Mass Metrology

An introduction

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RNM Red Nacional de Metrología de Chile

THIS LECTURE SCOPE

I will concentrate this lecture in the fundamentals of mass metrology

Number of CMCs in the KCDB that are covered by SIM MWG7 sub-working groups							
Country	**Mass**	**Density**	**Pressure**	**Force**	**Torque**	**Hardness**	**Total**
USA	38	7	22	18		17	**102**
México	28	20	18	15	4		**85**
Argentina	29	16	13	11			**69**
Brazil	27	4	14	16	1		**62**
Canada	32	8	9				**49**
Uruguay	35	11					**46**
Chile	24			9			**33**
Costa Rica	24						**24**
Paraguay	23						**23**
Jamaica	22						**22**
Ecuador	20						**20**
Panamá	20						**20**
Perú	19	1					**20**
Bolivia							**0**
Colombia							**0**
CARICOM							**0**
Total	**341**	**67**	**76**	**69**	**5**	**17**	**575**

- SIM is composed of national metrology institutes from 34 OAS member nations.
- 16 CIPM MRA signatories (28 national metrology institutes since CARICOM is included)
- But only 13 have CMCs in mass, less than 40%.
- Mass metrology is fundamental for the rest of mechanical quantities.

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I will concentrate this lecture in the calibration of weights by direct comparison

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- 16 CIPM MRA signatories (28 NMIs since CARICOM is included)
- But only 13 NMIs have CMCs in mass, less than 40%.
- Mass metrology is fundamental for the rest of mechanical quantities.

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Learning objectives:

- Know the relevance of mass measurements in commerce
- Discuss how mass measurements can be relevant for a country exporter of raw materials
- To understand how the traceability in mass measurements is realized

MASS MEASUREMENTS IN THE INDUSTRY AND TRACEABILITY

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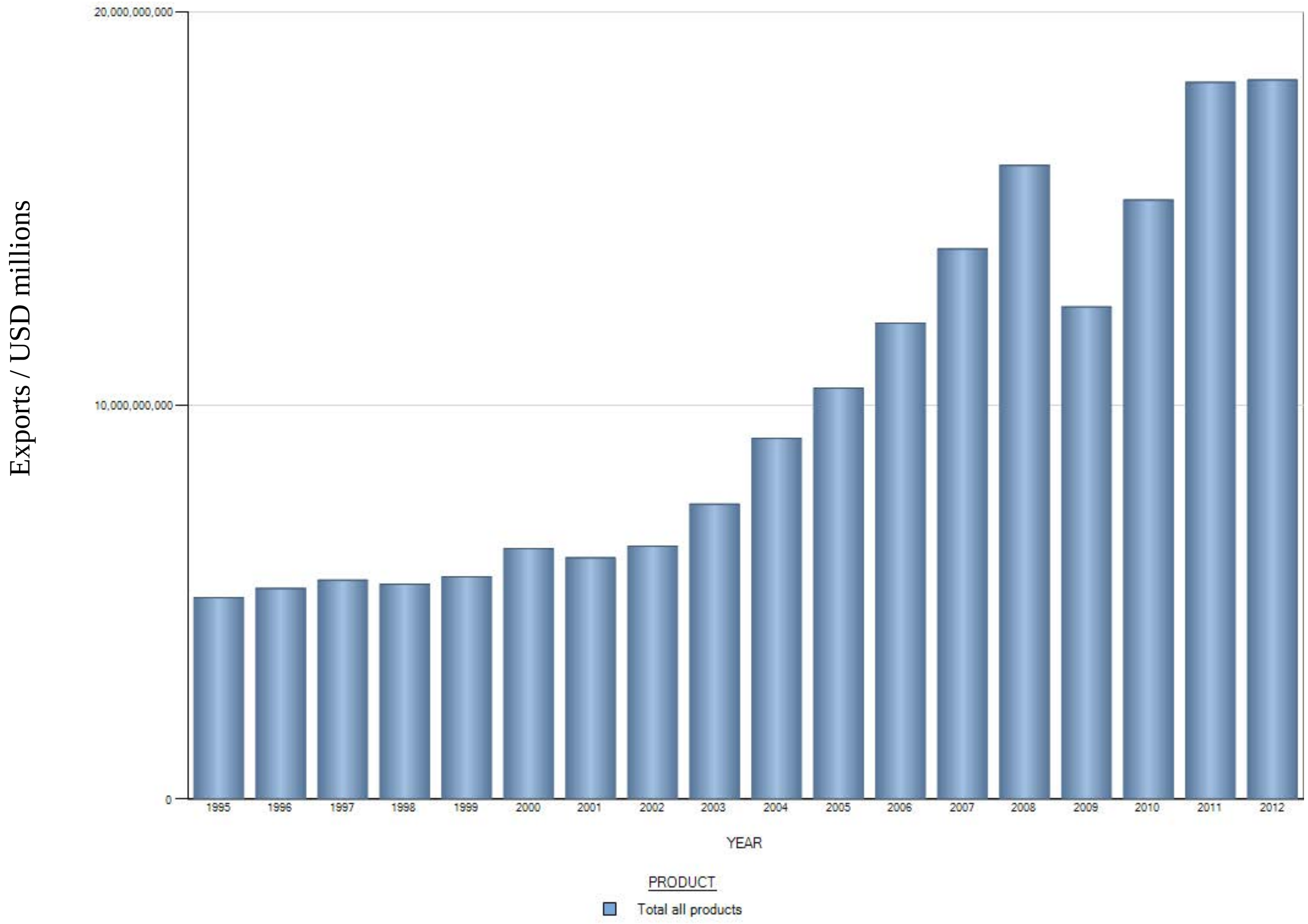
In 2012 world exports were equal to USD $1{,}8\times10^{13}$ = USD 18 000 000 000 000



Source: unctadstat.unctad.org

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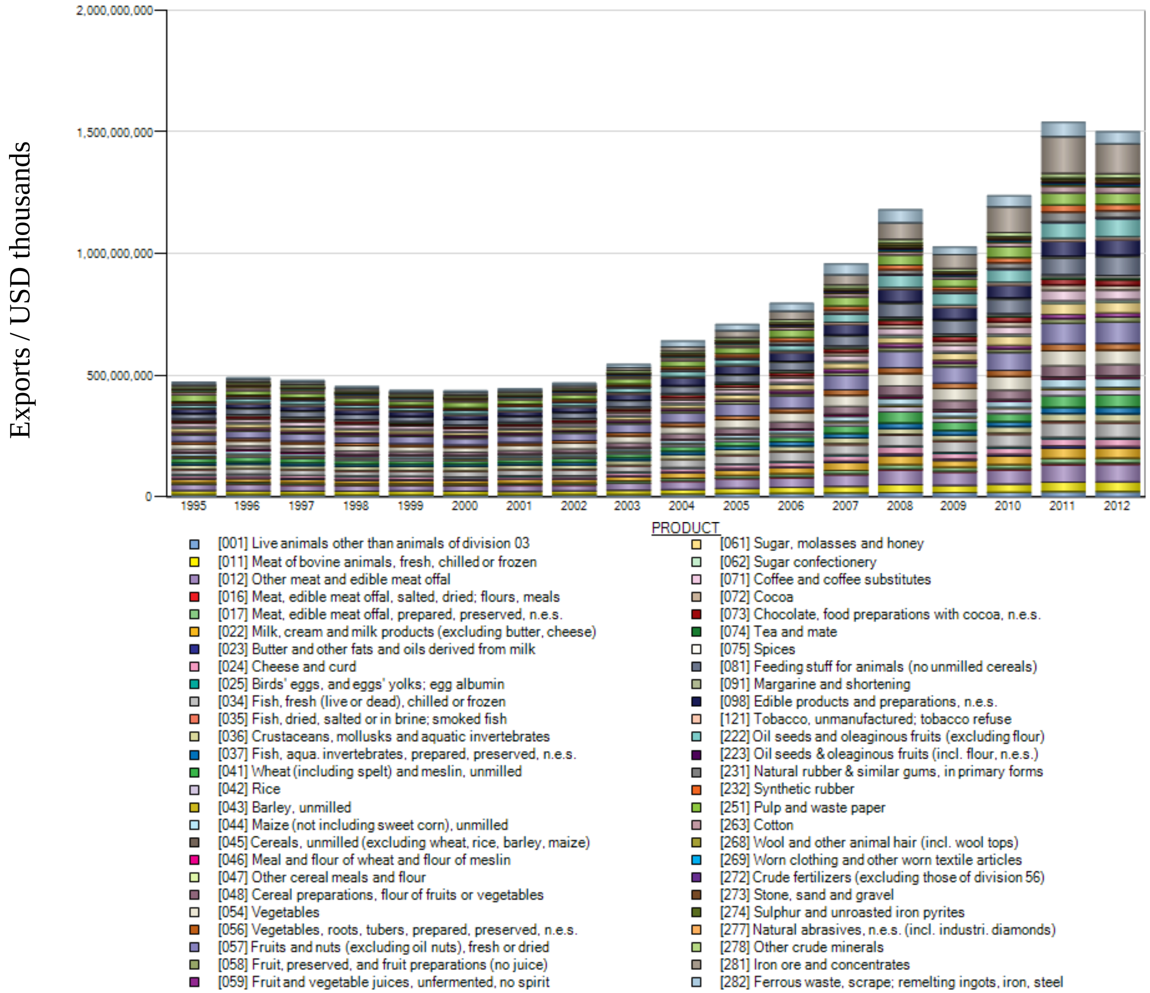
"Roughly 80 percent of global merchandise trade is affected by standards and by regulations that embody standards".

Source: National Institute of Standards and TechnologyTestimony before the U.S. House of Representatives – Committee on Science, Subcommittee on Technology September 13, 2000

World exports are equal to USD $1{,}8 \times 10^{13}$
80% of USD $1{,}8 \times 10^{13}$ = USD $1{,}4 \times 10^{13}$…. almost the same!

Conformity assessment (World Trade Organization glossary): "[…]procedures are used to determine that relevant requirements in technical regulations or standards are fulfilled. Typical conformity assessment procedures include testing, inspection and certification. They aim to increase confidence in the safety and quality of products — which is important in international trade. For example, they are used widely to determine whether goods such as toys, electronics, food, and beverages fulfil the requirements established in government regulations".

In 2012 exports directly sold in mass units were equal (at least) to USD $1,5 \times 10^{12}$ = USD 1 500 000 000 000



Source: unctadstat.unctad.org

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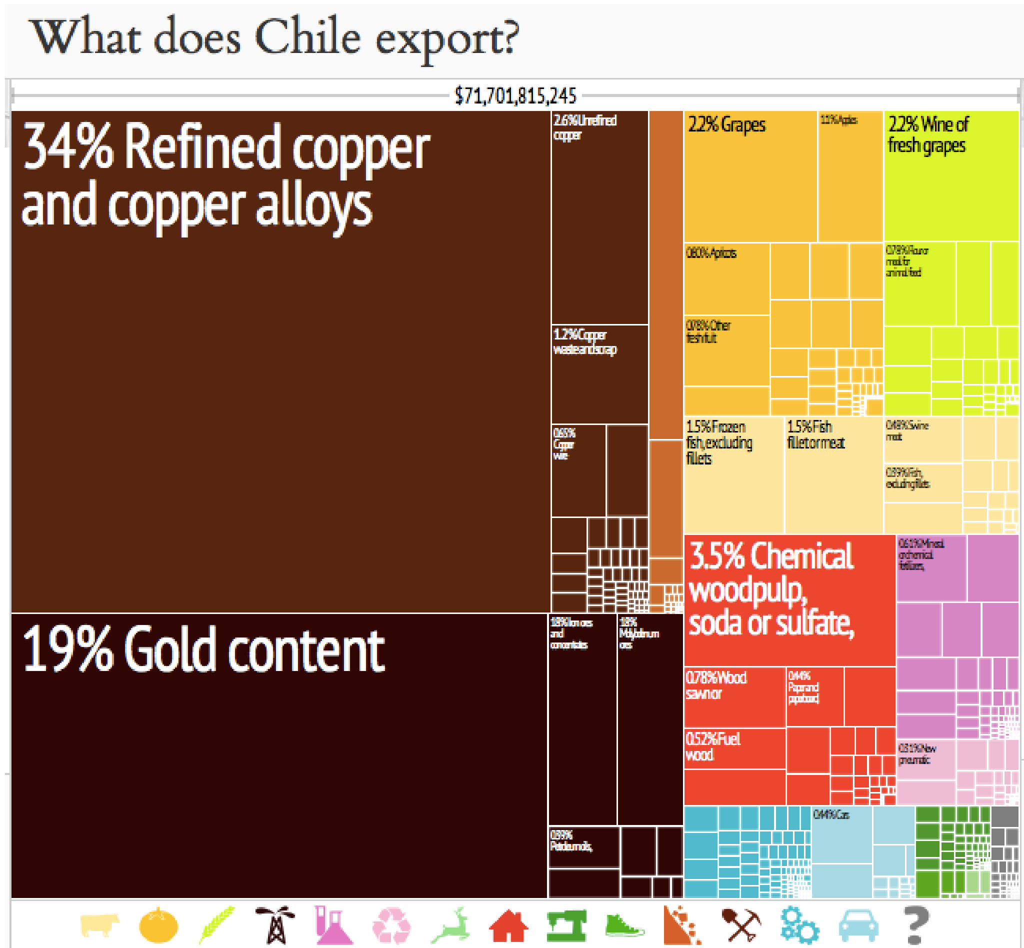
At least 90% of chilean exports are traded in mass. (USD $6,3 \times 10^{10}$ = USD 63 000 000 000)



Source: atlas.media.mit.edu

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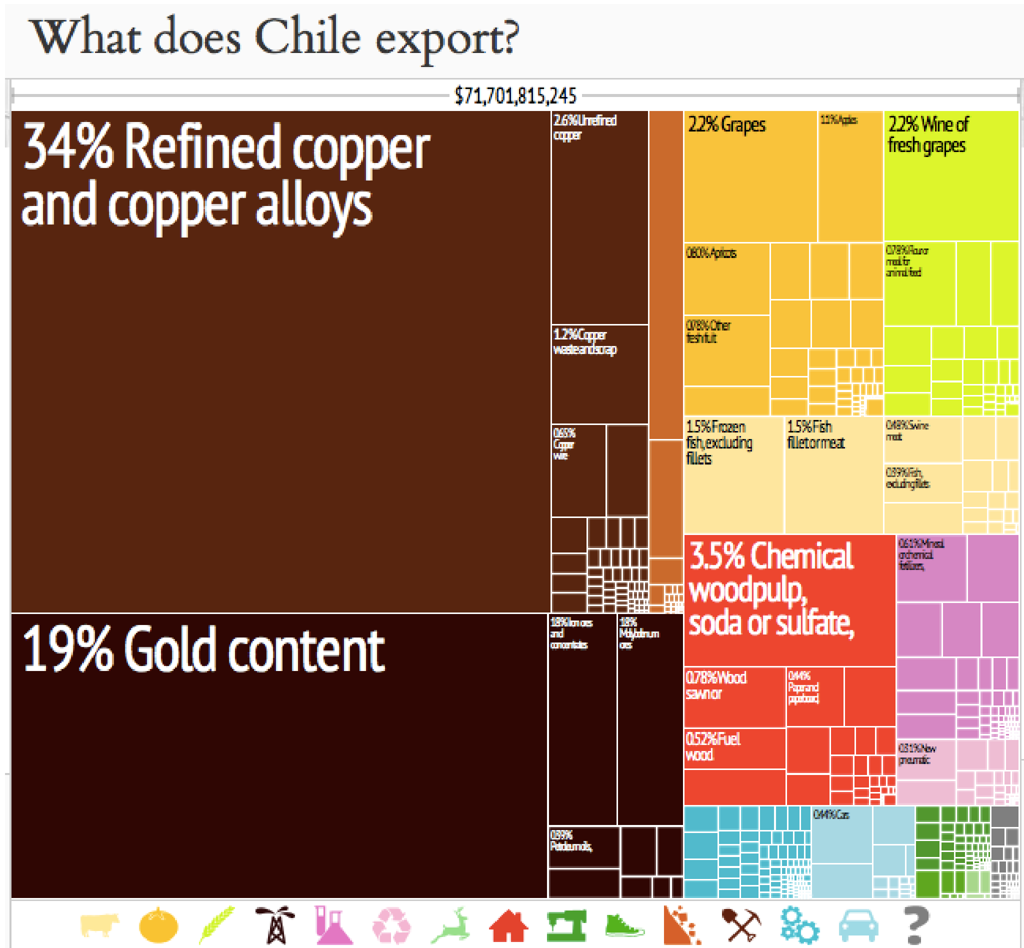
At least 65% of chilean exports related to the mining industry (USD $4{,}6 \times 10^{10}$ = USD 46 000 000 000)



Source: atlas.media.mit.edu

The Observatory of Economic Complexity

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In 2010 Chile exported 5,4 million t
4×10^{10} =USD 40 000 000 000

Static weighing

Belt conveyors

YEAR 2010 (COPPER)	Exports / t		Exports / USD		USD/kg
REFINED (1)	3.160.180,3	58%	21.403.409.864	53%	6,8
BLISTER(2)	418.491,6	8%	3.423.995.080	9%	8,2
GRANELES(3)	1.863.440,8	34%	15.326.971.095	38%	8,2
TOTAL	5.442.112,8		40.154.376.039		7,4

(1) Includes cathodes, semis, and fire-refined.
(2) Includes blister copper and copper anodes.
(3) Includes cements, concentrates, and secondary copper.

Fuente / Source: Chilean Copper Commision, based on Customs data.

For mining companies, mass measurements are critical

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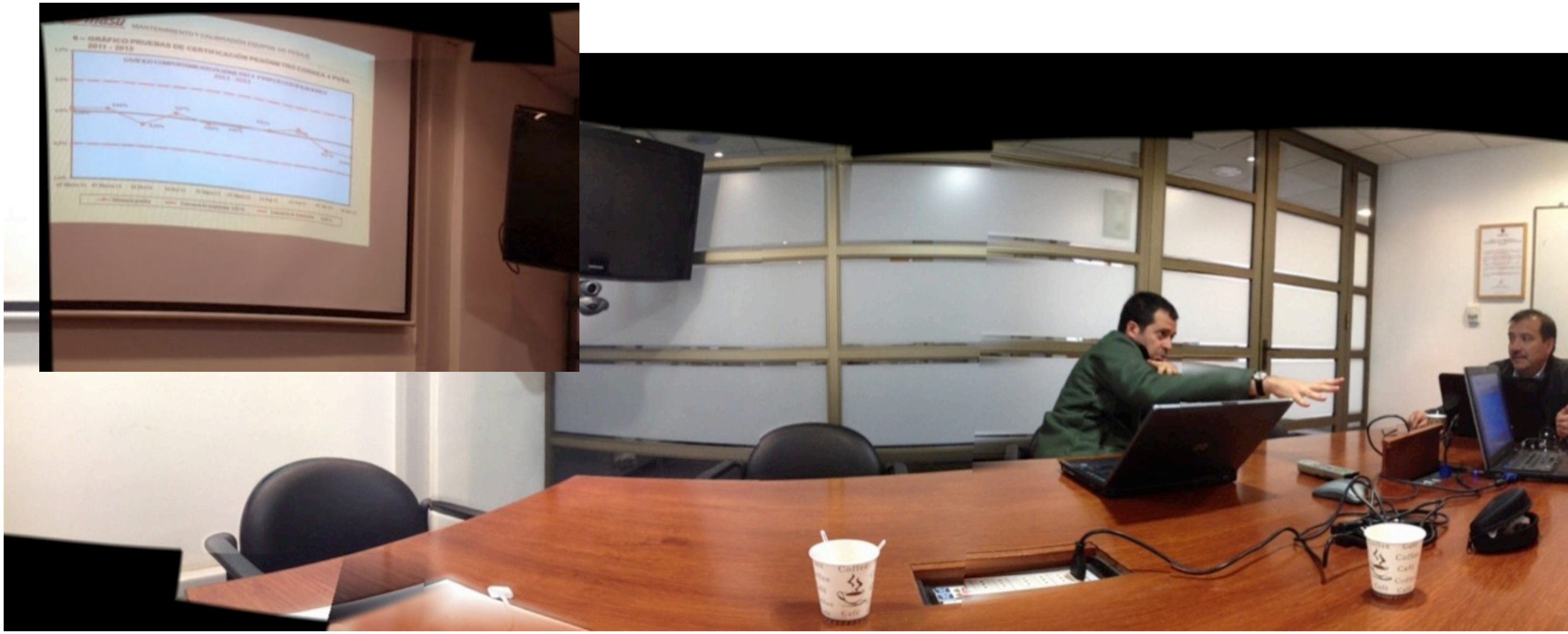
For mining companies, mass measurements are critical



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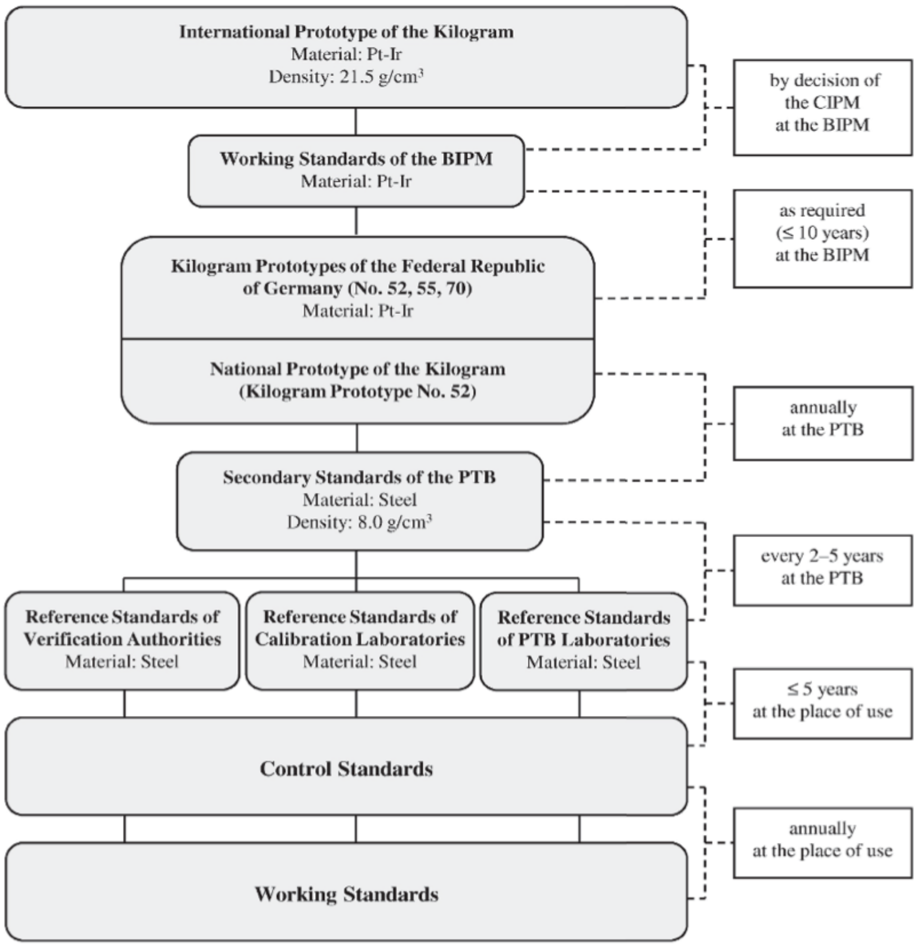
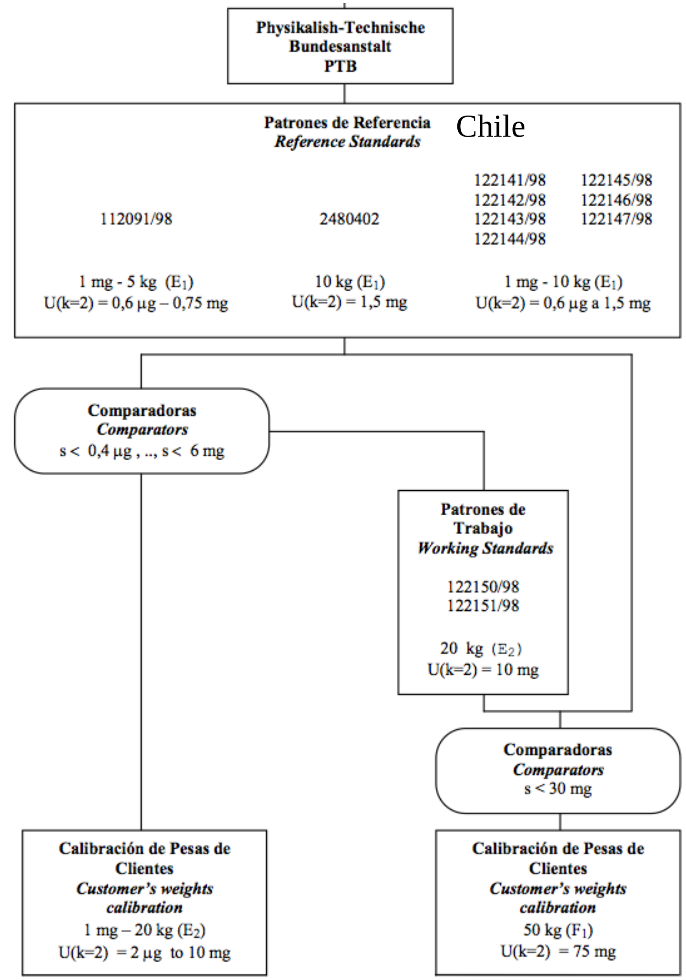
Traceability in mass (Chile)



Reference: Fundamentals of mass determinations



Reference: Quality Manual of the LCPN-M at CESMEC

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Learning objectives:

- Explain what is conventional mass.
- Describe the numerical difference between mass and conventional mass.

MASS AND CONVENTIONAL MASS

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Conventional mass is defined from mass in conventionionally chosen conditions.

- The conventional mass value of a body is equal to the mass m_c of a standard that balances this body under conventionally chosen conditions. The unit of the quantity “conventional mass” is the kilogram. The conventionally chosen conditions are: $t_{ref} = 20\ °C$; $\rho_0 = 1.2\ kg\ m^{-3}$; $\rho_c = 8\ 000\ kg\ m^{-3}$
- This is a theoretical definition that implies a change in a variable as will see in the next section.

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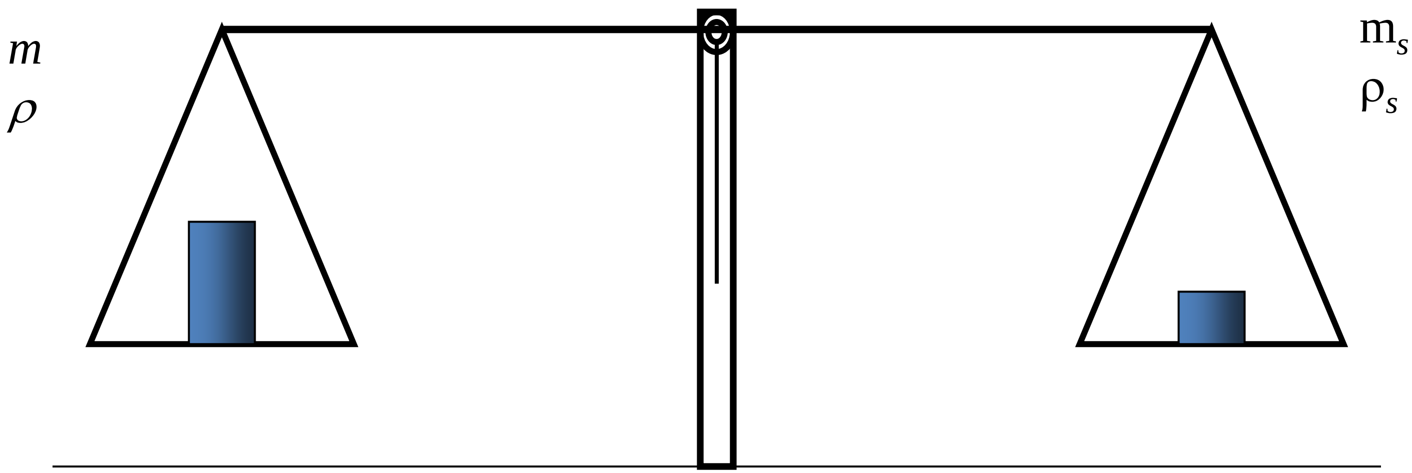
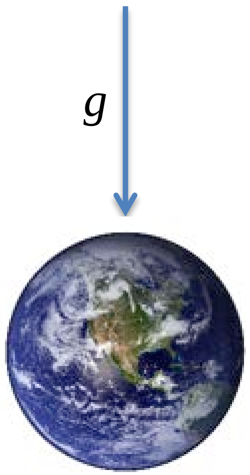
We can do an imaginary experiment to realise conventional mass with an equal arms balance





$$(mg - \rho_a V g)l = (m_s g - \rho_a V_s g)l$$

$$mg - \rho_a \frac{m}{\rho} g = m_s g - \rho_a \frac{m_s}{\rho_s} g$$

$$mg\left(1 - \frac{\rho_a}{\rho}\right) = m_s g\left(1 - \frac{\rho_a}{\rho_s}\right)$$

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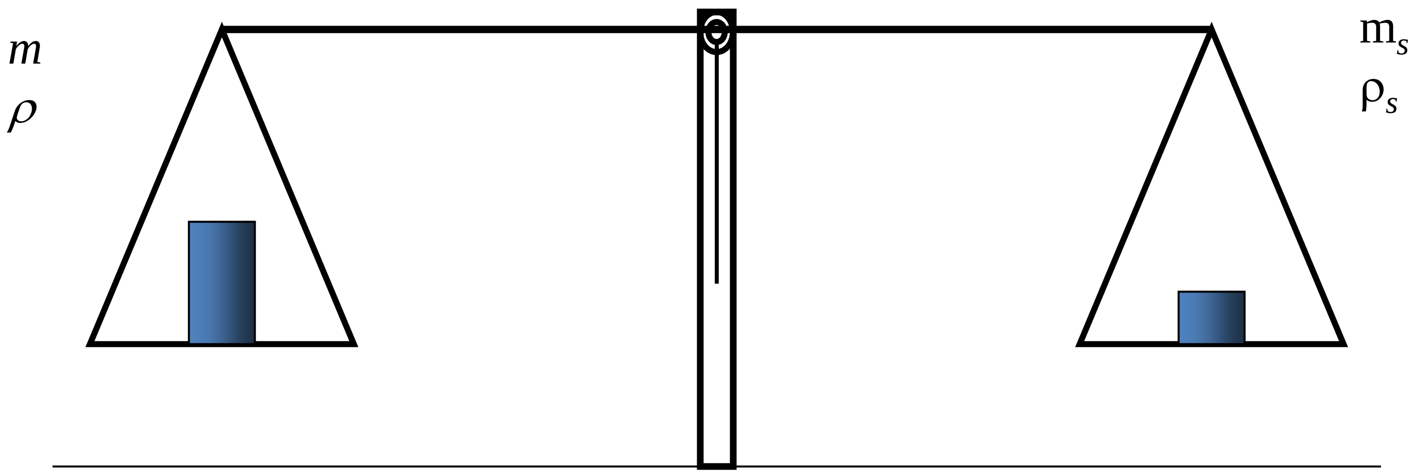
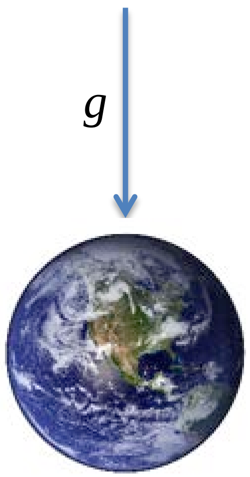
We can do an imaginary experiment to realise conventional mass





$$mg\left(1-\frac{\rho_a}{\rho}\right)=m_s g\left(1-\frac{\rho_a}{\rho_s}\right)$$

$$m\left(1-\frac{\rho_a}{\rho}\right)=m_s\left(1-\frac{\rho_a}{\rho_s}\right)$$

Observation: if $\rho_0 = \rho_s$ then $m=m_s$

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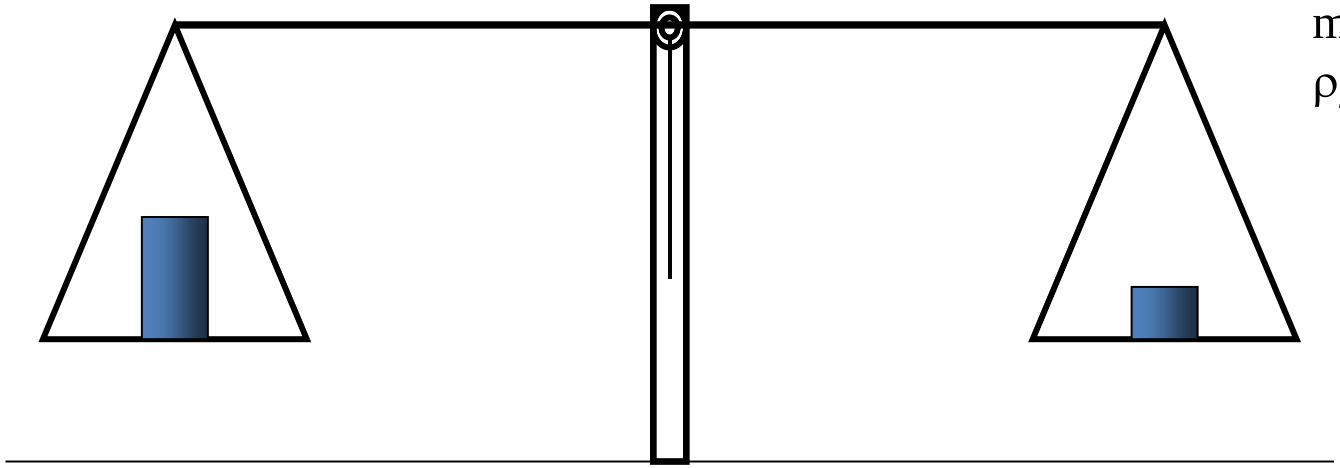
We can do an imaginary experiment to realise conventional mass



$$m\left(1-\frac{\rho_a}{\rho}\right)=m_s\left(1-\frac{\rho_a}{\rho_s}\right) \Rightarrow m\left(1-\frac{\rho_0}{\rho}\right)=m_c\left(1-\frac{\rho_0}{\rho_c}\right)$$

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Conventional mass depends on the mass of the object and is a change of variable.

$$m\left(1-\frac{\rho_0}{\rho}\right)=m_c\left(1-\frac{\rho_0}{\rho_c}\right)$$

$$m_c = m\frac{\left(1-\frac{\rho_0}{\rho}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)}$$

Note m_c is a function of the density of the object with mass *m*. The rest of the variables are fixed by definition.

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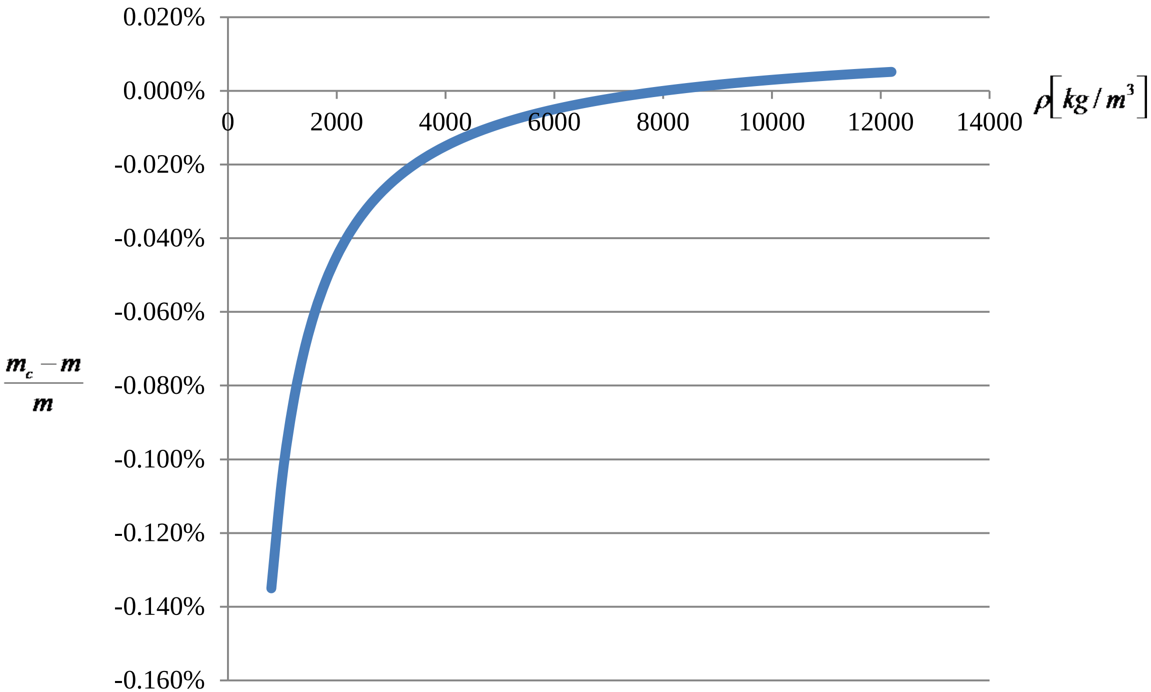
Conventional mass is equal to mass when the density of the object is equal to the conventional density of the reference standard (8000 kg m^{-3})

$$\frac{m_c - m}{m} = \frac{m\dfrac{\left(1-\dfrac{\rho_0}{\rho}\right)}{\left(1-\dfrac{\rho_0}{\rho_c}\right)} - m}{m} = \frac{\left(1-\dfrac{\rho_0}{\rho}\right)}{\left(1-\dfrac{\rho_0}{\rho_c}\right)} - 1$$



$$\frac{m_c - m}{m} \xrightarrow[\rho \to \infty]{} 0,015\%$$

Convetional mass values are close to mass values in the case of metals

Material	Density @ 20 ºC, 1 atm [kg/m^3]	$(m_c\text{-}m)/m$ [%]
Water	998	-0,105%
Glass	3140	-0,023%
Air	1	-100,000%
Chestnut wood	560	-0,199%
Emeralds	2700	-0,029%
High Density Polyethylene (HDP)	960	-0,110%
Shelled peanuts	641	-0,172%
Marble	2500	-0,033%

Material	Density @ 20 ºC, 1 atm [kg/m^3]	$(m_c\text{-}m)/m$ [%]
Platinum	21400	0,009%
Nickel silver	8600	0,001%
Brass	8400	0,001%
Stainless steel	7950	0,000%
Carbon steel	7700	-0,001%
Iron	7800	0,000%
Cast iron (white)	7700	-0,001%
Cast iron (grey)	7100	-0,002%
Aluminum	2700	-0,029%

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Learning objectives:

- Understand why conventional mass is a useful quantity.
- Know the nature of balance readings
- Know the basic equations that relate balance readings with mass and conventional mass.
- Derivate measurement models for other mass related measurements

BALANCE READINGS VS. MASS VALUES AND CONVENTIONAL MASS VALUES

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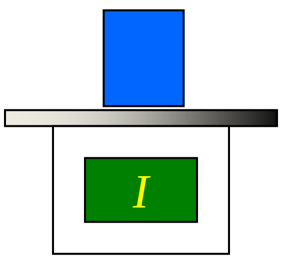
First we are going to model the indications of a balance in terms of mass and decide if this has practical sense

- The reading or indication of a balance is proportional to the force applied on the pan.



$$I \ \alpha \ m_J g - \rho_{aJ} V_J g$$

$$I = c_J\left(m_J g - \rho_{aJ} V_J g\right) = c_J\left(m_J g - \rho_{aJ}\frac{m_J}{\rho_J} g\right)$$

$$I = m_J c_J g\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)$$

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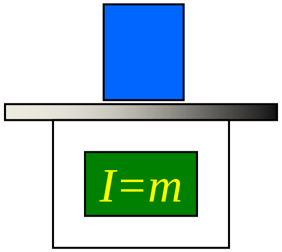
If we want the balance to indicate mass, then an adjustment factor c_J has to be applied

The adjusment factor can be introduced through the electronics of the balance.



$$I = m c_J g \left(1 - \frac{\rho_{aJ}}{\rho}\right)$$

If $c_J = \left[g \left(1 - \frac{\rho_{aJ}}{\rho}\right) \right]^{-1}$ then

$$I = m$$

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The problem of this approximation is that the adjustment factor c_J depends on the density of the object.

$$c_J = c_J\left(\rho, \rho_{aJ}, g\right) = \left[g\left(1 - \frac{\rho_{aJ}}{\rho}\right)\right]^{-1}$$

We would need a different factor for measuring the mass of objects with different densities in order to achieve accuracy.

On the other hand, we would need mass standards with different densities for each nominal values. In practice, mass standards are made of some metals.

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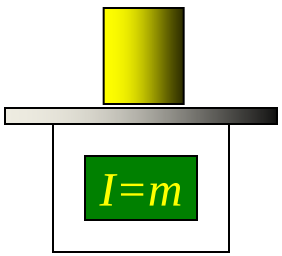
In order to understand this effect, lets consider that the balance is adjusted with a mass standard made of stainless steel before use.

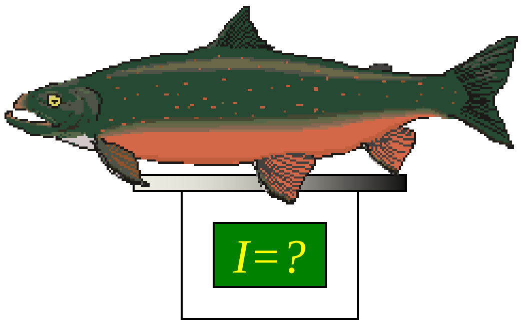


$$I = m_J c_J g\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)$$

$$\text{If } c_J = \left[g\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)\right]^{-1} \text{ then}$$

$$I = m_J$$

In order to understand this effect, lets consider that the balance is adjusted with a mass standard made of stainless steel before use.



$$I = m_{fish} c_J g \left(1 - \frac{\rho_a}{\rho_{fish}}\right)$$

$$I = m_{fish} \left[g \left(1 - \frac{\rho_{aJ}}{\rho_J}\right)\right]^{-1} g \left(1 - \frac{\rho_a}{\rho_{fish}}\right)$$

$$I = m_{fish} \left(1 - \frac{\rho_a}{\rho_{fish}}\right)\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)^{-1}$$

If $\begin{cases} \rho_{aJ} = \rho_a = 1{,}13 \text{ kg/m}^3 \\ \rho_J = 7950 \text{ kg/m}^3 \\ m_{fish} = 1 \text{ kg} \\ \rho_{fish} = 1200 \text{ kg/m}^3 \end{cases}$ → $I = 0{,}9992 \text{ kg}$

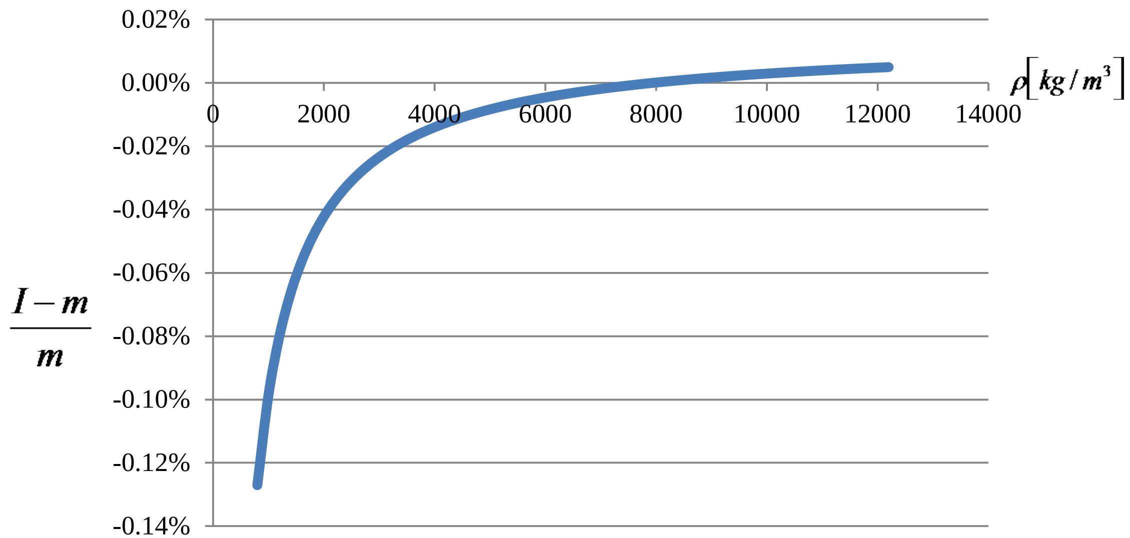
0,08% percent difference between the balance indication and the mass

In order to understand this effect, lets consider that the balance is adjusted with a mass standard made of stainless steel before use.

$$\frac{I-m}{m}=\frac{m\left[g\left(1-\frac{\rho_{aJ}}{\rho_J}\right)\right]^{-1}g\left(1-\frac{\rho_a}{\rho}\right)-m}{m}$$

$$\frac{I-m}{m}=\left[g\left(1-\frac{\rho_{aJ}}{\rho_J}\right)\right]^{-1}g\left(1-\frac{\rho_a}{\rho}\right)-1$$

$$\frac{I-m}{m}=\frac{\left(1-\frac{\rho_a}{\rho}\right)}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}-1$$

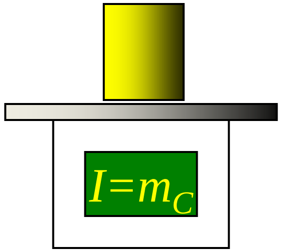


Do you remember a similar curve?

$$\rho_a=\rho_{aJ}=1{,}13\ \mathrm{kg/m^3}$$

$$\rho_J=7950\ \mathrm{kg/m^3}$$

For improving accuracy, conventional mass is used for adjusting balances. Now we are going to find a new adjustment factor c_J



What is the adjusment factor that would allow us to get a balance indication equal to the conventional mass of the mass standard?

$$I = m_J c_J g\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)$$

$$m_c = m\frac{\left(1-\frac{\rho_0}{\rho}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)} \rightarrow m = m_c\frac{\left(1-\frac{\rho_0}{\rho_c}\right)}{\left(1-\frac{\rho_0}{\rho}\right)}$$

$$I = m_{Jc}\frac{\left(1-\frac{\rho_0}{\rho_c}\right)}{\left(1-\frac{\rho_0}{\rho_J}\right)} c_J g\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)$$

$$I = m_{Jc} \text{ if } \quad c_J = \frac{1}{g}\frac{1}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}\frac{\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)}$$

With the new adjustment factor c_J let's find the indication fo the balance for an object of mass m and conventional mass m_C

For any object with mass *m*:

$$I = m c_J g \left(1 - \frac{\rho_a}{\rho}\right) \qquad c_J = \frac{1}{g} \frac{1}{\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)} \frac{\left(1 - \frac{\rho_0}{\rho_J}\right)}{\left(1 - \frac{\rho_0}{\rho_c}\right)}$$

$$I = m \frac{1}{\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)} \frac{\left(1 - \frac{\rho_0}{\rho_J}\right)}{\left(1 - \frac{\rho_0}{\rho_c}\right)} \left(1 - \frac{\rho_a}{\rho}\right)$$

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With the new adjustment factor c_J let's find the indication fo the balance for an object of mass m and conventional mass m_C

$$I = m\frac{1}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}\frac{\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)}\left(1-\frac{\rho_a}{\rho}\right)$$

$$m_c = m\frac{\left(1-\frac{\rho_0}{\rho}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)} \rightarrow m = m_c\frac{\left(1-\frac{\rho_0}{\rho_c}\right)}{\left(1-\frac{\rho_0}{\rho}\right)}$$

$$I = m_c\frac{\left(1-\frac{\rho_0}{\rho_c}\right)}{\left(1-\frac{\rho_0}{\rho}\right)}\frac{1}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}\frac{\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)}\left(1-\frac{\rho_a}{\rho}\right)$$

$$\boxed{I = m_c\frac{\left(1-\frac{\rho_a}{\rho}\right)}{\left(1-\frac{\rho_0}{\rho}\right)}\frac{\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}}$$

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The difference between the readings of a balance and the conventional mass of an object is much more smaller than the difference between the readings of a balance and the mass of the object.

If $$\begin{cases} \rho_{aJ} = \rho_a = 1{,}13 \text{ kg/m}^3 \\ \rho_J = 7950 \text{ kg/m}^3 \\ m_{fish} = 1 \text{ kg} \\ \rho_{fish} = 1200 \text{ kg/m}^3 \end{cases}$$

Conventional mass of the fish

$$m_c = m\frac{\left(1-\frac{\rho_0}{\rho}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)} = 1 \text{ kg}\frac{\left(1-\frac{1{,}2}{1200}\right)}{\left(1-\frac{1{,}2}{8000}\right)} = 0{,}999\ 149\ 872 \text{ kg}$$

$$I = m\frac{1}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}\frac{\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)}\left(1-\frac{\rho_a}{\rho}\right) = 1\frac{1}{\left(1-\frac{1{,}13}{7950}\right)}\frac{\left(1-\frac{1{,}2}{7950}\right)}{\left(1-\frac{1{,}2}{8000}\right)}\left(1-\frac{1{,}13}{1200}\right) = 0{,}999\ 199\ 415 \text{ kg}$$

0,005% difference between the balance indication and the conventional mass. 16 times smaller than before

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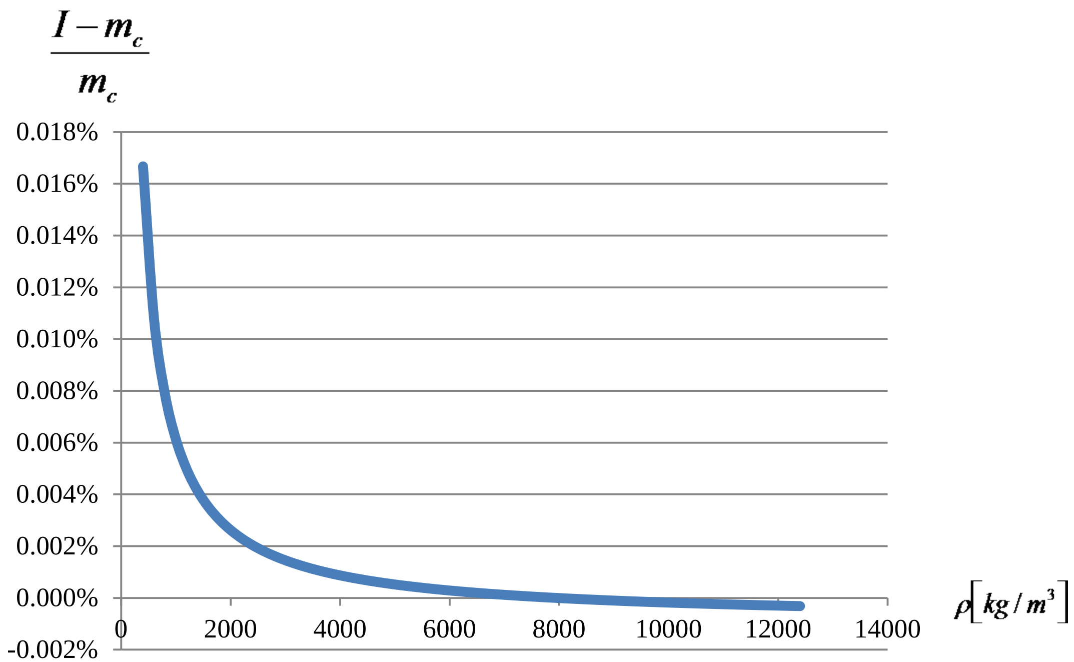
The difference between the readings of a balance and the conventional mass of an object is much more smaller than the difference between the readings of a balance and the mass of the object.

$$\frac{I-m_c}{m_c}=\frac{m_c\dfrac{\left(1-\dfrac{\rho_a}{\rho}\right)\left(1-\dfrac{\rho_0}{\rho_J}\right)}{\left(1-\dfrac{\rho_0}{\rho}\right)\left(1-\dfrac{\rho_{aJ}}{\rho_J}\right)}-m_c}{m_c}$$

$$\frac{I-m_c}{m_c}=\frac{\left(1-\dfrac{\rho_a}{\rho}\right)\left(1-\dfrac{\rho_0}{\rho_J}\right)}{\left(1-\dfrac{\rho_0}{\rho}\right)\left(1-\dfrac{\rho_{aJ}}{\rho_J}\right)}-1$$



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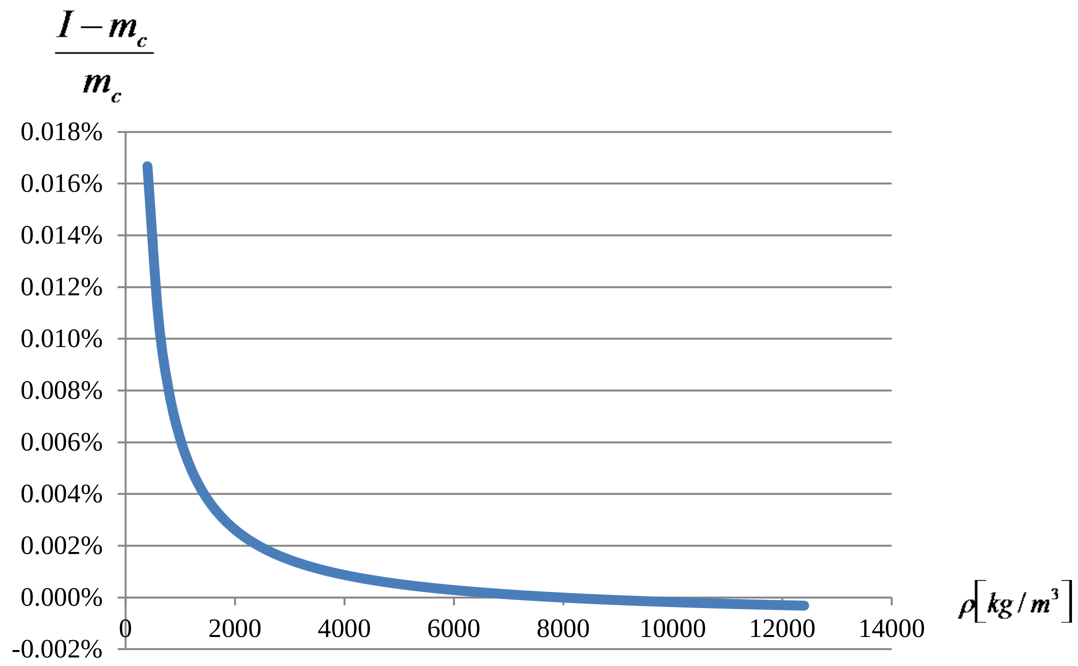
The difference between the readings of a balance and the conventional mass of an object is much more smaller than the difference between the readings of a balance and the mass of the object.

$$\frac{I-m_c}{m_c}=\frac{m_c\dfrac{\left(1-\dfrac{\rho_a}{\rho}\right)\left(1-\dfrac{\rho_0}{\rho_J}\right)}{\left(1-\dfrac{\rho_0}{\rho}\right)\left(1-\dfrac{\rho_{aJ}}{\rho_J}\right)}-m_c}{m_c}$$

$$\frac{I-m_c}{m_c}=\frac{\left(1-\dfrac{\rho_a}{\rho}\right)\left(1-\dfrac{\rho_0}{\rho_J}\right)}{\left(1-\dfrac{\rho_0}{\rho}\right)\left(1-\dfrac{\rho_{aJ}}{\rho_J}\right)}-1$$



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Now, we know how to relate mass with a balance readings and are able to write the basic model equation for some measurements.

For example: Density measurement of liquid with a pycnometer

$$I_{\substack{filled\\pycnometer}} = m_{liquid} c_J g \left(1 - \frac{\rho_a}{\rho_{liquid}}\right) + m_{glass} c_J g \left(1 - \frac{\rho_a}{\rho_{glass}}\right)$$

$$I_{\substack{empty\\pycnometer}} = m_{glass} c_J g \left(1 - \frac{\rho_a}{\rho_{glass}}\right)$$

$$I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}} = m_{liquid} c_J g \left(1 - \frac{\rho_a}{\rho_{liquid}}\right)$$

Remember:

$$c_J = \frac{1}{g} \frac{1}{\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)} \frac{\left(1 - \frac{\rho_0}{\rho_J}\right)}{\left(1 - \frac{\rho_0}{\rho_c}\right)}$$

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Now, we know how to relate mass with a balance readings and are able to write the basic model equation for some measurements.

$$I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}} = \rho_{liquid} V_{liquid} c_J g - V_{liquid} c_J g \rho_a$$

$$\frac{I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}}}{V_{liquid} c_J g} = \rho_{liquid} - \rho_a$$

$$V_{liquid} \approx V_{\substack{pycnometer's\\certified\\volume}}$$

$$c_J g \approx 1$$

$$\rho_{liquid} \approx \frac{I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}}}{V_{\substack{pycnometer's\\certified\\volume}}} + \rho_a$$

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Now, we know how to relate mass with a balance readings and are able to write the basic model equation for some measurements.

$$I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}} = m_{liquid} c_J g \left(1 - \frac{\rho_a}{\rho_{liquid}}\right)$$

$$I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}} = \rho_{liquid} V_{liquid} c_J g \left(1 - \frac{\rho_a}{\rho_{liquid}}\right)$$

$$I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}} = \rho_{liquid} V_{liquid} c_J g - V_{liquid} c_J g \rho_a$$

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Now, we know how to relate mass with a balance readings and are able to write the basic model equation for some measurements.

$$I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}} = \rho_{liquid} V_{liquid} c_J g - V_{liquid} c_J g \rho_a$$

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$$V_{liquid} \approx V_{\substack{pycnometer's\\certified\\volume}}$$

$$c_J g \approx 1$$

$$\rho_{liquid} \approx \frac{I_{\substack{filled\\pycnometer}} - I_{\substack{empty\\pycnometer}}}{V_{\substack{pycnometer's\\certified\\volume}}} + \rho_a$$

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Learning objectives:

- Know the existance of different accuracy class of mass standards
- Know general requirements for mass standards

SOME COMMENTS ON MASS STANDARDS

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Requirements for mass standards are defined in the document OIML R 111-1:2004

- http://www.oiml.org/publications/R/R111-1-e04.pdf
- OIML R 111-1 provides methods for calibration, density measurement of weights, determination of magnetic properties, etc.
- It's free.
- OIML is the french acronym for International Organization of Legal Metrology

INTERNATIONAL RECOMMENDATION

OIML R 111-1

Edition 2004 (E)

Weights of classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} and M_3

Part 1: Metrological and technical requirements

Poids des classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} et M_3

Partie 1: Exigences métrologiques et techniques

OIML R 111-1 Edition 2004 (E)

OIML

ORGANISATION INTERNATIONALE DE MÉTROLOGIE LÉGALE

INTERNATIONAL ORGANIZATION OF LEGAL METROLOGY

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Mass standards are classified in “accuracy classes”, each class has error limits or tolerances called “maximum permisible errors”.

Table 1 Maximum permissible errors for weights (± δm in mg)

Nominal value*	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
5 000 kg			25 000	80 000	250 000	500 000	800 000	1 600 000	2 500 000
2 000 kg			10 000	30 000	100 000	200 000	300 000	600 000	1 000 000
1 000 kg		1 600	5 000	16 000	50 000	100 000	160 000	300 000	500 000
500 kg		800	2 500	8 000	25 000	50 000	80 000	160 000	250 000
200 kg		300	1 000	3 000	10 000	20 000	30 000	60 000	100 000
100 kg		160	500	1 600	5 000	10 000	16 000	30 000	50 000
50 kg	25	80	250	800	2 500	5 000	8 000	16 000	25 000
20 kg	10	30	100	300	1 000		3 000		10 000
10 kg	5.0	16	50	160	500		1 600		5 000
5 kg	2.5	8.0	25	80	250		800		2 500
2 kg	1.0	3.0	10	30	100		300		1 000
1 kg	0.5	1.6	5.0	16	50		160		500
500 g	0.25	0.8	2.5	8.0	25		80		250
200 g	0.10	0.3	1.0	3.0	10		30		100
100 g	0.05	0.16	0.5	1.6	5.0		16		50
50 g	0.03	0.10	0.3	1.0	3.0		10		30
20 g	0.025	0.08	0.25	0.8	2.5		8.0		25
10 g	0.020	0.06	0.20	0.6	2.0		6.0		20
5 g	0.016	0.05	0.16	0.5	1.6		5.0		16
2 g	0.012	0.04	0.12	0.4	1.2		4.0		12
1 g	0.010	0.03	0.10	0.3	1.0		3.0		10
500 mg	0.008	0.025	0.08	0.25	0.8		2.5		
200 mg	0.006	0.020	0.06	0.20	0.6		2.0		
100 mg	0.005	0.016	0.05	0.16	0.5		1.6		
50 mg	0.004	0.012	0.04	0.12	0.4				
20 mg	0.003	0.010	0.03	0.10	0.3				
10 mg	0.003	0.008	0.025	0.08	0.25				
5 mg	0.003	0.006	0.020	0.06	0.20				
2 mg	0.003	0.006	0.020	0.06	0.20				
1 mg	0.003	0.006	0.020	0.06	0.20				

- You can notice that nominal values are multiples of 1, 2 and 5

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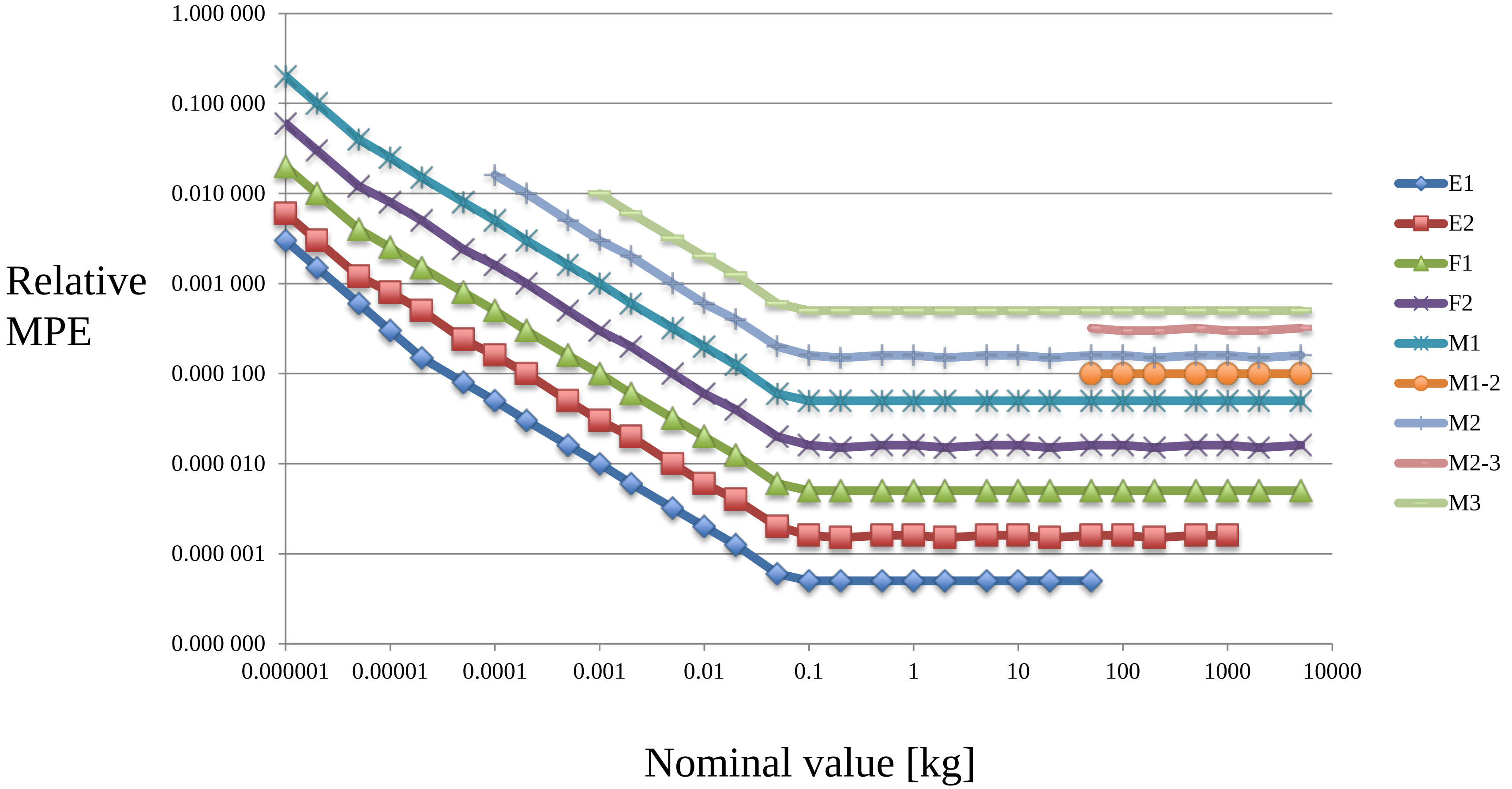
Relative maximum permissible errors increase for weights with nominal values smaller than 1 kg.



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The material shall corrosion resistant and such that the change in the mass of the weights shall be negligible in relation to MPE; the shape should assure stability and easy handling.

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The density of the materials shall be such that a deviation in the air density of 10 % of 1,2 kg m^{-3} does not produce an error exceeding one-quarter of the absolute value of the maximum permissible error.

Table 5 Minimum and maximum limits for density (ρ_{min}, ρ_{max})

Nominal value	ρ_{min}, ρ_{max} (10^3 kg m^{-3})							
	Class of weight (for class M_3, no value is specified)							
	E_1	**E_2**	**F_1**	**F_2**	**M_1**	**M_{1-2}**	**M_2**	**M_{2-3}**
≥ 100 g	7.934 – 8.067	7.81 – 8.21	7.39 – 8.73	6.4 – 10.7	≥ 4.4	> 3.0	≥ 2.3	≥ 1.5
50 g	7.92 – 8.08	7.74 – 8.28	7.27 – 8.89	6.0 – 12.0	≥ 4.0			
20 g	7.84 – 8.17	7.50 – 8.57	6.6 – 10.1	4.8 – 24.0	≥ 2.6			
10 g	7.74 – 8.28	7.27 – 8.89	6.0 – 12.0	≥ 4.0	≥ 2.0			
5 g	7.62 – 8.42	6.9 – 9.6	5.3 – 16.0	≥ 3.0				
2 g	7.27 – 8.89	6.0 – 12.0	≥ 4.0	≥ 2.0				
1 g	6.9 – 9.6	5.3 – 16.0	≥ 3.0					
500 mg	6.3 – 10.9	≥ 4.4	≥ 2.2					
200 mg	5.3 – 16.0	≥ 3.0						
100 mg	≥ 4.4							
50 mg	≥ 3.4							
20 mg	≥ 2.3							

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The density of the materials shall be such that a deviation in the air density of 10 % of 1,2 kg m^{-3} does not produce an error exceeding one-quarter of the absolute value of the maximum permissible error.

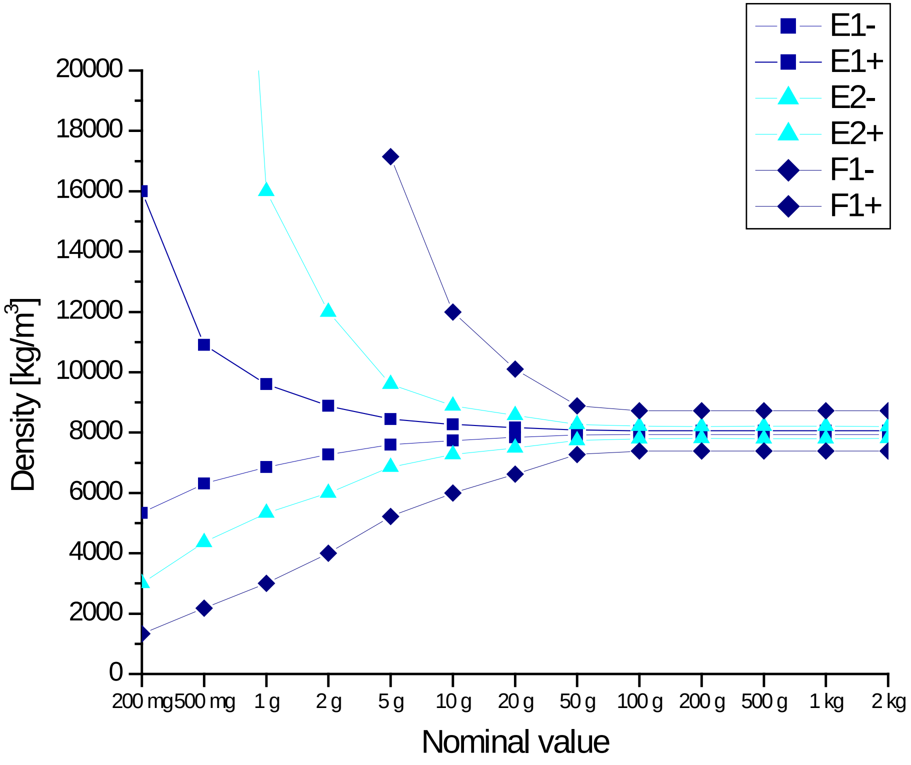
$$\left|(\rho_a - \rho_0)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r}\right)\right| \le \frac{1}{4}\frac{\delta m}{m_o}$$

If $\rho_a = 0{,}9\rho_0$

$\rho_r = \rho_c = 8000\, kg/m^3$

$$0{,}1\rho_0\left|\frac{1}{8000} - \frac{1}{\rho_t}\right| \le \frac{1}{4}\frac{\delta m}{m_o}$$

$$\begin{cases} \rho_t \le \dfrac{1}{\dfrac{1}{8000} - \dfrac{1}{4}\dfrac{\delta m}{m_o}\dfrac{1}{0{,}1\rho_o}} & \rho_t \ge 8000\, kg/m^3 \\ \rho_t > \dfrac{1}{\dfrac{1}{8000} + \dfrac{1}{4}\dfrac{\delta m}{m_o}\dfrac{1}{0{,}1\rho_o}} & \rho_t < 8000\, kg/m^3 \end{cases}$$



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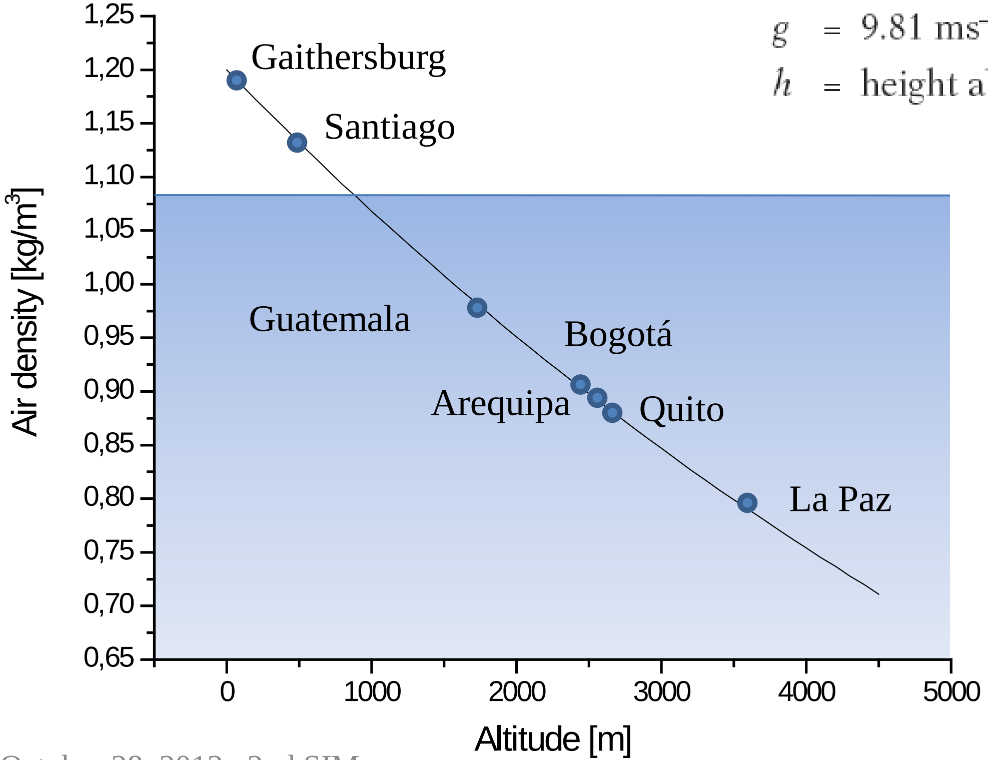
Have in mind that in some extreme cases Table 5 of OIML R111 may not apply

$$\rho_a = \rho_0 \times \exp\left(\frac{-\rho_0}{p_0} gh\right)$$

- E.3

Where: p_0 = 101 325 Pa;
ρ_0 = 1.2 kg m^{-3};
g = 9.81 ms^{-2}; and
h = height above sea level expressed in metres.



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Have in mind that in some extreme cases Table 5 of OIML R111 may not apply

- 1 kg F1 ($\delta m = 0{,}5\,mg$) en La Paz. $\rho_a \approx 0{,}8\,kg/m^3$

Densidad permitida $7390\,kg/m^3 - 8730\,kg/m^3$

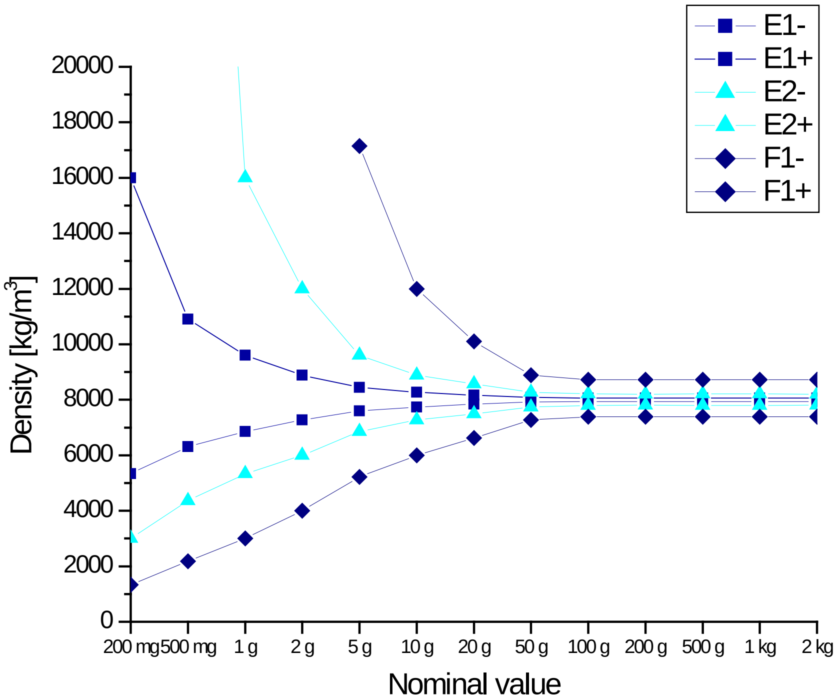
Método F: $\rho_t \approx 7950\,kg/m^3$

$U(\rho_t) \approx 140\,kg/m^3$ $7810\,kg/m^3 - 8090\,kg/m^3$

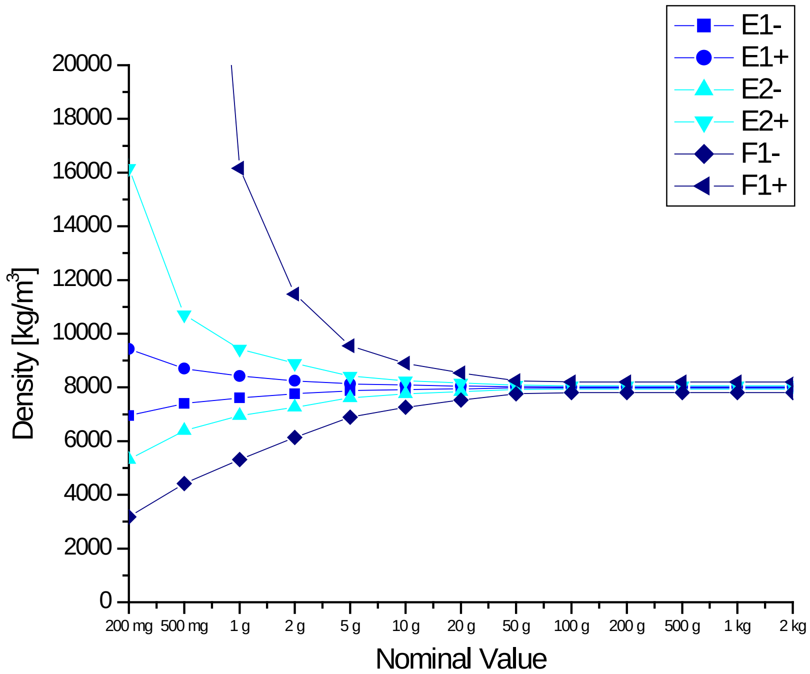
$$\left|(\rho_a - \rho_0)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r}\right)\right| = \left|(0{,}8 - 1{,}2)\left(\frac{1}{8090} - \frac{1}{8000}\right)\right| = 5{,}56 \times 10^{-7}$$

$$\frac{1}{4}\frac{\delta m}{m_o} = \frac{1}{4}\frac{1{,}6 \times 10^{-6}}{1} = 4{,}0 \times 10^{-7}$$

Have in mind that in some extreme cases Table 5 of OIML R111 may not apply



OIML R111



For La Paz

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Learning objectives:

- Understand the origin of OIML R111 equations.
- To explain why is easier to calibrate weights in conventional mass than in mass.

MEASUREMENT MODEL FOR MASS STANDARDS CALIBRATIONS (SUBSTITUTION WEIGHING IN AIR).

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The model equation for calibration of mass standards by direct comparison has some assumptions… as any model.

- Weights are non-magnetic.
- Weights and air are in thermal equilibrium.
- Measurements are done in air.
- The balance indication is linear and insensitive to eccentric loading.
- The weights' gravity centers are at the same height.

First we are going to derive an equation for mass. This is not the only way.

$$I_t = m_t c_J g \left(1 - \frac{\rho_a}{\rho_t}\right)$$

$$I_r = m_r c_J g \left(1 - \frac{\rho_a}{\rho_r}\right)$$

Remember:

$$c_J = \frac{1}{g} \frac{1}{\left(1 - \frac{\rho_{aJ}}{\rho_J}\right)} \frac{\left(1 - \frac{\rho_0}{\rho_J}\right)}{\left(1 - \frac{\rho_0}{\rho_c}\right)}$$

$$I_t - I_r = m_t c_J g \left(1 - \frac{\rho_a}{\rho_t}\right) - m_r c_J g \left(1 - \frac{\rho_a}{\rho_r}\right)$$

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First we are going to derive, step by step an equation for mass.

$$I_t - I_r = m_t c_J g\left(1-\frac{\rho_a}{\rho_t}\right) - m_r c_J g\left(1-\frac{\rho_a}{\rho_r}\right)$$

$$\frac{I_t - I_r}{c_J g} = m_t\left(1-\frac{\rho_a}{\rho_t}\right) - m_r\left(1-\frac{\rho_a}{\rho_r}\right)$$

$$m_t\left(1-\frac{\rho_a}{\rho_t}\right) = m_r\left(1-\frac{\rho_a}{\rho_r}\right) + \frac{I_t - I_r}{c_J g}$$

$$m_t = m_r\frac{\left(1-\frac{\rho_a}{\rho_r}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)} + \frac{I_t - I_r}{c_J g\left(1-\frac{\rho_a}{\rho_t}\right)}$$

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First we are going to derive, step by step an equation for mass.

$$m_t = m_r \frac{\left(1-\frac{\rho_a}{\rho_r}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)} + \frac{I_t - I_r}{c_J g\left(1-\frac{\rho_a}{\rho_t}\right)}$$

$$m_t = m_r \frac{\left(1-\frac{\rho_a}{\rho_r}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)} + (I_t - I_r)\frac{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)\left(1-\frac{\rho_0}{\rho_c}\right)}{\left(1-\frac{\rho_0}{\rho_J}\right)\left(1-\frac{\rho_a}{\rho_t}\right)}$$

Remember:

$$c_J = \frac{1}{g}\frac{1}{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}\frac{\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_c}\right)}$$

Too large…?

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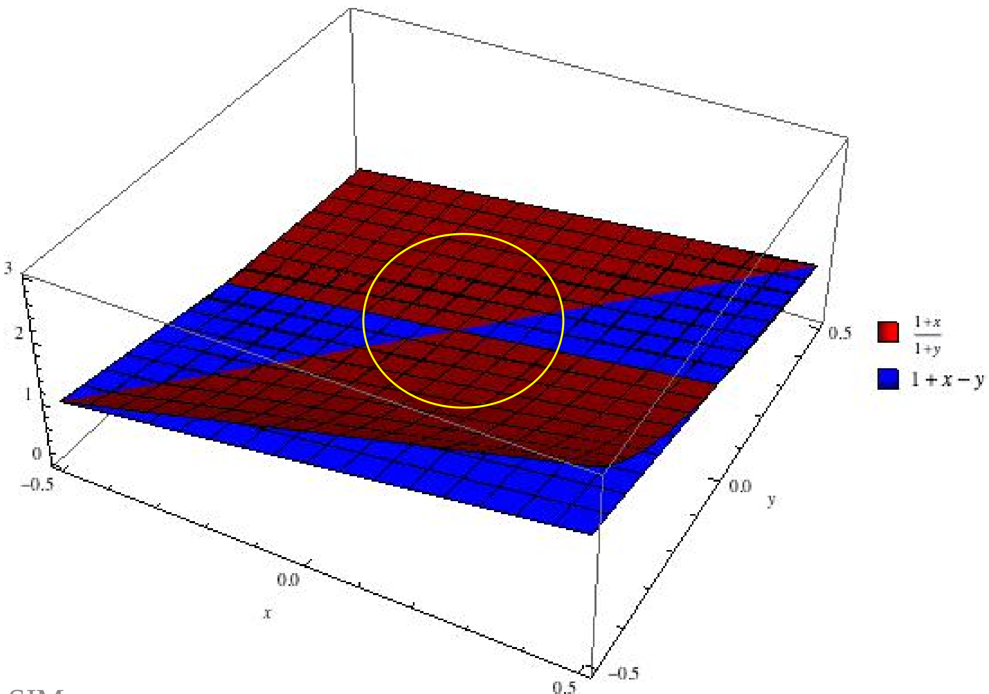
First we are going to derive, step by step an equation for mass.

If x and y <<1 then $$\frac{(1 \pm x)}{(1 \pm y)} \approx 1 \pm x \mp y$$



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First we are going to derive, step by step an equation for mass.

If x and y <<1 then $\dfrac{(1\pm x)}{(1\pm y)} \approx 1 \pm x \mp y$ $\qquad \dfrac{\rho_a}{\rho_r} \approx \dfrac{\rho_a}{\rho_t} \approx \dfrac{1}{8000}$

$$m_t = m_r\left(1-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_t}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_J}\right)$$

$$m_t = m_r\left(1-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_t}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_J}\right)$$

$$m_t = m_r\left(1-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_t}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_J}-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_{aJ}}{\rho_J}\frac{\rho_0}{\rho_c}+\frac{\rho_a}{\rho_t}-\frac{\rho_a}{\rho_t}\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_c}\frac{\rho_0}{\rho_J}\right)$$

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First we are going to derive, step by step an equation for mass.

$$m_t = m_r\left(1-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_t}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_J}-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_{aJ}}{\rho_J}\frac{\rho_0}{\rho_c}+\frac{\rho_a}{\rho_t}-\frac{\rho_a}{\rho_t}\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_c}\frac{\rho_0}{\rho_J}\right)$$

$$m_t = m_r\left(1+\rho_a\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_0}{\rho_c}+\frac{\rho_0}{\rho_J}-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_a}{\rho_t}\right) \qquad \rho_{aJ}\approx\rho_a$$

$$m_t = m_r\left(1+\rho_a\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\left(I_t-I_r\right)\left(1-\rho_0\left(\frac{1}{\rho_c}-\frac{1}{\rho_J}\right)-\rho_a\left(\frac{1}{\rho_J}-\frac{1}{\rho_t}\right)\right)$$

$$\boxed{m_t = m_r\left(1+\rho_a\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\left(I_t-I_r\right)}$$

Now we are going to derive, step by step an equation for conventional mass.

$$I_t = m_{ct} \frac{\left(1-\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_t}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)} \qquad I_r = m_{cr} \frac{\left(1-\frac{\rho_a}{\rho_r}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_r}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}$$

$$I_t - I_r = m_{ct} \frac{\left(1-\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_t}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)} - m_{cr} \frac{\left(1-\frac{\rho_a}{\rho_r}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_r}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}$$

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Now we are going to derive, step by step an equation for conventional mass.

$$I_t - I_r = m_{ct}\frac{\left(1-\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_t}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)} - m_{cr}\frac{\left(1-\frac{\rho_a}{\rho_r}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_r}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}$$

$$m_{ct}\frac{\left(1-\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_t}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)} = m_{cr}\frac{\left(1-\frac{\rho_a}{\rho_r}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_r}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)} + I_t - I_r$$

$$m_{ct} = m_{cr}\frac{\left(1-\frac{\rho_a}{\rho_r}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)}\frac{\left(1-\frac{\rho_0}{\rho_t}\right)}{\left(1-\frac{\rho_0}{\rho_r}\right)} + \left(I_t - I_r\right)\frac{\left(1-\frac{\rho_0}{\rho_t}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)}\frac{\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}{\left(1-\frac{\rho_0}{\rho_J}\right)}$$

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Now we are going to derive, step by step an equation for conventional mass.

$$m_{ct}=m_{cr}\frac{\left(1-\frac{\rho_a}{\rho_r}\right)\left(1-\frac{\rho_0}{\rho_t}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_r}\right)}+\left(I_t-I_r\right)\frac{\left(1-\frac{\rho_0}{\rho_t}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}\right)}{\left(1-\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_J}\right)}$$

$$\frac{(1\pm x)}{(1\pm y)}\approx 1\pm x\mp y$$

$$m_{ct}=m_{cr}\left(1-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_t}\right)\left(1-\frac{\rho_0}{\rho_t}+\frac{\rho_0}{\rho_r}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_0}{\rho_t}+\frac{\rho_0}{\rho_J}\right)\left(1-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_a}{\rho_t}\right)$$

$$m_{ct}=m_{cr}\left(1-\frac{\rho_0}{\rho_t}+\frac{\rho_0}{\rho_r}-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_r}\frac{\rho_0}{\rho_t}-\frac{\rho_a}{\rho_r}\frac{\rho_0}{\rho_r}+\frac{\rho_a}{\rho_t}-\frac{\rho_a}{\rho_t}\frac{\rho_0}{\rho_t}+\frac{\rho_a}{\rho_t}\frac{\rho_0}{\rho_r}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_a}{\rho_t}-\frac{\rho_0}{\rho_t}+\frac{\rho_0}{\rho_t}\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_0}{\rho_J}-\frac{\rho_0}{\rho_J}\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_0}{\rho_J}\frac{\rho_a}{\rho_t}\right)$$

$$m_{ct}=m_{cr}\left(1-\frac{\rho_0}{\rho_t}+\frac{\rho_0}{\rho_r}-\frac{\rho_a}{\rho_r}+\frac{\rho_a}{\rho_t}\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_{aJ}}{\rho_J}+\frac{\rho_a}{\rho_t}-\frac{\rho_0}{\rho_t}+\frac{\rho_0}{\rho_J}\right)$$

$$m_{ct}=m_{cr}\left(1-\rho_0\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)+\rho_a\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\left(I_t-I_r\right)\left(1-\frac{\rho_{aJ}-\rho_0}{\rho_J}+\frac{\rho_a-\rho_0}{\rho_t}\right)$$

$$\boxed{m_{ct}=m_{cr}\left(1+\left(\rho_a-\rho_0\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\left(I_t-I_r\right)}$$

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We can seen that the equations for mass and conventional mass similar, except for the factor (ρ_a-ρ_0)

$$m_{ct} = m_{cr}\left(1+(\rho_a-\rho_0)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\overline{\Delta m_c}$$

$$m_t = m_r\left(1+\rho_a\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\overline{\Delta m_c}$$

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Calibrations in conventional mass may not need the buoyancy correction due to the factor (ρ_a-ρ_0)

$$m_{ct} = m_{cr} \underbrace{\left(1 + (\rho_a - \rho_0)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r}\right)\right)}_{C_i} + \overline{\Delta m_c}$$

$$m_{ct} = m_{cr} + \overline{\Delta m_c}$$

Sometimes…

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The buoyancy correction can be neglected when is smaller than U/3 and its uncertainty can be neglected when is smaller than U/6

$$\left| m_{cr}\left(\rho_a-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right| \leq \frac{U}{3}$$

$$\left[\left[m_{cr}\frac{\rho_r-\rho_t}{\rho_r\rho_t}u_{\rho_a}\right]^2+\left[m_{cr}\left(\rho_a-\rho_o\right)^2\right]\left[\frac{u_{\rho_t}^2}{\rho_t^4}-\frac{u_{\rho_r}^2}{\rho_r^4}\right]\right]^{1/2}<\frac{U}{6}$$

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Air density an be determined for most applications using a simplyfied formula based on CIPM-1981/91 (Section E.3 of OIML R 111-1: 2004 (E))

$$\rho_a = \frac{0,0034848p - 0,009024hr \times e^{0.0612t}}{273,15 + t} \qquad [kg/m^3]$$

p is the atmospheric pressure in Pa

h is the air humidity in %

t, is the air temperature in ^oC

This equation is valid when used between the following ranges:

$90000\ Pa \leq p \leq 110000\ Pa$

$10\ ºC \leq t \leq 30\ ºC$

$hr \leq 80\ \%$

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Air density an be determined for most applications using a simplyfied formula based on CIPM-1981/91 which was updated as CIPM-2007

$$\rho_a = \frac{0{,}0034851p - 0{,}008863hr \times e^{0.062t}}{273{,}15 + t} \qquad [\mathrm{kg/m^3}]$$

p is the atmospheric pressure in Pa

h is the air humidity in %

t, is the air temperature in ^oC

This equation is valid when used between the following ranges:

$90000\ Pa \leq p \leq 110000\ Pa$

$10\ ºC \leq t \leq 30\ ºC$

$hr \leq 80\ \%$

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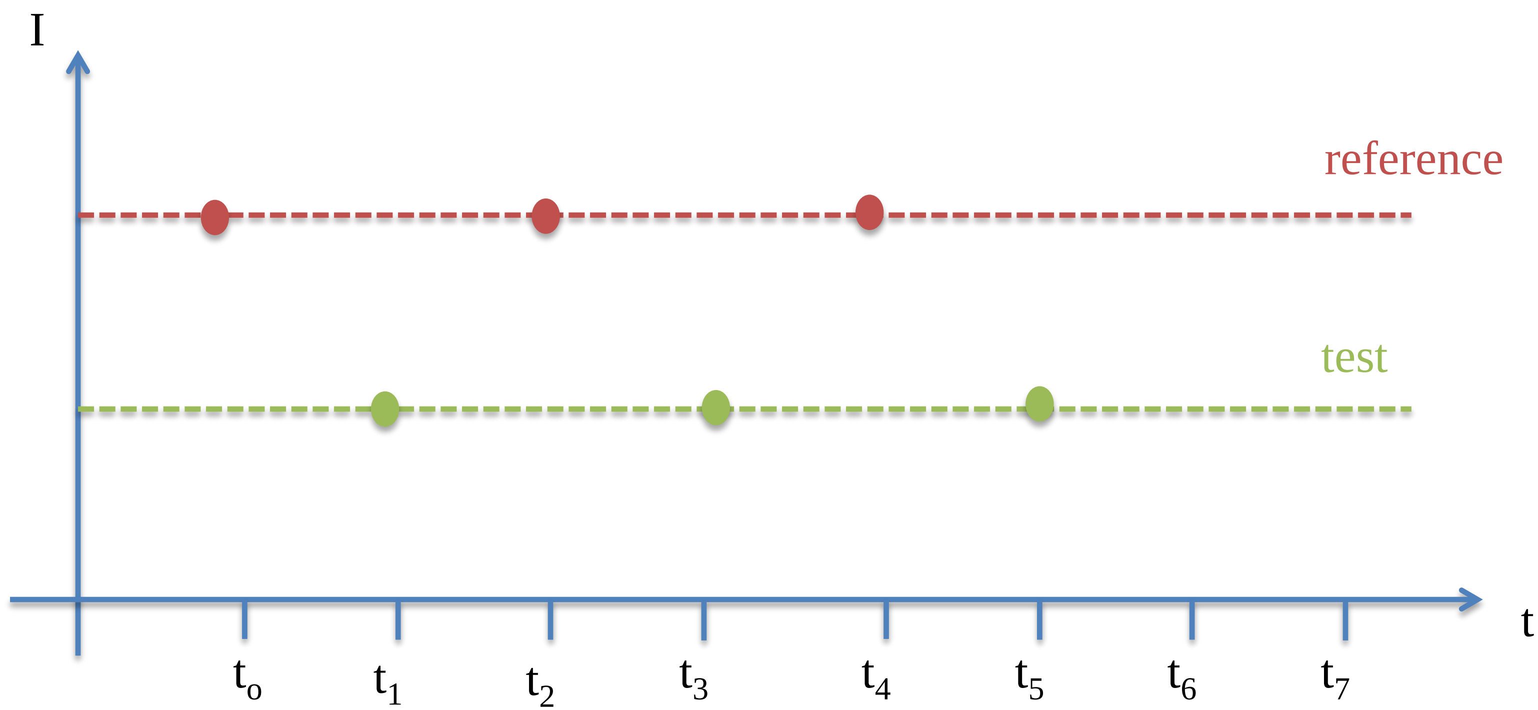
The determination of each balance readings differences need more than two values since balance indications are not stable during the weighing process



Ideal condition

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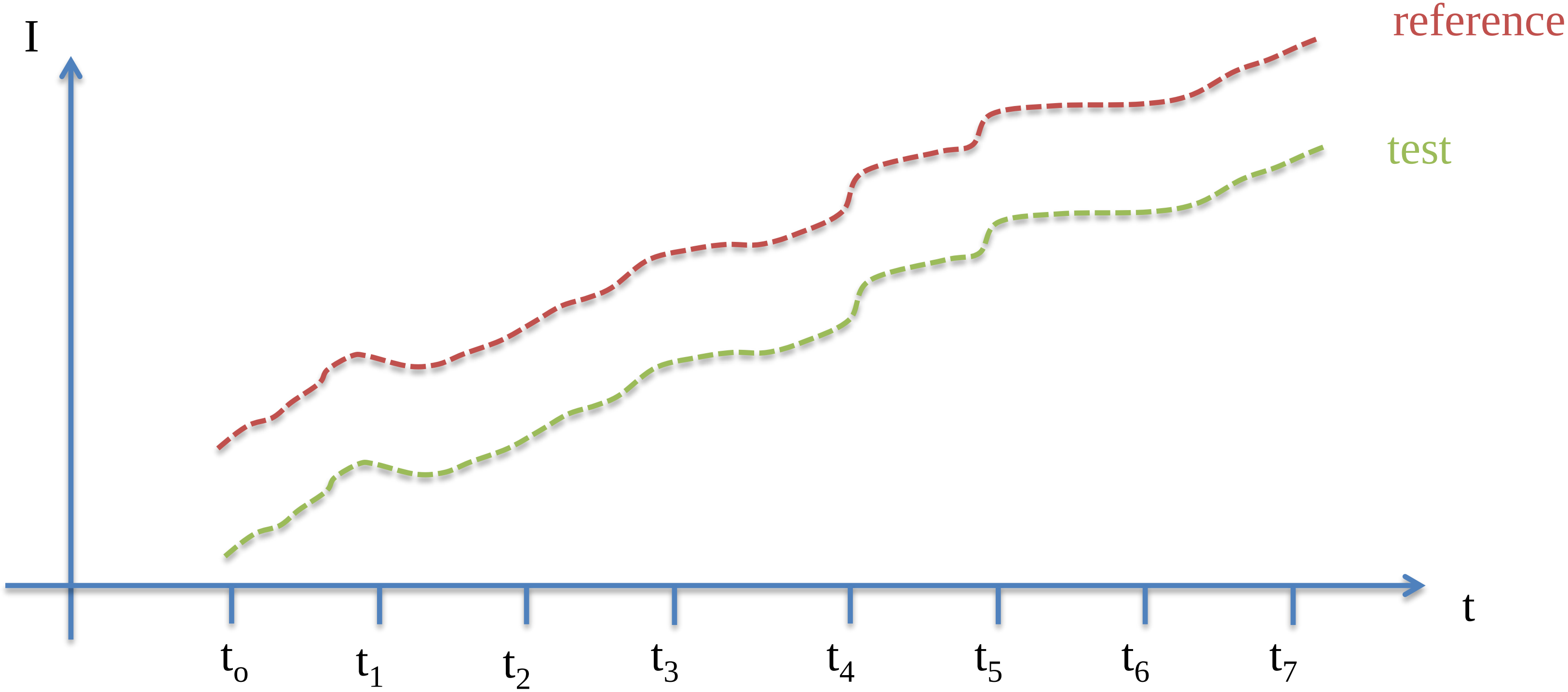
The determination of each balance readings differences need more than two values since balance indications are not stable during the weighing process



Reality

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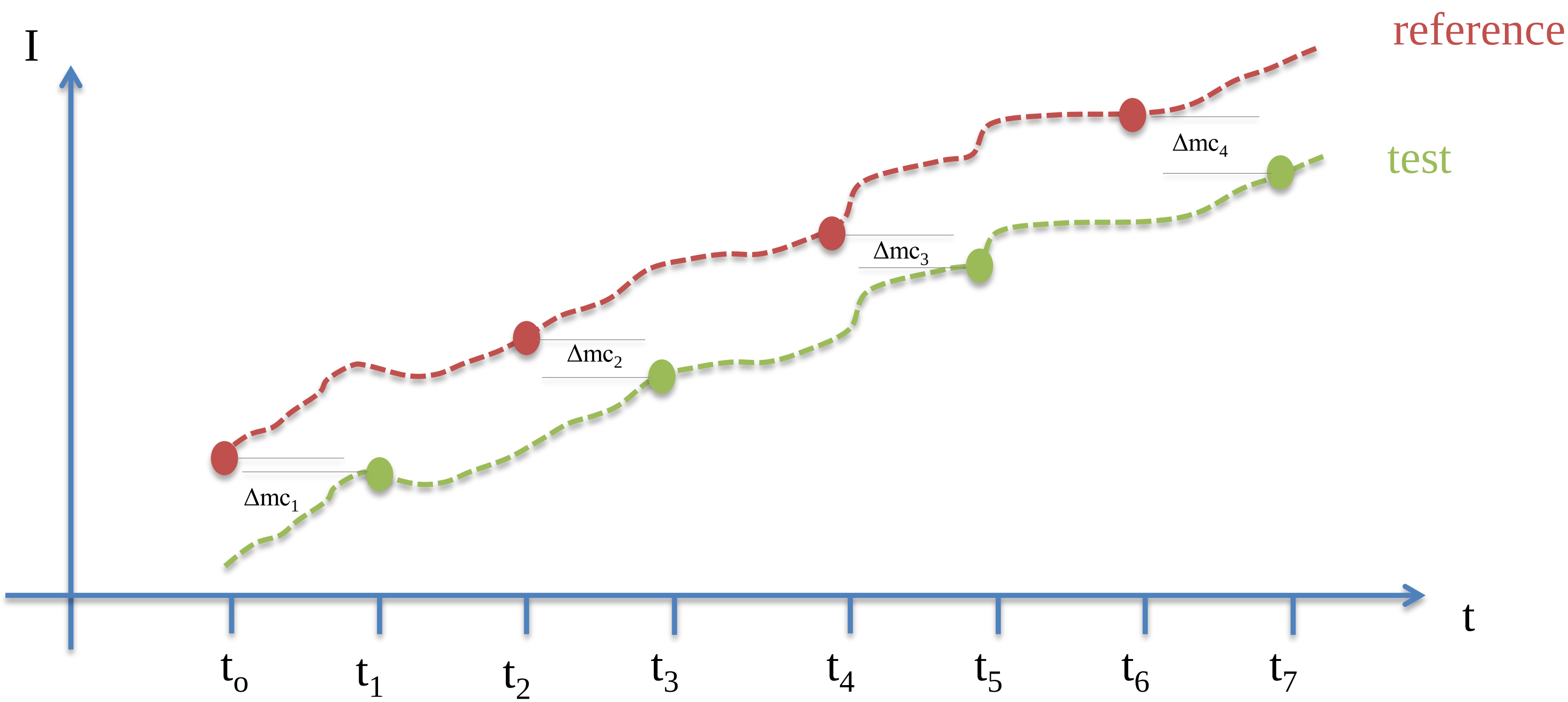
The determination of each balance readings differences need more than two values since balance indications are not stable during the weighing process



$$\Delta mc_1 = I_{t1} - I_{t0} = I_{t1} - I_{r1}$$

$$\Delta mc_4 = I_{t7} - I_{t6} = I_{t4} - I_{r4}$$

$$\overline{\Delta m_c} \neq \frac{1}{4}\sum_{i=1}^{4} \Delta m_{ci}$$

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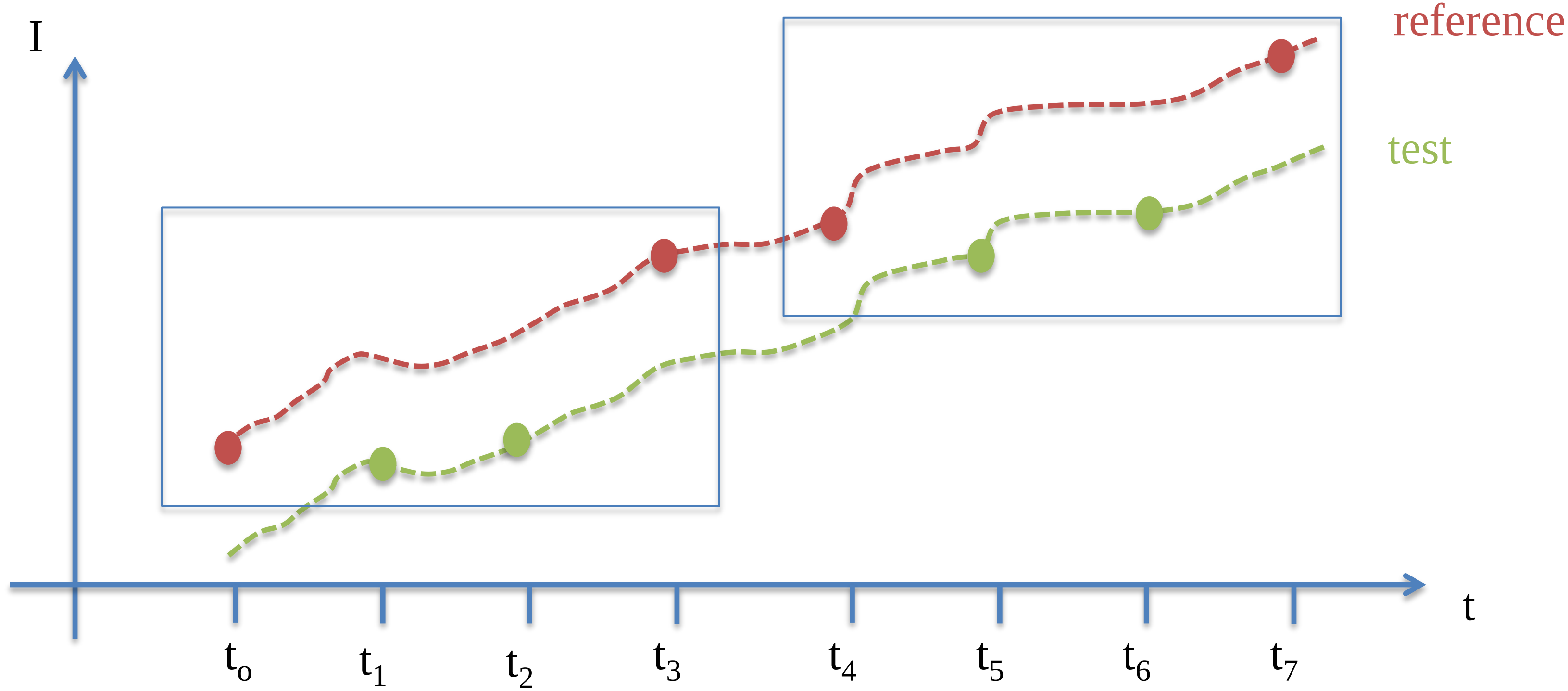
ABA, $A_1B_1 \ldots B_nA_n$ and ABBA methods applied.



ABBA

$$\Delta mc_1=(I_{t1}-I_{t0}+I_{t2}-I_{t3})/2=(I_{t1}-I_{r1}+I_{t2}-I_{r2})/2$$

$$\Delta mc_2=(I_{t5}-I_{t4}+I_{t6}-I_{t7})/2=(I_{t3}-I_{r3}+I_{t4}-I_{r4})/2$$

$$\overline{\Delta m_c} = \frac{1}{2}\sum_{i=1}^{2}\Delta m_{ci}$$

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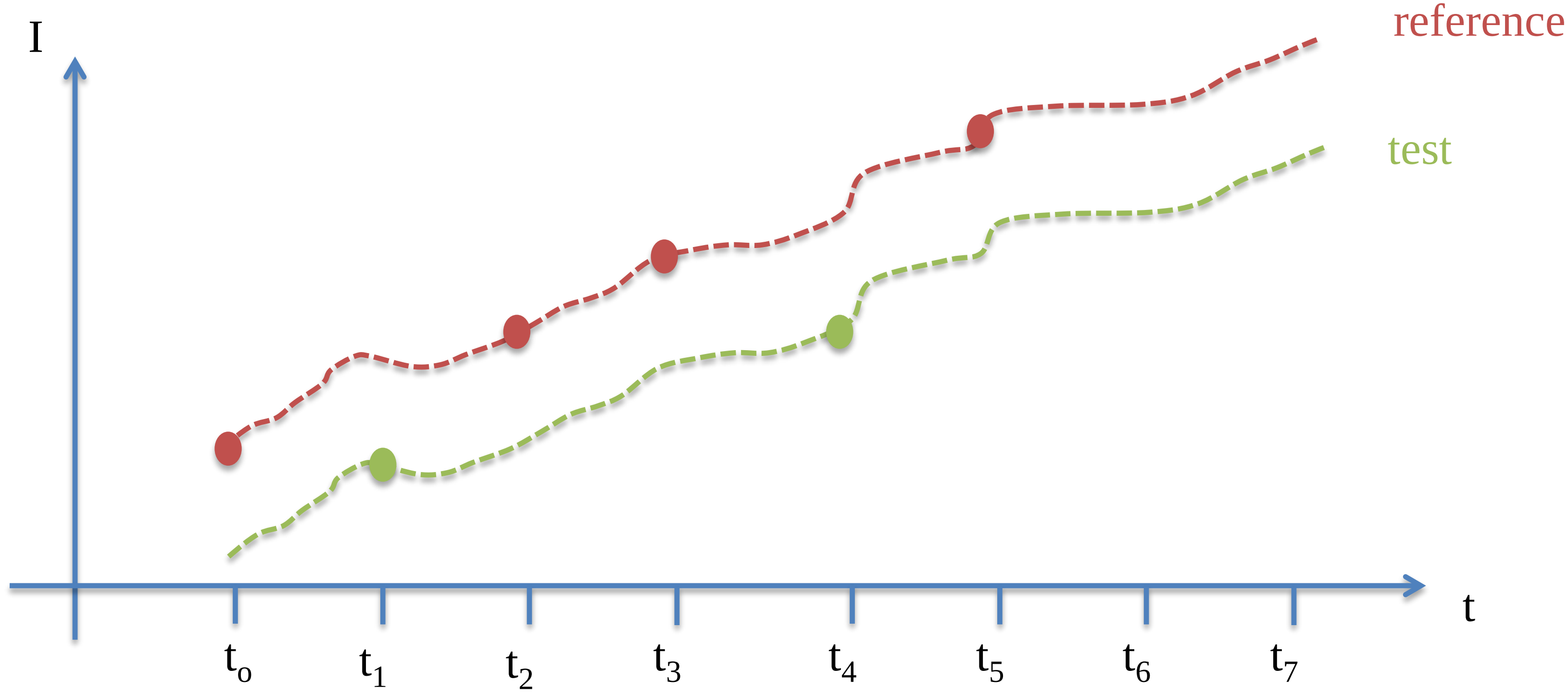
ABA, $A_1B_1 ... B_nA_n$ and ABBA methods are applied.



ABA

$$\Delta mc_1 = I_{t1} - (I_{t0} + I_{t2})/2 = I_{t1} - (I_{r1} + I_{r2})/2$$

$$\Delta mc_2 = I_{t4} - (I_{t3} + I_{t5})/2 = I_{t2} - (I_{r3} + I_{r4})/2$$

$$\overline{\Delta m_c} = \frac{1}{2} \sum_{i=1}^{2} \Delta m_{ci}$$

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Example of the determination of the conventional mass value of a weight (1 kg F2, calibrated with 1 kg F1)

ormation related to the environmental conditions measurement equipm				Test weight information			
	Correction	Expanded uncertainty k=2	Scale division interval	Marking	Nominal value [g]	Density [g/cm³]	Expanded uncertainty of the density value [g/cm³] k = 2
Temperature equipment [ºC]	0	0,1	0,1		1000	8,400	0,170
Relative humidity equipment [%]	0	1,0	1	Material	Serial number	Shape	Manufacturer
Pressure equipment [Pa]	0	100	1	Brass			
							Model
Comparator							
Manufacturer	Model	Interval scale division [mg]	Eccentricity [mg]	d1/d2	Expanded uncertainty of the adjustment		
		0,01	0,01	0	0		
Reference standard							
Identification	Nominal value [g]	Correcction for the nominal value [mg]	Expanded uncertainty [mg] k = 2	Density [g/cm³]	Expanded uncertainty of the density [g/cm³] k = 2	Institute that issue the calibration certificate number	Calibration certificate number
	1000	1,4	1,6	8000	0,047		
Readings R = Reference T = Test							
	r	t	t	r	Differences	Units	Factor
1	0,00	0,09	0,10	0,01	0,09	mg	0,001
2	0,01	0,10	0,12	0,02	0,10		
3	0,03	0,11	0,12	0,03	0,09		
				Mean	0,09		
				standard deviation (calculated)	0,04		
				standard deviation (pooled)	0,03		
Environmental conditions							
Variable	Begining	End	Mean	Corrected mean	Standard uncertainty	U (k=2)	
Temperature [ºC]	19,7	19,9	19,8	19,8	0,08	0,2	
Humidity[%]	55	53	54	54,0	0,82	1,6	
Pressure [Pa]	101950	101960	101955	101955,0	50,08	100,2	

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Example of the determination of the conventional mass value of a weight

- First, we will evaluate the air density

Environmental conditions							
Variable	Begining	End	Mean	Corrected mean	Standard uncertainty	U (k=2)	
Temperature [ºC]	19,7	19,9	19,8	19,8	0,08	0,2	
Humidity[%]	55	53	54	54,0	0,82	1,6	
Pressure [Pa]	101950	101960	101955	101955,0	50,08	100,2	

$$\rho_a = \frac{0,0034851 p - 0,008863 hr \times e^{0.062t}}{273,15 + t}$$

$$\rho_a = \frac{0,0034851 \times 101955 - 0,008863 \times 54 \times e^{0.062 \times 19,8}}{273,15 + 19,8} = 1,20734 \ kg/m^3$$

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Example of the determination of the conventional mass value of a weight

- Then we will evaluate the conventional mass value of the weight (obs.: the information of the value will be complete with the uncertainty)

$$m_{ct}=m_{cr}\left(1+\left(\rho_a-\rho_0\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\overline{\Delta m_c}$$

$$m_{ct}=\left(1{,}0000014\ kg\right)\left(1+\left(1{,}20734\ kg/m^3-1{,}2\ kg/m^3\right)\left(\frac{1}{8400\ kg/m^3}-\frac{1}{8000\ kg/m^3}\right)\right)+0{,}00000009kg$$

$$m_{ct}=\left(1{,}0000014\ kg\right)\left(1+0{,}00734\left(\frac{1}{8400}-\frac{1}{8000}\right)\right)+0{,}00000009\ kg=\underbrace{1{,}0000014\ kg\times 0{,}9999999583}_{\text{implies a buoyancy correction of about -0,04 mg}}+0{,}00000009\ kg$$

$$m_{ct}=1{,}00000145\ kg=1\ kg+1{,}45\ mg$$

Learning objectives:

- To understand the concept of measurement uncertainty.
- To know what contributes to the measurement uncertainty in the determination of conventional mass of weights.
- To be able to read a calibration certificate for a set of weights.

UNCERTAINTY OF MASS STANDARDS CALIBRATIONS IN CONVENTIONAL MASS (SUBSTITUTION WEIGHING IN AIR).

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Measurement uncertainty: “Caution. Handle with care”.

- “Do not confuse statistical significance with practical significance” Douglas Montgomery , Design and Analysis of Experiments”, Wiley, 2005

- “*Pluralitas non est ponenda sine neccesitate*'‘ ó “entities must not be multiplied beyond necessity”. William of Ockham ,14th century franciscan friar and english logician.

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Uncertainty has many uses in mass metrology and metrology in general.

- Reporting measurement results
- Conformity assessment
- Expressing calibration and measurement capabilities
- Comparisons of measurement results

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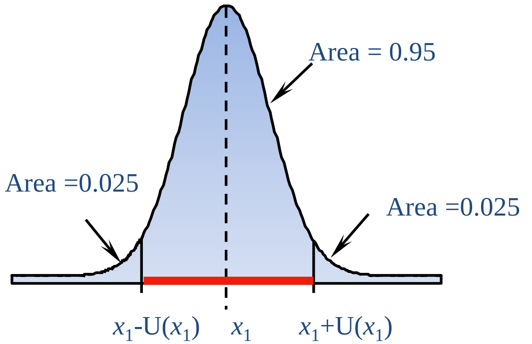
Reporting measurement results with uncertainty

- $x_1 \pm \mathrm{U}(x_1)$, k=2



- In metrology the following interpretation is very common: "The true value is within the interval $[x_1 - \mathrm{U}(x_1), x_1 + \mathrm{U}(x_1)]$ with a probability of 95%, associated to k=2, asuming a normal distribution.

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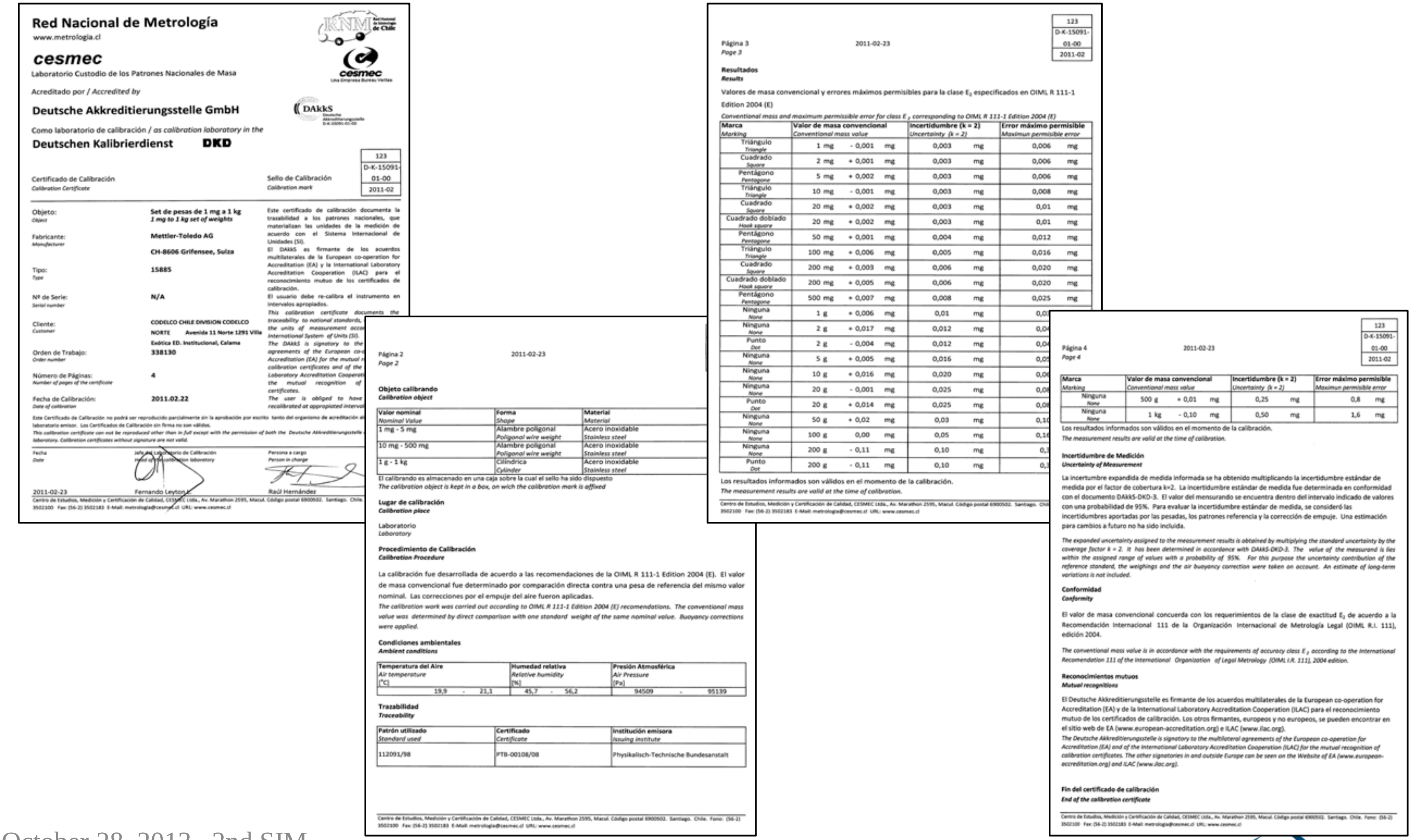
How to read a calibration certificate for mass standards with reported uncertainties?



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Laboratorio Custodio de los Patrones Nacionales de Masa

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Como laboratorio de calibración / *as calibration laboratory in the*

Deutschen Kalibrierdienst DKD

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2011-02

Certificado de Calibración
Calibration Certificate

Sello de Calibración
Calibration mark

Objeto: *Object*	**Set de pesas de 1 mg a 1 kg** *1 mg to 1 kg set of weights*
Fabricante: *Manufacturer*	**Mettler-Toledo AG** **CH-8606 Grifensee, Suiza**
Tipo: *Type*	**15885**
Nº de Serie: *Serial number*	**N/A**
Cliente: *Customer*	**CODELCO CHILE DIVISION CODELCO NORTE Avenida 11 Norte 1291 Villa Exótica ED. Institucional, Calama**
Orden de Trabajo: *Order number*	**338130**
Número de Páginas: *Number of pages of the certificate*	**4**
Fecha de Calibración: *Date of calibration*	**2011.02.22**

Este certificado de calibración documenta la trazabilidad a los patrones nacionales, que materializan las unidades de la medición de acuerdo con el Sistema Internacional de Unidades (SI).
El DAkkS es firmante de los acuerdos multilaterales de la European co-operation for Accreditation (EA) y la International Laboratory Accreditation Cooperation (ILAC) para el reconocimiento mutuo de los certificados de calibración.
El usuario debe re-calibra el instrumento en intervalos apropiados.
This calibration certificate documents the traceability to national standards, [illegible] the units of measurement acco[illegible] International System of Units (SI).
The DAkkS is signatory to the [illegible] agreements of the European co-[illegible] Accreditation (EA) for the mutual [illegible] calibration certificates and of the [illegible] Laboratory Accreditation Cooperati[illegible] the mutual recognition of [illegible] certificates.
The user is obliged to have [illegible] recalibrated at appropiated interval[illegible]

Este Certificado de Calibración no podrá ser reproducido parcialmente sin la aprobación por escrito tanto del organismo de acreditación [illegible] laboratorio emisor. Los Certificados de Calibración sin firma no son válidos.
This calibration certificate can not be reproduced other than in full except with the permission of both the Deutsche Akkreditierungsstelle [illegible] laboratory. Calibration certificates without signature are not valid.

Fecha *Date*	Jefe del Laboratorio de Calibración *Head of the calibration laboratory*	Persona a cargo *Person in charge*
2011-02-23	Fernando Leyton L.	Raúl Hernández

Centro de Estudios, Medición y Certificación de Calidad, CESMEC Ltda., Av. Marathon 2595, Macul. Código postal 6900502. Santiago. Chile. [illegible] 3502100 Fax: (56-2) 3502183 E-Mail: metrologia@cesmec.cl URL: www.cesmec.cl

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Objeto calibrando
Calibration object

Valor nominal *Nominal Value*	Forma *Shape*	Material *Material*
1 mg - 5 mg	Alambre poligonal *Poligonal wire weight*	Acero inoxidable *Stainless steel*
10 mg - 500 mg	Alambre poligonal *Poligonal wire weight*	Acero inoxidable *Stainless steel*
1 g - 1 kg	Cilíndrica *Cylinder*	Acero inoxidable *Stainless steel*

El calibrando es almacenado en una caja sobre la cual el sello ha sido dispuesto
The calibration object is kept in a box, on wich the calibration mark is affixed

Lugar de calibración
Calibration place

Laboratorio
Laboratory

Procedimiento de Calibración
Calibration Procedure

La calibración fue desarrollada de acuerdo a las recomendaciones de la OIML R 111-1 Edition 2004 (E). El valor de masa convencional fue determinado por comparación directa contra una pesa de referencia del mismo valor nominal. Las correcciones por el empuje del aire fueron aplicadas.
The calibration work was carried out according to OIML R 111-1 Edition 2004 (E) recomendations. The conventional mass value was determined by direct comparison with one standard weight of the same nominal value. Buoyancy corrections were applied.

Condiciones ambientales
Ambient conditions

Temperatura del Aire *Air temperature* [°C]	Humedad relativa *Relative humidity* [%]	Presión Atmosférica *Air Pressure* [Pa]
19,9 - 21,1	45,7 - 56,2	94509 - 95139

Trazabilidad
Traceability

Patrón utilizado *Standard used*	Certificado *Certificate*	Institución emisora *Issuing institute*
112091/98	PTB-00108/08	Physikalisch-Technische Bundesanstalt

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Resultados
Results

Valores de masa convencional y errores máximos permisibles para la clase E_2 especificados en OIML R 111-1 Edition 2004 (E)
Conventional mass and maximum permissible error for class E_2 corresponding to OIML R 111-1 Edition 2004 (E)

Marca *Marking*	Valor de masa convencional *Conventional mass value*	Incertidumbre (k = 2) *Uncertainty (k = 2)*	Error máximo permisible *Maximun permissible error*
Triángulo *Triangle*	1 mg - 0,001 mg	0,003 mg	0,006 mg
Cuadrado *Square*	2 mg + 0,001 mg	0,003 mg	0,006 mg
Pentágono *Pentagone*	5 mg + 0,002 mg	0,003 mg	0,006 mg
Triángulo *Triangle*	10 mg - 0,001 mg	0,003 mg	0,008 mg
Cuadrado *Square*	20 mg + 0,002 mg	0,003 mg	0,01 mg
Cuadrado doblado *Hook square*	20 mg + 0,002 mg	0,003 mg	0,01 mg
Pentágono *Pentagone*	50 mg + 0,001 mg	0,004 mg	0,012 mg
Triángulo *Triangle*	100 mg + 0,006 mg	0,005 mg	0,016 mg
Cuadrado *Square*	200 mg + 0,003 mg	0,006 mg	0,020 mg
Cuadrado doblado *Hook square*	200 mg + 0,005 mg	0,006 mg	0,020 mg
Pentágono *Pentagone*	500 mg + 0,007 mg	0,008 mg	0,025 mg
Ninguna *None*	1 g + 0,006 mg	0,01 mg	[illegible]
Ninguna *None*	2 g + 0,017 mg	0,012 mg	[illegible]
Punto *Dot*	2 g - 0,004 mg	0,012 mg	[illegible]
Ninguna *None*	5 g + 0,005 mg	0,016 mg	[illegible]
Ninguna *None*	10 g + 0,016 mg	0,020 mg	[illegible]
Ninguna *None*	20 g - 0,001 mg	0,025 mg	[illegible]
Punto *Dot*	20 g + 0,014 mg	0,025 mg	[illegible]
Ninguna *None*	50 g + 0,02 mg	0,03 mg	[illegible]
Ninguna *None*	100 g 0,00 mg	0,05 mg	[illegible]
Ninguna *None*	200 g - 0,11 mg	0,10 mg	[illegible]
Punto *Dot*	200 g - 0,11 mg	0,10 mg	[illegible]

Los resultados informados son válidos en el momento de la calibración.
The measurement results are valid at the time of calibration.

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Marca *Marking*	Valor de masa convencional *Conventional mass value*	Incertidumbre (k = 2) *Uncertainty (k = 2)*	Error máximo permisible *Maximun permissible error*
Ninguna *None*	500 g + 0,01 mg	0,25 mg	0,8 mg
Ninguna *None*	1 kg - 0,10 mg	0,50 mg	1,6 mg

Los resultados informados son válidos en el momento de la calibración.
The measurement results are valid at the time of calibration.

Incertidumbre de Medición
Uncertainty of Measurement

La incertumbre expandida de medida informada se ha obtenido multiplicando la incertidumbre estándar de medida por el factor de cobertura k=2. La incertidumbre estándar de medida fue determinada en conformidad con el documento DAkkS-DKD-3. El valor del mensurando se encuentra dentro del intervalo indicado de valores con una probabilidad de 95%. Para evaluar la incertidumbre estándar de medida, se consideró las incertidumbres aportadas por las pesadas, los patrones referencia y la corrección de empuje. Una estimación para cambios a futuro no ha sido incluida.

The expanded uncertainty assigned to the measurement results is obtained by multiplying the standard uncertainty by the coverage factor k = 2. It has been determined in accordance with DAkkS-DKD-3. The value of the measurand is lies within the assigned range of values with a probability of 95%. For this purpose the uncertainty contribution of the reference standard, the weighings and the air buoyancy correction were taken on account. An estimate of long-term variations is not included.

Conformidad
Conformity

El valor de masa convencional concuerda con los requerimientos de la clase de exactitud E_2 de acuerdo a la Recomendación Internacional 111 de la Organización Internacional de Metrología Legal (OIML R.I. 111), edición 2004.

The conventional mass value is in accordance with the requirements of accuracy class E_2 according to the International Recomendation 111 of the International Organization of Legal Metrology (OIML I.R. 111), 2004 edition.

Reconocimientos mutuos
Mutual recognitions

El Deutsche Akkreditierungsstelle es firmante de los acuerdos multilaterales de la European co-operation for Accreditation (EA) y de la International Laboratory Accreditation Cooperation (ILAC) para el reconocimiento mutuo de los certificados de calibración. Los otros firmantes, europeos y no europeos, se pueden encontrar en el sitio web de EA (www.european-accreditation.org) e ILAC (www.ilac.org).
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Fin del certificado de calibración
End of the calibration certificate

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Calibration Certificate

Sello de Calibración
Calibration mark

Objeto: *Object*	**Set de pesas de 1 mg a 1 kg** *1 mg to 1 kg set of weights*
Fabricante: *Manufacturer*	**Mettler-Toledo AG** **CH-8606 Grifensee, Suiza**
Tipo: *Type*	**15885**
Nº de Serie: *Serial number*	**N/A**
Cliente: *Customer*	**CODELCO CHILE DIVISION CODELCO NORTE Avenida 11 Norte 1291 Villa Exótica ED. Institucional, Calama**
Orden de Trabajo: *Order number*	**338130**
Número de Páginas: *Number of pages of the certificate*	**4**
Fecha de Calibración: *Date of calibration*	**2011.02.22**

Este certificado de calibración documenta la trazabilidad a los patrones nacionales, que materializan las unidades de la medición de acuerdo con el Sistema Internacional de Unidades (SI).
El DAkkS es firmante de los acuerdos multilaterales de la European co-operation for Accreditation (EA) y la International Laboratory Accreditation Cooperation (ILAC) para el reconocimiento mutuo de los certificados de calibración.
El usuario debe re-calibra el instrumento en intervalos apropiados.

This calibration certificate documents the traceability to national standards, which realize the units of measurement according to the International System of Units (SI).
The DAkkS is signatory to the multilateral agreements of the European co-operation for Accreditation (EA) for the mutual recognition of calibration certificates and of the International Laboratory Accreditation Cooperation (ILAC) for the mutual recognition of calibration certificates.
The user is obliged to have the object recalibrated at appropiated intervals.

Este Certificado de Calibración no podrá ser reproducido parcialmente sin la aprobación por escrito tanto del organismo de acreditación alemán como del laboratorio emisor. Los Certificados de Calibración sin firma no son válidos.
This calibration certificate can not be reproduced other than in full except with the permission of both the Deutsche Akkreditierungsstelle and the issuing laboratory. Calibration certificates without signature are not valid.

Fecha *Date*	Jefe del Laboratorio de Calibración *Head of the calibration laboratory*	Persona a cargo *Person in charge*
2011-02-23	Fernando Leyton L.	Raúl Hernández

Centro de Estudios, Medición y Certificación de Calidad, CESMEC Ltda., Av. Marathon 2595, Macul. Código postal 6900502. Santiago. Chile. Fono: (56-2) 3502100 Fax: (56-2) 3502183 E-Mail: metrologia@cesmec.cl URL: www.cesmec.cl

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Objeto calibrando
Calibration object

Valor nominal *Nominal Value*	Forma *Shape*	Material *Material*
1 mg - 5 mg	Alambre poligonal *Poligonal wire weight*	Acero inoxidable *Stainless steel*
10 mg - 500 mg	Alambre poligonal *Poligonal wire weight*	Acero inoxidable *Stainless steel*
1 g - 1 kg	Cilíndrica *Cylinder*	Acero inoxidable *Stainless steel*

El calibrando es almacenado en una caja sobre la cual el sello ha sido dispuesto
The calibration object is kept in a box, on wich the calibration mark is affixed

Lugar de calibración
Calibration place

Laboratorio
Laboratory

Procedimiento de Calibración
Calibration Procedure

La calibración fue desarrollada de acuerdo a las recomendaciones de la OIML R 111-1 Edition 2004 (E). El valor de masa convencional fue determinado por comparación directa contra una pesa de referencia del mismo valor nominal. Las correcciones por el empuje del aire fueron aplicadas.
The calibration work was carried out according to OIML R 111-1 Edition 2004 (E) recomendations. The conventional mass value was determined by direct comparison with one standard weight of the same nominal value. Buoyancy corrections were applied.

Condiciones ambientales
Ambient conditions

Temperatura del Aire *Air temperature* [°C]	Humedad relativa *Relative humidity* [%]	Presión Atmosférica *Air Pressure* [Pa]
19,9 - 21,1	45,7 - 56,2	94509 - 95139

Trazabilidad
Traceability

Patrón utilizado *Standard used*	Certificado *Certificate*	Institución emisora *Issuing institute*
112091/98	PTB-00108/08	Physikalisch-Technische Bundesanstalt

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Resultados
Results

Valores de masa convencional y errores máximos permisibles para la clase E_2 especificados en OIML R 111-1 Edition 2004 (E)

Conventional mass and maximum permissible error for class E_2 corresponding to OIML R 111-1 Edition 2004 (E)

Marca *Marking*	Valor de masa convencional *Conventional mass value*	Incertidumbre (k = 2) *Uncertainty (k = 2)*	Error máximo permisible *Maximun permisible error*
Triángulo *Triangle*	1 mg - 0,001 mg	0,003 mg	0,006 mg
Cuadrado *Square*	2 mg + 0,001 mg	0,003 mg	0,006 mg
Pentágono *Pentagone*	5 mg + 0,002 mg	0,003 mg	0,006 mg
Triángulo *Triangle*	10 mg - 0,001 mg	0,003 mg	0,008 mg
Cuadrado *Square*	20 mg + 0,002 mg	0,003 mg	0,01 mg
Cuadrado doblado *Hook square*	20 mg + 0,002 mg	0,003 mg	0,01 mg
Pentágono *Pentagone*	50 mg + 0,001 mg	0,004 mg	0,012 mg
Triángulo *Triangle*	100 mg + 0,006 mg	0,005 mg	0,016 mg
Cuadrado *Square*	200 mg + 0,003 mg	0,006 mg	0,020 mg
Cuadrado doblado *Hook square*	200 mg + 0,005 mg	0,006 mg	0,020 mg
Pentágono *Pentagone*	500 mg + 0,007 mg	0,008 mg	0,025 mg
Ninguna *None*	1 g + 0,006 mg	0,01 mg	0,03 mg
Ninguna *None*	2 g + 0,017 mg	0,012 mg	0,04 mg
Punto *Dot*	2 g - 0,004 mg	0,012 mg	0,04 mg
Ninguna *None*	5 g + 0,005 mg	0,016 mg	0,05 mg
Ninguna *None*	10 g + 0,016 mg	0,020 mg	0,06 mg
Ninguna *None*	20 g - 0,001 mg	0,025 mg	0,08 mg
Punto *Dot*	20 g + 0,014 mg	0,025 mg	0,08 mg
Ninguna *None*	50 g + 0,02 mg	0,03 mg	0,10 mg
Ninguna *None*	100 g 0,00 mg	0,05 mg	0,16 mg
Ninguna *None*	200 g - 0,11 mg	0,10 mg	0,3 mg
Punto *Dot*	200 g - 0,11 mg	0,10 mg	0,3 mg

Los resultados informados son válidos en el momento de la calibración.
The measurement results are valid at the time of calibration.

Marca *Marking*	Valor de masa convencional *Conventional mass value*	Incertidumbre (k = 2) *Uncertainty (k = 2)*	Error máximo permisible *Maximun permisible error*
Ninguna *None*	500 g + 0,01 mg	0,25 mg	0,8 mg
Ninguna *None*	1 kg - 0,10 mg	0,50 mg	1,6 mg

Los resultados informados son válidos en el momento de la calibración.
The measurement results are valid at the time of calibration.

Incertidumbre de Medición
Uncertainty of Measurement

La incertumbre expandida de medida informada se ha obtenido multiplicando la incertidumbre estándar de medida por el factor de cobertura k=2. La incertidumbre estándar de medida fue determinada en conformidad con el documento DAkkS-DKD-3. El valor del mensurando se encuentra dentro del intervalo indicado de valores con una probabilidad de 95%. Para evaluar la incertidumbre estándar de medida, se consideró las incertidumbres aportadas por las pesadas, los patrones referencia y la corrección de empuje. Una estimación para cambios a futuro no ha sido incluida.

The expanded uncertainty assigned to the measurement results is obtained by multiplying the standard uncertainty by the coverage factor k = 2. It has been determined in accordance with DAkkS-DKD-3. The value of the measurand is lies within the assigned range of values with a probability of 95%. For this purpose the uncertainty contribution of the reference standard, the weighings and the air buoyancy correction were taken on account. An estimate of long-term variations is not included.

Conformidad
Conformity

El valor de masa convencional concuerda con los requerimientos de la clase de exactitud E_2 de acuerdo a la Recomendación Internacional 111 de la Organización Internacional de Metrología Legal (OIML R.I. 111), edición 2004.

The conventional mass value is in accordance with the requirements of accuracy class E_2 according to the International Recomendation 111 of the International Organization of Legal Metrology (OIML I.R. 111), 2004 edition.

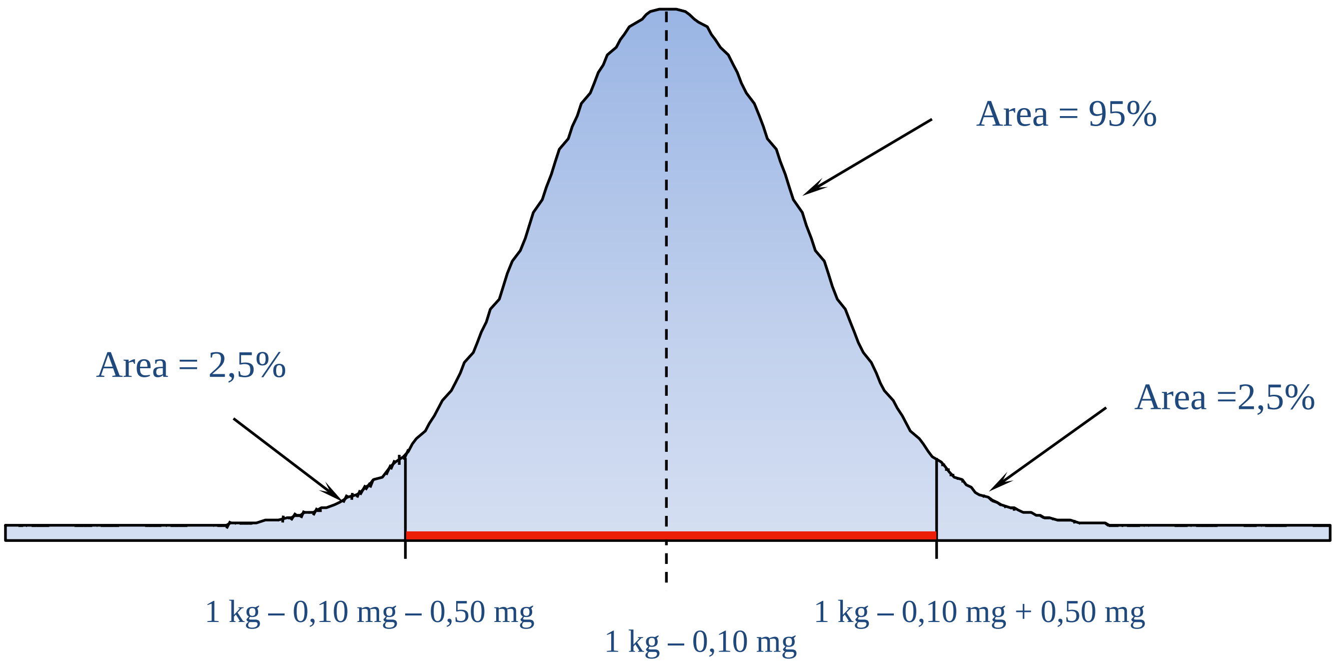
Reconocimientos mutuos
Mutual recognitions

El Deutsche Akkreditierungsstelle es firmante de los acuerdos multilaterales de la European co-operation for Accreditation (EA) y de la International Laboratory Accreditation Cooperation (ILAC) para el reconocimiento mutuo de los certificados de calibración. Los otros firmantes, europeos y no europeos, se pueden encontrar en el sitio web de EA (www.european-accreditation.org) e ILAC (www.ilac.org).

The Deutsche Akkreditierungsstelle is signatory to the multilateral agreements of the European co-operation for Accreditation (EA) and of the International Laboratory Accreditation Cooperation (ILAC) for the mutual recognition of calibration certificates. The other signatories in and outside Europe can be seen on the Website of EA (www.european-accreditation.org) and ILAC (www.ilac.org).

Fin del certificado de calibración
End of the calibration certificate

How to read a calibration certificate for mass standards with reported uncertainties?



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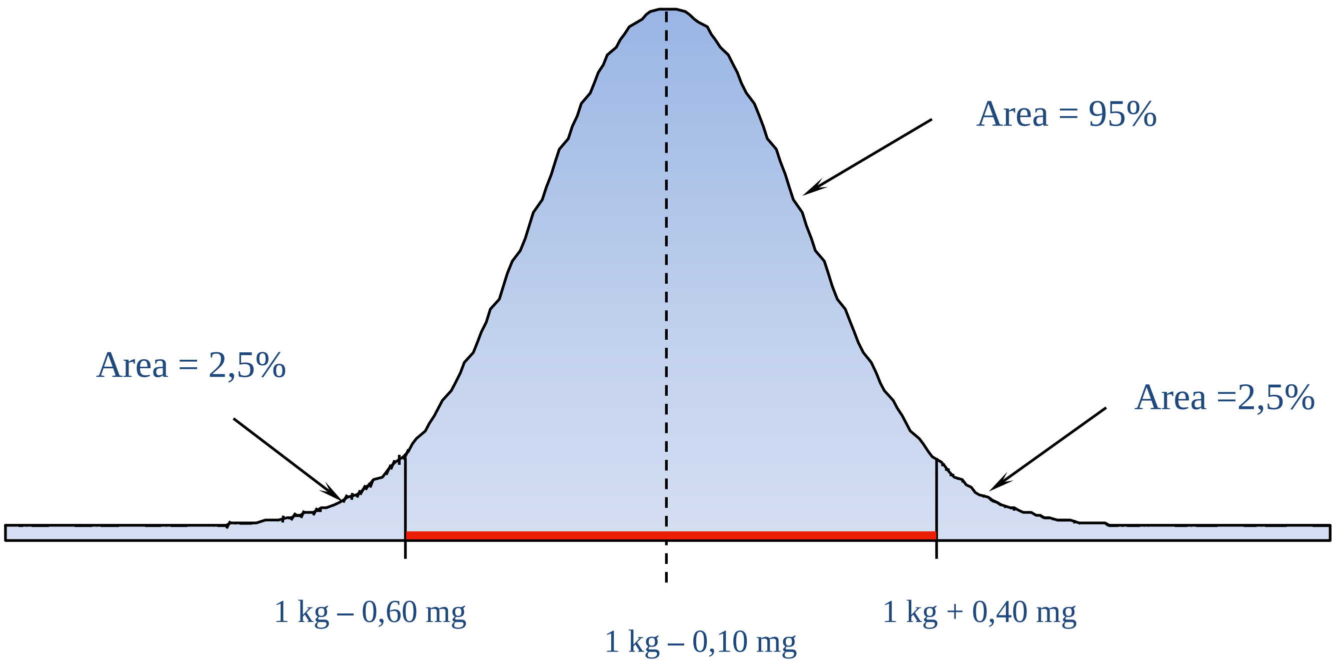
How to read a calibration certificate for mass standards with reported uncertainties?



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Measurement uncertainty is also used for conformity assessment

If

$$\begin{cases} VN + c + U \le VN + T \\ \text{and} \\ VN + c - U \le VN - T \end{cases}$$

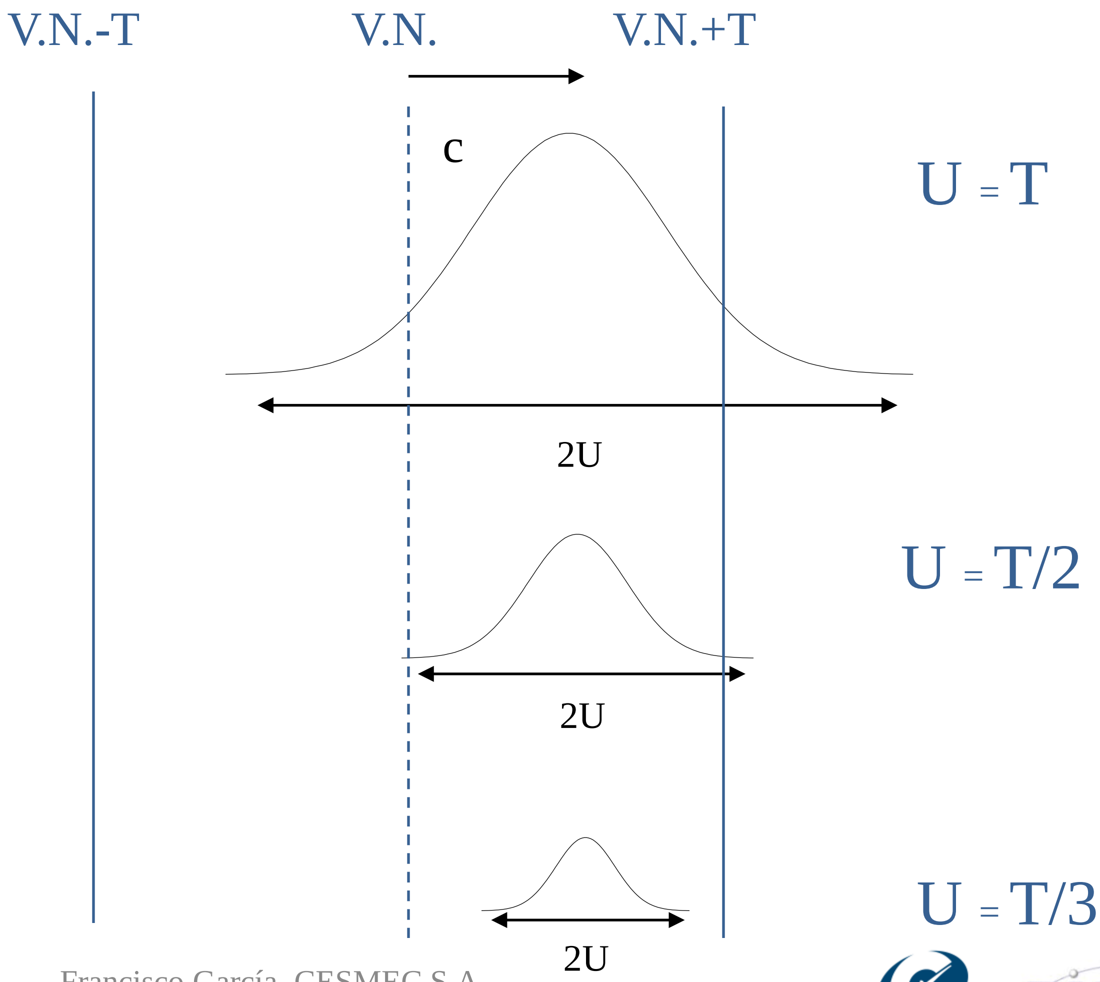
" the property value
is in agreement to requirement..."

o

Si

$$|c| \le T - U$$

" the property value
is in agreement to requirement..."



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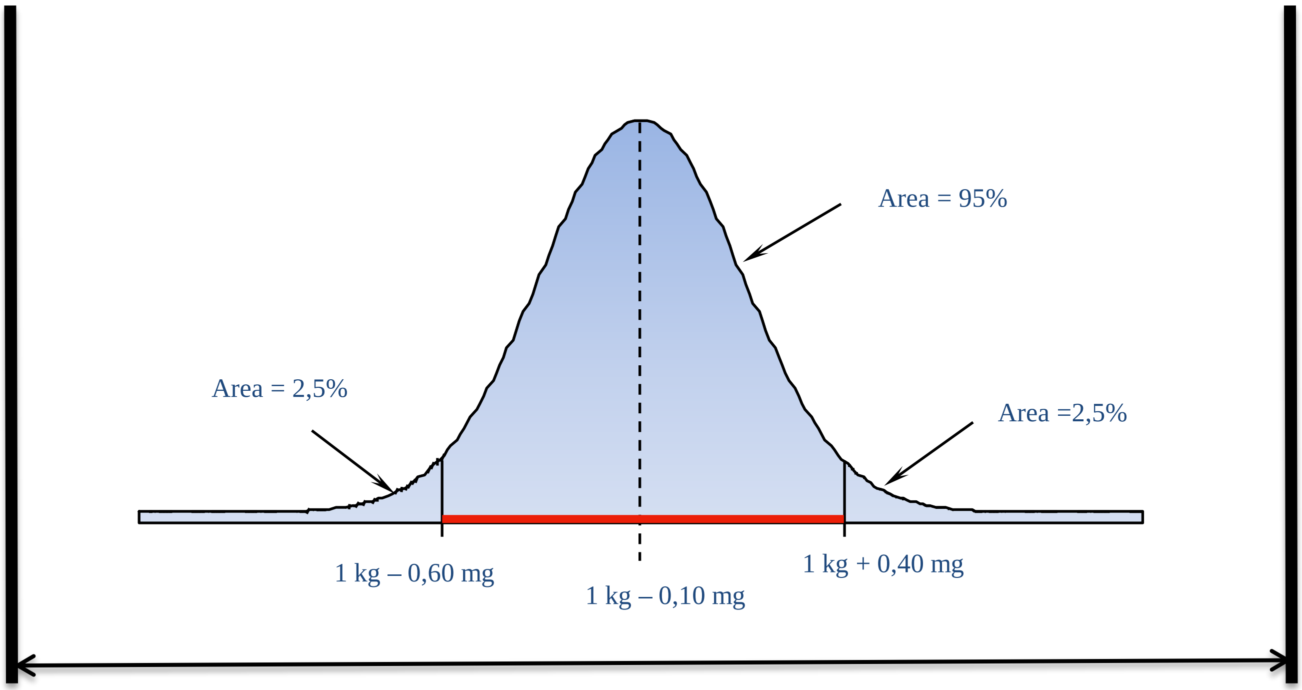
Measurement uncertainty is also used for conformity assessment



1,6 mg x 2 = 3,2 mg

$$|0{,}10\ \text{mg}| \le T - U$$

" the property value

is in agreement to requirement"

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Measurement uncertainty is also used for conformity assessment

Example:

From a calibration certificate we got the following in formation:

Conventional mass	Uncertainty k=2	Maximun permisible error for OIML R111 class F1
500 g – 3,6 mg	0,8 mg	2,5 mg

- Evaluate an interval for the conventional mass value.
- Does the weight’s conventional mass value agree with OIML R111 for class F1?

Table 1 Maximum permissible errors for weights (± δm in

Nominal value*	Class E_1	Class E_2	Class F_1	Cl
5 000 kg			25 000	8
2 000 kg			10 000	3
1 000 kg		1 600	5 000	1
500 kg		800	2 500	8
200 kg		300	1 000	3
100 kg		160	500	1
50 kg	25	80	250	
20 kg	10	30	100	
10 kg	5.0	16	50	
5 kg	2.5	8.0	25	
2 kg	1.0	3.0	10	
1 kg	0.5	1.6	5.0	
500 g	0.25	0.8	2.5	
200 g	0.10	0.3	1.0	
100 g	0.05	0.16	0.5	
50 g	0.03	0.10	0.3	
20 g	0.025	0.08	0.25	
10 g	0.020	0.06	0.20	
5 g	0.016	0.05	0.16	
2 g	0.012	0.04	0.12	
1 g	0.010	0.03	0.10	
500 mg	0.008	0.025	0.08	
200 mg	0.006	0.020	0.06	
100 mg	0.005	0.016	0.05	
50 mg	0.004	0.012	0.04	
20 mg	0.003	0.010	0.03	
10 mg	0.003	0.008	0.025	
5 mg	0.003	0.006	0.020	
2 mg	0.003	0.006	0.020	
1 mg	0.003	0.006	0.020	

Measurement uncertainty is also used for expressing calibration and measurement capabilities

Measured quantity / Calibration item	Range	Measurement conditions / procedure	Best measurement capability [1)]	Remarks
Mass Conventional Mass	1 mg, 2 mg, 5 mg, 10 mg 20 mg 50 mg 100 mg 200 mg 500 mg		0,002 mg 0,003 mg 0,004 mg 0,005 mg 0,006 mg 0,008 mg	OIML recommendation R111, class E_2
	1 g 2 g 5 g 10 g 20 g 50 g 100 g 200 g 500 g		0,010 mg 0,012 mg 0,015 mg 0,020 mg 0,025 mg 0,030 mg 0,05 mg 0,10 mg 0,25 mg	
	1 kg 2 kg 5 kg 10 kg 20 kg		0,5 mg 1,0 mg 2,5 mg 5 mg 10 mg	
	50 kg		75 mg	Klasse F_1

In accreditation

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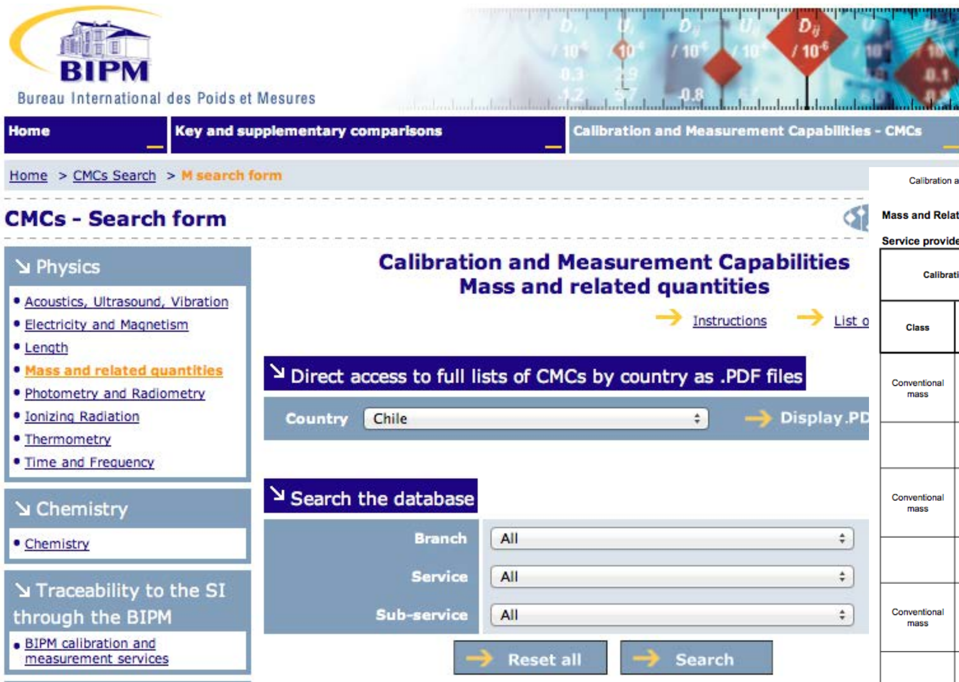
Measurement uncertainty is also used for expressing calibration and measurement capabilities



Calibration and Measurement Capabilities

Mass and Related Quantities, Chile, INN (Instituto Nacional de Normalizacion)

KCDB

Service providers: CESMEC (Centro de Estudios, Medicion y Certificacion de Calidad) and IDIC (Instituto de Investigaciones y Control)

Calibration or Measurement Service			Measurand Level or Range			Measurement Conditions/Independent Variable		Expanded Uncertainty						
Class	Instrument or Artifact	Instrument Type or Method	Minimum value	Maximum value	Units	Parameter	Specifications	Value	Units	Coverage Factor	Level of Confidence	Is the expanded uncertainty a relative one?	Service Provider	Internal identifier
Conventional mass	Standard weights	Substitution weighing with buoyancy correction	1	1	g	Laboratory temperature	18 °C to 27 °C, the allowed temperature change in 1 hour is equal to ± 0.5 °C, in 4 hours: ± 0.7 °C and in 24 hours: ± 1 °C	0.01	mg	2	95%	No	CESMEC	131-750
						Humidity	40 % to 60 %, the allowed humidity change in 4 hours is equal to ± 10 %							
Conventional mass	Standard weights	Substitution weighing with buoyancy correction	2	2	g	Laboratory temperature	18 °C to 27 °C, the allowed temperature change in 1 hour is equal to ± 0.5 °C, in 4 hours: ± 0.7 °C and in 24 hours: ± 1 °C	0.012	mg	2	95%	No	CESMEC	131-750
						Humidity	40 % to 60 %, the allowed humidity change in 4 hours is equal to ± 10 %							
Conventional mass	Standard weights	Substitution weighing with buoyancy correction	5	5	g	Laboratory temperature	18 °C to 27 °C, the allowed temperature change in 1 hour is equal to ± 0.5 °C, in 4 hours: ± 0.7 °C and in 24 hours: ± 1 °C	0.015	mg	2	95%	No	CESMEC	131-750
						Humidity	40 % to 60 %, the allowed humidity change in 4 hours is equal to ± 10 %							

The BIPM key comparison database, May 2006 4/10

In the KCDB: kcdb.bipm.org

Measurement uncertainty is used for evaluating measurement results

$$\left(x_1 - x_2, \sqrt{U^2(x_1) + U^2(x_2)}\right)$$

Degree of equivalence

$$E_n = \frac{x_1 - x_2}{\sqrt{U^2(x_1) + U^2(x_2)}}$$

Level of measurement agreement

$$0 \le |E_n| \le 1$$ Agree

$$1 < |E_n| < 2$$ Doubts

$$2 \le |E_n|$$ Disagree

This is out of the scope of this lecture but it's good to be aware of this

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GUM framework used in mass metrology

- In general, this method can be easily to applied when measuring physical quantities.
- It is necessary to specify a model that relates input quantities with the output quantity(ies). This model is provided by the definition of the measurand or physics that explains the output quantity(ies)
- It is necessary to identify and quantify the contribution of each input quantity to the measurement uncertainty.
- http://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf

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GUM framework is based on the linearization of the measurement model

$$Y = f\left(X_1,\dots,X_n\right)$$

$$Y = f\left(x_1,\dots,x_n\right) + \left.\frac{\partial f}{\partial X_1}\right|_{X_1=x_1}\left(X_1 - x_1\right) + \dots + \left.\frac{\partial f}{\partial X_n}\right|_{X_n=x_n}\left(X_n - x_n\right) + \dots$$

$$E\left(X_i\right) = x_i$$

$$Y = \left\{ f\left(x_1,\dots,x_n\right) - \left.\frac{\partial f}{\partial X_1}\right|_{X_1=x_1} x_1 - \dots - \left.\frac{\partial f}{\partial X_n}\right|_{X_n=x_n} x_n \right\} + \left\{ \left.\frac{\partial f}{\partial X_1}\right|_{X_1=x_1} X_1 + \dots + \left.\frac{\partial f}{\partial X_n}\right|_{X_n=x_n} X_n \right\} + \dots$$

$$Var\left(Y\right) = \left(\left.\frac{\partial f}{\partial X_1}\right|_{X_1=x_1} \right)^2 Var\left(X_1\right) + \dots + \left(\left.\frac{\partial f}{\partial X_n}\right|_{X_n=x_n} \right)^2 Var\left(X_n\right) + \sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial f}{\partial X_i}\right|_{X_i=x_i} \left.\frac{\partial f}{\partial X_j}\right|_{X_j=x_j} Cov\left(X_i, X_j\right) + \dots$$

$$u\left(Y\right) = \sqrt{ \left(\left.\frac{\partial f}{\partial X_1}\right|_{X_1=x_1} \right)^2 u^2\left(X_1\right) + \dots + \left(\left.\frac{\partial f}{\partial X_n}\right|_{X_n=x_n} \right)^2 u^2\left(X_n\right) + \sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial f}{\partial X_i}\right|_{X_i=x_i} \left.\frac{\partial f}{\partial X_j}\right|_{X_j=x_j} Cov\left(X_i, X_j\right) + \dots }$$

GUM framework is based on the linearization of the measurement model and on the use of two concepts of probability at the same time. But for practical purposes it works.

$$u(Y)=\sqrt{\left(\left.\frac{\partial f}{\partial X_1}\right|_{X_1=x_1}\right)^2 u^2(X_1)+\ldots+\left(\left.\frac{\partial f}{\partial X_n}\right|_{X_n=x_n}\right)^2 u^2(X_n)+\sum_{i=1}^{n}\sum_{j=1}^{n}\left.\frac{\partial f}{\partial X_i}\right|_{X_i=x_i}\left.\frac{\partial f}{\partial X_j}\right|_{X_j=x_j} Cov(X_i,X_j)+\ldots}$$

From measurements or prior information

- From measurements $u(X)=\frac{s}{\sqrt{n}}$

- From prior information: some pdfs are assumed

This generates a numerical conflict that for practical purposes do not affect the evaluation of the expanded uncertainty.

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For the calibration of mass standards in conventional mass, the following expressions apply.

$$u(m_{ct})=\sqrt{u^2\left(\overline{\Delta m_c}\right)+u^2\left(m_{cr}\right)+\left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2u^2\left(\rho_a\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2u^2\left(\rho_t\right)+m_{cr}^2\left(\rho_a-\rho_0\right)\left[\left(\rho_a-\rho_0\right)-2\left(\rho_{a1}-\rho_0\right)\right]\frac{u^2\left(\rho_r\right)}{\rho_r^4}+u_{ba}^2}$$

$$u(m_{ct})=\sqrt{u^2\left(\overline{\Delta m_c}\right)+u^2\left(m_{cr}\right)+\left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2u^2\left(\rho_a\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2u^2\left(\rho_t\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_r^2}\right)^2u^2\left(\rho_r\right)+u_{ba}^2}$$

Remember that the model equation is:

$$m_{ct}=m_{cr}\left(1+\left(\rho_a-\rho_0\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r}\right)\right)+\overline{\Delta m_c}$$

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Now, we'll review the evaluation of each uncertainty component, step y step

$$u(m_{ct}) = \sqrt{u^2\left(\overline{\Delta m_c}\right) + u^2\left(m_{cr}\right) + \left(m_{cr}\frac{\rho_t - \rho_r}{\rho_r \rho_t}\right)^2 u^2\left(\rho_a\right) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_t^2}\right)^2 u^2\left(\rho_t\right) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_r^2}\right)^2 u^2\left(\rho_r\right) + u_{ba}^2}$$

1 2 3 4 5 6

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The standard uncertainty of the weighing process (ABBA) is given by the experimental standard deviation

$$u(m_{ct}) = \sqrt{u^2\left(\overline{\Delta m_c}\right) + u^2(m_{cr}) + \left(m_{cr}\frac{\rho_t - \rho_r}{\rho_r \rho_t}\right)^2 u^2(\rho_a) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_t^2}\right)^2 u^2(\rho_t) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_r^2}\right)^2 u^2(\rho_r) + u_{ba}^2}$$

$$u^2\left(\overline{\Delta m_c}\right) = \frac{s^2(\Delta m_{ci})}{n}$$

$$s^2 = \frac{1}{n-1}\sum_i^n \left(\Delta m_{ci} - \overline{\Delta m_c}\right)^2$$

A “pooled standard deviation” can also be used. This is specially useful when few weighing cycles are done

Table C.3 Minimum number of weighing cycles

Class	E_1	E_2	F_1	F_2	M_1, M_2, M_3
Minimum number of ABBA	3	2	1	1	1
Minimum number of ABA	5	3	2	1	1
Minimum number of $AB_1 \ldots B_nA$	5	3	2	1	1

$$s^2(\Delta m_c) = \frac{1}{J}\sum_i^n s_j^2(\Delta m_{ci})$$

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The standard uncertainty of the reference standard is given by the calibration certificate, but other considerations made me done according to the information available

$$u(m_{ct}) = \sqrt{u^2\left(\overline{\Delta m_c}\right) + u^2\left(m_{cr}\right) + \left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2 u^2\left(\rho_a\right) + \left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2 u^2\left(\rho_t\right) + \left(m_{cr}\frac{\rho_a-\rho_0}{\rho_r^2}\right)^2 u^2\left(\rho_r\right) + u_{ba}^2}$$

$$u(m_{cr}) = \sqrt{\left(\frac{U}{k}\right)^2 + u_{inst}^2\left(m_{cr}\right)}$$

Most common situation

$$u(m_{cr}) = \sqrt{\frac{\delta m^2}{3} + u_{inst}^2\left(m_{cr}\right)}$$

This can be used for F1 and lower classes

$$u(m_{cr}) = \sum_i u\left(m_{cri}\right)$$

You can apply this if you have a set of weights

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The standard uncertainty of the reference standard is given by the calibration certificate, but other considerations made me done according to the information available

$$u(m_{ct}) = \sqrt{u^2\left(\overline{\Delta m_c}\right) + u^2\left(m_{cr}\right) + \left(m_{cr}\frac{\rho_t - \rho_r}{\rho_r \rho_t}\right)^2 u^2\left(\rho_a\right) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_t^2}\right)^2 u^2\left(\rho_t\right) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_r^2}\right)^2 u^2\left(\rho_r\right) + u_{ba}^2}$$

$$u(m_{cr}) = \sqrt{\left(\frac{U}{k}\right)^2 + u_{inst}^2\left(m_{cr}\right)}$$

Most common situation

$$u(m_{cr}) = \sqrt{\frac{\delta m^2}{3} + u_{inst}^2\left(m_{cr}\right)}$$

This can be used for F1 and lower classes

$$u(m_{cr}) = \sum_i u\left(m_{cri}\right)$$

You can apply this if you have a set of weights

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The standard uncertainty of the air density can be evaluated form the approximated formula

$$u(m_{ct}) = \sqrt{u^2\left(\overline{\Delta m_c}\right) + u^2\left(m_{cr}\right) + \left(m_{cr}\frac{\rho_t - \rho_r}{\rho_r \rho_t}\right)^2 u^2\left(\rho_a\right) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_t^2}\right)^2 u^2\left(\rho_t\right) + \left(m_{cr}\frac{\rho_a - \rho_0}{\rho_r^2}\right)^2 u^2\left(\rho_r\right) + u_{ba}^2}$$

$$u(\rho_{\mathrm{a}}) = \left[u_F^2 + \left(\frac{\partial \rho_a}{\partial p}\right)^2 u^2(p) + \left(\frac{\partial \rho_a}{\partial t}\right)^2 u^2(t) + \left(\frac{\partial \rho_a}{\partial h_r}\right)^2 u^2(h_r)\right]^{1/2}$$

$$\frac{\partial \rho_a}{\partial p} \approx \rho_a x 10^{-5}\, Pa^{-1}$$

$$\frac{\partial \rho_a}{\partial T} \approx -\rho_a x 4 x 10^{-3}\, K^{-1}$$

$$\frac{\partial \rho_a}{\partial h_r} \approx -\rho_a x 9 x 10^{-5}$$

The test weight density and its standard uncertainty can be determined by OIML method F for most practical cases

$$u(m_{ct}) = \sqrt{u^2\left(\overline{\Delta m_c}\right) + u^2(m_{cr}) + \left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2 u^2(\rho_a) + \left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2 u^2(\rho_t) + \left(m_{cr}\frac{\rho_a-\rho_0}{\rho_r^2}\right)^2 u^2(\rho_r) + u_{ba}^2}$$

Table B.8 Recommended methods for the density determination for class of weights

Weight	Class E_1	Class E_2	Class F_1	Classes F_2, M_1, M_2
5 000 kg			E, F	F
2 000 kg				
1 000 kg		E, F		
500 kg				
200 kg				
100 kg				
50 kg	A, C, D	D, E, F	D, E, F	
20 kg				
10 kg	A, B1*, C, D			
5 kg				
2 kg				
1 kg	A, B*, C	B, F	B, C, F	
500 g				
200 g				
100 g				
50 g	A, B1*	B, C, F		
20 g			F	
10 g				
5 g		F		
2 g	B*, F1			
1 g				
500 mg	F1			
200 mg				
100 mg				
50 mg				
20 mg				

This is outside of the scope of this lecture but you can deduce by yourself the equations according to what we saw in section "Balance readings vs. Mass values and Conventional Mass values"

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The test weight density and its standard uncertainty can be determined by OIML method F for most practical cases

- *Method F1: If it is known that the supplier consistently uses the same alloy for a particular class of weights, and its density is known from previous tests, then the known density should be applied using an uncertainty of one third of that giv- en in Table B.7 for the same alloy.*
- *Method F2: Obtain the composition of the alloy from the supplier of the weight in question. Find the density value from a physics/chemistry handbook that has tables of density as a function of the concentration of alloying elements. Use the handbook density value and apply the uncertainty value from Table B.7. For class E2 to M2 weights the "as- sumed density" values in Table B.7 below are adequate. The density of class M3 weights is usually of no concern.*

Table B.7 Method F2 - List of alloys most commonly used for weights

Alloy/material	Assumed density	Uncertainty (k = 2)
Platinum	21 400 kg m^{-3}	± 150 kg m^{-3}
Nickel silver	8 600 kg m^{-3}	± 170 kg m^{-3}
Brass	8 400 kg m^{-3}	± 170 kg m^{-3}
Stainless steel	7 950 kg m^{-3}	± 140 kg m^{-3}
Carbon steel	7 700 kg m^{-3}	± 200 kg m^{-3}
Iron	7 800 kg m^{-3}	± 200 kg m^{-3}
Cast iron (white)	7 700 kg m^{-3}	± 400 kg m^{-3}
Cast iron (grey)	7 100 kg m^{-3}	± 600 kg m^{-3}
Aluminum	2 700 kg m^{-3}	± 130 kg m^{-3}

$$u(\rho_t) = \frac{1}{3}\left[\frac{U(\rho_t)}{2}\right]$$ Method F1

$$u(\rho_t) = \frac{U(\rho_t)}{2}$$ Method F2

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The reference weight density and its standard uncertainty can be determined by OIML method F for most practical cases, too.

$$u(m_{ct}) = \sqrt{\begin{aligned}&u^2\left(\overline{\Delta m_c}\right)+u^2\left(m_{cr}\right)+\left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2 u^2\left(\rho_a\right)+\\ &+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2 u^2\left(\rho_t\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_r^2}\right)^2 u^2\left(\rho_r\right)+\\ &+u_{ba}^2\end{aligned}}$$

Table B.8 Recommended methods for the density determination for class of weights

Weight	Class E_1	Class E_2	Class F_1	Classes F_2, M_1, M_2
5 000 kg			E, F	F
2 000 kg				
1 000 kg		E, F		
500 kg				
200 kg				
100 kg				
50 kg	A, C, D	D, E, F	D, E, F	
20 kg				
10 kg	A, B1*, C, D			
5 kg				
2 kg				
1 kg	A, B*, C	B, F	B, C, F	
500 g				
200 g				
100 g				
50 g	A, B1*	B, C, F		
20 g			F	
10 g				
5 g		F		
2 g	B*, F1			
1 g				
500 mg	F1			
200 mg				
100 mg				
50 mg				
20 mg				

This is outside of the scope of this lecture but you can deduce by yourself the equations according to what we saw in section "Balance readings vs. Mass values and Conventional Mass values"

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The standard uncertainty of the balance is given by 4 contributions most of them can normally be neglected or vey small.

$$u(m_{ct})=\sqrt{u^2\left(\overline{\Delta m_c}\right)+u^2\left(m_{cr}\right)+\left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2u^2\left(\rho_a\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2u^2\left(\rho_t\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_r^2}\right)^2u^2\left(\rho_r\right)+u_{ba}^2}$$

$$u_{ba}=\sqrt{u_s^2+u_d^2+u_E^2+u_{ma}^2}$$

$$u_{ba}=\sqrt{\left(\overline{\Delta m_c}\right)^2\underbrace{\left(\frac{u^2\left(m_s\right)}{m_s^2}+\frac{u^2\left(\Delta I_s\right)}{\Delta I_s^2}\right)}_{\approx 10^{-8}}+\left(\frac{d/2}{\sqrt{3}}\sqrt{2}\right)^2+\left(\frac{\frac{d_1}{d_2}D}{2\sqrt{3}}\right)^2+\underbrace{u_{ma}^2}_{\approx 0}}$$

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The expanded uncertainty is obtained by multiplying the standard combined uncertainty by a coverage factor

- In mass metrology the coverage factor k=2 is normally applied.
- In addition the reported uncertainty is equal to the MPE of the reference standard and calibrations are done using an standard of higher accuracy.

$$U(m_{ct}) = 2u(m_{ct})$$

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Example of the determination of the measurement uncertainty of the conventional mass value of a weight (1 kg F2, calibrated with 1 kg F1)

ormation related to the environmental conditions measurement equipm				Test weight information			
	Correction	Expanded uncertainty k=2	Scale division interval	Marking	Nominal value [g]	Density [g/cm³]	Expanded uncertainty of the density value [g/cm³] k = 2
Temperature equipment [ºC]	0	0,1	0,1		1000	8,400	0,170
Relative humidity equipment [%]	0	1,0	1	Material	Serial number	Shape	Manufacturer
Pressure equipment [Pa]	0	100	1	Brass			
							Model
Comparator							
Manufacturer	Model	Interval scale division [mg]	Eccentricity [mg]	d1/d2	Expanded uncertainty of the adjustment		
		0,01	0,01	0	0		
Reference standard							
Identification	Nominal value [g]	Correcction for the nominal value [mg]	Expanded uncertainty [mg] k = 2	Density [g/cm³]	Expanded uncertainty of the density [g/cm³] k = 2	Institute that issue the calibration certificate number	Calibration certificate number
	1000	1,4	1,6	8000	0,047		
Readings				R = Reference T = Test			
	r	t	t	r	Differences	Units	Factor
1	0,00	0,09	0,10	0,01	0,09	mg	0,001
2	0,01	0,10	0,12	0,02	0,10		
3	0,03	0,11	0,12	0,03	0,09		
				Mean	0,09		
				standard deviation (calculated)	0,04		
				standard deviation (pooled)	0,03		
Environmental conditions							
Variable	Begining	End	Mean	Corrected mean	Standard uncertainty	U (k=2)	
Temperature [ºC]	19,7	19,9	19,8	19,8	0,08	0,2	
Humidity[%]	55	53	54	54,0	0,82	1,6	
Pressure [Pa]	101950	101960	101955	101955,0	50,08	100,2	

Example of the determination of the conventional mass value of a weight

- First, we will evaluate the air density uncertainty

Environmental conditions							
Variable	Begining	End	Mean	Corrected mean	Standard uncertainty	U (k=2)	
Temperature [ºC]	19,7	19,9	19,8	19,8	0,08	0,2	
Humidity[%]	55	53	54	54,0	0,82	1,6	
Pressure [Pa]	101950	101960	101955	101955,0	50,08	100,2	

Air density							
Variable	Units	Distribution	Estimator x_i	Standard uncertainty $u(x_i)$	c_i	Z_i [kg/m³]	%
t	°C	Normal	19,8	0,08	-0,00446703	-0,00036	26,0981
h_r	%	Normal	54,0	0,82	-0,00010326	-0,00008	1,3945
p	Pa	Normal	101955	50	0,00001190	0,00060	69,6477
f	kg/m³	Normal	0	0,000121	1,00000000	0,00012	2,8597
ρ_a	kg/m³	Normal	1,2073388	0,0007140			
ρ_a	g/cm³	Normal	0,0012073	0,0000007			

$$u(\rho_a) = \left[0{,}000121^2 + (0{,}0000119)^2\, 50^2 + (-0{,}004467)^2\, 0{,}08^2 + (-0{,}00010326)^2\, 0{,}82^2\right]^{1/2}$$

$$u(\rho_a) = 0{,}00071\, kg/m^3$$

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Example of the determination of the conventional mass value of a weight

- Second, we will evaluate the comparator uncertainty

Comparator							
Variable	Units	Distribution	Estimator x_i	Standard uncertainty $u(x_i)$	c_i	Z_i [g]	%
Interval scale division	g	Normal	0,00000000	0,00000408	1,000000000	0,000004082	99,9879
Eccentricity	g	Normal	0,000000	0,000000	1,000000000	0,000000000	0,0000
Adjustment / sensitivity	g	Normal	0,000000000	0,000000045	1,000000000	0,000000045	0,0121
Comparator	g	Normal	0,000000	0,0000041			

$$u_{ba}=\sqrt{\left(\overline{\Delta m_c}\right)^2\underbrace{\left(\frac{u^2(m_s)}{m_s^2}+\frac{u^2(\Delta I_s)}{\Delta I_s^2}\right)}_{\approx(5x10^{-4})^2}+\left(\frac{d/2}{\sqrt{3}}\sqrt{2}\right)^2+\left(\frac{\frac{d_1}{d_2}D}{2\sqrt{3}}\right)^2+\underbrace{u_{ma}^2}_{\approx 0}}=\sqrt{0{,}00009^2\left(5x10^{-4}\right)^2+\left(\frac{0{,}00001/2}{\sqrt{3}}\sqrt{2}\right)^2+\left(\frac{\frac{d_1}{d_2}0}{2\sqrt{3}}\right)^2+0^2}$$

$$u_{ba}=0{,}0000041\ \text{g}=0{,}0000000041\ \text{kg}$$

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Example of the determination of the conventional mass value of a weight

- Third, we will evaluate the combined standard uncertainty

$$u(m_{ct})=\sqrt{u^2\left(\overline{\Delta m_c}\right)+u^2\left(m_{cr}\right)+\left(m_{cr}\frac{\rho_t-\rho_r}{\rho_r\rho_t}\right)^2u^2\left(\rho_a\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_t^2}\right)^2u^2\left(\rho_t\right)+\left(m_{cr}\frac{\rho_a-\rho_0}{\rho_r^2}\right)^2u^2\left(\rho_r\right)+u_{ba}^2}$$

$$u(m_{ct})=\sqrt{\begin{aligned}&\left(\frac{0{,}00003}{\sqrt{3}}g\right)^2+\left(\frac{0{,}0016}{2}g\right)^2+\left(1000{,}0000014\,g\times\frac{8{,}4\,g/cm^3-8\,g/cm^3}{8\,g/cm^3\times 8{,}4\,g/cm^3}\right)^2\left(\frac{0{,}00071}{1000}g/cm^3\right)^2+\\&+\left(1000{,}0000014\,g\,\frac{0{,}00120734\,g/cm^3-0{,}0012\,g/cm^3}{\left(8{,}4\,g/cm^3\right)^2}\right)^2\left(0{,}170\,g/cm^3\right)^2+\\&+\left(1000{,}0000014\,g\,\frac{0{,}00120734\,g/cm^3-0{,}0012\,g/cm^3}{\left(8\,g/cm^3\right)^2}\right)^2\left(0{,}047\,g/cm^3\right)^2+\\&+\left(0{,}000018\,g\right)^2\end{aligned}}$$

$$u(m_{ct})=0{,}0008003\,g$$

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Example of the determination of the conventional mass value of a weight

- Finally, we will evaluate the evaluate the expanded uncertainty and the expanded uncertainty to be reported.

$$U(m_{ct}) = 2 \times u(m_{ct}) = 2 \times 0{,}0008003\ g = 0{,}001600525\ g = 1{,}600525\ mg$$

Table 1 Maximum permissible errors for weights (± δm in mg)

Nominal value*	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
5 000 kg			25 000	80 000	250 000	500 000	800 000	1 600 000	2 500 000
2 000 kg			10 000	30 000	100 000	200 000	300 000	600 000	1 000 000
1 000 kg		1 600	5 000	16 000	50 000	100 000	160 000	300 000	500 000
500 kg		800	2 500	8 000	25 000	50 000	80 000	160 000	250 000
200 kg		300	1 000	3 000	10 000	20 000	30 000	60 000	100 000
100 kg		160	500	1 600	5 000	10 000	16 000	30 000	50 000
50 kg	25	80	250	800	2 500	5 000	8 000	16 000	25 000
20 kg	10	30	100	300	1 000		3 000		10 000
10 kg	5.0	16	50	160	500		1 600		5 000
5 kg	2.5	8.0	25	80	250		800		2 500
2 kg	1.0	3.0	10	30	100		300		1 000
1 kg	0.5	1.6	5.0	16	50		160		500
500 g	0.25	0.8	2.5	8.0	25		80		250

$$U_{reported}(m_{ct}) = 5{,}0\ mg$$

Conventional mass value					Expanded uncertainty (k=2)	
1000	**g**	**+**	**1,4**	**mg**	**5,0**	**mg**

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Example of the determination of the conventional mass value of a weight

- Finally, we will evaluate the evaluate the expanded uncertainty and the expanded uncertainty to be reported.

$$U(m_{ct}) = 2 \times u(m_{ct}) = 2 \times 0{,}0008003\ g = 0{,}001600525\ g = 1{,}600525\ mg$$

Table 1 Maximum permissible errors for weights (± δm in mg)

Nominal value*	Class E_1	Class E_2	Class F_1	Class F_2	Class M_1	Class M_{1-2}	Class M_2	Class M_{2-3}	Class M_3
5 000 kg			25 000	80 000	250 000	500 000	800 000	1 600 000	2 500 000
2 000 kg			10 000	30 000	100 000	200 000	300 000	600 000	1 000 000
1 000 kg		1 600	5 000	16 000	50 000	100 000	160 000	300 000	500 000
500 kg		800	2 500	8 000	25 000	50 000	80 000	160 000	250 000
200 kg		300	1 000	3 000	10 000	20 000	30 000	60 000	100 000
100 kg		160	500	1 600	5 000	10 000	16 000	30 000	50 000
50 kg	25	80	250	800	2 500	5 000	8 000	16 000	25 000
20 kg	10	30	100	300	1 000		3 000		10 000
10 kg	5.0	16	50	160	500		1 600		5 000
5 kg	2.5	8.0	25	80	250		800		2 500
2 kg	1.0	3.0	10	30	100		300		1 000
1 kg	0.5	1.6	5.0	16	50		160		500
500 g	0.25	0.8	2.5	8.0	25		80		250

$$U_{reported}(m_{ct}) = 5{,}0\ mg$$

Conventional mass value					Expanded uncertainty (k=2)	
1000	**g**	**+**	**1,4**	**mg**	**5,0**	**mg**

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Learning objectives:

- Know how to use a check standard
- Know how to evaluate the precision of the comparator

STATISTICAL CONTROL

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Check standard

D.1.2 The purpose of the check standard is to assure the goodness of individual calibrations. A history of values on the check standard is required for this purpose. The accepted value of the mass difference, $\overline{m}_{diff}$, for the check standard (usually an average) is computed from the historical data and is based on at least 10–15 measurements. The value of the check standard for any new calibration, m_{diff}, is tested for agreement with the accepted value using a statistical control technique. The test is based on the t-statistic:

$$t = \frac{\left| m_{diff} - \overline{m}_{diff} \right|}{S} \qquad (D.1.2\text{-}1)$$

Where: S is the standard deviation of n historical values of the mass difference, which is estimated with $\nu = n-1$ degrees of freedom by:

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(m_{diff_i} - \overline{m}_{diff}\right)^2} \qquad (D.1.2\text{-}2)$$

The calibration process is judged to be in control if:

$t \leq$ critical value of Student's t-distribution with ν degrees of freedom.

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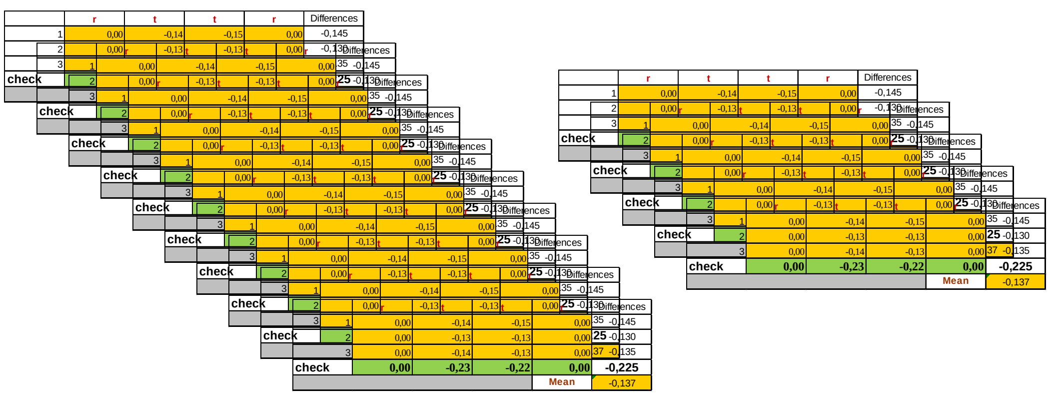
Check standard

Implementación:

Any calibration data

	r	t	t	r	Differences
1	0,00	-0,14	-0,15	0,00	-0,145
2	0,00	-0,13	-0,13	0,00	-0,130
3	0,00	-0,14	-0,13	0,00	-0,135
check	**0,00**	**-0,23**	**-0,22**	**0,00**	**-0,225**
				Mean	-0,137



-0,226
-0,223
-0,222
-0,226
-0,227
-0,225
-0,224
-0,227
-0,225
-0,222
-0,228
-0,222
-0,227
-0,226
-0,226
-0,225
0,002

n = 15
f.d.: 15-1

$\overline{m}_{diff}$

s

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Check standard

New calibration data once $\overline{m}_{diff}$ and had been determined

	r	t	t	r	Differences
1	0,00	0,04	0,05	0,00	0,045
2	0,00	0,05	0,04	0,00	0,045
3	0,00	0,04	0,04	0,00	0,040
check	**0,00**	**-0,24**	**-0,23**	**0,00**	**-0,235**
				Mean	0,043

$$\overline{m}_{diff} = 0,225$$

$$S = 0,002$$

$$t = \frac{\left| m_{diff} - \overline{m}_{diff} \right|}{S} = \frac{\left| -0,235 - (-0,225) \right|}{0,002} = 5$$

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Check standard

$$t = \frac{\left| m_{diff} - \overline{m}_{diff} \right|}{S} = \frac{\left| -0,235 - (-0,225) \right|}{0,002} = 5$$

Table D.1 Critical values of Student's t-distribution for a two-sided test with $\alpha = 0.05$

Note : ν = degrees of freedom

ν	Critical value	ν	Critical value	ν	Critical value	ν	Critical value	ν	Critical value
1	12.706	**11**	2.201	**21**	2.080	**31**	2.040	**41**	2.020
2	4.303	**12**	2.179	**22**	2.074	**32**	2.037	**42**	2.018
3	3.182	**13**	2.160	**23**	2.069	**33**	2.035	**43**	2.017
4	2.776	**14**	2.145	**24**	2.064	**34**	2.032	**44**	2.015
5	2.571	**15**	2.131	**25**	2.060	**35**	2.030	**45**	2.014
6	2.447	**16**	2.120	**26**	2.056	**36**	2.028	**46**	2.013
7	2.365	**17**	2.110	**27**	2.052	**37**	2.026	**47**	2.012
8	2.306	**18**	2.101	**28**	2.048	**38**	2.024	**48**	2.011
9	2.262	**19**	2.093	**29**	2.045	**39**	2.023	**49**	2.010
10	2.228	**20**	2.086	**30**	2.042	**40**	2.021	**50**	2.009

Conclusion: process is out of control

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Check standard

D.1.4 The accepted value of the check standard is updated as data on it are accumulated. Several approaches could be followed, however the data should always be plotted and examined for drift or change. The check standard value has changed from its "old" value, $\overline{m}_{\text{diff}}$ to a "new" value, $\overline{m}'_{\text{diff}}$, based on the most recent 10–15 measurements, if:

$$t = \frac{\left|\overline{m}_{\text{diff}} - \overline{m}'_{\text{diff}}\right|}{\sqrt{\frac{s_{\text{old}}^2}{J} - \frac{s_{\text{new}}^2}{K}}} > t_{\alpha/2}(v) \qquad (D.1.4\text{-}1)$$

Where J and K are the number of "old" and "new" measurements respectively, and $v = J + K - 2$.

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Check standard

OLD

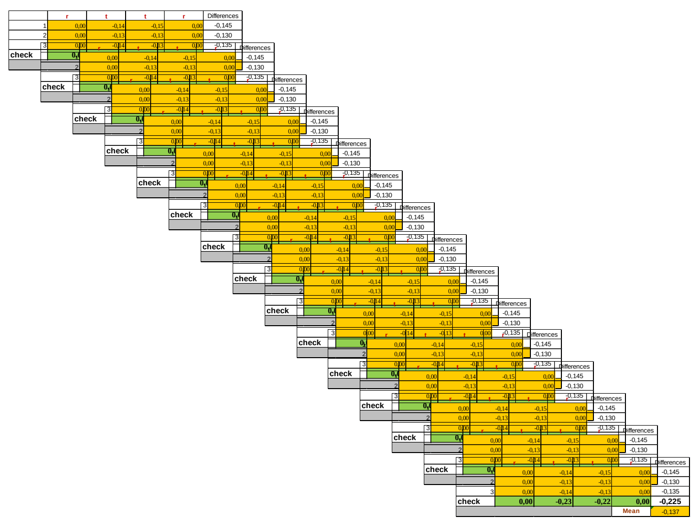
-0,226
-0,223
-0,222
-0,226
-0,227
-0,225
-0,224
-0,227
-0,225
-0,222
-0,228
-0,222
-0,227
-0,226
-0,226
-0,225
0,002

n = 15
f.d.: 15-1

$\overline{m}_{diff}$

s

NEW



-0,231
-0,225
-0,224
-0,225
-0,225
-0,225
-0,227
-0,224
-0,223
-0,226
-0,224
-0,227
-0,227
-0,229
-0,225
-0,226
0,002

n = 15
f.d.: 15-1

$\overline{m}_{diff}$

s

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Check standard

-0,231
-0,225
-0,224
-0,225
-0,225
-0,225
-0,227
-0,224
-0,223
-0,226
-0,224
-0,227
-0,227
-0,229
-0,225
-0,226
0,002

n = 15
f.d.: 15-1

$\overline{m}_{diff}$ ← -0,226

S ← 0,002

$$t = \frac{\left| m_{diff} - \overline{m}'_{diff} \right|}{\sqrt{\frac{s_{old}^{2}}{J} - \frac{s_{new}^{2}}{K}}} = \frac{\left| -0,225 - (-0,226) \right|}{\sqrt{\frac{(0,002)^2}{15} - \frac{(0,002)^2}{15}}}$$

Table D.1 Critical values of Student's t-distribution for a two-sided test with $\alpha = 0.05$
Note : V = degrees of freedom

V	Critical value	V	Critical value	V	Critical value	V	Critical value	V	Critical value
1	12.706	**11**	2.201	**21**	2.080	**31**	2.040	**41**	2.020
2	4.303	**12**	2.179	**22**	2.074	**32**	2.037	**42**	2.018
3	3.182	**13**	2.160	**23**	2.069	**33**	2.035	**43**	2.017
4	2.776	**14**	2.145	**24**	2.064	**34**	2.032	**44**	2.015
5	2.571	**15**	2.131	**25**	2.060	**35**	2.030	**45**	2.014
6	2.447	**16**	2.120	**26**	2.056	**36**	2.028	**46**	2.013
7	2.365	**17**	2.110	**27**	2.052	**37**	2.026	**47**	2.012
8	2.306	**18**	2.101	**28**	2.048	**38**	2.024	**48**	2.011
9	2.262	**19**	2.093	**29**	2.045	**39**	2.023	**49**	2.010
10	2.228	**20**	2.086	**30**	2.042	**40**	2.021	**50**	2.009

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Precision of the comparator

D.2 Precision of the balance

The precision of the balance can also be monitored using a statistical control technique. The residual standard deviation from a weighing design or a standard deviation of repeated measurements on a single weight is the basis for the test. Again, the test relies on a past history of standard deviations on the same balance. If there are m standard deviations, s_1, ..., s_m, from historical data, a pooled standard deviation:

$$s_P = \sqrt{\frac{1}{m}\sum s_i^2} \qquad (D.2\text{-}1)$$

is the best estimate of the balance standard deviation. The equation above assumes that the individual standard deviations have ν degrees of freedom, in which case the pooled standard deviation has $m \cdot \nu$ degrees of freedom. For each new design or series of measurements, the residual standard deviation, s_{new}, can be tested against the pooled value. The test statistic is:

$$F = \frac{s_{new}^2}{s_P^2} \qquad (D.2\text{-}2)$$

D.2.1 Normally, only the degradation in precision is tested. The precision of the balance is judged to be in control if:

$$F \leq \textit{critical value from the F-distribution}$$

with ν degrees of freedom for s_{new} and $m \cdot \nu$ degrees of freedom for s_P. Critical values of F for a one-sided test at the $\alpha = 0.05$ significance level are listed in Table D.2. If the standard deviation is judged to have degraded, then the cause must be investigated and rectified.

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Precision of the comparator

$$H_o : \sigma_{new} = \sigma_P$$

$$H_1 : \sigma_{new} > \sigma_P$$

$$\text{If } F = \frac{s_{new}^2}{s_P^2} > F_{\alpha,\nu_{new},\nu_P} \quad H_0 \text{ is rejected}$$

Table D.2 Critical values of F distribution for a one-sided test that s_{new} (ν degrees of freedom) does not exceed s_p ($m{\cdot}\nu$, ν) at a significance level of $\alpha = 0.05$

$F(\alpha, \nu, \nu{\cdot}m)$ $\alpha = 0.05$	ν									
m	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	161.448	19.000	9.277	6.388	5.050	4.284	3.787	3.438	3.179	2.978
2	18.513	6.944	4.757	3.838	3.326	2.996	2.764	2.591	2.456	2.348
3	10.128	5.143	3.863	3.259	2.901	2.661	2.488	2.355	2.250	2.165
4	7.709	4.459	3.490	3.007	2.711	2.508	2.359	2.244	2.153	2.077
5	6.608	4.103	3.287	2.866	2.603	2.421	2.285	2.180	2.096	2.026
6	5.987	3.885	3.160	2.776	2.534	2.364	2.237	2.138	2.059	1.993
7	5.591	3.739	3.072	2.714	2.485	2.324	2.203	2.109	2.032	1.969
8	5.318	3.634	3.009	2.668	2.449	2.295	2.178	2.087	2.013	1.951
9	5.117	3.555	2.960	2.634	2.422	2.272	2.159	2.070	1.998	1.938
10	4.965	3.493	2.922	2.606	2.400	2.254	2.143	2.056	1.986	1.927
11	4.844	3.443	2.892	2.584	2.383	2.239	2.131	2.045	1.976	1.918
12	4.747	3.403	2.866	2.565	2.368	2.227	2.121	2.036	1.968	1.910
13	4.667	3.369	2.845	2.550	2.356	2.217	2.112	2.029	1.961	1.904
14	4.600	3.340	2.827	2.537	2.346	2.209	2.104	2.022	1.955	1.899
15	4.543	3.316	2.812	2.525	2.337	2.201	2.098	2.016	1.950	1.894
16	4.494	3.295	2.798	2.515	2.329	2.195	2.092	2.011	1.945	1.890
17	4.451	3.276	2.786	2.507	2.322	2.189	2.087	2.007	1.942	1.887
18	4.414	3.259	2.776	2.499	2.316	2.184	2.083	2.003	1.938	1.884
19	4.381	3.245	2.766	2.492	2.310	2.179	2.079	2.000	1.935	1.881
20	4.351	3.232	2.758	2.486	2.305	2.175	2.076	1.997	1.932	1.878
30	4.171	3.150	2.706	2.447	2.274	2.149	2.053	1.977	1.915	1.862
40	4.085	3.111	2.680	2.428	2.259	2.136	2.042	1.967	1.906	1.854
50	4.034	3.087	2.665	2.417	2.250	2.129	2.036	1.962	1.901	1.850
60	4.001	3.072	2.655	2.409	2.244	2.124	2.031	1.958	1.897	1.846
70	3.978	3.061	2.648	2.404	2.240	2.120	2.028	1.955	1.895	1.844
80	3.960	3.053	2.642	2.400	2.237	2.117	2.026	1.953	1.893	1.843
90	3.947	3.046	2.638	2.397	2.234	2.115	2.024	1.951	1.891	1.841
100	3.936	3.041	2.635	2.394	2.232	2.114	2.023	1.950	1.890	1.840
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831

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Precision of the comparator, example 1

$s_P = 0{,}15\ \mu g \qquad \nu_P = 9$

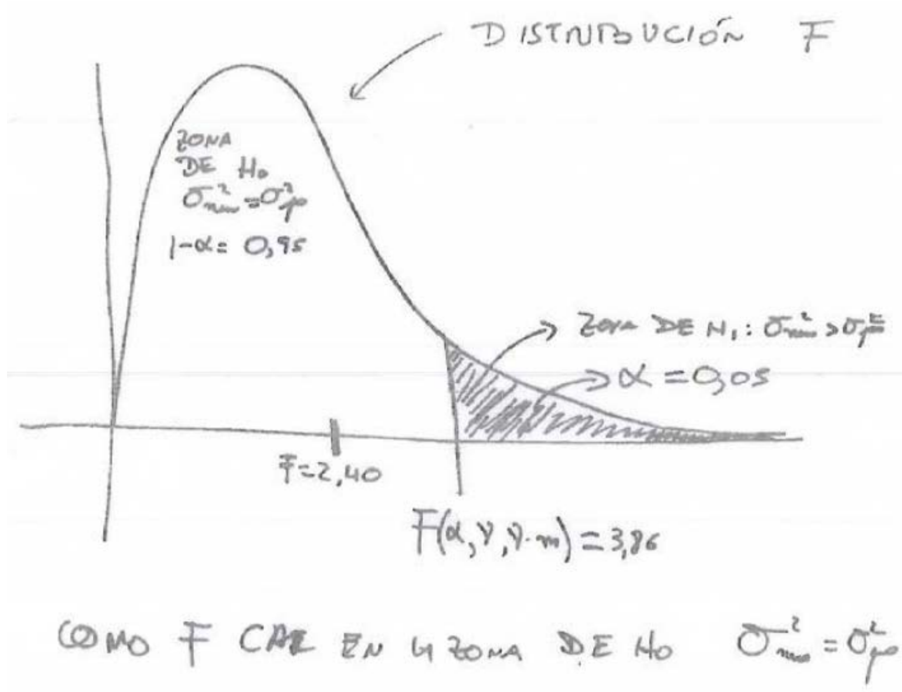
$s_{new} = 0{,}23\ \mu g \qquad \nu_{new} = 3$

$$F = \frac{(0{,}23)^2}{(0{,}15)^2} = 2{,}4$$

$$F_{\alpha,\nu_{new},\nu_P} = F_{0{,}05;3{,}9} = F_{0{,}05;3;3\cdot 3} = 3{,}86$$

⇓

H_0 *is accepted*



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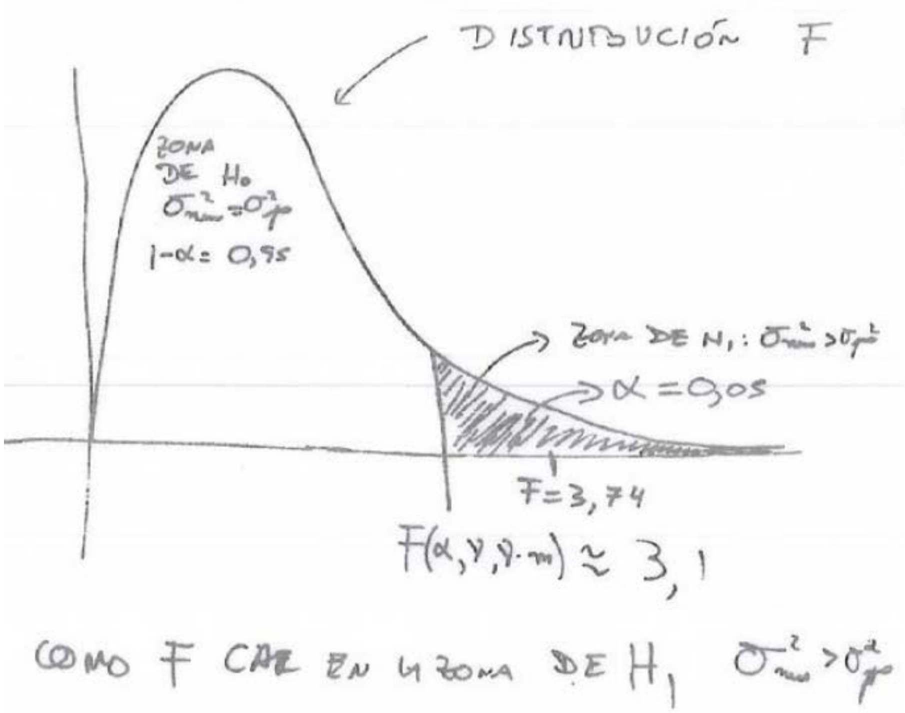
Precision of the comparator, example 2

$$s_P = 0{,}15\ \mu g \qquad \nu_{\mathrm{P}} = 14$$

$$s_{new} = 0{,}29\ \mu g \qquad \nu_{\mathrm{new}} = 4$$

$$\mathrm{F} = \frac{(0{,}29)^2}{(0{,}15)^2} = 3{,}74$$

$$\mathrm{F}_{\alpha,\nu_{new},\nu_P} = \mathrm{F}_{0{,}05;4;14} = \mathrm{F}_{0{,}05;4;\underbrace{3{,}5\cdot 4}_{\mathrm{m}\cdot\nu}} \approx 3{,}1$$



⇓

H_0 *is rejected*

RECOMMENDED READING

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Recommended reading

INTERNATIONAL RECOMMENDATION

OIML R 111-1

Edition 2004 (E)

Weights of classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} and M_3

Part 1: Metrological and technical requirements

Poids des classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} et M_3

Partie 1: Exigences métrologiques et techniques

OIML R 111-1 Edition 2004 (E)

ORGANISATION INTERNATIONALE DE MÉTROLOGIE LÉGALE

INTERNATIONAL ORGANIZATION OF LEGAL METROLOGY

NISTIR 6969

Selected Laboratory and Measurement Practices, and Procedures, to Support Basic Mass Calibrations (2012 Ed)

Georgia L. Harris

Office of Weights and Measures

Physical Measurement Laboratory

NIST National Institute of Standards and Technology U.S. Department of Commerce

NISTIR 5672

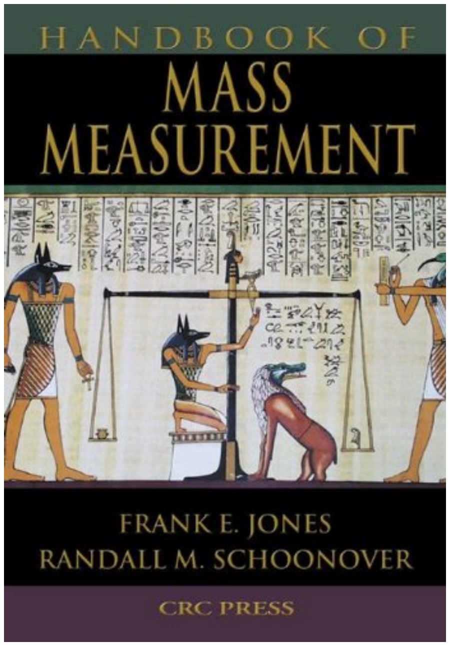
Advanced Mass Calibration and Measurement Assurance Program for State Calibration Laboratories (2012 Ed)

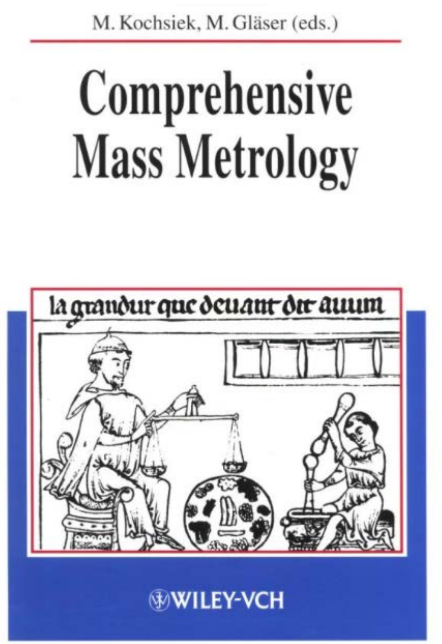
Fraley, Ken L.

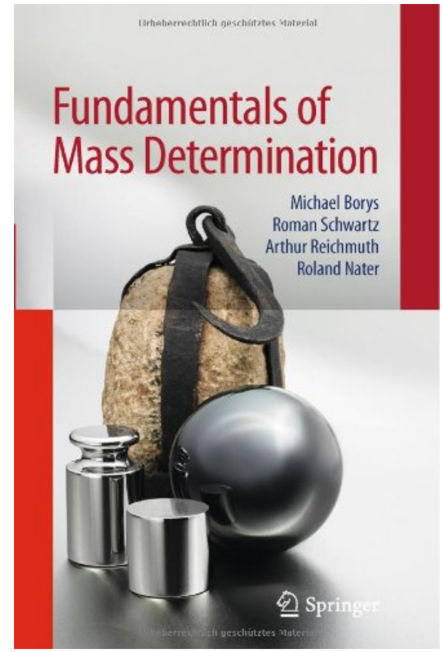
Harris, Georgia L.

NIST National Institute of Standards and Technology U.S. Department of Commerce

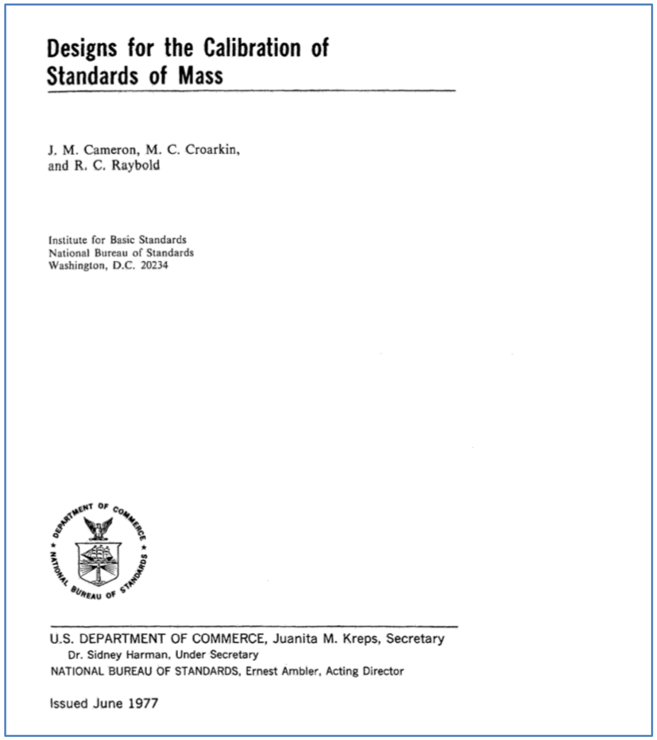
Recommended reading







Recommended reading



Designs for the Calibration of Standards of Mass

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