Characterized Generator Extends Phase Meter Calibrations from 50 kHz to 20 MHz

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***Abstract*—A phase-angle generator made by phase locking two function generators is described. The generator produces two sine waves that are programmable in phase (0-360°), amplitude (0-40 V_{rms}), and frequency (<1 Hz – 20 MHz). Phase linearities within ±0.1° are achieved without external phase standards.**

I. Introduction

PROGRAMMABLE phase angle standards that digitally synthesize two phase-adjustable sine waves [1], [2] are used to calibrate phase angle meters below 50 kHz. However, a new class of wideband counters, known generically as time interval analyzers (TIA's), can be configured as phase meters that operate up to 20 MHz and beyond. These instruments are being used to enhance the measurement precision of heterodyne interferometers [3], but there are no established standards to support them. Existing phase standards (based on digitally synthesized waveforms) operate from 1 Hz to 50 kHz with uncertainties from ±0.005° to ±0.05° [4]. A simple method has been developed to extend this capability from 50 kHz out to 20 MHz while maintaining uncertainties from ±0.05 to ±0.25°.

II. Phase Angle Measurement

The phase angle measurement, shown in Fig. 1, employs two commercial generators that are phase locked by connecting the frequency reference output, "clock out," of one of the generators to the frequency reference input, "clock in," of both generators. This technique works particularly well for generators of the same type where the operating principle is the same. Attempts to phase lock different types of generators resulted in slight frequency differences which produced large phase drifts.

The output signal of each generator can be phase shifted relative to the common clock. In this manner, it is possible to set any phase angle between 0 and 360° to within the phase resolution of the generator. Moreover, it is possible to set a particular phase angle a number of different ways. For example, a 30° phase angle between the two output signals can be programmed by setting one generator to 0° and the other to 30°, or by setting one to 60° and the other to 90°.

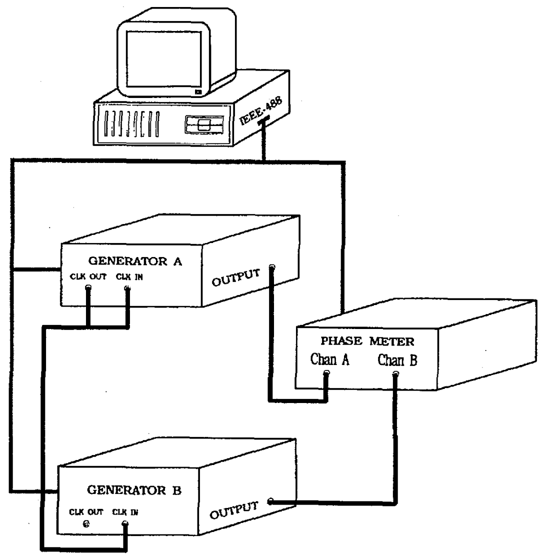

Fig. 1. Functional diagram of the measurement setup.

This property makes it possible to compute the errors of specific phase settings for each generator. The differences between settings that produce a particular phase angle can be measured using a phase meter only as a detector for that angle. The phase meter accuracy is not critical; however, it must have sufficient differential linearity to measure the differences to within the required accuracy. As long as the differences are small, this is not a stringent requirement. A series of equations describing the phase errors of each generator and the measured differences can then be expressed as

$$A_i - B_j + C_k = R_{ij} \tag{1}$$

where:

1) A_i is the error of generator A at the i phase setting,
2) B_j is the error of generator B at the j phase setting,
3) C_k is the error of the phase meter at the i-j phase setting, and

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4) R_{ij} is the phase-angle error (nominal–reading) at the i-j phase setting.

If n measurements are made at each of n different phase angles (equally spaced between 0° and 360°), there will be n^2 independent measurements and $3n$ unknowns. A solution for this set of equations requires that n be greater than or equal to 3.

III. Matrix Solution

A simplified example describing measurements at three phase angles (0°, 120°, and 240°) is shown below.

$$
\begin{aligned}
A_{000} - B_{000} + C_{000} &= R_1 \\
A_{000} - B_{120} + C_{120} &= R_2 \\
A_{000} - B_{240} + C_{240} &= R_3 \\
A_{120} - B_{000} + C_{240} &= R_4 \\
A_{120} - B_{120} + C_{000} &= R_5 \\
A_{120} - B_{240} + C_{120} &= R_6 \\
A_{240} - B_{000} + C_{120} &= R_7 \\
A_{240} - B_{120} + C_{240} &= R_8 \\
A_{240} - B_{240} + C_{000} &= R_9.
\end{aligned}
\tag{2}
$$

However, (2) are not linearly independent. To demonstrate this, consider that both sources have fixed phase offsets. If the offsets are equal, the readings would always be the same no matter how large the offset. There is no way to determine whether these offset errors are due to the sources or the meter. If the offsets are independent of phase angle, the problem is solved by adding two equations which assign all offset errors to the phase meter by setting the average of the offsets of each generator to zero

$$
\begin{aligned}
A_{000} + A_{120} + A_{240} &= 0 \\
B_{000} + B_{120} + B_{240} &= 0.
\end{aligned}
\tag{3}
$$

Fig. 2 shows the complete system of equations in matrix form. The matrix is an over-determined set of 11 equations containing nine unknowns and can be solved by the matrix equation:

$$X = (A^T A)^{-1} A^T R \tag{4}$$

where

A is the system matrix,

R is the vector containing the phase angle errors, and

X is the coefficient vector $[A_{000} \cdots C_{240}]$.

$$
\begin{array}{ccccc}
A & \times & X & = & R \\
\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0
\end{bmatrix}
& \times &
\begin{bmatrix}
A_{000} \\ A_{120} \\ A_{240} \\ B_{000} \\ B_{120} \\ B_{240} \\ C_{000} \\ C_{120} \\ C_{240}
\end{bmatrix}
& = &
\begin{bmatrix}
R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \\ 0.0 \\ 0.0
\end{bmatrix}
\end{array}
$$

Fig. 2. A matrix equation with nine unknowns for determining the errors of the generators and phase meter at three phase angles.

IV. Test Results

Measurements were made using two different types of function generators. Type I generators synthesize signal frequencies using a phase-locked loop (PLL), while type II generators use direct digital synthesis (DDS), a technique that has become popular in the past several years. Tests were performed using two type I or two type II generators. A TIA configured as a phase meter was employed as a detector for each set of tests. Results at 1 MHz, shown in Fig. 3, are based on the average of 1000 phase measurements at each test point. Results indicate that the maximum linearity errors at 1 MHz were less than ± 0.1° for type I generators and the TIA, and less than ±0.15° for type II generators.

Similar tests were performed at 10 MHz on both generator types. These data are plotted in Fig. 4. At 10 MHz the nonlinearity of the type I generators and the TIA was approximately ±0.1°. This figure is of particular interest because it implies that the technique used to shift the phase in these instruments has time nonlinearity of ±30 ps. A similar figure was observed for this instrument by Souders *et al.* using a sampling voltage tracker [5]. The type II generator's nonlinearity was ±0.8°, which represents a time nonlinearity of ±220 ps.

In most applications, the phase linearity is more important than the actual phase error, which includes a phase offset term that can generally be removed at the time of the measurement. The offset of the phase meter is measured by reading the meter with the same signal applied to both input channels. Once the phase meter is corrected, it can be used to measure the offsets of each generator. In several instances, the measured linearity errors of these generators have sufficient short-term stability to justify applying a correction at fixed phase angles, thus improving accuracy by a factor of 2 or 3.

V. Conclusions

A simple technique of verifying the phase angle linearity of a pair of synchronized function generators has been described. The approach uses a commercial time interval analyzer as a phase meter between 50 kHz and 10 MHz. Results for one type of generator and a TIA demonstrate time linearities less than ±30 ps.

The approach offers an economical means of extending conventional digitally synthesized phase angle standards

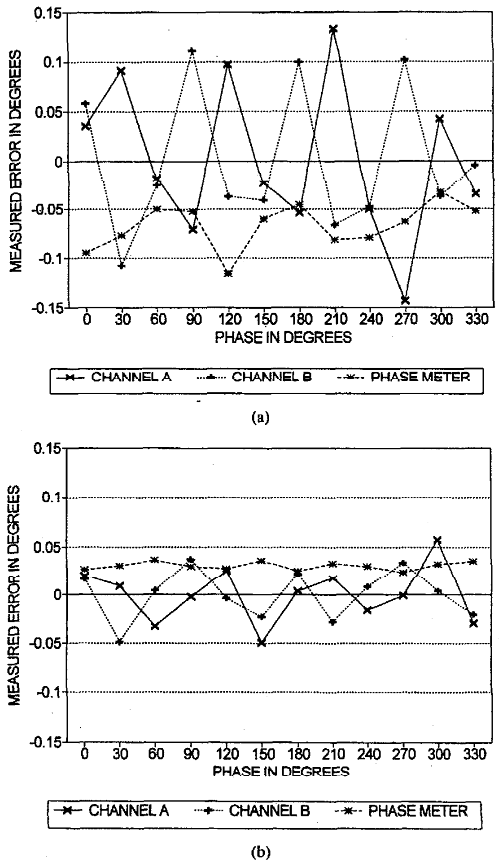

Fig. 3. Plots of the phase nonlinearity of the generators and phase meter at 1 MHz. (a) Type I generators. (b) Type II generators.

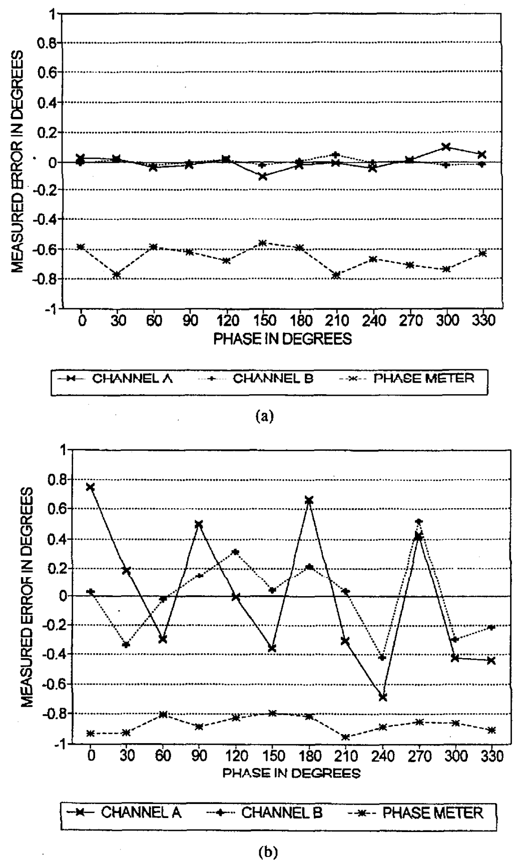

Fig. 4. Plots of phase nonlinearity of the generators and phase meter at 10 MHz. (a) Type I generators. (b) Type II generators.

up to 10 or 20 MHz. In addition, commercial TIA's can be configured as linear phase meters in this frequency range. Applications for precise electrical phase angle measurements include optical interferometry where (in commercial heterodyne interferometers) a 1° phase error translates approximately to a 1 nm displacement error.

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