6)		+	-			-	+
Y(7)		+	-				-
Y(8)		+		-		-	
Y(9)			+	-			-
Y(10)			+		-	-	
Y(11)				+	-		-
Y(12)					+	-	-
RESTRAINT A	+						
RESTRAINT B							+

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	10	6	5	4	3	2	1
1	.4129	.0000							+
2	.4229	.8556						+	
3	.4250	1.3841					+		
4	[illegible]	[illegible]				+			
5	.4552	2.0860			+				
6	.4889	2.4656		+					
7	.6696	2.4656		+					+
7	.5818	2.8467			+			+	
7	.6607	3.1442				+	+		
8	.6977	3.2365		+				+	
8	.6597	3.4224			+		+		
9	.6819	3.7975		+			+		
9	.6090	3.8493			+	+			
10	[illegible]	[illegible]		+		+			

DESIGN E.1 FOUR EQUAL WEIGHTS *K= 5

Trend Elimination N= 8

D.F.= 4

OBSERVATIONS	1	1	1	1	
Y(1)	+				-
Y(2)		+			-
Y(3)			+		-
Y(4)				+	-
Y(5)				+	-
Y(6)			+		-
Y(7)		+			-
Y(8)	+				
RESTRAINT A	+	+			
RESTRAINT B				+	

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	1	1	1	1
1	.8660	.0000				+
1	.8660	1.0000			+	
1	.5000	1.0000		+		
1	.5000	1.0000	+			
2	.0000	1.7321	+	+		
3	.8660	2.4495	+	+	+	
4	1.4142	2.4495	+	+	+	+

*See page 13.

DESIGN E.2 2,2,1,1 *K= 6

N= 10

Trend Elimination D.F.= 5

OBSERVATIONS	2	2	1	1		
Y(1)	+				-	
Y(2)		+			-	
Y(3)			+	+	-	
Y(4)			+			-
Y(5)				+		-
Y(6)				+		-
Y(7)			+			-
Y(8)			+	+	-	
Y(9)		+			-	
Y(10)	+				-	
RESTRAINT A	+	+				
RESTRAINT B	+	+	+			

FACTORS FOR COMPUTING ST DEVS

WT	RESTRAINTS A	RESTRAINTS B	2	2	1	1
1	.5514	.0928				+
1	.6614	.5292			+	
2	.5000	.5657		+		
2	.5000	.5657	+			
3	.8253	.9592	+			+
4	.0000	.5292	+	+		
5	.6614	1.0000	+	+		-

*See page 13.

APPENDIX B

LIST OF TABLES

TABLE 1: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN A.1.2

DESIGN A.1.2 FOUR EQUAL WEIGHTS K= 4 N= 6 D.F.= 3

OBSERVATIONS	1	1	1	1
Y(1)	+	−		
Y(2)	+		−	
Y(3)	+			−
Y(4)		+	−	
Y(5)		+		−
Y(6)			+	−
RESTRAINT	+	+		

PARAMETER VALUES DIVISOR = 8

OBSERVATIONS	1	1	1	1
Y(1)	2	−2	0	0
Y(2)	1	−1	−3	−1
Y(3)	1	−1	−1	−3
Y(4)	−1	1	−3	−1
Y(5)	−1	1	−1	−3
Y(6)	0	0	2	−2
M(1)	4	4	4	4

DEVIATIONS DIVISOR = 4

	1	2	3	4	5	6
Y(1)	2	−1	−1	1	1	0
Y(2)	1	2	−1	−1	0	1
Y(3)	−1	−1	2	0	−1	−1
Y(4)	1	−1	0	2	−1	1
Y(5)	1	0	−1	−1	2	−1
Y(6)	0	1	−1	1	−1	2

MATRIX OF NORMAL EQUATIONS

3	−1	−1	−1	1
−1	3	−1	−1	1
−1	−1	3	−1	0
−1	−1	−1	3	0
1	1	0	0	0

INVERSE DIVISOR = 8

1	−1	0	0	4
−1	1	0	0	4
0	0	3	1	4
0	0	1	3	4
4	4	4	4	0

TABLE 2: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN A.1.2

DESIGN A.1.2 FOUR EQUAL WEIGHTS

K= 4
N= 6
D.F.= 3

OBSERVATIONS	1	1	1	1
Y(1)	+	-		
Y(2)	+		-	
Y(3)	+			-
Y(4)		+	-	
Y(5)		+		-
Y(6)			+	-
RESTRAINT				+

PARAMETER VALUES
DIVISOR = 4

OBSERVATIONS	1	1	1	1
Y(1)	1	-1	0	0
Y(2)	1	0	-1	0
Y(3)	2	1	1	0
Y(4)	0	1	-1	0
Y(5)	1	2	1	0
Y(6)	1	1	2	0
R(1)	4	4	4	4

DEVIATIONS
DIVISOR = 4

	1	2	3	4	5	6
Y(1)	2	-1	-1	1	1	0
Y(2)	-1	2	-1	-1	0	1
Y(3)	-1	-1	2	0	-1	-1
Y(4)	1	-1	0	2	-1	1
Y(5)	1	0	-1	-1	2	-1
Y(6)	0	1	-1	1	-1	2

MATRIX OF NORMAL EQUATIONS

3	-1	-1	-1	0
-1	3	-1	-1	0
-1	-1	3	-1	0
-1	-1	-1	3	1
0	0	0	1	0

INVERSE DIVISOR = 4

2	1	1	0	4
1	2	1	0	4
1	1	2	0	4
0	0	0	0	4
4	4	4	4	0

TABLE 3: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN B.3

DESIGN B.3   2,2,1,1   K= 4
N= 6
D.F.= 3

OBSERVATIONS	2	2	1	1
Y(1)	+	-	+	-
Y(2)	+	-	-	+
Y(3)	+	-		
Y(4)	+		-	-
Y(5)		+	-	-
Y(6)			+	-
RESTRAINT	+	+		

PARAMETER VALUES
DIVISOR = 84

OBSERVATIONS	2	2	1	1
Y(1)	12	-12	14	-14
Y(2)	12	-12	-14	14
Y(3)	12	-12	0	0
Y(4)	6	-6	-21	-21
Y(5)	-6	6	-21	-21
Y(6)	0	0	14	-14
M(1)	42	42	21	21

DEVIATIONS
DIVISOR = 21

	1	2	3	4	5	6
Y(1)	0	1	-6	-3	3	-7
Y(2)	1	8	-6	-3	3	7
Y(3)	-6	-6	15	-3	3	0
Y(4)	-3	-3	-3	9	-9	0
Y(5)	3	3	3	-9	9	0
Y(6)	-7	7	0	0	0	14

MATRIX OF NORMAL EQUATIONS

4	-3	-1	-1	1
-3	4	-1	-1	1
-1	-1	5	-1	0
-1	-1	-1	5	0
1	1	0	0	0

INVERSE   DIVISOR = 168

12	-12	0	0	84
-12	12	0	0	84
0	0	35	7	42
0	0	7	35	42
84	84	42	42	0

TABLE 4: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN B.3

DESIGN B.3   2,2,1,1

K= 4
N= 6
D.F.= 3

OBSERVATIONS	2	2	1	1
Y(1)	+	−	+	−
Y(2)	+	−	−	+
Y(3)	+	−		
Y(4)	+		−	−
Y(5)		+	−	−
Y(6)			+	−
RESTRAINT				+

PARAMETER VALUES
DIVISOR = 21

OBSERVATIONS	2	2	1	1
Y(1)	10	4	7	0
Y(2)	−4	−10	−7	0
Y(3)	3	−3	0	0
Y(4)	12	9	0	0
Y(5)	9	12	0	0
Y(6)	7	7	7	0
M(I)	42	42	21	21

DEVIATIONS
DIVISOR = 21

	1	2	3	4	5	6
Y(1)	8	1	−6	−3	3	−7
Y(2)	1	8	−6	−3	3	7
Y(3)	−6	−6	15	−3	3	0
Y(4)	−3	−3	−3	9	−9	0
Y(5)	3	3	3	−9	9	0
Y(6)	−7	7	0	0	0	14

MATRIX OF NORMAL EQUATIONS

4	−3	−1	−1	0
−3	4	−1	−1	0
−1	−1	5	−1	0
−1	−1	−1	5	1
0	0	0	1	0

INVERSE   DIVISOR = 21

19	16	7	0	42
16	19	7	0	42
7	7	7	0	21
0	0	0	0	21
42	42	21	21	0

TABLE 5: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN C.2

DESIGN C.2 5,3,2,1,1,1 K= 6 N=11 D.F.= 6

OBSERVATIONS	5	3	2	1	1	1
Y(1)	+	-	-	+	-	
Y(2)	+	-	-		+	-
Y(3)	+	-	-	-		+
Y(4)	+	-	-			
Y(5)	+		-	-	-	-
Y(6)		+	-	+	-	-
Y(7)		+	-	-	+	-
Y(8)		+		-	-	+
Y(9)			+	-	-	
Y(10)			+	-		-
Y(11)			+		-	-
RESTRAINT	+	+	+			

PARAMETER VALUES
DIVISOR = 920

OBSERVATIONS	5	3	2	1	1	1
Y(1)	100	-68	-32	119	-111	4
Y(2)	100	-68	-32	4	119	-111
Y(3)	100	-68	-32	-111	4	119
Y(4)	100	-68	-32	4	4	4
Y(5)	60	-4	-66	-108	-108	-108
Y(6)	-20	124	-104	128	-102	-102
Y(7)	-20	124	-104	-102	128	-102
Y(8)	-20	124	-104	-102	-102	128
Y(9)	-20	-60	50	-125	-125	-10
Y(10)	-20	-60	50	-125	-10	-125
Y(11)	-20	-60	50	-10	-125	-125
R(1)	460	276	184	92	92	92

DEVIATIONS
DIVISOR = 184

	1	2	3	4	5	6	7	8	9	10	11
Y(1)	90	-17	-17	-40	-24	-38	54	8	8	31	-15
Y(2)	-17	90	-17	-40	-24	8	-38	54	31	-15	31
Y(3)	-17	-17	90	-40	-24	54	8	-38	-15	8	8
Y(4)	-40	-40	-40	144	-24	8	8	8	8	8	8
Y(5)	-24	-24	-24	-24	96	-32	-32	-32	-32	-32	-32
Y(6)	-38	8	54	8	-32	72	-20	-20	26	26	-20
Y(7)	54	-38	8	8	-32	-20	72	-20	26	-20	26
Y(8)	8	54	-38	8	-32	-20	-20	72	-20	26	26
Y(9)	8	31	-15	8	-32	26	26	-20	118	-43	-43
Y(10)	31	-15	8	8	-32	26	-20	26	-43	118	-43
Y(11)	-15	8	31	8	-32	-20	26	26	-43	-43	118

MATRIX OF NORMAL EQUATIONS

5	-4	-5	-1	-1	-1	1
-4	7	1	-1	-1	-1	1
-5	1	11	0	0	0	1
-1	-1	0	8	0	0	0
-1	-1	0	0	8	0	0
-1	-1	0	0	0	8	0
1	1	1	0	0	0	0

INVERSE DIVISOR = 920

50	-34	-16	2	2	2	460
-34	62	-48	6	6	6	276
-16	-48	64	-8	-8	-8	184
2	6	-8	116	1	1	92
2	6	-8	1	116	1	92
2	6	-8	1	1	116	92
460	276	184	92	92	92	0

TABLE 6: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN C.2

DESIGN C.2 5,3,2,1,1,1

K= 6
N=11
D.F.= 6

OBSERVATIONS	5	3	2	1	1	1
Y(1)	+	-	-	+	-	
Y(2)	+	-	-		+	-
Y(3)	+	-	-	-		+
Y(4)	+	-	-			
Y(5)	+		-	-	-	-
Y(6)		-	-	+	-	-
Y(7)		-	-	-	+	-
Y(8)		-	-	-	-	+
Y(9)			+	-	-	
Y(10)			+	-		-
Y(11)			+		-	-
RESTRAINT						+

PARAMETER VALUES
DIVISOR = 184

OBSERVATIONS	5	3	2	1	1	1
Y(1)	16	-16	-8	23	-23	0
Y(2)	131	53	38	23	46	0
Y(3)	-99	-65	-54	-46	-23	0
Y(4)	16	-16	-8	0	0	0
Y(5)	120	64	32	0	0	0
Y(6)	50	66	20	46	0	0
Y(7)	56	66	20	0	46	0
Y(8)	-132	-52	-72	-46	-46	0
Y(9)	6	-6	20	-23	-23	0
Y(10)	131	63	66	0	23	0
Y(11)	121	63	66	23	0	0
R(1)	920	552	368	184	184	184

DEVIATIONS
DIVISOR = 184

	1	2	3	4	5	6	7	8	9	10	11
Y(1)	58	-17	-17	-40	-24	-38	54	8	8	31	-15
Y(2)	-17	98	-17	-40	-24	8	-38	54	31	-15	8
Y(3)	-17	-17	98	-40	-24	54	8	-38	-15	8	31
Y(4)	-40	-40	-40	44	-24	8	0	8	8	8	8
Y(5)	-24	-24	-24	-24	96	-32	-32	-32	-32	-32	-32
Y(6)	-38	8	54	8	-32	72	-20	-20	26	26	-20
Y(7)	54	-38	8	8	-32	-20	72	-20	26	-20	26
Y(8)	8	54	-38	8	-32	-20	-20	72	-20	26	26
Y(9)	8	31	-15	8	-32	26	26	-20	118	-43	-43
Y(10)	31	-15	8	8	-32	26	-20	26	-43	110	-43
Y(11)	-15	8	31	8	-32	-20	26	26	-43	-43	118

MATRIX OF NORMAL EQUATIONS

5	-4	-5	-1	-1	-1	0
-4	7	1	-1	-1	-1	0
-5	1	11	0	0	0	0
-1	-1	0	8	0	0	0
-1	-1	0	0	8	0	0
-1	-1	0	0	0	8	1
0	0	0	0	0	1	0

INVERSE DIVISOR = 184

506	334	236	115	115	0	920
334	218	132	69	69	0	552
236	132	112	46	46	0	368
115	69	46	46	23	0	184
115	69	46	23	46	0	184
0	0	0	0	0	0	184
920	552	368	184	184	184	0

TABLE 7. MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN C.10

DESIGN C.10 5,2,2,1,1,1

K= 6
N= 8
D.F.= 3

OBSERVATIONS	5	2	2	1	1	1
Y(1)	+	−	−	−		+
Y(2)	+	−	−	−	+	−
Y(3)	+	−	−	+	−	−
Y(4)	+	−		−	−	−
Y(5)	+		−	−	−	−
Y(6)		+	−	+		
Y(7)		+	−	−		+
Y(8)		+	−		+	−
RESTRAINT	+	+	+	+		

PARAMETER VALUES
DIVISOR = 70

OBSERVATIONS	5	2	2	1	1	1
Y(1)	15	−8	−8	1	1	21
Y(2)	15	−8	−8	1	21	1
Y(3)	5	−12	−12	19	−1	−1
Y(4)	0	2	12	−14	−14	−14
Y(5)	0	12	2	−14	−14	−14
Y(6)	−5	8	−12	9	−11	−1
Y(7)	5	12	−8	−9	1	11
Y(8)	0	10	−10	0	10	−10
R(1)	35	14	14	7	7	7

DEVIATIONS
DIVISOR = 7

	1	2	3	4	5	6	7	8
Y(1)	2	−1	−1	0	0	0	−2	2
Y(2)	−1	2	−1	0	0	2	0	−2
Y(3)	−1	−1	2	0	0	−2	2	0
Y(4)	0	0	0	3	−3	1	1	1
Y(5)	0	0	0	−3	3	−1	−1	−1
Y(6)	0	2	−2	1	−1	3	−1	−1
Y(7)	−2	0	2	1	−1	−1	3	−1
Y(8)	2	−2	0	1	−1	−1	−1	3

MATRIX OF NORMAL EQUATIONS

5	−4	−4	−3	−3	−3	1
−4	7	0	2	2	2	1
−4	0	7	2	2	2	1
−3	2	2	7	0	0	1
−3	2	2	0	7	0	0
−3	2	2	0	0	7	0
1	1	1	1	0	0	0

INVERSE DIVISOR = 700

75	−40	−40	5	55	55	350
−40	130	4	−68	−48	−48	140
−40	4	134	−68	−48	−48	140
5	−68	−68	131	41	41	70
55	−48	−48	41	151	51	70
55	−48	−48	41	51	151	70
350	140	140	70	70	70	0

TABLE 8 MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN C.10

DESIGN C.10 5,2,2,1,1,1

K= 6
N= 8
D.F.= 3

OBSERVATIONS	5	2	2	1	1	1
Y(1)	+	-	-	-	-	-
Y(2)	+	-	-	-	+	-
Y(3)	+	-	-	+	-	-
Y(4)	+	-		-	-	-
Y(5)	+		-	-	-	-
Y(6)		+	-	+	-	
Y(7)		+	-	-		+
Y(8)		+	-		+	-
RESTRAINT						+

PARAMETER VALUES
DIVISOR = 7

OBSERVATIONS	5	2	2	1	1	1
Y(1)	-9	-5	-5	-2	-2	0
Y(2)	1	-1	-1	0	2	0
Y(3)	1	-1	-1	2	0	0
Y(4)	7	3	4	0	0	0
Y(5)	7	4	5	0	0	0
Y(6)	0	1	-1	1	-1	0
Y(7)	-5	-1	-3	-2	-1	0
Y(8)	5	3	1	1	2	0
M(1)	35	14	14	7	7	7

DEVIATIONS
DIVISOR = 7

	1	2	3	4	5	6	7	8
Y(1)	2	-1	-1	0	0	0	-2	2
Y(2)	-1	2	-1	0	0	0	0	-2
Y(3)	-1	-1	2	0	0	-2	2	0
Y(4)	0	0	0	4	-3	1	1	1
Y(5)	0	0	0	-3	4	-1	-1	-1
Y(6)	0	2	-2	1	-1	3	-1	-1
Y(7)	-2	0	2	1	-1	-1	3	-1
Y(8)	2	-2	0	1	-1	-1	-1	3

MATRIX OF NORMAL EQUATIONS

5	-4	-4	-3	-3	-3	0
-4	7	0	2	2	2	0
-4	0	7	2	2	2	0
-3	2	2	7	0	0	0
-3	2	2	0	7	0	0
-3	2	2	0	0	7	1
0	0	0	0	0	1	0

INVERSE DIVISOR = 7

21	16	16	5	5	0	35
16	9	8	2	2	0	14
16	8	9	2	2	0	14
5	2	2	2	1	0	7
5	2	2	1	2	0	7
0	0	0	0	0	0	7
35	14	14	7	7	7	0

TABLE 9: MULTIPLIERS OF THE OBSERVATIONS FOR DETERMINING PARAMETER VALUES AND DEVIATIONS FOR DESIGN E.1

DESIGN E.1 FOUR EQUAL WEIGHTS K= 6
N= 8
D.F.= 3

OBSERVATIONS	1	1	1	1	Tare*	Trend**
Y(1)	1	0	0	0	-1	-7
Y(2)	0	1	0	0	-1	-5
Y(3)	0	0	1	0	-1	-3
Y(4)	0	0	0	1	-1	-1
Y(5)	0	0	0	1	-1	1
Y(6)	0	0	1	0	-1	3
Y(7)	0	1	0	0	-1	5
Y(8)	1	0	0	0	-1	7
RESTRAINT	+			+		

*The observation is on the difference between the listed weight and the built-in or tare weight of the balance.

**For explanation of trend, see page 13.

PARAMETER VALUES
DIVISOR = 168

OBSERVATIONS	1	1	1	1	Tare*	Trend**
Y(1)	42	-42	-42	-42	-42	-7
Y(2)	0	84	0	0	0	-5
Y(3)	0	0	84	0	0	-3
Y(4)	-42	-42	-42	42	-42	-1
Y(5)	-42	-42	-42	42	-42	1
Y(6)	0	0	84	0	0	3
Y(7)	0	84	0	0	0	5
Y(8)	42	-42	-42	-42	-42	7
R(1)	84	84	84	84	84	0

DEVIATIONS
DIVISOR = 168

	1	2	3	4	5	6	7	8
Y(1)	36	-35	-21	-7	7	21	35	-35
Y(2)	-35	59	-15	-5	5	15	-59	35
Y(3)	21	-15	75	-3	3	-75	15	21
Y(4)	-7	-5	-3	83	-83	3	5	7
Y(5)	7	5	3	83	83	-3	-5	-7
Y(6)	21	15	-75	3	-3	75	-15	-21
Y(7)	35	-59	15	5	-5	-15	59	-35
Y(8)	-35	35	21	7	-7	-21	-35	35

MATRIX OF NORMAL EQUATIONS

2	0	0	0	-2	0	1
0	2	0	0	-2	0	0
0	0	2	0	-2	0	0
0	0	0	2	-2	0	1
-2	-2	-2	-2	8	0	0
0	0	0	0	0	168	0
1	0	0	1	0	0	0

INVERSE DIVISOR = 168

42	0	0	-42	0	0	84
0	126	42	0	42	0	84
0	42	126	0	42	0	84
-42	0	0	42	0	0	84
0	42	42	0	42	0	84
0	0	0	0	0	1	0
84	84	84	84	84	0	0

DESIGN E.1 FOUR EQUAL WEIGHTS

K= 6
N= 8
D.F.= 3

OBSERVATIONS	1	1	1	1		
Y(1)	1	0	0	0	-1	-7
Y(2)	0	1	0	0	-1	-5
Y(3)	0	0	1	0	-1	-3
Y(4)	0	0	0	1	-1	-1
Y(5)	0	0	0	1	-1	1
Y(6)	0	0	1	0	-1	3
Y(7)	0	1	0	0	-1	5
Y(8)	1	0	0	0	-1	7
RESTRAINT			+			

PARAMETER VALUES
DIVISOR = 168

OBSERVATIONS	1	1	1	1		
Y(1)	84	0	0	0	0	-7
Y(2)	0	84	0	0	0	-5
Y(3)	-84	-84	0	-84	-84	-3
Y(4)	0	0	0	84	0	-1
Y(5)	0	0	0	84	0	1
Y(6)	-84	-84	0	-84	-84	3
Y(7)	0	84	0	0	0	5
Y(8)	84	0	0	0	0	7
M(1)	168	168	168	168	168	0

DEVIATIONS
DIVISOR = 168

	1	2	3	4	5	6	7	8
Y(1)	35	-35	-21	-7	7	21	35	-35
Y(2)	-35	59	-15	-5	5	15	-59	35
Y(3)	-21	-15	75	-3	3	-75	15	21
Y(4)	-7	-5	-3	83	-83	3	5	7
Y(5)	7	5	3	-83	83	-3	-5	-7
Y(6)	21	15	-75	3	-3	75	-15	-21
Y(7)	35	-59	15	5	-5	-15	59	-35
Y(8)	-35	35	21	7	-7	-21	-35	35

ERRATA FOR

DESIGNS FOR THE CALIBRATION OF STANDARDS OF MASS,
J. M. CAMERON, M. C. CROARKIN, R. C. RAYBOLD
NBS TECHNICAL NOTE 952, JUNE 1977

pp. 17-19

The numerical example given for design E.1 is based on the following table and not on Table 10 as stated in the text. Note that the example assumes the restraint is on the third weight, and the restraint in Table 10 is on the fourth weight.

MATRIX OF NORMAL EQUATIONS

2	0	0	0	-2	0	0
0	2	0	0	-2	0	0
0	0	2	0	-2	0	1
0	0	0	2	-2	0	0
-2	-2	-2	-2	8	0	0
0	0	0	0	0	168	0
0	0	1	0	0	0	0

INVERSE DIVISOR = 168

168	84	0	84	84	0	168
84	168	0	84	84	0	168
0	0	0	0	0	0	168
84	84	0	168	84	0	168
84	84	0	84	84	0	168
0	0	0	0	0	1	0
168	168	168	168	168	0	0

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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

This report presents a collection of designs for the intercomparison of sets of weights for use in precision calibration of standards of mass. These include a number of previously unpublished designs which have an additional weight in each set to serve as the check standard for monitoring the performance of the weighing process. Also included are the classical designs of Benoit and Hayford. The complete least squares analysis is presented in integer form (i.e., with a common division) for the most widely used designs; and for the others, the standard deviations are given for various weight combinations when used as an ascending or as a descending series. Designs for sets of nominally equal objects, the 2 2 . . . 1 1 . . . series, the binary sequences, the 5 2 2 1 1 series, and the 5 3 2 1 1 and some miscellaneous series are given.

17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)

Design of experiments; least squares; mass calibration; statistical design; weighing design

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