Dissipation Factors of 1 pF, 10 pF, and 100 pF Fused-Silica Capacitors

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Abstract — We describe dissipation factor measurements of 1 pF, 10 pF, and 100 pF fused-silica capacitance standards from 50 Hz to 20 kHz, using a toroidal cross capacitor and a 10 pF nitrogen-filled capacitor as the references. We also describe an analytical model that relates the dissipation factor of a capacitor to the frequency dependence of its capacitance.

Keywords — Capacitance, cross-capacitor, dielectric loss, dissipation factor, farad, frequency dependence, fused-silica capacitor.

I. INTRODUCTION

We have recently reported progress on determining the frequency dependence of capacitance standards in the audio frequency range [1]. This effort is in part a response to industrial needs. Recently, ultra-precision multi-frequency (from 50 Hz to 20 kHz) capacitance bridges have become commercially available and secondary calibration laboratories have started using this new type of bridge for their impedance calibrations. Another closely related calibration need is the determination of dissipation factors of capacitance standards, needed not only for calibrations of capacitance bridges, but also for LCR meters and network analyzers.

Dissipation factors of capacitors have previously been studied at NIST for various applications. Astin [2] studied loss mechanisms of air capacitors at 60 Hz, 200 Hz, and 1000 Hz, achieving uncertainties as low as 0.5×10^{-6}. Shields [3] established a dissipation factor standard using a 0.5 pF toroidal cross capacitor, $C_{0.5}$, with an estimated uncertainty of 0.02×10^{-6} at 1592 Hz. So and Shields [4] used a variable parallel-plate guard-ring capacitor as the reference for dissipation factor measurements, achieving uncertainties as low as 0.01×10^{-6} at 1592 Hz. Absolute determinations of the dissipation factor of capacitors have also been carried out in other National Metrology Institutes. Inglis [5] performed a thorough study of electrode surface film effects on the frequency dependence and dissipation factor of parallel-plate capacitors in the frequency range from 11 Hz to 52 kHz. He found that the dissipation factor of a well cleaned parallel-plate capacitor with a 1 mm vacuum gap is less than $1.5\text{x}10^{-7}$ over the frequency range. With a variable parallel-plate guard-ring capacitor as the reference, Eklund [6] has determined the dissipation factors of 100 pF capacitors at 1 kHz, 4 kHz, and 10 kHz. Using a combination of equivalent circuit modeling for capacitor and ac resistor, a programmable two-channel ac voltage source and a sampling voltmeter, Ramm and Moser [7] have developed a multi-frequency method for determining the dissipation factor of capacitors and the time constant of resistors simultaneously below a few kHz with uncertainties of $6\text{x}10^{-7}$.

II. KRONIG-KRAMERS RELATIONS

For a simple parallel-plate capacitor, the dominant loss mechanisms include the dielectric loss between the electrodes and the resistance of the electrodes and their leads. Conductivity of a dielectric can be written as two components: $\sigma_o + \omega\varepsilon''(\omega)$, where the first term results from the dc conductivity and the second term is due to dielectric relaxation with ε'' being the imaginary part of the dielectric constant. When the dc conductivity is negligible as is the case for fused-silica capacitors, the dissipation factor

$$\tan\delta = \frac{G}{\omega C} = \frac{\varepsilon''}{\varepsilon'}, \tag{1}$$

where C is the capacitance and G is the conductance of the dielectric and ε' is the real part of the dielectric constant. ε'' and ε' are related via the Kronig-Kramers relations:

$$\varepsilon'(\omega) - \varepsilon_\infty = \frac{2}{\pi}\int_0^\infty \frac{\varepsilon''(u)u^2}{\omega^2 - u^2} d\ln u\,. \tag{2}$$

Approximating the integral factor $u^2/(\omega^2\text{-}u^2)$ by the unit-step function [8], we obtain

$$\varepsilon'(\omega) - \varepsilon_\infty = \frac{2}{\pi}\int_\omega^\infty \varepsilon''(u) d\ln u\,. \tag{3}$$

This derivation effectively assumes that the distribution of the dielectric relaxation times is very broad, leading to relatively flat curves of $\varepsilon''(\omega)$ and $\varepsilon'(\omega)$. Differentiating Eq. (3), we have

$$\tan\delta = -\frac{\pi}{2}\frac{d\varepsilon'(\omega)}{\varepsilon' d\ln\omega}. \qquad (4)$$

Eq. (4) is useful for estimating the dissipation factor of a capacitance standard from its frequency dependence of capacitance.

III. EXPERIMENT

The ultimate reference for dissipation factor measurements at NIST is the toroidal cross capacitor $C_{0.5}$ which is made of stainless steel and sealed in a vacuum housing [3]. It has been shown that the net contributions of thin dielectric films on the electrodes of such a cross capacitor to the dissipation factors of the two cross capacitances are negligible to the first order. The toroidal arrangement also contains another cylindrical 10 pF capacitance, C_{10}, between two of the four active electrodes. The electrode separation of C_{10} is about 3 mm. Since all electrodes were made from the same material and were finished and cleaned in the same manner, Shields was able to determine that C_{10} has a dissipation factor less than 0.02×10^{-6} at 1592 Hz with comparable uncertainty. The dissipation factor due to the dielectric films is inversely proportional to the electrode separation and is less than 0.15×10^{-6} in the audio frequency range when the electrodes are well cleaned and their separation is 1 mm or more [3, 5]. Comparing C_{10} with an identically made capacitor shows that the difference of their dissipation factors is within the detection limit in the audio frequency range. Comparing with another 10 pF nitrogen-filled cylindrical capacitor whose frequency dependence of capacitance had been determined earlier with respect to a 1 pF cross capacitor shows that the frequency dependence of C_{10} is no more than 0.2×10^{-6} per decade change in frequency. Using this estimate of frequency dependence in Eq. (4), we conclude that the dissipation factor of C_{10} is less than 0.14×10^{-6} in the audio frequency range. Simple substitution techniques are employed to measure dissipation factors of 10 pF fused-silica capacitors with respect to C_{10}, using ac bridges which have been described previously [1]. These ac bridges also allow us to measure dissipation factors of 1 pF and 100 pF capacitors with respect to the 10 pF standards.

IV. RESULTS AND UNCERTAINTY ANALYSIS

Shown in Fig. 1 is the measured dissipation factor of a 10 pF fused-silica transfer standard, C_{112}, as a function of frequency from 50 Hz to 20 kHz. The main sources of uncertainties for the measurements are listed on Table 1 for four representative frequencies. The Type A uncertainty, which is directly linked to the signal-to-noise ratio of the ac bridge systems and the stabilities of the standards, dominates at low frequencies. The reference standard C_{10} is a four-terminal-pair capacitor, and its loss due to the leads and contacts is negligible in the frequency range. However, C_{112} is a three terminal capacitor and its lead resistance is the dominant loss mechanism at high frequencies. In the frequency range from 300 Hz to 6 kHz, the uncertainties of the reference standard dominates. The relative combined standard uncertainties are shown in Fig. 1 together with the dissipation factor data.

Also shown in Fig. 1 is the estimated dissipation factor below 1592 Hz using Eq. (4) and the frequency dependence of capacitance of C_{112} measured earlier [1]. The comparison is restricted to the low frequency region where the dominant source of frequency dependence results from dielectric relaxation. The leads effect becomes significant above 1592 Hz, and we have not attempted to separate the contributions from the two sources.

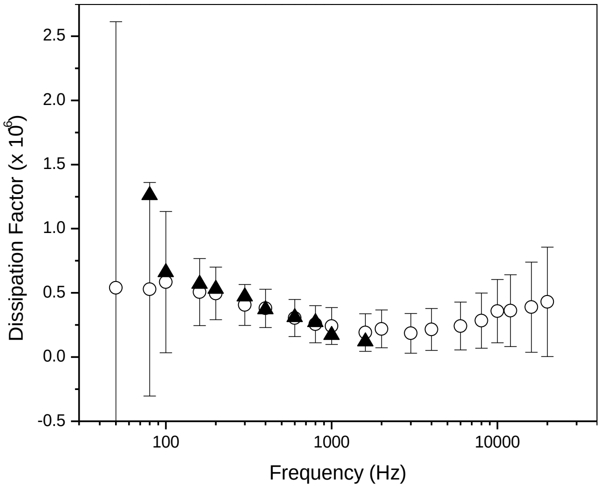

Figure 1. Measured dissipation factor of C_{112} as a function of frequency (open circles), with 1σ uncertainty bars, and calculated dissipation factor (solid triangles).

Source of uncertainty	Relative standard uncertainty ($\times 10^{-6}$)			
	100 Hz	400 Hz	1 kHz	10 kHz
Type A	0.53	0.05	0.03	0.03
Reference capacitor C_{10}	0.14	0.14	0.14	0.14
Contact resistance of C_{112}	0.01	0.01	0.02	0.2
Bridge linearity errors	0.05	0.05	0.05	0.05
Relative combined standard uncertainty	0.56	0.16	0.16	0.26

Table 1. Contribution of component uncertainties to the total uncertainty at four representative frequencies for C_{112}.

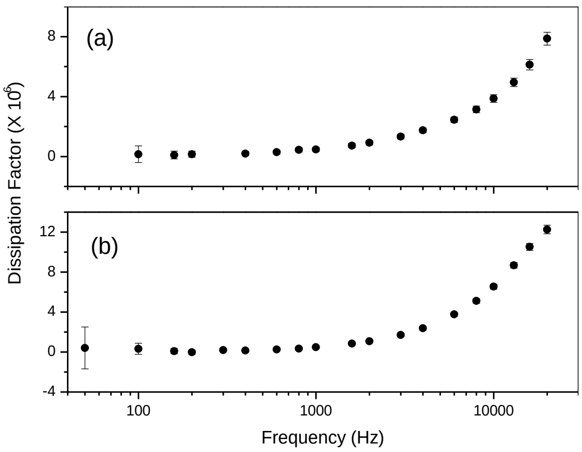

Figure 2. Measured dissipation factors of fused-silica capacitors with 1σ uncertainty bars: (a), 100 pF; (b), 1 pF.

Shown in Fig. 2 are the measured dissipation factors of commercial 1 pF and 100 pF fused-silica capacitance standards as a function of frequency from 50 Hz to 20 kHz. The dissipation factors of these standards are within a few parts in 10^7 below 1 kHz; they increase with frequency to a few parts in 10^6 at higher frequencies. This increase is largely due to the series lead and electrode resistance.

V. CONCLUSION

References have been established for measuring dissipation factor of 1 pF, 10 pF, 100 pF capacitance standards from 50 Hz to 20 kHz. Measurements of both NIST-fabricated fused-silica capacitors as well as similar commercial standards have shown that the dissipation factors of these standards are typically within a few parts in 10^7 below 1 kHz; their dissipation factors may increase with frequency to a few parts at higher frequencies in the audio range, due to their lead and electrode resistances.

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