Simulation of non-Adiabaticity in Surface Electrode Traps

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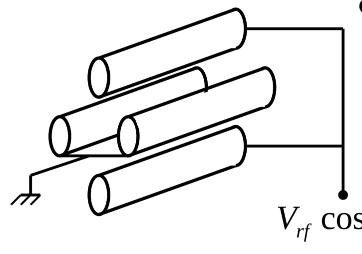
Abstract:

Radio frequency (RF) surface electrode traps where all the RF and static control electrodes lie in a single plane are being developed for scaling ion trap quantum information processing to larger numbers of ions [1]. In such traps, ions are confined a small distance above the electrode surface. The RF ponderomotive potential is anharmonic, rising steeply for ion excursions towards the electrode surface but much more gradually for excursions away from the surface. Multipole RF traps can exhibit a significant reduction in the effective trapping depth from the calculated psuedo-potential well depth due to non-adiabatic ion motion in the anharmonic potential [2]. We simulate whether anharmonic contributions to surface electrode trap potentials can reduce the effective well depth of surface electrode traps. Specifically we simulate the motion of a charged particle in a 4-wire surface electrode trap. By starting the particle in the center of the trap with different energies we determine a safe or effective well depth as a function of the q parameter of the trap. We find significant reduction in the effective well depth for q > 0.3, resulting in a maximum well depth for q < 0.3.

1. Microfabricated Chip Traps for Ions, J. M. Amini, J. Britton, D. Leibfried, and D.J. Wineland. Chapter in the upcoming book "Atom chips" edited by J. Reichel and V. Vuletic (to be published by WILEY-VCH); J.H. Wesenberg, Phys. Rev. A 78, 063410 (2008).
2. J. Mikosh et al., Phys. Rev. Lett. 98, 223001 (2007).

Linear quadrupole vs linear multipole traps

linear rf trap:

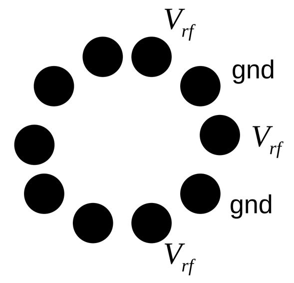


$$\Phi_{rf}(r)=\frac{1}{2}V_{rf}\cos(\Omega t)\left(1+\frac{x^2-y^2}{d^2}\right)$$ $d =$ characteristic trap dimension

$\omega_r \equiv$ secular frequency $\qquad \omega_r=\frac{q}{2\sqrt{2}}\Omega$

$$q\equiv\frac{2eV_{rf}}{m\Omega^2 d^2}$$ confinement for $0<q<\sim 0.9$

multipole rf trap:



an n rod trap produces a pseudo-potential $\sim r^{n/2}$

equations of motion have no analytical solution; regions of stability not well defined; stability of ion motion strongly depends on initial position, velocity, and rf phase

safe confinement if effective (pseudo) potential approx. valid

condition of adiabaticity determines validity of pseudo-potential approx.

Definition of adiabaticity

Dieter Gerlich, Advances in Chemical Physics: State-Selected and State-to-State Ion-Molecule reaction Dynamics, Part 1, Experiment, Volume 82, 1992

defines adiabaticity as validity of expansion:

$$\vec{E}_{rf}(\vec{R}_0-\vec{a}\cos(\Omega t))\approx\vec{E}_{rf}(\vec{R}_0)-(\vec{a}\cdot\vec{\nabla})\vec{E}_{rf}(\vec{R}_0)\cos(\Omega t)$$

where ion motion $\vec{r}(t)=\vec{R}_0(t)-\vec{a}(t)\cos(\Omega t)$

$$\vec{a}(t)=\frac{e\vec{E}_{rf}}{m\Omega^2}\quad\text{(the micro-motion)}$$

term in expansion is small when adiabaticity parameter $\eta \ll 1$

$$\eta\equiv\frac{2e\left|\vec{\nabla}E_{rf}\right|}{m\Omega^2},\ E_{rf}\equiv\left|\vec{E}_{rf}\right|,\ \text{safe confinement for } \eta<0.3$$

for quadrupole trap $\eta = q$; for higher order multipoles, $\eta = \eta(r)$

An interesting example:

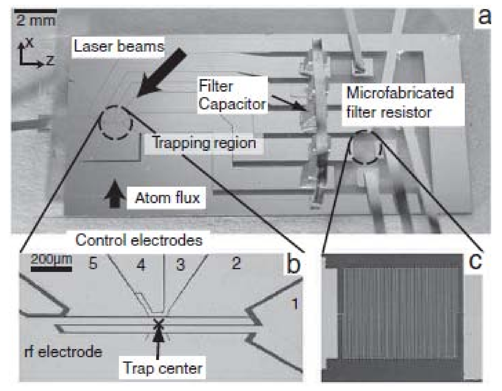
$^{24}Mg^+$ ion

$\frac{\Omega}{2\pi}=87$ MHz

$V_{rf}=103.2$ V

$d=40\ \mu$m

$$q_{4w}=\frac{2eV_{rf}}{m\Omega^2(\pi d^2)}\approx 0.55$$



Seidelin et al., PRL **96** (2006)

pseudo-potential well depth $E_{well}\approx(3.6\times10^{-3})q_{4w}V_{rf}\sim 204$ meV

simulated adiabatic well depth $E_{safe}\sim 20$ meV

Adiabaticity requirement in multipole traps:

Evaporation of Buffer-Gas-Thermalized Anions out of a Multipole rf Ion Trap

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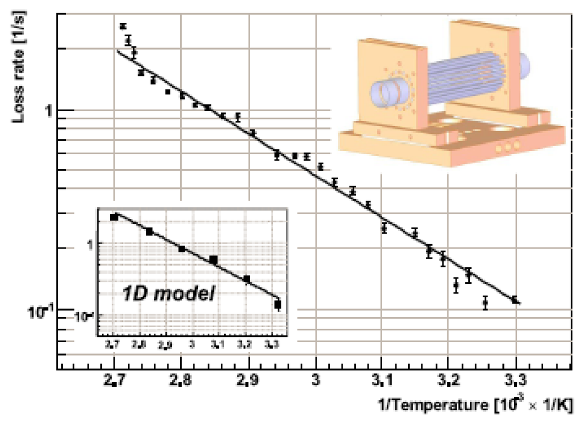
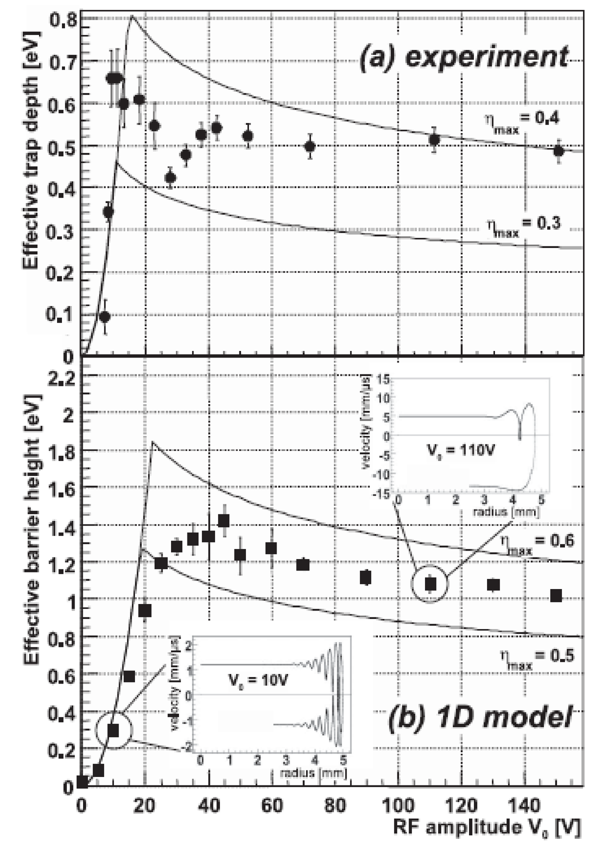
(Received 22 December 2006; published 30 May 2007)

Physical Review Letters **98**, 223001 (2007)





Cl^- ion temperature varied through heated/cooled He buffer gas

measure loss rate of thermalized Cl^-

extract effective well depth from loss rate

adiabaticity can limit effective well depth of high order multipole traps

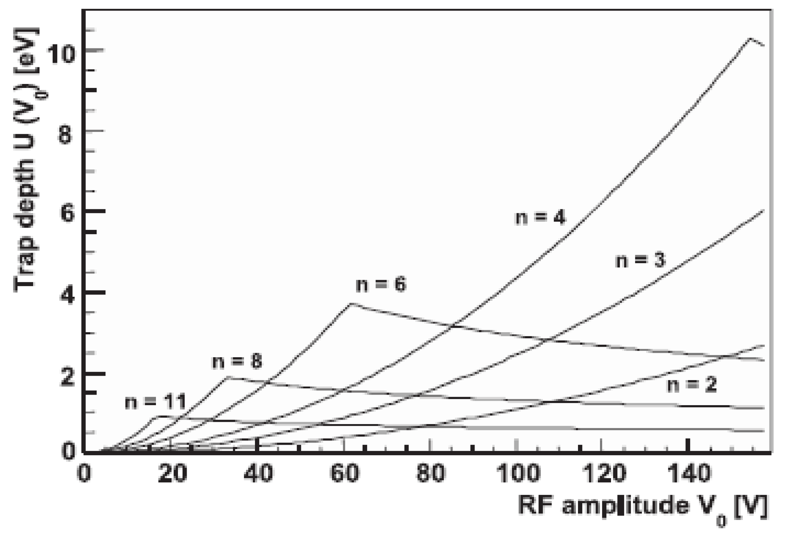
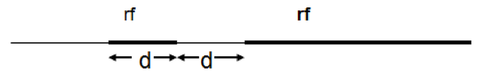


FIG. 3. Effective trap depths for multipoles of different order n as derived from the analytical model assuming a maximum adiabaticity parameter $\eta_{max}=0.4$ (calculation for Cl^- ions, $r_0 = 5$ mm, $\omega = 2\pi\times 5$ MHz).

Can adiabaticity limit the effective well depth of surface electrode traps?

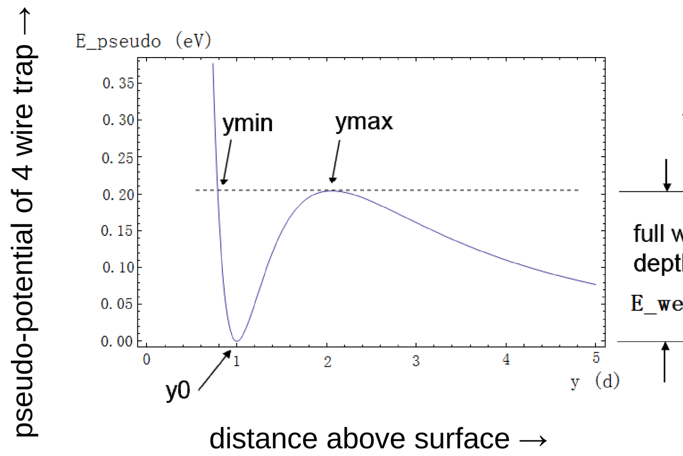
four-wire trap:

$\Phi_{4w}=\Phi_s(-d,0)+\Phi_s(d,\infty)$



$$\Phi_s(a,b)=\frac{U_s}{\pi}\times\begin{cases}\tan^{-1}\left(\frac{x-a}{y}\right)-\tan^{-1}\left(\frac{x-b}{y}\right), & -\infty<a<b<\infty\\ \frac{\pi}{2}-\tan^{-1}\left(\frac{x-b}{y}\right), & a=-\infty\\ \frac{\pi}{2}+\tan^{-1}\left(\frac{x-a}{y}\right), & b=\infty\end{cases}$$

$$\omega_{4w}=\frac{eV_{rf}}{\sqrt{2}m\pi\Omega d^2}\qquad q_{4w}=\frac{2eV_{rf}}{m\Omega^2(\pi d^2)}$$



$E_{well}\approx(3.6\times10^{-3})q_{4w}V_{rf}$

$\eta norm$(x,y); pseudo-potential ϕ4wPS(x,y)

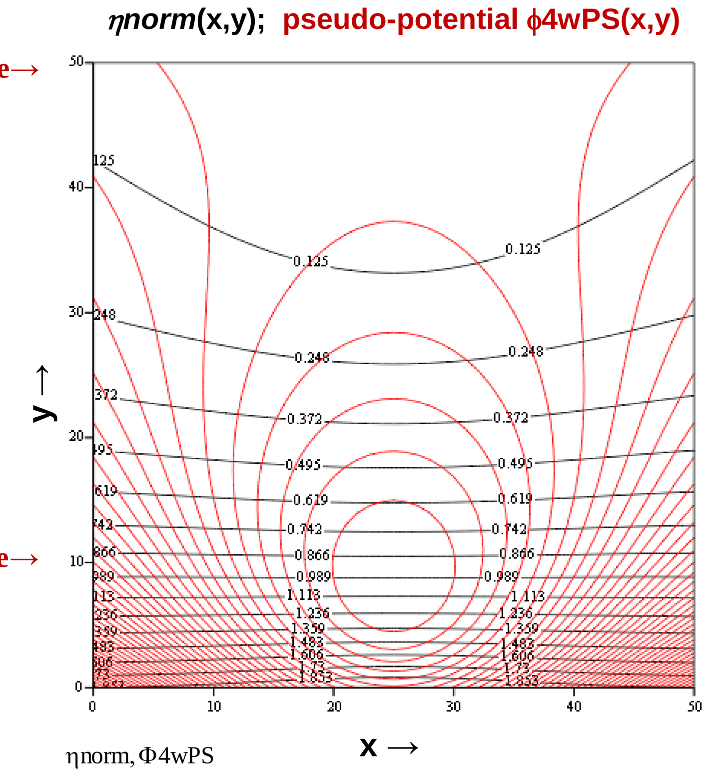
saddle ~2d above trap surface→

normalize η to its value at the trap center:

$$\eta norm(x,y)\equiv\frac{\eta(x,y)}{\eta(0,d)}$$

where $(0,d)$ is the position of zero rf field (trap center)

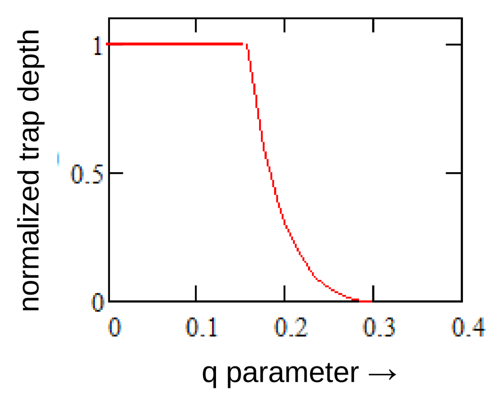
ion located d above trap surface→



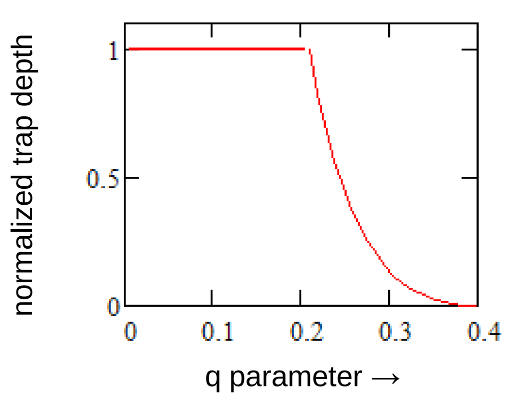
for trap with $q=\eta(0,d)$, adiabatic well depth determined from $q\cdot\eta norm>\eta_{max}$ as $y\to 0$

adiabatic trap depth (normalized to E_well)

$\eta_{max} = 0.3$



$\eta_{max} = 0.4$



maximum trap q for full well depth with η_{max} =0.3: **0.16**

maximum trap q for full well depth with η_{max} =0.4: **0.21**

Similar results for 5-wire traps:

maximum trap q for full well depth with η_{max} =0.3: **0.17**

maximum trap q for full well depth with η_{max} =0.4: **0.23**

Simulation of reduced well depth in a 4-wire trap

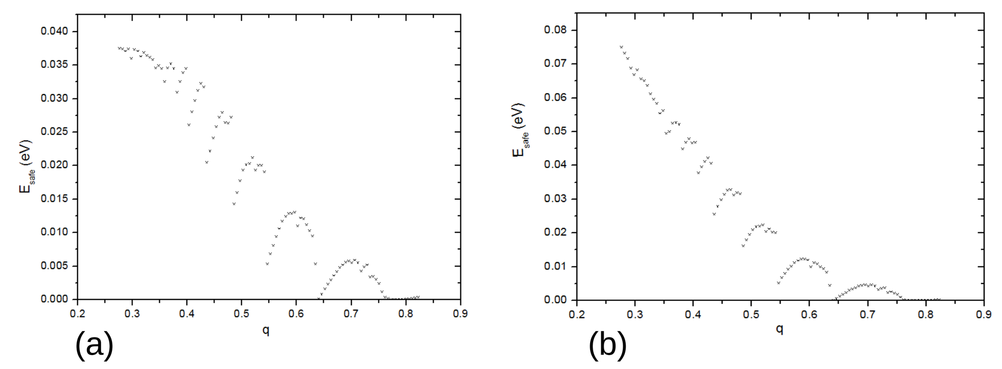
(preliminary results)

Procedure for determining E_{safe}

- Set an initial guess for the energy E_i
- Set the initial positions (x_i, y_i) at the center of the trap
- Calculate the magnitude of initial velocity $v_0=\sqrt{2E_i/m}$
- Vary the angle ψ of the velocity from 0 to $15\pi/16$ with an interval of $\pi/16$: $v_{xi}=v_0\cos\psi$; $v_{yi}=v_0\sin\psi$
- Choose the initial phase for the electric field ϕ from 0 to $15\pi/16$ with an interval of $\pi/16$
- Update the position, velocity and energy at each time step by solving the equations of motion
- If the energy is larger than full well depth, the particle is unbounded, go back to the very beginning with a slightly lower E_i, (i.e. 10^{-5} eV)
- If the ion trajectory is trapped for a sufficiently long time (10^5 rf cycles, i.e. more than 1 ms), we assume this initial condition gives a trapped orbit
- Repeat the above steps for all the initial conditions (16 velocity angles ψ and 16 electric field phases ϕ)
- For each initial condition, record the boundary energy to safely confine the trap
- From these boundary energies, choose the minimal value as E_{safe}

Numerical Simulation

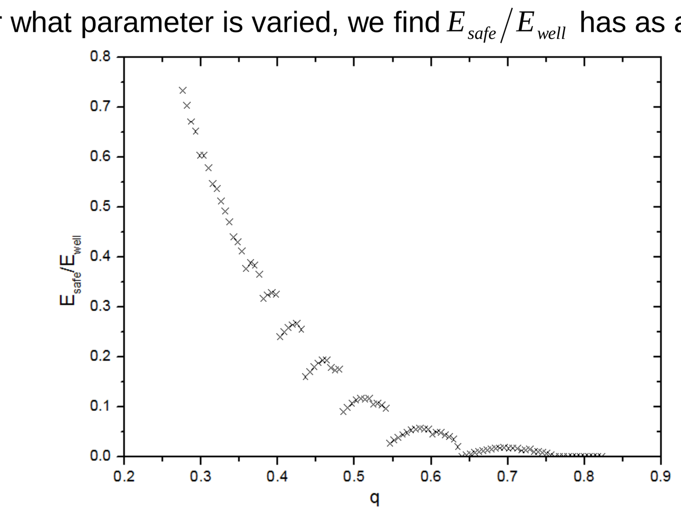
- Equation of motion: $m\ddot{\vec{r}}=e\vec{E}_{rf}\cos(\Omega t)$
- Numerical integration using the 4th order Runge-Kutta method
- Time step: $\Delta t=0.01(2\pi/\Omega)$. The time step was reduced by a factor of 10 and then 100 with no change in the results to less than 1%.



E_{safe} as a function of $q\equiv\eta(0,d)$: (a) vary V_{rf}; (b) vary $1/md^2\Omega^2$. The simulation clearly shows the reduction in effective well depth as expected from adiabaticity theory, but also at a fine level of detail a "periodic" dependence which we currently do not have a theory for.

Conclusion

However, no matter what parameter is varied, we find E_{safe}/E_{well} has as a direct relationship with q:



These simulations involving a single ion show the importance of non-adiabatic effects is surface electrode traps. Simulations with two ions can investigate whether the Coulomb interaction can amplify non-adiabatic effects. For example, experiments have observed significantly shorter lifetimes with two ions than a single ion. Simulations can investigate whether this is due to non-adiabatic effects and whether these effects can be minimized with particular trap operating parameters.