a numerical example let $\alpha = 10°$, $V_{FS} = 10.24$ V, $= 12$, and $Vp = 1$ V; this leads to $\phi_{e2}(\max) = 0.072°$. order to reduce this error we should increase M and Vp, decrease α. Table II illustrates the change of $\alpha/\sin\alpha$ th α. It is obvious that we have to choose a reasonably gh sampling frequency which is at least, say, four times signal frequency.

IV. Overall Error

The phase error due to mislocating a single zero-crossing is $\pm\phi_{e1} \pm \phi_{e2}$. However, the overall error ϕ_e is due to slocating the two zero-crossings of the signals $x(t)$ and and is given by

$$\phi_e = \pm\phi_{e1x} \pm \phi_{e2x} \pm \phi_{e1y} \pm \phi_{e2y}. \tag{16}$$

om the previous analysis it is obvious that ϕ_{e1} of Table has a double hump shape in the range $0 \le \theta \le \alpha$, ereas ϕ_{e2} is maximal when $\theta = 0$ or α. Consequently, overall worst-case θ_e is not the sum of the worst-case and ϕ_{e2}, but depends on the various parameters involved.

V. Conclusions

Calculating the phase difference between two signals, using synchronous real-time sampling, has been proposed. The method is based on linear interpolation, which gives the least measurement accuracy. It was shown that this accuracy can be improved by increasing the sampling frequency, the signal amplitudes, and the number of A/D quantization levels. However, more accurate results are expected if we resort to computer-based nonlinear interpolation, since the sampled values can be fitted into a sinusoidal function.

Acknowledgment

As a first author, I would like to present the research of my life to the memory of my mother, Hekmat Mahmoud Fawzy, who passed away in Cairo, Egypt, on December 13, 1984, during the preparation of this paper.

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A Precision Phase Angle Calibration Standard for Frequencies up to 50 kHz

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bstract—A Phase Angle Calibration Standard covering a frequency ge from 2 Hz to 50 kHz has been designed and constructed. Digital veform generation is used to provide sinusoidal analog outputs hav- precisely settable phase angles. Output voltages are independently justable from 0.5 to 100 V rms on both channels. An auto-zero feed k loop compensates for differential phase errors of the output am- fiers.

TO ATTAIN consistent and traceable audiofrequency phase angle measurements, it is desirable to have a nvenient means for calibrating phase meters. The Phase ngle Calibration Standard, to be described, provides a ual-channel signal source in which the phase relationship the two output sine waves can be accurately set to any esired phase angle. The output of the Phase Angle Calibration Standard can, therefore, be connected directly to the inputs of a phase meter for the purpose of calibration. The frequency range extends from 2 Hz to 50 kHz, and the amplitudes of each output signal can be varied independently from 0.5 to 100 V.

II. Principle of Operation

Exceptional signal stability and rapid response when changing operating parameters can be achieved by using a method of digital waveform generation for the analog output [1]. With this method two sets of calculated values, representing sample points at equal time intervals along two waveforms, are converted to voltages using dual-channel analog-to-digital converters. The resulting stepped sine waves (see Fig. 1) are then passed through low-pass filters to remove the frequency components introduced by the sampling process.

Similar approaches have been tried elsewhere, but the present design differs in the way the phase angle between the two sinusoidal output signals is determined [2]. In-

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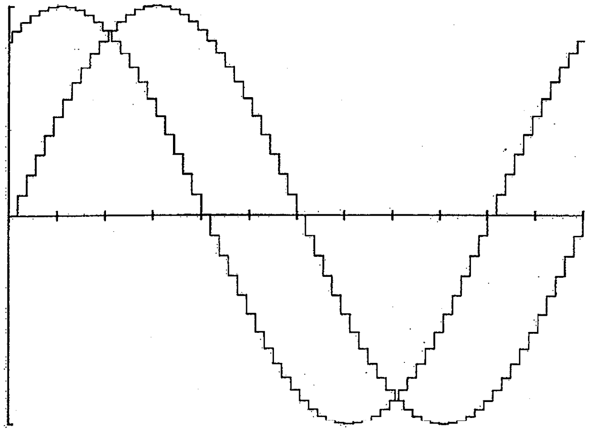


Fig. 1. Stepped sine waves, phase difference 60°, 64 steps per cycle.

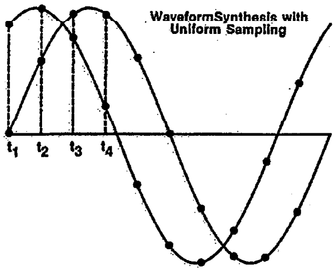


Fig. 2. Waveform synthesis with uniform sampling.

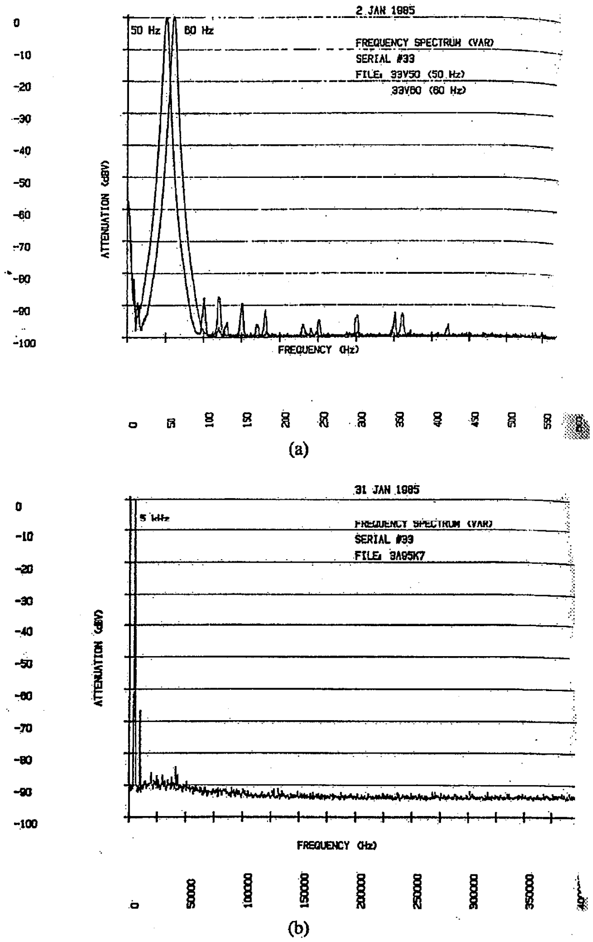


Fig. 3. Spectrum of Phase Angle Calibration Standard. (a) 50/60 Hz. (b) 5 kHz.

stead of shifting the phase of the two output signals using some form of time delay, such as counting clock pulses or using delay lines, the sets of instantaneous amplitudes that define the two sine waves are computed in such a way that the sine waves they generate have the desired phase difference when corresponding sample points are converted simultaneously. As illustrated in Fig. 2, at time t_1, the two points generated correspond to Sin 0° and Sin 60°, those at time t_2 to Sin(0° + 360°/N), etc., where N is equal to the number of sample points per cycle of the output waveform.

The phase angle, therefore, depends only on the calculated two sets of the sampling values and is independent of frequency and timing considerations. The resolution obtainable with this method is a function of both the number of sample points per cycle and the number of amplitude levels corresponding to the number of bits of the digital-to-analog converters. It does permit phase resolution of the order of millidegrees at frequencies where the equivalent time delay is in the nanosecond range without the problems of generating accurate strobe pulses in that time range.

Even though converter characteristics depart from the ideal, the spectral purity of the output sine wave can be made relatively high by using a reasonable large number of sample points per waveform. The lower limit of the harmonic content, assuming that the sampling harmonics have been filtered out, is given by the quantization noise which for an ideal 16-bit converter is -97.8 dB, and -73.8 dB for a 12-bit converter [4]. Fig. 3 shows measured spectra for 50/60 kHz using 16-bit conversion.

The quantization noise is more evident for the 12-bit conversion at 50 kHz and appears in the form of larger harmonics (Fig. 4). The measured harmonic components which are above the theoretical limit, are attributed to imperfect converter and amplifier characteristics.

III. System Description

A block diagram of the system is shown in Fig. 5. The general physical arrangement is similar to the 5-kHz Phase Angle Calibration Standard developed earlier [1], [3]. A 20-bit bit-slice microprocessor computes the data needed for generation of the waveforms and is aided by a conventional 8-bit microprocessor which handles control functions, keyboard entry, IEEE-488 Bus communication and the display.

For output frequencies up to 5 kHz the bit-slice processor calculates two 16-bit values in every sampling time interval while the previous pair of values are applied to

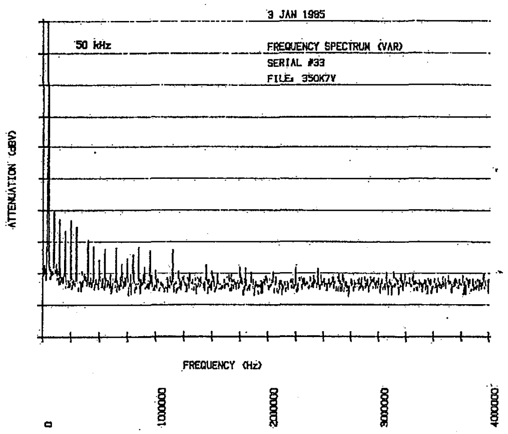


Fig. 4. Spectrum of Phase Angle Calibration Standard at 50 kHz.

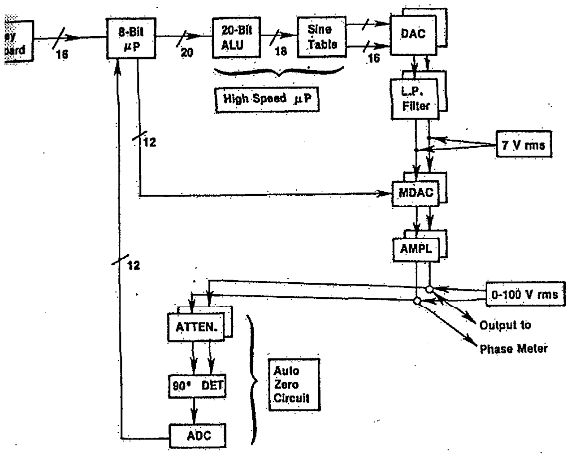


Fig. 5. Block diagram of Phase Angle Calibration Standard system.

the inputs of the 16-bit digital-to-analog converters. The sampling time strobe pulse is generated from an independent crystal-controlled time base and the strobe rate can be varied from 0.2 to 0.4 MHz. Output frequencies from 2 Hz to 5 kHz in 1-Hz steps are obtained by setting the time base within its range and also by changing the number of sample points per cycle in a binary sequence ranging from 64 points at 5 kHz to 131072 points at 2 Hz. With this scheme only one fixed-frequency (25 kHz) low-pass filter is required, and the converters are used near their maximum conversion rates. The spectral purity is particularly good at lower frequencies where the number of sample points is large (see Fig. 3).

Above 5 kHz the mode of operation is modified. Although in theory a sine wave can be reconstructed from as few as two sample points per waveform, in practice a small number of points limits the possible phase resolution and places severe demands on the low-pass filter to obtain spectral purity. For the Phase Angle Calibration Standard it was decided to retain a minimum of 64 sample points at the top frequency (50 kHz). To achieve this higher sampling rate, resolution is traded for conversion speed with faster 12-bit converters in place of the 16-bit converters used at lower frequencies. Active low-pass filters with a corner frequency of approximately 0.35 MHz reduce the harmonics generated by sampling. The choice of the corner frequency is a compromise between phase shift below 50 kHz and attenuation at the sampling frequencies (see Fig. 6).

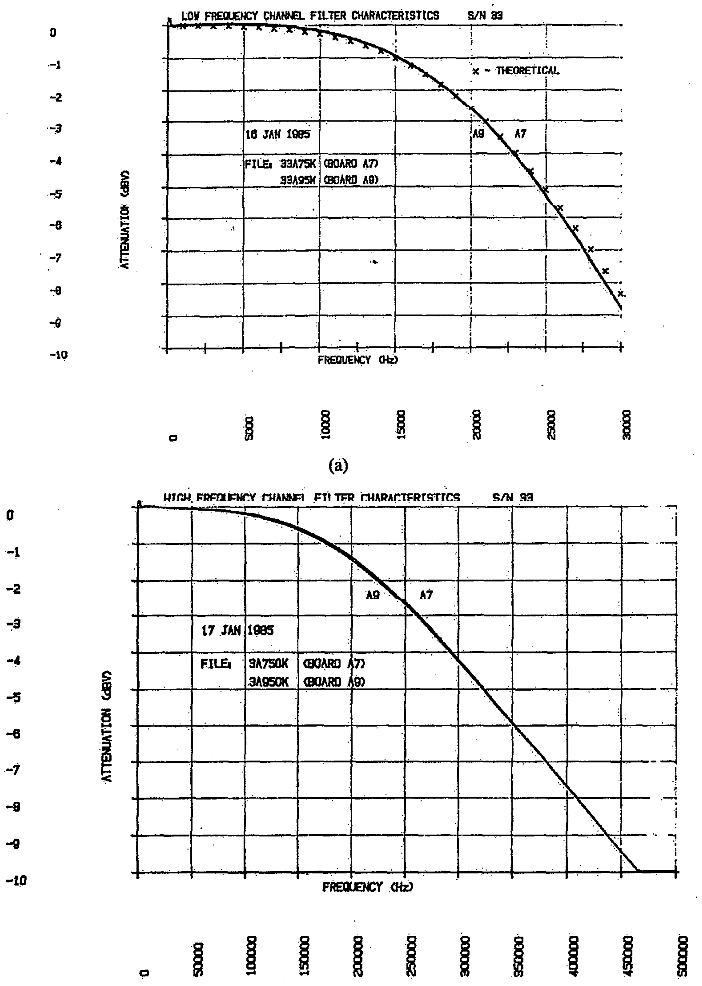
Because the shorter sampling interval is no longer sufficient for real-time computation of successive pairs of values, the desired sets of values must be calculated ahead of time. A two-range time base provides strobe pulses at a low rate for computation and storage and at a rate ten times faster for retrieval from memory and for digital-to-analog conversion. Over the range of sine wave frequencies from 5 to 50 kHz (in 10-Hz steps) the number of stored sample points is varied from 512 to 64,,and stobe frequencies are selected in a range from 2 to 4 MHz. Thus for instance, the data points for a 50-kHz output signal are calculated and stored with a strobe frequency of 0.32 MHz, as if generating a 5-kHz sine wave, and are subsequently transferred from memory to the 12-bit converters at a 3.2-MHz rate. For this particular example the time for computation and storage is 1.6 ms. In addition, several milliseconds are needed by the 8-bit microprocessor to set up signal routing and strobe pulse rates.

The algorithm for the waveform generations permits updating the signal at the beginning of every cycle of the output frequency using phase and frequency parameters entered via the keyboard. Consequently, the response to changes can be almost immediate, and even at higher frequencies, when a new set of values must first be stored, the setup time is barely perceptible by the user.

IV. AUTO-ZERO CORRECTION

Closely matched amplifiers and filter circuits still have residual differential phase shifts which, in practice, cannot be eliminated by circuit adjustment. Since such phase shifts cause a systematic error in the calibration of the Phase Angle Calibration Standard, an auto-zero compensation circuit is incorporated in the system. Its action is based on the characteristic of a quadrature phase detector which has zero output for nominal inputs with 90° phase difference and has a positive or negative dc output voltage if the input phase angle deviates from 90°. The detector output voltage, after being digitized, is fed back to apply a correction to the microprocessor generating the waveforms. Two-range gain-controlled preamplifiers and 7-bit binary inductive dividers, with very low intrinsic phase errors, condition the two input signals to the quadrature detector. The circuit, as well as the auto-zero measurement procedure, has been designed to minimize the effect of internal errors in the phase detector. A more detailed description can be found in [3].

The auto-zero procedure can be thought of as establishing a fixed point on the angle scale. It is an iterative procedure which is terminated when the incremental correc-



(a)

(b)

Fig. 6. Measured low-pass filter characteristics. (a) Low-frequency channel. (b) High-frequency channel.

tion falls below a preset tolerance. Because the correction process is also subject to noise, it introduces a small random error into the output of the Phase Angle Calibration Standard. This error varies from less than 0.002° at low frequencies to about 0.010° at the upper frequency limit. The auto-zero correction must be applied whenever the output voltage on either channel is changed, or when going from a frequency below 5 kHz (16-bit conversion) to a frequency above 5 kHz (12-bit conversion), or vice versa. Within each frequency range, however, the corrections vary approximately linear with the frequency and are adjusted by the software for the particular frequency in use. Once the correction has been determined, it is automatically applied for any phase angle setting of the standard. Fig. 7 shows the residual phase offset errors for both frequency ranges after applying the auto-zero procedure with output amplitudes of 5 and 70 V on both channels. The low-frequency plot also shows offset errors measured at the output of the low-pass filter (7-V mode).

V. RESOLUTION AND LINEARITY

The dependence of angular resolution on both the number of sampling points per cycle and the number of possible output levels of the digital-to-analog converters has already been briefly mentioned. Since the output sine wave and its phase is defined by a set of sample values, changing the magnitude of a few of the sample values by a small amount will result in a new (slightly distorted) sine wave. To preserve symmetry, the sample points have to be chosen in pairs 180° apart. This change in values can be thought of as a superposition on the original sine wave of a component with the same fundamental frequency plus some harmonics. The superposition may result in a small but noticeable phase shift, depending on the location of

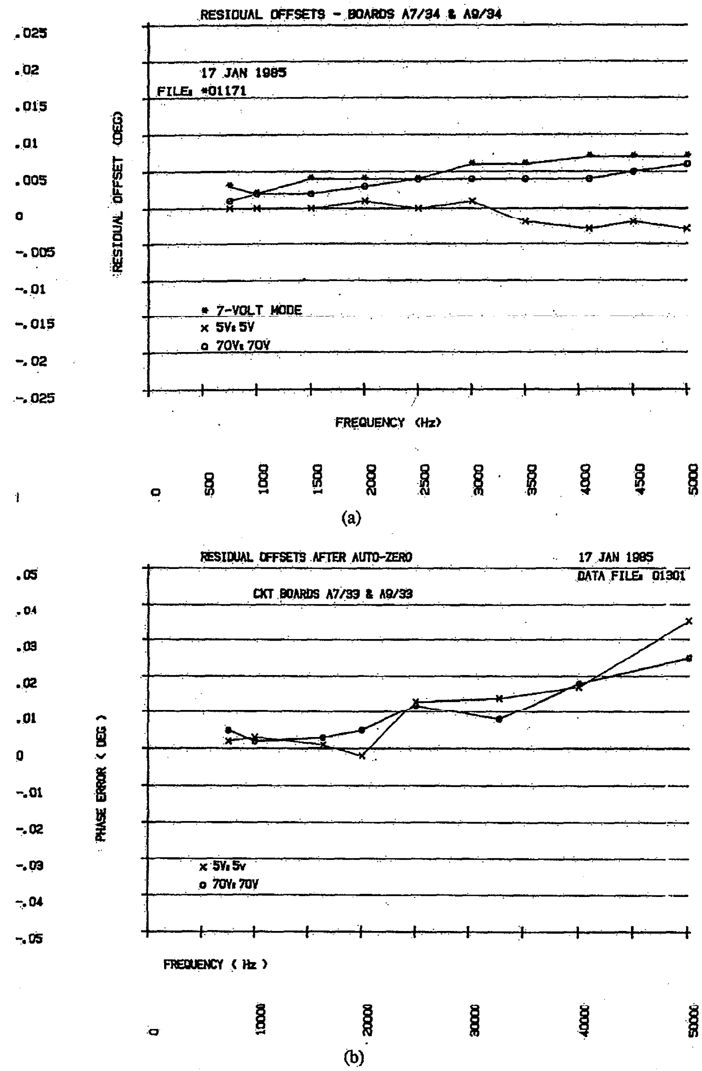


Fig. 7. Residual phase offset after applying auto-zero correction. (a) 0–5 kHz. (b) 5 kHz.

the changed sample values with respect to the phase of the fundamental. It is possible, therefore, to produce a phase shift of the digitally generated signal by changing the magnitude of only two values out of the set of 64 or more. To investigate the minimum theoretical phase resolution possible with this method of generating sine waves, all possible sets to sample values have to be compared. To be distinguishable, a phase change must be associated with a new set of values in which at least one pair of members is different from corresponding members of the old set. This is the basis for investigating the theoretical resolution using computer simulation.

To simulate the reconstruction of sine waves from 64 samples with 2^{12} quantized levels, sets of sample values are calculated for a series of sine waves each shifted in phase relative to its prodecessor by 0.001° over an interval of, for instance, (360/64) = 5.625°. Every member of the initial set of 64 points is then compared with the corresponding member of a following set until a difference in at least one pair is detected. The incremental phase shift of the sine wave associated with that set is recorded, and the comparison process is restarted using the new set of values in place of the initial set.

Using this method of comparison, a histogram of the number and size of incremental phase shifts between neighboring nonidentical sample sets can be constructed (see Fig. 8). As shown in Fig. 8, for 12-bit converters the theoretically attainable phase angle resolution is nonuniform due to the restriction imposed by the limited number of output levels. However, with 64 sample points per cycle

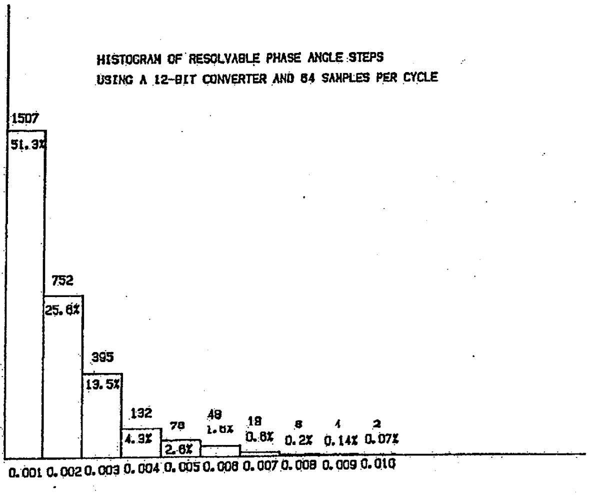


Fig. 8. Resolvable phase angle steps using a 12-bit converter and 64 samples per cycle.

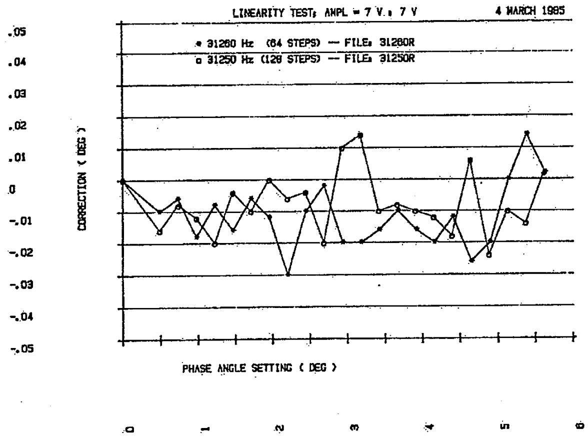


Fig. 9. Phase linearity tests of Phase Angle Calibration Standard output (12-bits).

97 percent of the possible phase steps will be less than or equal to 0.005°. As the number of sample points is increased to 128, approximately 95 percent of the resolvable phase angle steps are 0.003° or less. Simulations for ideal digital-to-analog converters show that for 16-bit converters attainable resolution is better than 0.001°.

Actual converters, however, depart from the ideal, and Fig. 9 shows the experimentally determined linearity of the Phase Angle Calibration Standard using 12-bit data for two approximately equal output frequencies, one of which is generated with 128 samples per cycle and the other with 64 samples. For comparison, a linearity plot for 16-bit data at one tenth the output frequency is shown in Fig. 10.

Exhaustive experimental tests for linearity are impractical because of the very large number of possible measurements that would have to be made. For N sample points per cycle, the sample values will be identical in magnitude if the phase is shifted by $360/N$ degrees, although the order of the points will have been rotated. The two sets will generate identically shaped sine waves displaced by exactly $360/N$ degrees. From the point of view of linearity, the pattern of minimum resolvable phase angles will repeat in each $360/N$ ° interval, and so only one of N such intervals needs to be investigated. Nevertheless, the number of possible nonidentical sample sets, and, therefore, corresponding resolvable phase angle steps, is still too large even for one repeat interval (approximately 3000, if $N = 64$). However, it is likely that limits of linearity errors can be arrived at by making measurements at a small number of angles within one repeat interval (between 20 and 30), selected so that they do not fall on binary subdivisions of the interval which could bias the test results.

An experimental check of linearity of the phase angle scale can be performed with the help of the 180° bridge described below. A design feature of the Phase Angle Calibration Standard permits the phase of each output to be varied independently with respect to an internal reference marker that is common to both signal channels. Thus if it is desired to set the phase difference between the two output signals to 180°, this can be accomplished by setting the "reference" channel to any desired phase angle and adjusting the "variable" channel so that the difference is 180°. For example, choices might be 0.000° and 180.000°, 2.002° and 182.002°, etc.

As pointed out earlier, each angular setting is associated with a particular set of sample values defining a sine wave. The linearity test is designed to verify how closely the phase angle determined by the particular set of sample values come to its theoretical (nominal) value. Experimentally, the test established the phase adjustment of the Phase Angle Calibration Standard necessary to balance the bridge at selected test points and compares the variation in that adjustment over the interval. The graphs in Figs. 9 and 10 show the departure from the ideal 180° difference. To make comparisons easier, the points on the plots have been shifted so that the first point falls on the zero axis.

VI. Experimental Methods

The accuracy of the auto-zero correction, as well as the linearity data, were checked experimentally using a 180° bridge method described in more detail in [3]. The two output circuits of the Phase Angle Calibration Standard, with the signals set to a nominal 180° phase angle, constitute one-half of the bridge, the other half is formed by an inductive or capacitive divider. The adjustable center tap of the divider is connected to a null detector tuned to the output frequency. The bridge is balanced by nulling the magnitude, using the divider, and nulling the phase, and by adjusting the phase angle setting of the Phase Angle Calibration Standard. A detector sensitivity of 100 μV full scale is adequate to detect phase errors of less than 0.001° when the input signals are of the order of 7 V rms. A semiautomatic experimental procedure utilizes the IEEE-488 bus to control the Phase Angle Calibration Standard and to record and plot the data. Full automation

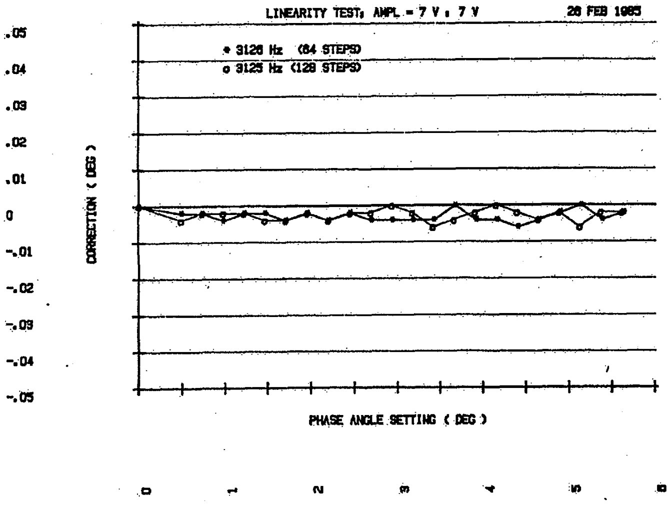


Fig. 10. Phase linearity tests of Phase Angle Calibration Standard output (16-bits).

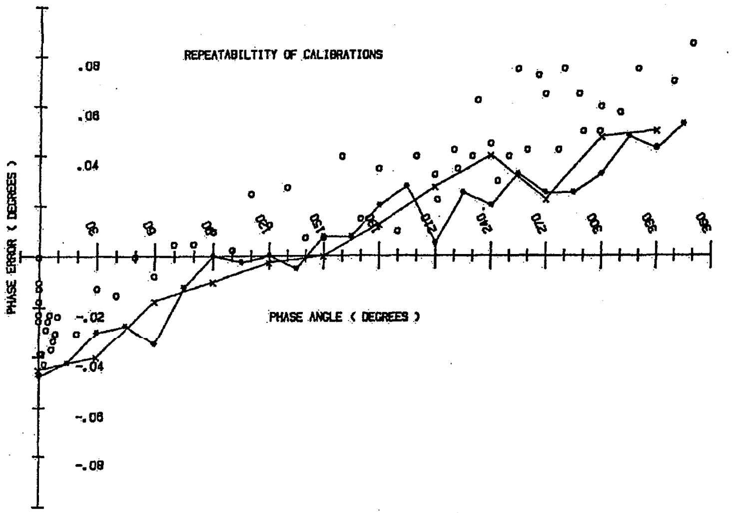


Fig. 11. Repeatability of calibrations.

is theoretically possible, but some judgement is required to decide when the balance condition or the bridge has been reached. The algorithm for this decision is not simple to implement.

At low frequencies a five- or six-dial inductive divider replaces the two high-quality adjustable air capacitors used at higher frequencies. Tests in the intermediate frequency range show good agreement between the two methods, as well as with an entirely different third method using an external quadrature detector [5].

VII. PERFORMANCE

The uncertainty associated with the output phase angle is a function of frequency, amplitude, and amplitude ratio. The main error sources are the random fluctuations in determining the auto-zero correction, the departure from linearity of the phase angle scale, and to a lesser extent the slight nonlinearity of the auto-zero frequency adjustment. An additional error component is introduced when the signal amplitudes are unequal and is due to small systematic offsets in the phase detector signal conditioning circuitry that may not be fully compensated. The following table provides an estimate of these uncertainties:

ESTIMATED MAXIMUM PHASE ANGLE UNCERTAINTY (IN MILLIDEGREES)

	60 Hz	400 Hz	5 kHz	15 kHz	30 kHz	50 kHz
Auto-Zero Corr.*	±1	±2	±5	±6	±12	±20
Linearity	±2	±2	±3	±10	±15	±20
7:1 Voltage Ratio	±1	±2	±3	**	**	±40

*The uncertainty due to frequency adjustment is included.
**Data not available.

As an example of the calibrations of a commercial phase meter that can be easily carried out using the Phase Angle Calibration Standard, Fig. 11 shows a plot of corrections determined for the same meter, one set (open circles) taken 11 months prior to the two others (solid lines), which were taken one day apart. Each test point shown represents the average of four readings. The day-to-day variations evident on the graph were due to fluctuations in the phase meter response. Readings taken at the same time on another meter show no correlation. The estimated uncertainty of the Phase Angle Calibration Standard at the test frequency (60 Hz) was almost an order of magnitude better than the specification limits of the meter under test.

VIII. Conclusion

A phase angle calibration standard has been designed and constructed that provides a convenient way to calibrate audiofrequency phase measuring equipment for frequencies from 2 Hz to 50 kHz. The digital techniques used to generate the test signals provide stable and highly repeatable results. The instrument lends itself to other applications such as providing a signal source for power and energy measurements [6].

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Automatic Direct Volts Stability System

STEVEN G. HAYNES

***Abstract*—The introduction of automatic test equipment to the production environment represented a milestone in manufacturing technology. The marriage of computer automation and precision measurement technology has produced many systems with capability never before thought possible, either economically or quantitatively.**

An automated test system using precision direct voltage measurement techniques, extended IEEE-488 bus capability, and software algorithms for data processing has made possible the Automatic Direct Volts Stability System, used in the production of the Fluke 5440A Direct Voltage Calibrator and 732A DC Reference Standard. With a capacity of sixty units per month, and absolute uncertainty better than one ppm, this system has demonstrated the cost effectivity, reliability, and accuracy necessary to ensure the stability specifications of these Fluke products.

This paper will detail the hardware and software of the system design, the system's capabilities and specifications, and the test data available.

I. Introduction

THE concurrent development of the Fluke 732A DC Reference and 5440A Direct Voltage Calibrator presented some unique challenges in both the fields of metrology and manufacturing engineering. The precise accuracy and stability specifications of these two instruments required traceable uncertainties several times better than anything previously used on a production basis. And the long-term stability requirements necessitated a stability measurement system that was accurate, fail safe, and did not contribute excessively to the instruments' factory cost.

The key to solving the problem of a supporting standard has been provided by the highly stable solid-state references used in the 732A Reference Standard. A family of selected 732A's verified monthly against The Fluke Primary Standards Laboratory's own family of solid-state references, has become the production transfer standard providing the high confidence, low uncertainties, and excellent long-term stability characteristics required for large scale traceable transfers.

The recent advances in measurement technology provided a supporting standard, but a cost-effective solution to making precise voltage measurements over long period of time on a large scale had to be found before the stability specifications could be guaranteed. Manual measurements were obviously out of the question—the labor costs were prohibitive and the introduction of human error was inevitable. A fully automated system with the software and hardware to provide instrument control, signal path switching, references and data measurement, recording capability, as well as self-verification has provided the solution to this measurement problem.

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