ISA TRANSACTIONS: 9 (201–209) 1970

Two Stage, Guarded Inductive Voltage Divider for Use at 100 kHz*

DONALD N. HOMAN†
and THOMAS L. ZAPF‡
Radio Standards Engineering Division
National Bureau of Standards
Boulder, Colorado

▶ A single decade inductive voltage divider with output tap corrections less than 5×10^{-7} of input voltage at a frequency of 100 kHz is described and evaluated both experimentally and theoretically. The divider is a two-stage device which is also guarded section by section. These two techniques greatly reduce errors caused by unequal leakage inductance and interwinding capacitances, which otherwise cause significant errors at 100 kHz. Design and construction details are presented for the dividers and for the bridge circuit used to make the measurements of the output tap corrections to the divider.

INTRODUCTION

INDUCTIVE voltage dividers are widely used for precise electrical measurements because they are rugged stable, and highly accurate devices. Their high accuracy, however, has been limited to audio frequencies. This paper describes work directed toward extending the high accuracy of inductive dividers into the radio frequency region. In particular, there is a growing need for highly accurate inductive dividers at 30, 50, and 100 kHz. Most of the work discussed here was done at 100 kHz, since the problems at 30 and 50 kHz are similar but of smaller magnitude.

Errors in voltage ratio of inductive dividers arise from differences in the currents in the leakage inductances of the sections and variations among the leakage inductances. Although the excitation current required to energize the magnetic core is relatively uniform in each section, the leakage inductances are different, and give rise to errors proportional to the excitation current. Also, the current in the interwinding capacitances varies from section to section. This paper describes a method for reducing the errors which involves a reduction in the excitation current, as well as a reduction in the interwinding capacitance. The excitation current is reduced by the use of a two-stage[1,2] technique and the interwinding capacitance is reduced by guarding. The two techniques combine to yield errors at the taps of unloaded inductive voltage dividers at a frequency of 100 kHz of less than 0.5 ppm of input.

The bridge circuit used in this work is basically the same as that described by Sze[3] for use at audio frequencies; however, the circuit has been modified considerably for use at higher frequencies. Some of the details of the bridge are given in this paper.

TWO-STAGE TECHNIQUE

The two-stage transformer has been examined by Deacon and Hill[4] for its application to inductive voltage dividers. In this paper the two-stage inductive voltage divider will be very briefly reviewed, and equations will be developed from the inductive voltage divider circuit rather than from the transformer circuit.

The two-stage divider is constructed using two cores and two windings. One of the cores is wound with a single layer of insulated wire. The second core is placed adjacent to the first (as two rings on a finger). The second

*Presented at the 1969 ISA Annual Metrology Symposium; revised April, 1970.
†Physicist.
‡Physicist, ISA Senior Member.

winding is then wound onto the two cores with the same number of turns as in the first winding. Thus the second winding envelops both cores, whereas the first winding envelops only one core. The ends of the windings are connected to the voltage source. The result of this winding method is that nearly all of the core excitation current is in the winding that envelops the single core. Thus the winding that envelops both cores has very little current, and is tapped for use as the voltage divider winding. Since it has very little current, errors caused by current in leakage inductances (which, in general, are different for different sections of the divider) are correspondingly small.

Figure 1 represents the circuit of a two-stage inductive voltage divider. The impedances in the first and second windings under no-load conditions are Z and Z' in two fictitious loops. To show the reduction in the error from that in a single stage divider, the following seven equations are written from Figure 1, and solved for the ratio e_K/E (where e_K is the voltage at the Kth output tap). Interwinding capacitance is omitted and its omission will be justified in the next section.

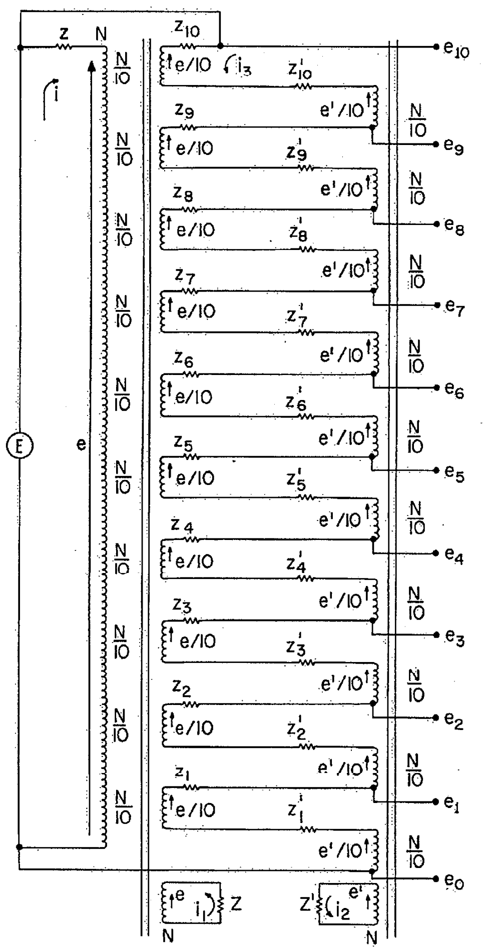


Figure 1. An equivalent circuit for a two-stage inductive voltage divider. The guard circuit that effectively eliminates the interwinding capacitances is not shown.

For the loop containing i, Kirchoff's law gives

$$E - e = iZ$$

and for the loop containing i_3

$$E - 10(e/10 + e'/10) = i_3 \sum_{n=1}^{10} (z_n + z'_n)$$

Also, for the Kth output voltage, e_K, a similar equation can be written for part of the i_3 loop

$$e_K - K\left(\frac{e + e'}{10}\right) = i_3 \sum_{n=1}^{K} (z_n + z'_n)$$

For the fictitious loops

$$e = -i_1 z$$

and

$$e' = i_2 Z'$$

From flux linkages in the two cores we have

$$Ni + Ni_1 + Ni_3 = 0$$

and

$$Ni_2 = Ni_3$$

Solving the equations for the ratio of output voltage to input voltage yields

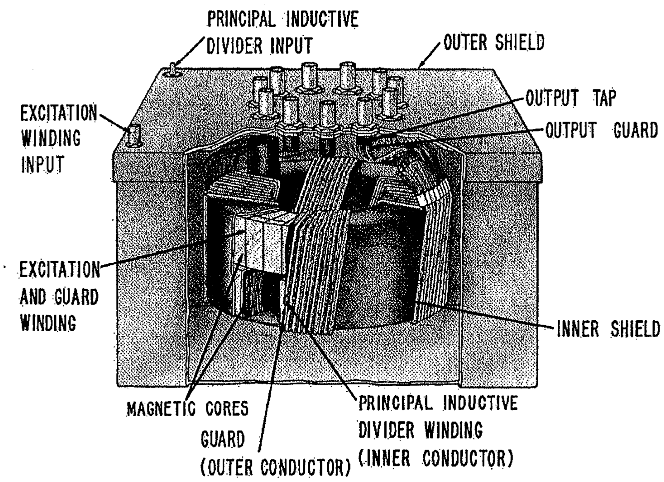
$$\frac{e_K}{E} = \frac{K}{10}\left[\frac{1 + \frac{z}{Z'} + \frac{z}{Z} + \frac{\sum_{n=1}^{10} (z_n + z'_n)}{Z'} + \frac{10z \sum_{n=1}^{K} (z_n + z'_n)}{KZZ'}}{1 + \frac{z}{Z'} + \frac{z}{Z} + \frac{\sum_{n=1}^{10} (z_n + z'_n)}{Z'} + \frac{z \sum_{n=1}^{10} (z_n + z'_n)}{ZZ'}}\right]$$

If second order terms are omitted, the last equation becomes:

$$\begin{aligned}\frac{e_K}{E} &= \frac{K}{10}\left[1 - \frac{z \sum_{n=1}^{10} (z_n + z'_n)}{ZZ'} + \frac{10z \sum_{n=1}^{K} (z_n + z'_n)}{KZZ'}\right] \\ &= \frac{K}{10} + \frac{z}{ZZ'}\left[\sum_{n=1}^{K} (z_n + z'_n) - \frac{K}{10} \sum_{n=1}^{10} (z_n + z'_n)\right] \\ &= \frac{K}{10} + \frac{z}{ZZ'}\left[\sum_{n=1}^{K} z_n + \sum_{n=1}^{K} z'_n - Kz_{\text{avg.}} - Kz'_{\text{avg.}}\right] \\ &= \frac{K}{10} + \frac{z}{ZZ'}\left[\sum_{n=1}^{K} z_n - Kz_{\text{avg.}} + \sum_{n=1}^{K} z'_n - Kz'_{\text{avg.}}\right]\end{aligned}$$

Which is of the form

$$\frac{e_K}{E} = \frac{K}{10} + \frac{z(\Delta z_a + \Delta z_b)}{ZZ'}$$



For a single core divider, the equation is of the form

$$\frac{e_K}{E} = \frac{K}{10} + \frac{\Delta z}{Z}$$

So the error term in the two-stage divider is reduced by a factor of the order of z/Z.

GUARDING

Stray admittances between sections of an inductive voltage divider can cause errors because: (1) the currents in these admittances pass through the leakage inductances of the sections, and (2) the admittances are, in general, not equal.

The troublesome stray admittance current can be nearly eliminated by guarding the windings. The method of guarding used here employs coaxial cable for the core windings. The outer conductor of the coaxial cable is the guard for the center conductor. The center conductor is the principal inductive divider winding. The center conductor is tapped at nine points, and the outer conductor (the guard) is cut at each tap so that the guard is separated into ten parts. Each of the ten parts of the guard is driven at guard voltage by a guard voltage source. The guard voltages are connected to the centers of each guard section in an attempt to equalize the admittances between each half of each guard and nearby conductors. If, in a given guard section, the admittances were not balanced, the resulting unbalanced current would induce an unwanted voltage in the center conductor (the principal inductive divider winding).

The guard source is obtained by tapping the first stage winding at the appropriate points, which are equivalent to $K = 0.05, 0.15, 0.25, \ldots 0.95$.

DESIGN AND CONSTRUCTION

Figure 2 depicts the construction of the divider, and Figure 3 shows a circuit diagram. The cores are $2\frac{1}{2}$ in. I.D., 3 in. O.D., and $\frac{1}{2}$ in. high and are made of high permeability, one mil tape. The winding around the single core (the energizing winding) consists of 40 turns of four #20, Formvar-coated copper wires connected in parallel and wound on the core to form a flat winding. The winding is tapped at 19 points, and the taps, together with the two ends, are gathered and bundled together. A similar core is then placed next to the energizing core and both cores are enclosed by a $\frac{1}{8}$ in.-thick brass shield. The bundle of tap wires from the energizing winding is fed through a hole in the shield.

The inductive divider winding is the center conductor of RG 174/U coaxial cable; the outer conductor is the guard.

The winding method is as follows: ten lengths of RG 174/U coaxial cable are wound four times around the brass shield containing the cores, and the ends are connected in series to create a 40-turn winding. The winding is tapped at nine points. The outer shield of

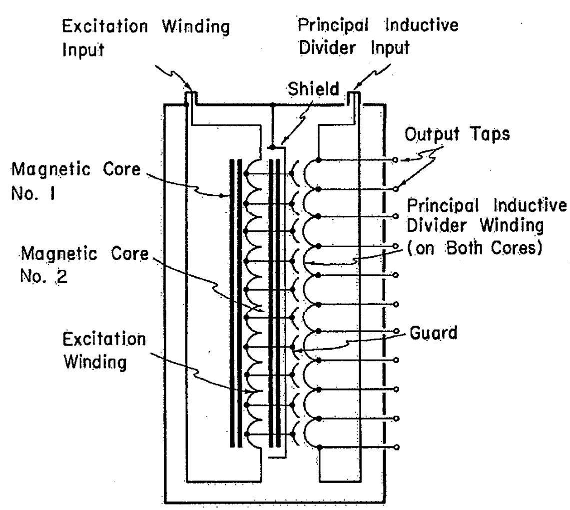


Figure 3. Schematic diagram showing the arrangement of cores, windings, and guarding.

the cable is cut at each tap, and separate guard tap wires from the first stage are connected to the centers of each section of coaxial cable. The guard voltages are 0.05 V, 0.15 V, 0.25 V... 0.95 V (assuming 1 V input). In addition to the guarding for the inductive divider winding itself, each tap of the inductive divider winding is guarded. The guard voltages for the tap guards are supplied by taps from the first stage winding also, and assuming a 1 V input, the guard voltages are 0.1 V, 0.2 V,... 1.0 V. The brass shield is set at the mid-tap potential (taken again from the first stage winding). The divider is enclosed in an aluminum case which is connected to ground potential. The divider has two coaxial input terminals—one for the first stage winding and the other for the second stage winding.

The outer conductor of the coaxial connector for the first stage winding is connected to the aluminum case, and provides the ground for the case. The connector for the second stage winding is insulated from the case. The design of the divider described here is typical of those produced for experimental purposes, but is not necessarily optimum for 100 kHz operation.

MEASUREMENT CIRCUIT DESIGN

Figure 4 illustrates the basic circuit used. Since this bridge has been described in the literature,[3] most of the discussion in this section will deal with the problems encountered in adapting the bridge to 100 kHz measurements.

Figure 5 is a schematic diagram of the bridge, which has been simplified by omission of shielding, the lower decades of the inductive voltage dividers in the bridge, and other details. Box *B* shows the circuit that provides the adjustable in-phase and quadrature voltages required to balance the bridge. These small voltages are derived from *E*, the input voltage to the dividers under test, *U* and *Y*.

Resistors R_1 and R_2 form a voltage divider for the in-phase component, and capacitor *C* and resistor R_2 form a voltage divider to create the quadrature component. The bridge is balanced by turning dials on the decade inductive dividers *I* and *Q*, thereby adjusting currents in R_1 and *C*, respectively, to the proper values. The values of R_1 and *C* are approximately 10 kilohms and 159.2 pF (10 kilohms at 100 kHz). Resistor $R_2 \cong$ 10 ohms so that the voltage step down is 10 kilohms/10

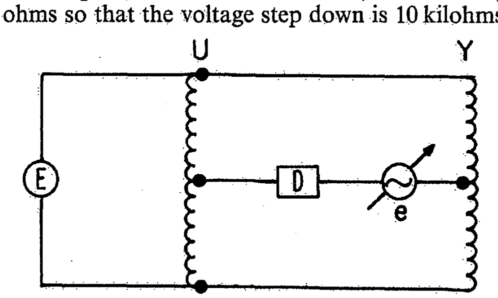


Figure 4. Simplified circuit diagram of a bridge using two dividers, *U* and *Y*.

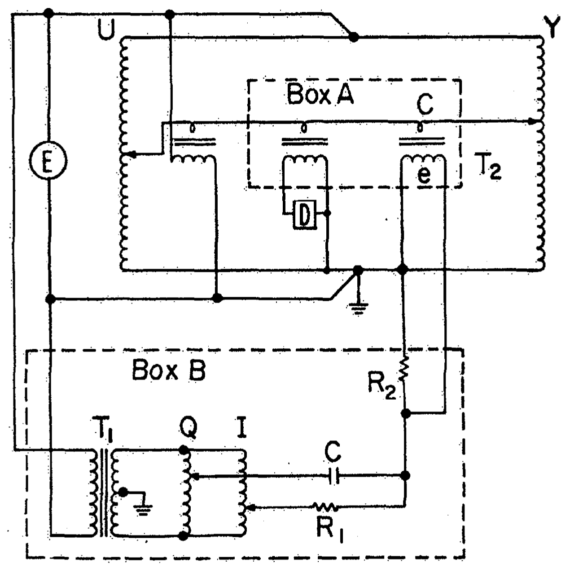


Figure 5. Circuit diagram showing the elements of the voltage-injector and detector devices in Box A, and the in-phase and quadrature control circuits in Box B.

ohms = 1000. Box A contains two 100:1 transformers: one inserts the voltage, *e*, between the outputs of the dividers under test; the other provides the detector input. The total step down from fixed components in Boxes A and B is then 100,000. The inductive dividers, *Q* and *I*, are cascaded decade dividers, each with four dials, so that one step of the last dial of each is 1/1000 of input. Thus the least count is 1 in 10^9, and *e* can be changed from 10^{-5} to 10^{-9} of *E*. If greater range is needed, R_2 may be replaced by 100 ohms, making it possible to change *e*/*E* from 10^{-4} to 10^{-8}.

In Figure 5, transformer T_1 has 100 turns of #22 wire on the primary and 100 turns on the secondary. A brass shield encloses the primary winding. The inductive dividers *Q* and *I* are connected across the transformer secondary winding through reversing switches not shown in the figure. The reversing switches are marked appropriately to enable the operator to keep track of signs. The inductive dividers are cascaded decade dividers, each with four dials. The capacitor, *C*, is an air capacitor with a value of 159.155 pF so that $X_c = 10^4$ ohms at 100 kHz. R_1 is a 10 kilohm wirewound resistor designed for high frequency use. Resistor R_2 is either 10.01001 ohms or 101.0101 ohms, depending on the resolution desired. These resistors are carbon.

Box A in Figure 5 is the injector-detector device. Several designs of this part of the circuit were tried before errors in the bridge were deemed sufficiently small. Triaxial cable is used between the output taps of the two inductive dividers. The center conductor is a single turn winding of the injector, detector, and auxiliary transformers. (The auxiliary transformer is used in a "bootstrap calibration" discussed later in this paper.) The conductor around the center conductor is the guard,

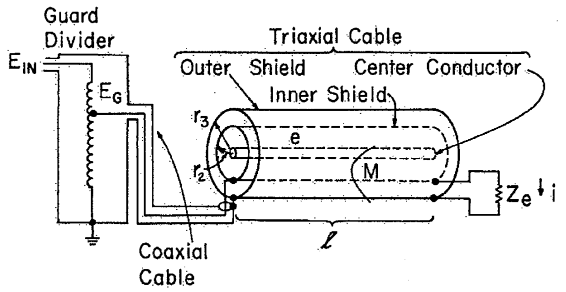


Figure 6. Schematic diagram showing mutual inductance in a triaxial cable consisting of center conductor, guard shield, and grounded outer shield.

and the outer conductor is grounded. Figure 6 shows a portion of the triaxial cable and a guard divider (which is not shown in Figure 5).

ERRORS

Consider first the equal go-and-return current in the guard and ground shields through admittance between the shields (Figure 6). The inner shield of the triaxial cable is driven at guard potential through a coaxial cable, and the outer shield is grounded through the outer conductor of the coaxial cable. The guard divider supplies the current in the dielectric between the shields. The inner and outer shields form a go-and-return path for the current in the dielectric. The current paths in the two shields are nearly coaxial. With coaxial go-and-return current paths there is almost no coupling from the double-shield circuit to the detector or to other parts of the measurement circuit external to the outside shield. However, coupling from the shield circuit to the center conductor circuit can cause a significant error, and must be compensated, balanced, or otherwise corrected. The mutual inductance of a thin straight tube of length, l, and radius, r_2, (when r_2/l is very small) to a conductor within it[5] is

$$M_2 = 2l\left(\ln\frac{2l}{r_2} - 1\right)$$

For concentric cylinders of equal lengths and of radii r_3 and r_2 (where $r_3 > r_2$) which form a go-and-return circuit, the mutual inductance to a third, central conductor circuit is

$$M = M_2 - M_3 = 2l\left(\ln\frac{2l}{r_2} - 1\right) - 2l\left(\ln\frac{2l}{r_3} - 1\right)$$

$$= 2l\left(\ln\frac{r_3}{r_2}\right) \times 10^{-9}\ \text{Henry}$$

where l is in cm.

The error in voltage ratio caused by this mutual inductance can be expressed as

$$\delta = \frac{e}{E_{IN}}$$

Where $e = j\omega Mi$ is the voltage induced in the center conductor circuit by the current i in the shield circuit.

The current $i = E_G/Z_e$, where Z_e is the effective lumped impedance at the end of the triaxial cable representing the distributed shunt impedance between the two shields. This impedance is predominantly capacitive; hence, $Z_e = 1/j\omega C_e$, where C_e is the effective lumped capacitance at the end of the cable. The voltages E_G and E_{IN} are the output and input voltages at the terminals of the guard divider having a ratio

$$A \equiv \frac{E_G}{E_{IN}}$$

Now, the formula for δ above can be expressed as

$$\delta = -\omega^2 AMC_e$$

By substituting reasonable values of M and C_e into this formula, the value of δ for any voltage ratio, A, can be estimated. At $f = 100$ kHz, $l = 10$ cm, $r_3 = 3$ mm and $r_2 = 2$ mm, then $M = 8$ nanohenries. If 90 picofarads is linearly distributed in a ten cm length, the effective lumped capacitance, C_e, at the end of the triaxial circuit (Figure 6) is 45 picofarads, and

$$\delta = -\omega^2 MC_e A = -140 \times 10^{-9}\ \text{A}$$

This would be a significant error.

Both M and C are functions of length, l. It is impractical to make the total length of the triaxial cable short enough to keep this error less than a part in 10^7. However, by connecting the coaxial cable to the guard and ground shields of the triaxial cable at the center of a section of the triaxial cable, the net emf induced in the center conductor can be made very small, since the emfs induced in the two parts are approximately equal and opposite.

To prevent currents in the guard shield and ground shield from passing through any transformer cores, a gap is made in the shields at each transformer, and at the connection to the output taps of the inductive voltage dividers. Separate coaxial connections to a guard driver are made. The final design of this part of the bridge is shown in Figure 7.

Another error is created at the connections of the triaxial cable to the output taps of the inductive voltage

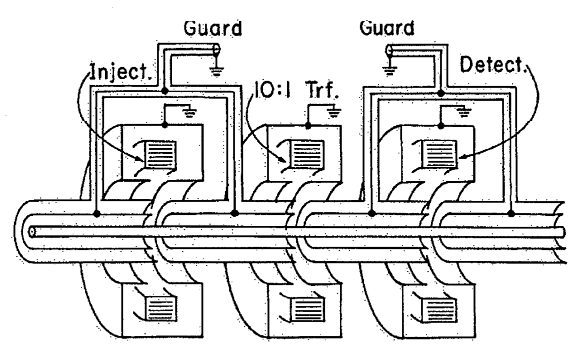


Figure 7. Cross section of comparison circuit.

dividers. There is a gap in the shields at this point, for the reason given above. The error arises from the capacitance from the guard shield in the triaxial cable across the gap to the grounded part of the connector at the output of the inductive voltage divider. The capacitance current induces an error voltage in the center conductor

$$\delta' = \omega^2 A M_2 C_G$$

Where M_2 is the mutual inductance between the guard shield and the center conductor, and C_G is the capacitance from the ground on the connector to the guard shield of the triaxial cable. For a length of 10 cm and $r_2 = 0.2$ cm,

$$M_2 = 2l\left(\ln\frac{2l}{r_2} - 1\right) = 20\left(\ln\frac{20}{0.2} - 1\right)$$
$$= 72 \times 10^{-9} \text{ Henry}$$

If C_G is 1 picofarad, the above formula yields, at 100 kHz,

$$\delta' = 29 \times 10^{-9} \text{ A}$$

A slightly smaller error, of opposite sign, occurs if the other cross capacitance (to the outer shield) is considered. Thus, the net error should be much less than that calculated above.

The best arrangement of the comparison circuit components is believed to be as shown in Figure 7. Even so, there are errors that appear as fixed errors in the 10:1 transformer ratio. When the 10:1 transformer is energized, a voltage of $0.1E_{IN}$ appears across the gap in the transformer case and across the gap in the outer shield. These voltages can cause currents in the capacitances shown in Figure 8. These currents should not be permitted to pass through the injector core or the detector core. If they did, they would create significant emfs in the shields and in the central conductor. The inductance of a single turn around one core of area, A_c, mean length, l, and permeability, μ, is

$$L = \frac{4\pi A_c \mu}{l} 10^{-9} \text{ henries}$$

For a core having $A_c = 0.1$ cm^2, $l = 9$ cm, and $\mu = 5000$, $L = 1$ microhenry. The emf generated in the center conductor by current in an equivalent capacitance, C, is approximately $e = 0.1E_{IN}\omega^2 LC$. The error in voltage

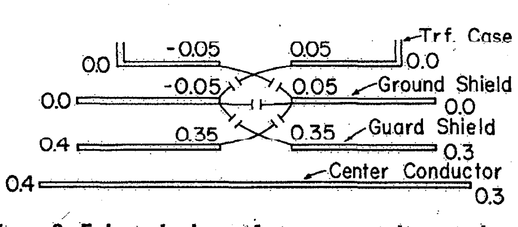


Figure 8. Enlarged view of cross capacitances in the middle gap of Figure 7 showing voltages (normalized, with unity input) when 10:1 transformer is energized, and with one divider set at 0.3 and the other at 0.4.

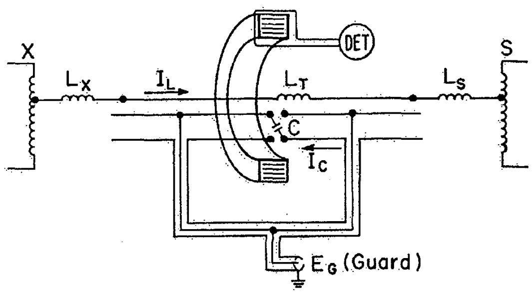


Figure 9. Partial schematic diagram showing a source of error from the current I_L in L_T. When the detector indicates null, $I_L = I_C$. Capacitance C is the net cross capacitance at the gap in the shields. The remedy for reducing the error is to make $C = 0$ by adjustment of the cross capacitances, with the center conductor open-circuited.

ratio is $\delta'' = e/E_{IN}$ or, approximately

$$\delta'' = 0.1\omega^2 LC$$

At a frequency of 100 kHz, if C is 1 picofarad, $\delta'' = 40 \times 10^{-9}$. When the transformer is not energized the error is nonexistent. Such an error does not appear in the results of calculations of inductive voltage divider ratios by the step-up method, but it does make the calculated 10:1 transformer ratio appear poorer than it would be otherwise.

Errors from voltages induced in the center conductor of the triaxial cable from currents in the shields are discussed above. Now we shall consider errors from current in the equivalent output impedances of the dividers and in the self inductance of the center conductor which is augmented by the high permeability cores of the injector, detector, and transformer.

When comparison measurements are being made, the guard voltage, E_G, at a tap is impressed across a net unbalanced cross-capacitance, C, (Figure 9) resulting in a current, in the shields through the core (or cores) of magnitude

$$I_C = E_G \omega C$$

When the detector indicates null, an equal current, I_L, in the inductance, L_T, causes, in the center conductor, a voltage drop

$$e = I_L \omega L_T = E_G \omega^2 L_T C$$

Dividing by the input voltage, E_{IN},

$$\frac{e}{E_{IN}} = \frac{E_G}{E_{IN}} \omega^2 L_T C$$

The ratio error, $\delta''' = e/E_{IN}$, is then

$$\delta''' = A\omega^2 L_T C$$

At a frequency of 100 kHz, if $L_T = 1$ microhenry, and $C = 1$ picofarad, $a = 0.4 \times 10^{-6}\, A$. This is significant. An error of approximately this magnitude can arise at each of the cores in the comparison circuit, and errors of smaller magnitude, but similar in nature, can occur at L_X and L_S, both of which are usually smaller than L_T.

It is apparent that any current in the center conductor is undesirable. Thus, it is necessary to ensure that there be no current in the center conductor when the detector indicates null.

If provision is made for adjusting one or more of the cross-capacitances shown in Figure 8, it is possible to use such an adjustment to obtain cancellation of errors. The adjustment for cancellation must be made with the center conductor disconnected from the dividers, and with the ends open circuited and capped at guard potential. The cross-capacitances in the gap can then be adjusted to obtain a null on the detector, with assurance that there is no current in the center conductor. When the ends of the center conductor are then reconnected to the output terminals of the dividers, a null will indicate zero current in the center conductor.

JUNCTIONS

In all, eight coaxial connections to a voltage source are required: the first and second stages of both dividers under test (four connections), the first and second stages of the 10:1 transformer (two connections), the input to Box B (one connection), and the input to the guard divider (one connection). Thus, including the input from the voltage source, a junction block with nine coaxial connections is required.

It was difficult to make a junction box that would give identical results if connections to the junction were rearranged. A large part of the problem was caused by mutual inductance between the wires near the junction point. Finally, miniature connectors were mounted on printed circuit boards to form a strip-line junction. (Figure 10.) These junctions work quite well; the errors due to the junction are reduced to negligibly small values.

CALIBRATION PROCEDURE

Let the dividers to be calibrated be called U and Y, and let the calibration begin with a setting of zero on both dividers. After the bridge is balanced, readings α and β are recorded, corresponding to the in-phase and quadrature components of the small adjustable voltage. Next, the Y divider is set to 0.1, and the 10:1 standard transformer is inserted.

New readings for α and β are recorded after the bridge is balanced. Next, the standard is removed, and the U divider is set to 0.1, the bridge is balanced giving new values of α and β. This process is continued up to and including comparing the tenth tap of Y with the tenth tap of U with the standard out of the circuit. In all, 21 sets of readings (α and β) are recorded for ten section dividers.

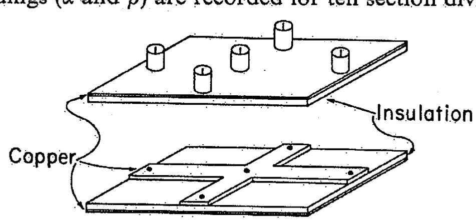


Figure 10. Diagram indicating construction of the slab-line junction.

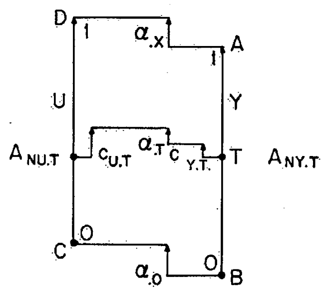


Figure 11. Diagram indicating the mathematical relationships for in-phase components between the input and output phasors of the *U* and *Y* dividers, the corrections to the nominal ratios, and the measurable differences.

EQUATIONS

By definition, let the voltage between B and A in Figure 11 be the reference voltage forcing the corrections at taps .0 and $.X$ to be zero. Represent this reference voltage with phasor AB. Represent the voltage between C and D with phasor CD. Then

$$A_{NU.T} + c_{U.T} = \frac{A_{NY.T} + C_{Y.T} + \alpha_{.T} - \alpha_{.0}}{1 + \alpha_{.X} - \alpha_{.0}}$$

Where $A_{NU.T}$ means: ratio, nominal, (of) U divider (at) tap "T", α means: reading of dials that balance bridge, and the subscripts $.X$ and .0 means :X tap and zero tap.

$$A_{NU.T} + c_{U.T}$$

$$= A_{NY.T}\left(1 + \frac{C_{Y.T}}{A_{NY.T}} + \frac{\alpha_{.T}}{A_{NY.T}} - \frac{\alpha_{.0}}{A_{NY.T}} + \alpha_{.0} - \alpha_{.X}\right)$$

$$A_{NU.T} = A_{NY.T} \equiv A$$

$$c_{U.T} = c_{Y.T} + \alpha_{.T} - \alpha_{.0} + A\alpha_{.0} - A\alpha_{.X}$$

$$c_{U.T} = c_{Y.T} + \alpha_{.T} - (1 - A)\alpha_{.0} - A\alpha_{.X}$$

Now a series of equations are written for a step-up. Let c_t be the correction to the transformer used as the standard that is switched in and out. Let α be the reading when the standard and the unknown are on the same tap and let α' be the reading when they are not, at which time the 10:1 standard transformer is in the circuit.

$$c_{U.0} = c_{Y.0} + \alpha_{.0} - (1 - 0)\alpha_{.0} - 0\alpha_{.X}$$

$$c_{U.0} = c_{Y.1} + \alpha'_{.1} - (1 - 0.1)\alpha_{.0} - 0.1\alpha_{.X} - c_t$$

$$c_{U.1} = c_{Y.1} + \alpha_{.1} - (1 - 0.1)\alpha_{.0} - 0.1\alpha_{.X}$$

$$c_{U.1} = c_{Y.2} + \alpha'_{.2} - (1 - 0.2)\alpha_{.0} - 0.2\alpha_{.X} - c_t$$

$$c_{U.2} = c_{Y.2} + \alpha_{.2} - (1 - 0.2)\alpha_{.0} - 0.2\alpha_{.X}$$

$$c_{U.2} = c_{Y.3} + \alpha'_{.3} - (1 - 0.3)\alpha_{.0} - 0.3\alpha_{.X} - c_t$$

$$\vdots$$

$$c_{U.8} = c_{Y.9} + \alpha'_{.9} - (1 - 0.9)\alpha_{.0} - 0.9\alpha_{.X} - c_t$$

$$c_{U.9} = c_{Y.9} + \alpha_{.9} - (1 - 0.9)\alpha_{.0} - 0.9\alpha_{.X}$$

$$c_{U.9} = c_{Y.X} + \alpha'_{.X} - (1 - 0.X)\alpha_{.0} - .X\alpha_{.X} - c_t$$

$$c_{U.X} = c_{Y.X} + \alpha_{.X} - (1 - 0.X)\alpha_{.0} - .X\alpha_{.X}$$

Eliminating $C_{Y.T}$ gives equations for $C_{U.T}$

$c_{U.1} = c_{U.0} + c_t + \alpha_{.1} - \alpha'_{.1}$ (Where $c_{U.0} = 0$ by definition)

$$c_{U.2} = c_{U.1} + c_t + \alpha_{.2} - \alpha'_{.2}$$

$$c_{U.3} = c_{U.2} + c_t + \alpha_{.3} - \alpha'_{.3}$$

$$\vdots$$

$$c_{U.X} = c_{U.9} + c_t + \alpha_{.X} - \alpha'_{.X}$$

$$\sum_{T=.1}^{.X} c_{U.T} = \sum_{T=.0}^{.9} c_{U.T} + 10c_t + \sum_{T=.1}^{.X} (\alpha_{.T} - \alpha'_{.T})$$

$$c_{U.X} - c_{U.0} = 10c_t + \sum_{T=.1}^{.X} (\alpha_{.T} - \alpha'_{.T})$$

$$c_{U.X} = c_{U.0} \equiv 0$$

and

$$c_t = -\frac{\sum_{T=.1}^{.X} (\alpha_{.T} - \alpha'_{.T})}{10}.$$

The general equation for the correction is

$$c_{U.T} = c_{U(.T-.1)} + c_t + (\alpha_{.T} - \alpha'_{.T})$$

The steps taken are as follows:

(1) Find and record the 10 differences $(\alpha_{.T} - \alpha'_{.T})$
(2) Sum: $(\alpha_{.T} - \alpha'_{.T})$ and calculate c_t
(3) Combine c_t with the difference found in (1)
(3) Sum accumulatively for the final answers
(5) Check on arithmetic: $C_{U.X}$ must = zero

Eliminating $C_{U.T}$ gives equations for $C_{Y.T}$ where $(C_{S.0} = 0)$

$$c_{Y.1} = c_{Y.0} + c_t + 0.1\alpha_{.X} - 0.1\alpha_{.0} + (\alpha_{.0} - \alpha'_{.1})$$

$$c_{Y.2} = c_{Y.1} + c_t + 0.1\alpha_{.X} - 0.1\alpha_{.0} + (\alpha_{.1} - \alpha'_{.2})$$

$$c_{Y.3} = c_{Y.2} + c_t + 0.1\alpha_{.X} - 0.1\alpha_{.0} + (\alpha_{.2} - \alpha'_{.3})$$

$$c_{Y.4} = c_{Y.3} + c_t + 0.1\alpha_{.X} - 0.1\alpha_{.0} + (\alpha_{.3} - \alpha'_{.4})$$

$$\vdots$$

$$c_{Y.X} = c_{Y.9} + c_t + 0.1\alpha_{.X} - 0.1\alpha_{.0} + (\alpha_{.9} - \alpha'_{.X})$$

$$\sum_{T=.1}^{.X} c_{Y.T} = \sum_{T=.0}^{.9} c_{Y.T} + 10c_t + \alpha_{.X} - \alpha_{.0} + \sum_{T=.1}^{.X} (\alpha_{.T-.1} - \alpha'_{.T})$$

$$c_{Y.X} - c_{Y.0} = 10c_t + \alpha_{.X} - \alpha_{.0} + \sum_{T=.1}^{.X} (\alpha_{.T=.1} - \alpha'_{.T})$$

$$c_{Y.X} = c_{Y.0} \equiv 0$$

And, therefore

$$c_t = -\frac{\sum_{T=.1}^{.X} (\alpha_{.T-.1} - \alpha'_{.T}) + \alpha_{.X} - \alpha_{.0}}{10}$$

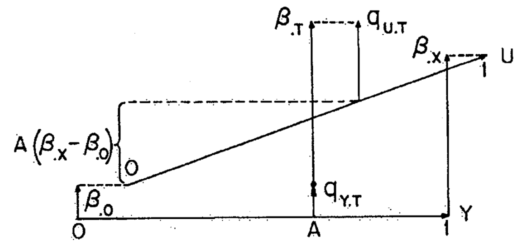


Figure 12. Diagram indicating the mathematical relationships for quadrature components.

The general equation for the correction is

$$c_{Y.T} = c_{Y(.T-.1)} + c_t + 0.1(\alpha_{.X} - \alpha_{.0}) + (\alpha_{.T-1} - \alpha'_{.T})$$

The general equation for a simple comparison of the unknown divider compared with a standard divider with known corrections is

$$c_{U.T} = c_{Y.T} + \alpha_{.T} - (1 - 0.T)\alpha_{.0} - 0.T\alpha_{.X}$$

The treatment of the quadrature components of voltage ratio is similar to that shown above. In Figure 12 the input phasors for dividers U and Y are separated from each other. The angular and linear displacements have been grossly exaggerated for clarity. Second order errors resulting from the very small angular differences can be ignored in the derivation of the equations for the calculation of the quadrature components of ratio for the U and Y dividers.

Inspection of Figure 12 will show that

$$q_{Y.T} + \beta_{.T} = \beta_{.0} + A(\beta_{.X} - \beta_{.0}) + q_{U.T}$$

Then,

$$q_{U.T} = q_{Y.T} + \beta_{.T} - \beta_{.0} + A\beta_{.0} - A\beta_{.X}$$

This equation is similar to the equation for the α component, except that β appears in the place of α. Thus, the analysis, the equations, and the procedures given for the calculation of the in-phase corrections may be used for the calculations of quadrature if α is replaced by β.

MEASUREMENT RESULTS AND CONCLUSION

The errors from the several sources were reduced, almost to the level of imprecision, from variations in connections, variations in mutual inductances between parts of the measurement circuit, and variations in stray capacitances, as connections in the circuit are changed in the course of the step-up procedure. Further effort will be required to determine more exactly the limitations of the design and the measurement method at 100 kHz. Typical measurement results at 100 kHz are shown in Table I. The uncertainty in the measurements of in-phase ratio can be described as a standard deviation of 0.2×10^{-7}, computed from the ranges of eight sets of two or three repetitions using two different dividers, and

TABLE 1

In-phase Corrections to the Nominal Ratio for a Single Two-stage, Guarded Inductive Voltage Divider. The Input Voltage was approximately 10 V, at 100 kHz

Nominal ratio	Measured corrections (ppm)
0.*X*	0.00
0.9	−0.01
0.8	+0.02
0.7	+0.04
0.6	+0.06
0.5	+0.07
0.4	+0.02
0.3	−0.04
0.2	−0.08
0.1	−0.08
0.0	0.00

the limits of systematic error are estimated as $\pm 1 \times 10^{-7}$, somewhat subjectively determined from experiments in which the dividers were tested with both normal and inverted connections to the measurement circuit.

Obviously, even in the present stage of development, two-stage, guarded dividers, with corrections determined by the step-up method, can serve as excellent standards for the calibration of first decade ratios of other dividers. Other uses might be as ratio arms of bridges for radio-frequency impedance measurements, or as standards of voltage for attenuator calibrations. To be truly useful for these purposes the two-stage, guarded design must be extended to multidecade dividers. Results of a few measurements at 1 MHz indicated that the design may prove advantageous much higher into the radio-frequency spectrum.

REFERENCES

1. Brooks, H. B., and Holtz, F. C. June 1922. "The Two-Stage Current Transformer." *AIEE Trans.* 41:382–393.
2. Cutkosky, R. D. 1964. "Active and Passive Direct-Reading Ratio Sets for the Comparison of Audio-Frequency Admittances." *IEEE Trans.* IM-13:243.
3. Sze, W. C. January–March 1968. "An Injection Method for Self-Calibration of Inductive Voltage Dividers." *J. Res. Nat. Bur. Stand.* 72C:49–50.
4. Deacon, T. A., and Hill, J. J. 1968. "Two-Stage Inductive Voltage Dividers." *IEE Proc.* 115:727.
5. Rosa, E. B., and Grover, F. W. 1948 revision. *Formulas and Tables for the Calculation of Mutual and Self-Inductance.* Nat. Bur. Stand. special publication 169, p. 158.