Generation of All Symmetric and All Total Angular Momentum Eigenstates in Photonic or Remote Qubits

Friedrich-Alexander-Universität Erlangen-Nürnberg

J. von Zanthier[1], A. Maser[1], R. Wiegner[1], U. Schilling[1], C. Thiel[1], T. Bastin[2], E. Solano[3], G. S. Agarwal[4]

[1]Institut für Optik, Information und Photonik, Friedrich-Alexander Universität Erlangen-Nürnberg, Erlangen, Germany
[2]Institut de Physique Nucléaire, Atomique et de Spectroscopie, Université de Liège au Sart Tilman, Liège, Belgium
[3]Departamento de Química-Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea, Bilbao, Spain
[4]Department of Physics, Oklahoma State University, Stillwater, Oklahoma, USA

We propose a method which allows to couple N remote single photon emitters with a Λ-configuration to form any symmetric N-qubit state [1,2] or any (symmetric or non-symmetric) total angular momentum eigenstates of the N-qubit compound [3]. Modifying slightly the setup, the same classes of states can also be generate among the polarization degrees of freedom of N photons [4].

References:

[1] C.Thiel *et al.*, Phys. Rev. Lett. **99**, 193602 (2007)

[2] T. Bastin *et al.*, Phys. Rev. Lett. **102**, 053601 (2009)

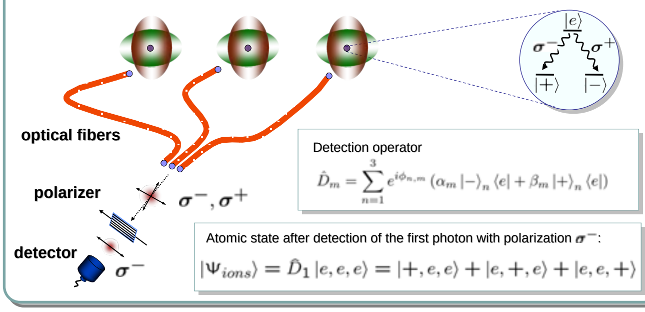
[3] A. Maser *et al.*, Phys. Rev. A **79**, 033833 (2009)

[4] A. Maser *et al.*, Phys. Rev. A **81**, 053842 (2010)

[5] T. Bastin *et al.*, Phys. Rev. Lett. **103**, 070503 (2009)

I. How to Entangle Remote Emitters by Projection

Example for three trapped ions



Detection operator

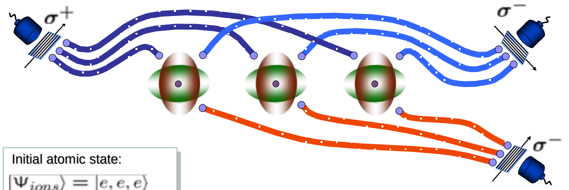
$$\hat{D}_m = \sum_{n=1}^{3} e^{i\phi_{n,m}} \left(\alpha_m |-\rangle_n \langle e| + \beta_m |+\rangle_n \langle e|\right)$$

Atomic state after detection of the first photon with polarization σ^-:

$$|\Psi_{ions}\rangle = \hat{D}_1 |e,e,e\rangle = |+,e,e\rangle + |e,+,e\rangle + |e,e,+\rangle$$

II. Generation of long-living W-state in Emitters

Detection of three photons



Initial atomic state:

$$|\Psi_{ions}\rangle = |e,e,e\rangle$$

Detection of a σ^-–polarized photon

$$|\Psi_{ions}\rangle = \hat{D}_1 |e,e,e\rangle = |+,e,e\rangle + |e,+,e\rangle + |e,e,+\rangle$$

Detection of a σ^-–polarized photon

$$|\Psi_{ions}\rangle = \hat{D}_2\hat{D}_1 |e,e,e\rangle = |+,+,e\rangle + |+,e,+\rangle + |e,+,+\rangle$$

Detection of a σ^+–polarized photon

$$|\Psi_{ions}\rangle = \hat{D}_3\hat{D}_2\hat{D}_1 |e,e,e\rangle = |+,+,-\rangle + |+,-,+\rangle + |-,+,+\rangle$$

III. Generation of any symmetric State in Emitters

Usage of elliptical polarizers allows the generation of any symmetric N-qubit state

Any symmetric N-qubit state $|\psi\rangle$ can be expressed as a sum of symmetric Dicke states $|D_N(k)\rangle$, with $|D_N(k)\rangle$ being the symmetric Dicke state with k $|+\rangle$ excitations.

$$|\psi\rangle = \sum_{k=0}^{N} c_k |D_N(k)\rangle$$

Example: 3-qubit Dicke states

$$|D_3(0)\rangle = |---\rangle$$
$$|D_3(1)\rangle = \frac{1}{\sqrt{3}}(|+--\rangle + |-+-\rangle + |--+\rangle)$$
$$|D_3(2)\rangle = \frac{1}{\sqrt{3}}(|++-\rangle + |+-+\rangle + |-++\rangle)$$
$$|D_3(3)\rangle = |+++\rangle$$

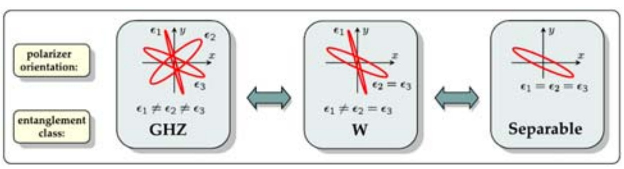
The coefficients c_k are used to construct the polynomial $P(z)$.

$$P(z) = \sum_{k=0}^{N} (-1)^{N-k} \binom{N}{k}^{\frac{1}{2}} c_k z^k$$

The roots z_i of this polynomial define the polarizer orientations $\epsilon_i = \alpha_i\sigma^+ + \beta_i\sigma^-$.

$$z_i = \frac{\alpha_i}{\beta_i}$$

In case of a 3-qubit state there is furthermore a simple correspondence between polarizer orientation and the entanglement class of the generated state [1,5].



IV. Generation of all Total Angular Momentum Eigenstates in Emitters

Explanation of the algorithm: mimicking the coupling of angular momentam

Total angular momentum (TAM) eigenstates $|S,m\rangle$ are defined as the eigenstates of the total angular momentum operator $\hat{\mathbf{S}}$ and its z-component $\hat{S}_z$.

$$\hat{\mathbf{S}}^2 |S,m\rangle = S(S+1)\hbar^2 |S,m\rangle$$
$$\hat{S}_z |S,m\rangle = m\hbar |S,m\rangle$$

Generally, N qubits can be coupled to 2^N different quantum states.

We exemplify our method for this state !

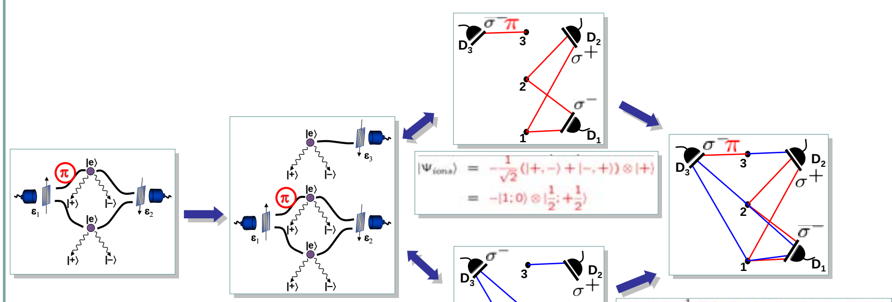
1. Example: 2-qubit system

spin-1 triplet	$\|S,m\rangle$
$\|++\rangle$	$\|1,1\rangle$
$\frac{1}{\sqrt{2}}(\|+-\rangle + \|-+\rangle)$	$\|1,0\rangle$
$\|--\rangle$	$\|1,-1\rangle$
spin-0 singlet	$\|S,m\rangle$
$\frac{1}{\sqrt{2}}(\|+-\rangle - \|-+\rangle)$	$\|0,0\rangle$

2. Example: 3-qubit system

spin-$\frac{3}{2}$ quartet	$\|S,m\rangle$
$\|+++\rangle$	$\|\frac{3}{2},+\frac{3}{2}\rangle$
$\frac{1}{\sqrt{3}}(\|++-\rangle + \|+-+\rangle + \|-++\rangle)$	$\|\frac{3}{2},+\frac{1}{2}\rangle$
$\frac{1}{\sqrt{3}}(\|+--\rangle + \|-+-\rangle + \|--+\rangle)$	$\|\frac{3}{2},-\frac{1}{2}\rangle$
$\|---\rangle$	$\|\frac{3}{2},-\frac{3}{2}\rangle$

first spin-$\frac{1}{2}$ doublet	$\|S,m\rangle$
$\frac{1}{\sqrt{6}}(2\|++-\rangle - \|+-+\rangle - \|-++\rangle)$	$\|\frac{1}{2},+\frac{1}{2}\rangle$
$\frac{1}{\sqrt{6}}(\|+--\rangle + \|-+-\rangle - 2\|--+\rangle)$	$\|\frac{1}{2},-\frac{1}{2}\rangle$
second spin-$\frac{1}{2}$ doublet	$\|S,m\rangle$
$\frac{1}{\sqrt{2}}(\|+-+\rangle - \|-++\rangle)$	$\|\frac{1}{2},+\frac{1}{2}\rangle$
$\frac{1}{\sqrt{2}}(\|+--\rangle - \|-+-\rangle)$	$\|\frac{1}{2},-\frac{1}{2}\rangle$



$$|\Psi_{ions}\rangle = -\frac{1}{\sqrt{2}}(|+,-\rangle + |-,+\rangle) \otimes |+\rangle = -|1;0\rangle \otimes |\tfrac{1}{2};+\tfrac{1}{2}\rangle$$

$$|\Psi_{ions}\rangle = 2|+,+\rangle \otimes |-\rangle = 2|1;+1\rangle \otimes |\tfrac{1}{2};-\tfrac{1}{2}\rangle$$

Pick out two particular setups

$$|\Psi_{ions}\rangle = \frac{1}{\sqrt{6}}(2|+,+,-\rangle - |+,-,+\rangle - |-,+,+\rangle) = \frac{\sqrt{2}}{\sqrt{3}}|1;+1\rangle \otimes |\tfrac{1}{2};-\tfrac{1}{2}\rangle - \frac{1}{\sqrt{3}}|1;0\rangle \otimes |\tfrac{1}{2};+\tfrac{1}{2}\rangle$$

2-particle system + ½-spin particle
=
3-particle system

Detection
(two photons measured)

ϵ_1	ϵ_2	$\|state\rangle$
σ^-	σ^-	$\|++\rangle$
σ^-	σ^+	$\|+-\rangle + \|-+\rangle$
σ^-	σ^+	$\|+-\rangle - \|-+\rangle$
σ^+	σ^+	$\|--\rangle$

Including π shift

Detection
(three photons measured)

ϵ_1	ϵ_2	ϵ_3	$\|state\rangle$
σ^-	σ^-	$\sigma^\mp$	$\|++\rangle \otimes \|\pm\rangle$
σ^-	σ^+	$\sigma^\mp$	$(\|+-\rangle + \|-+\rangle) \otimes \|\pm\rangle$
σ^-	σ^+	$\sigma^\mp$	$(\|+-\rangle - \|-+\rangle) \otimes \|\pm\rangle$
σ^+	σ^+	$\sigma^\mp$	$\|--\rangle \otimes \|\pm\rangle$

Including π shift

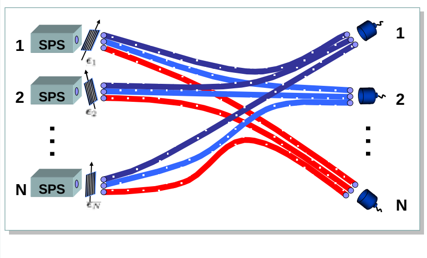
→ This approach corresponds to a successive coupling of angular momenta.

→ We have found an algorithm that transforms the coupling of angular momenta into explicit experimental setups.

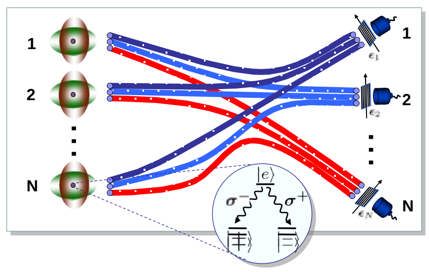
V. Generation of all above States in Photonic Qubits

Moving the polarizers from the detectors to the single photon sources allows to generate the equivalent states encoded in the polarization degrees of freedom of N photons [4].

Photonic qubits



Atomic qubits



State $|\psi_{\text{phot}}\rangle$ compatible with a successful detection event:

$$\hat{P}_n = \sum_{m=1}^{N} e^{i\phi_{n,m}} \left(\alpha_n |\sigma^+\rangle_m \langle 0| + \beta_n |\sigma^-\rangle_m \langle 0|\right)$$
$$|\psi_{\text{phot}}\rangle = \hat{P}_N \ldots \hat{P}_2\hat{P}_1 |0,0,\ldots,0\rangle .$$

$\cong$

State $|\psi_{\text{atom}}\rangle$ compatible with a successful detection event:

$$\hat{D}_m = \sum_{n=1}^{N} e^{i\phi_{n,m}} \left(\alpha_m |-\rangle_n \langle e| + \beta_m |+\rangle_n \langle e|\right)$$
$$|\psi_{\text{ions}}\rangle = \hat{D}_N \ldots \hat{D}_2\hat{D}_1 |e,e,\ldots,e\rangle .$$

→ Direct transfer of results obtained for the generation of entangled states in matter qubits is possible.