Dephasing of trapped-ion qubit due to Stark shift during shuttling

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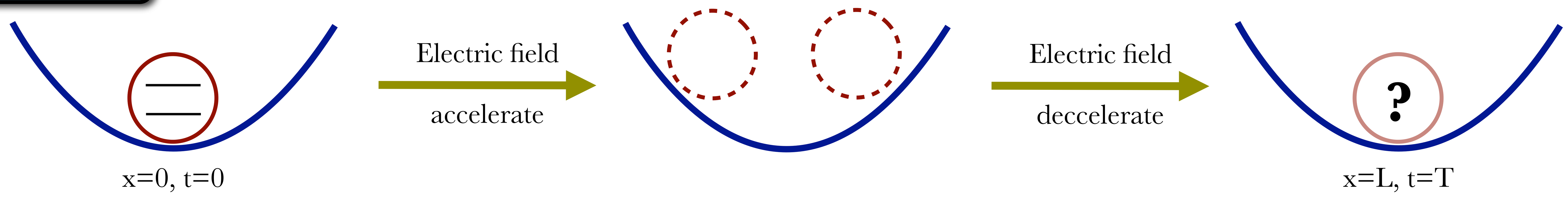
Question:



Faster ion transportation deteriorates quantum information more seriously

⇒ Speed limit of trapped ion quantum computer?

Idea:

- Fast transportation needs strong electric field
- Qubit states are disrupted by Stark effect
- Cannot track the path of ion precisely, Stark shift becomes dephasing and decoherence error

Motion of ion:

- Hamiltonian: $\frac{\hat{p}_x^2}{2m} + \frac{1}{2} m\omega^2 \left(\hat{x} - s(t)\right)^2$
- For any displacement of the potential well, s(t), the ion remains in a coherent state
- Classical path of the ion

$$q(t) = s(t) - \int_0^t \dot{s}(t) \cos\omega(t - t_1) dt_1$$

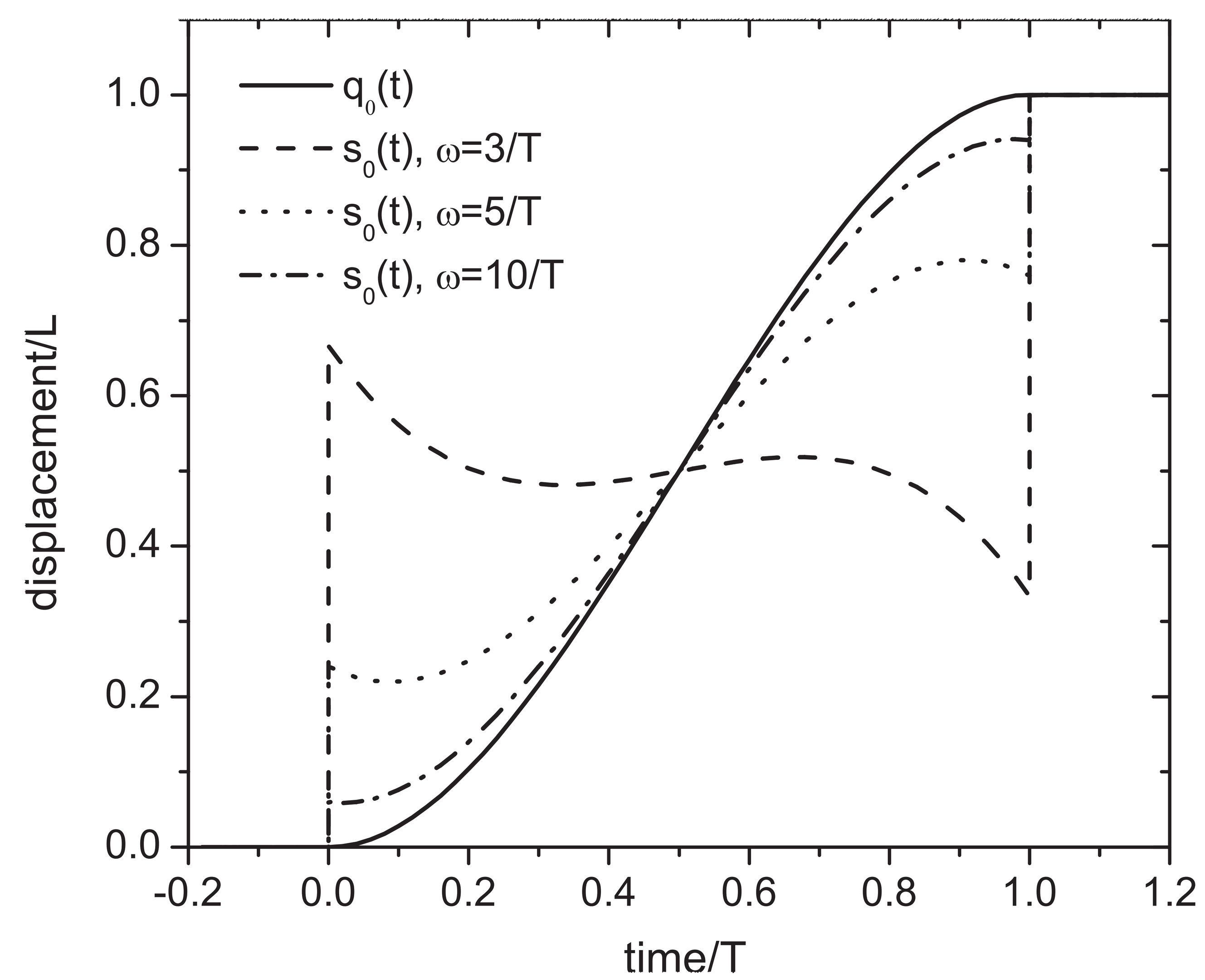
- Electric field strength experienced by the ion is related to its acceleration by Newton's second law
- Stark shift is a functional of path of electric potential well

Minimum phase shift:

- Dephase is the difference of phase shift between two qubit states

$$\Delta\phi = \frac{m^2}{\hbar}\left(\sum_{m\neq i}\frac{|\langle m|\hat{x}|i\rangle|^2}{\hbar\omega_{im}} - \sum_{m\neq f}\frac{|\langle m|\hat{x}|f\rangle|^2}{\hbar\omega_{fm}}\right)\int_0^T (\ddot{q}(t))^2\, dt$$

- Matrix elements are inherent to the electronic structure, determined by the choice of ion qubits.
- Path dependent part is non-vanishing, but we can find the minimum possible value



Results:

- Optimal path of the ion is

$$q_o(t) = L\left(-2\frac{t^3}{T^3} + 3\frac{t^2}{T^2}\right)$$

- Minimum phase shift:

$^{40}Ca^+$ ion, $4S_{1/2}$ and $D_{5/2}$ as qubit states

$$\phi_{\rm min}^{\rm Ca} = 9.86 \times 10^{-18}\frac{[L^2]}{[T^3]}$$

$^{9}Be^+$ ion, hyperfine states as qubit states

$$\phi_{\rm min}^{\rm Be} = 2.6 \times 10^{-25}\frac{[L^2]}{[T^3]}$$

- Speed limit of ion qubit shuttling across 100 μm before dephasing > π/100

$$T_{\rm min}^{\rm Ca} \gtrsim 14.6 \text{ ns}, \quad T_{\rm min}^{\rm Be} \gtrsim 0.044 \text{ ns}$$

- Decoherence error is less important than dephasing error, negligible for speed limit

Conclusions:

- We find a relation between ion shuttling speed and minimum dephasing caused by Stark effect
- For 100 μm length trap, error becomes important when operation rate is 100 MHz for Ca^+ ion qubit; 10 GHz for Be^+ ion
- Without very precise ion controlling techniques, trade-off between dephasing error and shuttling speed is unavoidable