A Method of Determining the Mismatch Correction in Microwave Power Measurements

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Abstract—With the increasing demands for accuracy in microwave measurements, mismatch corrections are assuming an increased importance. In the application of a terminating-type power meter, the appropriate mismatch factor involves the complex reflection coefficients of the generator, load, and power meter. Instead of measuring the individual reflection coefficients, which usually calls for the use of a slotted line, a directional coupler technique is described that obtains the mismatch factor directly.

Introduction

A LARGE FRACTION of the microwave power meters in current use are of the terminating type. The practical application of these devices requires their connection to the signal source in place of the load for which the power is required. Ideally, both load and power meter terminate the transmission line or waveguide in its characteristic impedance, and the power delivered to the load is given by the power meter reading.

With increasing accuracy requirements, it often becomes necessary to explicitly account for the difference in the load and power meter impedances. It has been shown [1] that, in general, the power P_l delivered to the load is given by

$$P_l = P_m \frac{|1 - \Gamma_g\Gamma_m|^2(1 - |\Gamma_l|^2)}{|1 - \Gamma_g\Gamma_l|^2(1 - |\Gamma_m|^2)}, \tag{1}$$

where P_m is the power meter indication and Γ_g, Γ_l, and Γ_m are, respectively, the reflection coefficients of the generator, load, and power meter. Thus, a complete evaluation of P_l requires, in addition to P_m, the complex values of the reflection coefficients Γ_g, Γ_l, Γ_m.

A substantial simplification in this expression results if $\Gamma_g = 0$. In the power calibration laboratory, the parameters of the generator are usually at one's disposal and such an adjustment is possible. In the typical field application, however, this is not possible and because the mismatch correction [the coefficient of P_m in (1)] involves the measurement of three complex reflection coefficients and a rather involved computation, this correction is often, if not usually ignored, a practice that is hardly in keeping with the increasing accuracy requirements.

It is the purpose of this paper to describe an alternative method, based upon directional coupler techniques, that provides a direct measurement of the mismatch correction.

Theory

Beginning with the circuit of Fig. 1, it is convenient to write

$$P_{gl} = P_g M_{gl}, \tag{2}$$

where P_{gl} is the power delivered by the generator to the load, P_g is the (maximum) available power from the generator, and M_{gl} is a mismatch factor whose value ranges between zero and unity, depending upon how well the conditions for maximum power transfer are satisfied.[1] In a similar manner, if the load is replaced by the power meter, it will indicate a power P_{gm} given by

$$P_{gm} = P_g M_{gm}, \tag{3}$$

where M_{gm} is the mismatch factor for the power meter.

Equations (2) and (3) may be combined to yield

$$P_{gl} = P_{gm} \frac{M_{gl}}{M_{gm}}. \tag{4}$$

The mismatch factor M_{gl} is given by[2]

$$M_{gl} = \frac{(1 - |\Gamma_g|^2)(1 - |\Gamma_l|^2)}{|1 - \Gamma_g\Gamma_l|^2} \tag{5}$$

with a similar expression for M_{gm}.

The measurement of M_{gl} and M_{gm} may be effected by means of a pair of directional couplers and tuning transformers connected as shown in Fig. 2. Although the order of the couplers has been reversed, the basic configuration will be recognized as that of a reflectometer. Historically [2], the tuning transformers T_x and T_y were introduced for the purpose of achieving an ideal reflectometer response. In time, the same basic configuration

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[1] The slightly more general terminology introduced here is in anticipation of a further development of the subject. Thus P_{gl} is a function of both generator and load parameters, while P_g is a function only of the former.

[2] This result has been recorded by several authors. It can also be obtained by noting that if $\Gamma_m = \Gamma_g^*$, then $P_m = P_g$. Substitution of these results in (1) and comparing with (2) yields (5).

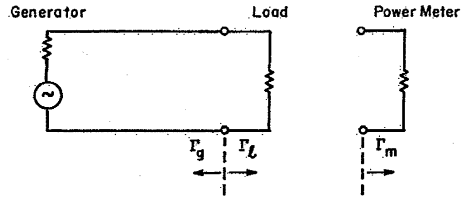

Fig. 1. Equivalent circuit for mismatch error evaluation.

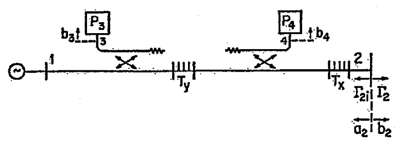

Fig. 2. Generalized reflectometer for measurement of mismatch factors.

of couplers and tuning transformers (but with different tuning adjustments) was proposed for several additional measurement applications [3], [4], [5]. Because of the widespread utility of this arrangement, a name is desirable and the term "generalized reflectometer" (g-reflectometer) is suggested as reflecting the similarity to, yet emphasizing the distinction from, the usual reflectometer.

The application of the g-reflectometer to the measurement of M_{gl} and M_{gm} will now be developed.

In accord with usual practice, it is assumed that the g-reflectometer is provided with uniform and lossless waveguide leads. The ratio between the emergent wave amplitudes b_3 and b_4 at ports 3 and 4 may then be expressed in terms of the incident and emergent waves at port 2 as follows[3]:

$$\frac{b_3}{b_4} = \frac{Aa_2 + Bb_2}{Ca_2 + Db_2}, \tag{6}$$

where A, B, C, and D are parameters of the g-reflectometer including the detectors. Provided that the junction is source free, this relationship is completely general. By use of the tuning transformers T_x, T_y, it is possible (as described later) to impose the following conditions:

$$BD^* - AC^* = 0 \tag{7}$$

$$C + D\Gamma_g = 0, \tag{8}$$

where (*) denotes the complex conjugate and Γ_g is the reflection coefficient of the generator of Fig. 1. Let Γ_2 represent the reflection coefficient of an arbitrary passive termination on arm 2 of the g-reflectometer. Then $\Gamma_2 = a_2/b_2$. Substitution of (7) and (8) into (6) yields

$$\frac{P_3}{P_4} = \frac{|A|^2}{|D|^2} \frac{|\Gamma_2 - \Gamma_g^*|^2}{|1 - \Gamma_2\Gamma_g|^2}, \tag{9}$$

[3] For a more complete discussion of this result, see [5].

where $P_3(=|b_3|^2)$ is the power indicated by the detector on arm 3 and similarly for P_4.

Now (5) for M_{gl} may be written in the form

$$M_{gl} = 1 - \frac{|\Gamma_l - \Gamma_g^*|^2}{|1 - \Gamma_g\Gamma_l|^2}, \tag{10}$$

while if port 2 is terminated by a short (of arbitrary phase) $|\Gamma_2| = 1$ and (9) becomes

$$\left.\frac{P_3}{P_4}\right|_{|\Gamma_2|=1} = \frac{|A|^2}{|D|^2}. \tag{11}$$

Finally

$$M_{gl} = 1 - \frac{\left.\frac{P_3}{P_4}\right|_{\Gamma_2=\Gamma_l}}{\left.\frac{P_3}{P_4}\right|_{|\Gamma_2|=1}} \tag{12}$$

and a similar equation obtains for M_{gm}.

The measurement procedure thus consists of tuning adjustment of T_x and T_y (to be described in a following paragraph), and then observing the ratio P_3/P_4 as the fixed short, the load, and the power meter are connected in turn to arm 2. From these observations, M_{gl} and M_{gm} may be calculated using (12), and P_{gl} is then computed from (4). If desired, the maximum available power may be determined from (3). In addition to providing a determination of P_{gl}, the parameters M_{gl}, M_{gm}, and P_g are usually of considerable interest in their own right.

Tuning Procedure

In order for the junction parameters to satisfy (7) and (8), two tuning adjustments are necessary. These are provided by the transformers T_x and T_y.

In order to adjust T_x, it is necessary to first replace the generator on arm 1 by a passive load. The g-reflectometer is next excited via arm 2 and tuner T_y adjusted[4] for a null in arm 4. Tuner T_x is then adjusted such that the reflection coefficient Γ_{2i} looking into port 2 is now equal to Γ_g.

This may be conveniently done, with the help of an auxiliary directional coupler and tuner, as follows. In Fig. 3, the active element in the generator is turned off such that it becomes a passive load.[5] It is then connected to the auxiliary coupler and tuner T_a is adjusted for a side-arm null. The generator is then replaced by the g-reflectometer, tuner T_y is adjusted for a null in arm 4 (of the g-reflectometer), and T_a is adjusted for a side-

[4] Alternatively, an adjustable load on arm 1 could be used to achieve this condition. However, since T_y is available, its use is convenient.

[5] It is assumed, as is usually the case, that enough isolation is present to ensure that the generator impedance is independent of the actual energy source.

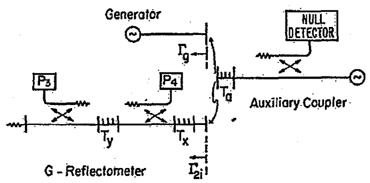

Fig. 3. Procedure for adjusting T_x.

arm null in the auxiliary coupler. This completes the adjustment of T_x and (8) is now satisfied. It remains to show that the adjustment of T_x is invariant to the adjustment of T_y, which follows. This is the subject of Appendix I.

The adjustment of T_y calls first for a reconnection of the usual signal generator to arm 1 of the g-reflectometer. Arm 2 is then terminated by a sliding short and tuner T_y is adjusted such that the ratio P_3/P_4 remains constant for all positions of the short. The g-reflectometer then satisfies (7).[6]

Conclusions

The described procedure thus provides a direct determination of the mismatch corrections that are required if the accuracy expectancy of today's power meters is to be realized. In complexity, the procedure is nominally equivalent to starting with an untuned reflectometer and making the appropriate tuning adjustments and measurements as required in the precise determination of reflection coefficient [2]. By contrast, if the mismatch factor is to be determined from (1) or (5), a measurement of the complex reflection coefficients is implied and this requires either the use of a slotted line or a modification of the usual reflectometer configuration such that the argument, as well as the amplitude of the reflection coefficient, is obtained. Following the measurement of the individual reflection coefficients, a rather awkward computation is also required that is avoided by the new technique. In addition, the new method requires neither a precision (slotted) section of waveguide nor a matched load.[7]

Although a detailed analysis has not been made, the accuracy expectancy of this technique is better than for the existing methods because of the smaller number of quantities to be measured. Finally, because it is the *difference* between M_{gl} and unity that is thus measured, and because this difference is usually small, M_{gl} is determined to a high degree of accuracy.

Appendix I

It will be shown that the adjustment of T_x, to achieve the $C+D\Gamma_g=0$ condition, is invariant to the adjustment of T_y.

[6] This condition has been required in all applications of the g-reflectometer proposed to date. Although this result has been previously quoted, its derivation has not been recorded. This is given in Appendix II.

[7] The implications of this will be developed in a paper to follow.

This could be done in a completely formal manner by finding the appropriate expressions for C and D in terms of the parameters of the two couplers and tuning transformers; this approach, however, is tedious. Instead, in Figs. 2 and 3, let an auxiliary terminal plane be temporarily inserted between the tuner T_y and the second coupler (the one on the right). Within this second coupler (including T_x), the electromagnetic fields are completely determined by a_2 and b_2, and, in particular, the field in arm 4 can be written

$$b_4 = C'a_2 + D'b_2, \tag{13}$$

where C' and D' are functions of the parameters of the second coupler and T_x (but not of T_y). If b_4 is eliminated between (6) and (13), it becomes apparent that a nontrivial linear relationship exists between b_3, b_2, and a_2, if, and only if, $C/D=C'/D'$. Since the ratio C'/D' is invariant to the adjustment of T_y, the same is true of C/D. Thus if this ratio is such that (8) is satisfied for one particular adjustment of T_y, the same is true for all possible adjustments.

Appendix II

Referring to Fig. 2, it will be shown that (7) is implied if the ratio P_3/P_4 remains constant for all positions of a sliding short on arm 2.

With arm 2 terminated by a sliding short, b_2 and a_2 are related by

$$a_2 = b_2 e^{i\theta}. \tag{14}$$

Substituting in (6) and taking the absolute value yields

$$\frac{P_3}{P_4} = \left| \frac{Ae^{i\theta} + B}{Ce^{i\theta} + D} \right|^2. \tag{15}$$

By hypothesis, the ratio P_3/P_4 is a constant that will be denoted by K. Substitution of K and expansion of (15) yields

$$|A|^2+|B|^2+AB^*e^{i\theta}+A^*Be^{-i\theta} \\ -K(|C|^2+|D|^2+CD^*e^{i\theta}+C^*De^{-i\theta})=0. \tag{16}$$

If this equation is to hold for arbitrary values of θ, the coefficients of the several powers of $e^{i\theta}$ must vanish independently. This leads to

$$AB^* - KCD^* = 0 \tag{17}$$

$$|A|^2 + |B|^2 - K(|C|^2 + |D|^2) = 0. \tag{18}$$

(The third equation is the complex conjugate of (17) and contains no additional information.)

Elimination of K between (17) and (18) yields

$$CD^*[|A|^2 + |B|^2] = AB^*[|C|^2 + |D|^2]. \tag{19}$$

Multiplication of this expression by B/B^* and using the result $A^*BCD^* = AB^*C^*D$ (which follows from (17) since K is real) leads to the result

$$A^2C^*D - AB(|C|^2 + |D|^2) + B^2CD^* = 0, \tag{20}$$

which may be factored to yield

$$(AD - BC)(AC^* - BD^*) = 0. \tag{21}$$

This equation admits of two solutions; however, the $AD-BC=0$ solution is trivial in the present context, since it yields a value for P_3/P_4 that is completely independent of the terminating load. (As a practical measure, this adjustment would result if one of the couplers were reversed, and is useful in a different context.) This leaves the condition $AC^*-BD^*=0$, which was to be proved.

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Broad-Band Microwave Transmission Characteristics from a Single Measurement of the Transient Response

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***Abstract*—A technique is presented for the measurement of the transmission and reflection scattering coefficients of microwave networks by Fourier analysis of the sampled transient response of the network to impulsive or steplike waveforms. It is demonstrated that the method can reduce errors due to mismatch of the components of the measurment system by sampling the response over a finite time domain window that excludes unwanted reflected components. A review is made of the errors introduced due to aliasing, truncation, and noiselike sources such as timing jitter and additive noise, and a description is then given of an experimental system to evaluate the technique, based on use of a 12-GHz sampling oscilloscope for measurement of the transient response and a suitable form of the fast Fourier transform algorithm. Measurements on some typical wideband components are presented, and it is concluded that for very broad-band measurements with moderate resolution the method has a potential accuracy of about ±0.1 dB and ±1°, with a significant reduction in mismatch errors.**

I. Introduction

Components for systems such as broad-band radars frequently must have a close tolerance over a wide frequency range of both the amplitude and phase of a system function in order to minimize waveform distortion. Measurements to accuracies like ±0.1 dB in amplitude and ±1° in phase with homodyne or slotted line techniques require considerable experimental sophistication and may become very time consuming; this has led to the advent of computer-controlled network analyzers [1]. The recent availability of sampling oscilloscopes allowing the display of frequency components of waveforms up to 12 GHz has suggested that an alternative method to obtain, for example, the transmission scattering coefficient of a component might be by a single measurement of its transient response to an impulselike or step waveform, followed by the use of appropriate Fourier analysis techniques to transform this into the system function. Measurement of the reflected response similarly can yield the reflection coefficient over a wide range of frequencies. With the availability of small instrumentation computers and the fast Fourier transform algorithm, real-time Fourier analysis could form the basis of an alternative frequency domain measurement tool. Various analog methods [2], [3] have been reported for low-frequency spectral analysis using swept oscillators or delay lines, but while these are suitable for display, they do not offer the potential flexibility and accuracy of a digital method.

In this paper we discuss the feasibility of a time domain technique and experimental work to demonstrate its capability. In Section II, the principles of the method are discussed for a general measurement system. The component parts of a practical microwave time domain measurement system are reviewed in Section III. The various sources of error peculiar to the time domain measurement are then discussed in Section IV, and some experimental results presented in Section V.

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