Dephasing of trapped-ion qubit due to Stark shift during shuttling

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Question:
Faster ion transportation deteriorates quantum information more seriously
⇒ Speed limit of trapped ion quantum computer?

Idea:
• Fast transportation needs strong electric field
• Qubit states are disrupted by Stark effect
• Cannot track the path of ion precisely, Stark shift becomes dephasing and decoherence error

Motion of ion:
• Hamiltonian: \( \frac{\hat{p}_x^2}{2m} + \frac{1}{2} m\omega^2 (\hat{x} - s(t))^2 \)
• For any displacement of the potential well, \( s(t) \), the ion remains in a coherent state
• Classical path of the ion
  \[ q(t) = s(t) - \int_0^t \dot{s}(t) \cos (\omega (t - t_1)) dt_1 \]
• Electric field strength experienced by the ion is related to its acceleration by Newton’s second law
• Stark shift is a functional of path of electric potential well

Minimum phase shift:
• Dephase is the difference of phase shift between two qubit states
  \[ \Delta \phi = \frac{m^2}{\hbar} \left( \sum_{m \neq i} \frac{|m| \langle \hat{x} | i \rangle |^2}{\hbar \omega_{im}} - \sum_{m \neq f} \frac{|m| \langle \hat{x} | f \rangle |^2}{\hbar \omega_{fm}} \right) \int_0^t (\ddot{q}(t))^2 dt \]
• Matrix elements are inherent to the electronic structure, determined by the choice of ion qubits.
• Path dependent part is non-vanishing, but we can find the minimum possible value

Results:
• Optimal path of the ion is
  \[ q_0(t) = L \left( -2 \frac{t^3}{T^3} + 3 \frac{t^2}{T^2} \right) \]
• Minimum phase shift:
  \(^{40}\text{Ca}^+\) ion, 4S\(_{1/2}\) and D\(_{5/2}\) as qubit states
  \[ \phi_{\text{min}}^{\text{Ca}} = 9.86 \times 10^{-18} \left[ \frac{L^2}{T^3} \right] \]
  \(^{9}\text{Be}^+\) ion, hyperfine states as qubit states
  \[ \phi_{\text{min}}^{\text{Be}} = 2.6 \times 10^{-25} \left[ \frac{L^2}{T^3} \right] \]
• Speed limit of ion qubit shuttling across 100 \( \mu \)m before dephasing > \( \pi/100 \)
  \[ T_{\text{min}}^{\text{Ca}} \gtrsim 14.6 \text{ ns} \]
  \[ T_{\text{min}}^{\text{Be}} \gtrsim 0.044 \text{ ns} \]
• Decoherence error is less important than dephasing error; negligible for speed limit

Conclusions:
• We find a relation between ion shuttling speed and minimum dephasing caused by Stark effect
• For 100 \( \mu \)m length trap, error becomes important when operation rate is 100 MHz for \(^{40}\text{Ca}^+\) ion qubit; 10 GHz for \(^{9}\text{Be}^+\) ion
• Without very precise ion control techniques, trade-off between dephasing error and shuttling speed is unavoidable