Robust Generation of Entanglement and Gate Operations with Trapped Ions Using Adiabatic Approaches

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Dicke state generation with RAP

Dicke states

Symmetric Dicke states:

Entangled states symmetric with regard to permutation of particles

$$\left|D_N^m\right\rangle = \frac{1}{\sqrt{{}_N C_m}} \sum_k P_k \left(\left| \underbrace{\uparrow \ldots \uparrow}_{m} \underbrace{\downarrow \ldots \downarrow}_{N-m} \right\rangle \right)$$

N : Number of particles, m : Number of excitations

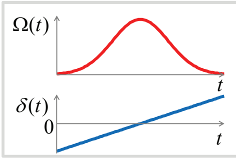
P_k : permutation of particles, ${}_N C_m = N!/[m!(N-m)!]$

$$\left|D_2^1\right\rangle = \frac{1}{\sqrt{2}} \left(\left|\downarrow\uparrow\right\rangle + \left|\uparrow\downarrow\right\rangle \right)$$ (One of the Bell states)

- Single photon generation by collective coupling
- Projective measurement on large Dicke state yields different classes of entangled states
  - use as entanglement resources for QIP

Rapid adiabatic passage(RAP/ARP)

Population transfer using optical pulse with time-dep. envelope/frequency



Advantages:

- Robustness against pulse area (intensity/time) fluctuation
- One step transfer over multi-level ladder

Disadvantages:

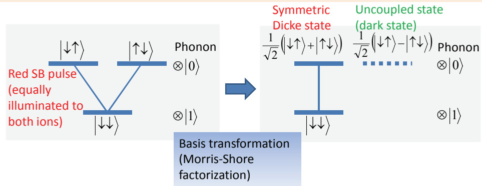
- Dynamical phase due to time-dependent pulse envelope leading to uncontrollable global phase

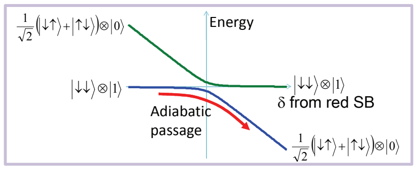
Applications:

- Large (entangled) state preparation

Dicke state gen. by RAP

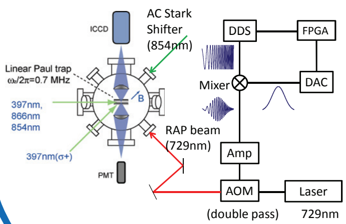
I. E. Linington, and N. V. Vitanov, Phys. Rev. A 77 010302 (2008).

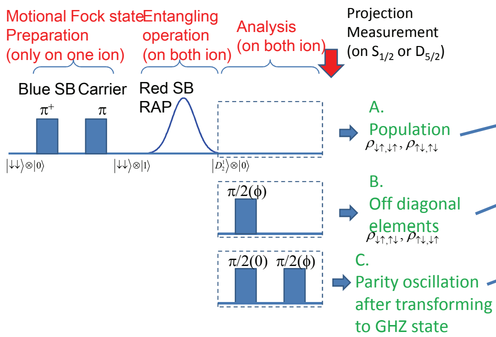


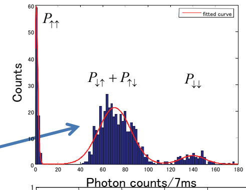


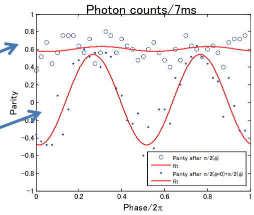
Dicke state generation with square pulse:
D. B. Hume, C. W. Chou, T. Rosenband, D. J. Wineland, Phys. Rev. A 80, 052302 (2009)

Setup









$\rho_{\downarrow\uparrow,\downarrow\uparrow} + \rho_{\uparrow\downarrow,\uparrow\downarrow} \cong 0.74 \pm 0.06$, $2\,\mathrm{Re}(\rho_{\downarrow\uparrow,\uparrow\downarrow}) \cong 0.58 \pm 0.02$

$\Rightarrow \quad F = 0.66 \pm 0.03$

Summary

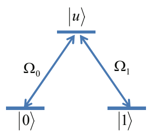
- Rapid adiabatic passage of two ions in sideband transitions
- Individual addressing using AC Stark shifter optical beam
- Entanglement generation by adiabatic passage Fidelity 0.66±0.03
- Effect of AC Stark shift due to time-dependent pulse envelope analyzed

Prospects

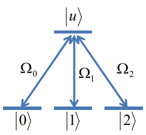
- Dicke state generation with more particles/excitations

Robust single-qubit gate with tripod-STIRAP

STIRAP in λ-type three-level system
Population transfer using a dark state

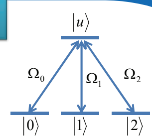


STIRAP in tripod-type four-level system
Two dark states
Geometric phase factors form 2x2 matrix
non-commutable(non-Abelian) unitary operations



Gate operations using tripod-STIRAP
Robust operations
Insensitive to pulse shapes (peak height, length)

Hamiltonian for the tripod system



$$\hat{H}(t) = -\hbar\delta|u\rangle\langle u| + \frac{\hbar}{2}\sum_{k=0}^{2}\left[\Omega_k(t)|k\rangle\langle u| + \mathrm{H.c.}\right]$$

where $\Omega_0(t) = \Omega(t)\cos\chi, \Omega_1(t) = \Omega(t)e^{i\eta}\sin\chi$ ($\Omega(t)$: real, $0 \le \chi \le \pi/2, 0 \le \eta < 2\pi$)

Dark (uncoupled) state and coupled state with field 0 and 1

$$|D_1\rangle = -\sin\chi|0\rangle + e^{i\eta}\cos\chi|1\rangle$$

$$|C\rangle = \cos\chi|0\rangle + e^{i\eta}\sin\chi|1\rangle$$

Hamiltonian rewritten

$$\hat{H}(t) = -\hbar\delta|u\rangle\langle u| + \frac{\hbar}{2}\left[\Omega(t)|C\rangle\langle u| + \Omega_2(t)|2\rangle\langle u| + \mathrm{H.c.}\right]$$

Second dark state

$$|D_2(t)\rangle = \frac{1}{\sqrt{\Omega^2(t) + |\Omega_2(t)|^2}}\left[\Omega_2^*(t)|C\rangle - \Omega(t)|2\rangle\right]$$

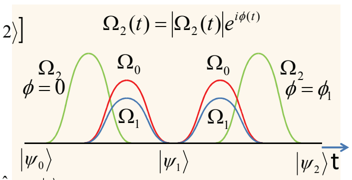
State change caused by two STIRAP

$$|i\rangle \equiv \alpha|0\rangle + \beta|1\rangle$$

$$|\psi_0\rangle = |i\rangle = \langle D_1|i\rangle|D_1\rangle + \langle C|i\rangle|C\rangle$$

$$|\psi_1\rangle = \langle D_1|i\rangle|D_1\rangle - \langle C|i\rangle|2\rangle$$

$$|\psi_2\rangle = |i\rangle = \langle D_1|i\rangle|D_1\rangle + e^{-i\phi_1}\langle C|i\rangle|C\rangle = e^{-i\phi_1/2}\hat{R}_{\mathbf{n}}(\phi_1)|i\rangle$$



$$\mathbf{n} = (\sin 2\chi\cos\eta, \sin 2\chi\sin\eta, \cos 2\chi), \quad \hat{R}_{\mathbf{n}}(\phi) \equiv \exp\left(-i\frac{\phi}{2}\mathbf{n}\cdot\hat{\boldsymbol{\sigma}}\right) = \cos\frac{\phi}{2}\hat{I} - i\mathbf{n}\cdot\hat{\boldsymbol{\sigma}}\sin\frac{\phi}{2}$$

References

Tripod system

- R. Unanyan et al., Opt. Comm. 155, 144 (1998); R. G. Unanyan et al., Phys. Rev. A 59, 2910 (1999).

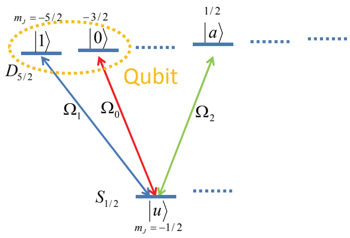
Gate operations/Holonomic QC

- L. M. Duan et al., Science 292, 1695 (2001); R. G. Unanyan et al., Phys. Rev. A 69, 050302 (2004);
- Z. Kis and F. Renzoni, Phys. Rev. A 65, 032318 (2002); D. Moller et al., Phys. Rev. A 75, 062302 (2007).

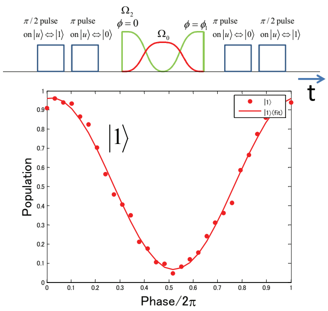
Applications of non-Abelian holonomy to cold atoms

- K. Osterloh et al., Phys Rev Lett 95, 010403 (2005); J. Ruseckas et al., Phys Rev Lett 95, 010404 (2005)

Encoding to levels in single $^{40}Ca^+$

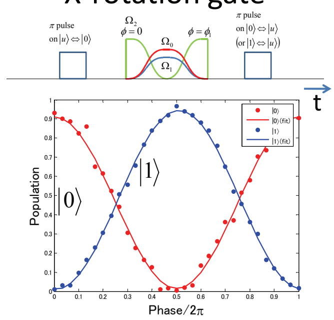


Z-rotation gate



ϕ_1 Fringe visibility: 0.90

X-rotation gate



ϕ_1 Fringe visibility for $|0\rangle$: 0.90
for $|1\rangle$: 0.93

Summary

X-rotation and Z-rotation with two STIRAP on a tripod system

Demonstrated using single $^{40}Ca^+$

Visibility≧0.9

Prospects

Verifying robustness

Implement to S-D optical qubit