Self-Study Manual on Optical Radiation Measurements: Part I—Concepts, Chapters 4 and 5

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Chapter 4. More on the Distribution of Optical Radiation with respect to Position and Direction, Fred E. Nicodemus

Chapter 5. An Introduction to the Measurement Equation, Henry J. Kostkowski and Fred E. Nicodemus

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Issued February 1978

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National Bureau of Standards Technical Note 910-2

Nat. Bur. Stand. (U.S.), Tech. Note 910-2, 118 pages (Feb. 1978)
CODEN: NBTNAE

U.S. GOVERNMENT PRINTING OFFICE
WASHINGTON: 1978

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402
Stock No. 003-003-01880-5; Price $3.00
(Add 25 percent additional for other than U.S. mailing)

PREFACE

This is the second in a series of Technical Notes (910-) entitled "Self-Study Manual on Optical Radiation Measurements." It contains the fourth and fifth chapters of that Manual. Along with Chapter 6, which has already been published in Technical Note 910-3 (June 1977), this completes the first six chapters of Part I--Concepts.

There may be confusion about a gap in the numbering of the appendices. There were two appendices in TN 910-1 and, when TN 910-3 went to press in June 1977, we were planning three more for this issue, so we designated those in TN 910-3 as numbers 6 and 7. However, the third appendix for this issue was to have been on "Apertures, Stops, Pupils, Windows, and Baffles in Radiometry," and it developed that this topic had more ramifications, not directly pertaining to chapters 4 and 5, that should be developed in greater detail. Accordingly, in order not to delay further the publication of these chapters and to provide still broader and more adequate coverage of the theory of stops in relation to radiometry, it was decided to leave out the appendix and revise and expand the material for a separate chapter in the near future. So *there is no Appendix 5.*

For background information on the whole project and on the plans for the "Self-Study Manual" (SSM), we reproduce here (immediately following) the Preface to the first issue, Technical Note 910-1, of March 1976. Although the exact details of the outline and plans for the individual chapters are still developing, our principal aims and overall plans are still as set forth in that Preface.

Here, in Chapter 4, we are primarily concerned with developing the rest of the important concepts relating to the distribution of optical radiation with respect to the spatial parameters of position and direction and the spectral parameter (wavelength, frequency ν, or wave number). All of the important radiometric quantities relating to these parameters are defined and discussed, and are related to the radiance or spectral radiance and the flux or spectral flux. Important interrelationships are also explored to clarify the significance of the different quantities and the distinctions between them.

In Chapter 5 we treat the measurement equation, the heart of our approach to the problems of radiometric measurements. The basic ideas are first introduced by deriving the measurement equation for each of three simple illustrative examples. This is followed by a more general discussion based on the points developed in the sample problems. A systematic step-by-step approach is presented for setting up and solving any measurement equation with respect to the spatial and spectral parameters. Considerations involving the other radiation parameters (polarization and time or frequency f) and the instrumental and environmental parameters will be developed in future chapters. The measurement equation provides the basis for a systematic approach to the difficult multidimensional problems of radiometric measurements. Such an approach is an aid for planning and carrying out the measurements and the analysis of the resulting data and for preventing possible neglect of significant parameters that affect those results.

We are grateful to many for valuable comments and criticisms, received both informally as well as in formal reviews, concerning all phases of the SSM. In connection with this Technical Note, we are particularly indebted to W. L. Wolfe and F. O. Bartell for informal reviews of portions of the text of Chapter 4, to L. V. Spencer for his helpful criticisms and discussion of Chapter 4, and to A. T. Hattenburg and J. B. Shumaker for valuable discussions and comments on all phases of the project. Mrs. Betty Castle has done an outstanding job of typing another difficult text, including the effective layout of text and figures. And again, we acknowledge H. J. Zoranski's able assistance with the preparation of the figures.

We continue to earnestly solicit your "feedback"--positive or negative. We realize that most of you are probably waiting for the applications chapters and we'll try to complete some of them as soon as possible. In the meantime, however, are you finding these first chapters on basic concepts useful and helpful in your work? Are they clear and complete enough? Have you found any inconsistencies or ambiguities in them, or between them and other material? Have they been useful for indoctrination of new employees or for solving particular problems? Have you suggestions for improving them? Please let us hear from you.

Fred E. Nicodemus, Editor

Henry J. Kostkowski, Chief,
Optical Radiation Section

PREFACE to NBS TN 910-1

This is the initial publication of a new series of Technical Notes (910) entitled "Self-Study Manual on Optical Radiation Measurements." It contains the first three chapters of this Manual. Additional chapters will be published, similarly, as they are completed. The Manual is being written by the Optical Radiation Section of NBS. In addition to writing some of the chapters, themselves, Fred E. Nicodemus is the Editor of the Manual and Henry J. Kostkowski, Chief of the Section, heads the overall project.

In recent years, the economic and social impact of radiometric measurements (including photometric measurements) has increased significantly. Such measurements are required in the manufacture of cameras, color TV's, copying machines, and solid-state lamps (LED's). Ultraviolet radiation is being used extensively for the polymerization of industrial coatings, and regulatory agencies are concerned with its effects on the eyes and skin of workers. On the other hand, phototherapy is usually the preferred method for the treatment of jaundice in the newborn. Considerable attention is being given to the widespread utilization of solar energy. These are just a few examples of present day applications of optical radiation. Most of these applications would benefit from simple measurements of one to a few per cent uncertainty and, in some cases, such accuracies are almost essential. But this is rarely possible. Measurements by different instruments or techniques commonly disagree by 10% to 50%, and resolving these discrepancies is time-consuming and costly.

There are two major reasons for the large discrepancies that occur. One is that optical radiation is one of the most difficult physical quantities to measure accurately. Radiant power varies with the radiation parameters of position, direction, wavelength, time, and polarization. The responsivity of most radiometers also varies with these same radiation parameters and with a number of environmental and instrumental parameters, as well. Thus, the accurate measurement of optical radiation is a difficult multi-dimensional problem. The second reason is that, in addition to this inherent difficulty, there are few measurement experts available. Most of the people wanting to make optical radiation measurements have not been trained to do so. Few schools have had programs in this area and tutorial and reference material that can be used for self-study is only partially available, is scattered throughout the literature, and is generally inadequate. Our purpose in preparing this Self-Study Manual is to make that information readily accessible in one place and in systematic, understandable form.

The idea of producing such a manual at NBS was developed by one of us (HJK) in the latter part of 1973. Detailed planning got under way in the summer of 1974 when a full-time editor (FEN) was appointed. The two of us worked together for about one year developing an approach and format while writing and rewriting several drafts of the first few chapters. These are particularly important because they will serve as a model for the rest of the Manual. During this period, a draft text for the first four chapters was distributed, along with a questionnaire, for comment and criticism to some 200 individuals representing virtually every technical area interested in the Manual. About 50 replies were received, varying widely in the reactions and suggestions expressed. Detailed discussions were also held with key individuals, including most of the Section staff, particularly those that will be writing some of the later chapters. In spite of the very wide range of opinions encountered, all of this feedback has provided valuable guidance for the final decisions about objectives, content, style, level of presentation, etc.

In particular, we have been able to arrive at a clear solution to difficult questions about the level of presentation. Both of us started out with the firm conviction that, with enough time and effort, we should be able to present the subject so that readers with the equivalent of just elementary college mathematics and science could easily follow it. That conviction was based on our experience of success in explaining the subtleties of radiometric measurements to technicians at that level. What we failed to consider, however, was that, in making such explanations to individuals we always were able to relate what we said to the particular background and immediate problem of the individual. That's just not possible in a text intended for broad use by workers in astronomy, mechanical heat-transfer engineering, illumination engineering, photometry, meteorology, photo-biology and photo-chemistry, optical pyrometry, remote sensing, military infrared applications, etc. To deal directly and explicitly with each individual's problems in a cook-book approach would require an impossibly large and unwieldy text. So we must fall back on general principles which immediately and unavoidably require more knowledge and familiarity with science and mathematics, at the level of a bachelor's degree in some branch of science or engineering, or the equivalent in other training and experience.

In its present form, the Manual is a definitive tutorial treatment of the subject that is complete enough for self instruction. This is what is meant by the phrase "self-study" in the title. The Manual does not contain explicitly programmed learning steps as that phrase sometimes denotes. In addition, through detailed, yet concise, chapter summaries, the Manual is designed to serve also as a convenient and authoritative reference source. Those already familiar with a topic should turn immediately to the summary at the end of the appropriate chapter. They can determine from that summary what, if any, of the body of the chapter they want to read for more details.

The basic approach and focal point of the treatment in this Manual is the measurement equation. We believe that every measurement problem should be addressed with an equation relating the quantity desired to the data obtained through a detailed characterization of the instruments used and the radiation field observed, in terms of all of the relevant parameters. The latter always include the radiation parameters, as well as environmental and instrumental parameters, as previously pointed out. The objective of the Manual is to develop the basic concepts and characteristics required so that the reader will be able to use this measurement-equation approach. It is our belief that this is the only way that uncertainties in the measurement of optical radiation can generally be limited to one, or at most a few, per cent.

Currently, the Manual deals only with the classical radiometry of incoherent radiation. The basic quantitative relations for the propagation of energy by coherent radiation (e.g., laser beams) are just being worked out [1,2,3,4].[1] Without that basic theory, a completely satisfactory general treatment of the measurement of coherent (including partially coherent) optical radiation is not possible. Accordingly, in spite of the urgent need for improved measurements of laser radiation, we won't attempt to deal with it now. Possibly this situation will be changed before the current effort has been completed and a supplement on laser measurements can be added.

As stated above, we first hoped to prepare this Manual on a more elementary level but found that it was impossible to avoid making use of both differential and integral calculus of more than one variable. However, to help those that might be a bit "rusty" with such mathematics, we go back to first principles each time a mathematical concept or procedure beyond those of simple algebra or trigonometry is introduced. This should also throw additional light on the physical and geometrical relationships involved. Where it seems inappropriate to do this in the text, we cover such mathematical considerations in appendices. It is also assumed that the reader has had an introductory college course in physics, or the equivalent.

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[1]Figures in brackets indicate literature references listed at the end of this Technical Note.

The Manual is being organized into three Parts, as follows:

Part I. Concepts

Step by step build up of the <u>measurement equation</u> in terms of the radiation parameters, the properties and characteristics of sources, optical paths, and receivers, and the environmental and instrumental parameters. Useful quantities are defined and discussed and their relevance to various applications in many different fields (photometry, heat-transfer engineering, astronomy, photo-biology, etc.) is indicated. However, discussions of actual devices and measurement situations in this Part are mainly for purposes of illustrating concepts and basic principles.

Part II. Instrumentation

Descriptions, properties, and other pertinent data concerning typical instruments, devices, and components involved in common measurement situations. Included is material dealing with sources, detectors, filters, atmospheric paths, choppers (and other types of optical modulators), prisms, gratings, polarizers, radiometers, photometers, spectroradiometers, spectrophotometers, etc.

Part III. Applications

Measurement techniques for achieving a desired level of, or improving, the accuracy of a measurement. Included will be a very wide variety of examples of environmental and instrumental parameters with discussion of their effects and how to deal with them. This is where we deal with real measurements in the real world. The examples will also be drawn from the widest possible variety of areas of application in illumination engineering, radiative heat transfer, military infrared devices, remote sensing, meteorology, astronomy, photo-chemistry and photo-biology, etc.

Individual chapter headings have been assigned only to the first five chapters:

Chapter 1. Introduction

Chapter 2. Distribution of Optical Radiation with respect to Position and Direction — Radiance

Chapter 3. Spectral Distribution of Optical Radiation

Chapter 4. Optical Radiation Measurements — a Measurement Equation

Chapter 5. More on the Distribution of Optical Radiation with respect to Position and Direction

Other subjects definitely planned for Part I are, thermal radiation, photometry, distribution with respect to time, polarization, diffraction, and detector concepts. It is not our intention, however, to try to complete all of Part I before going on to Parts II and III. In fact, because we realize that a great many readers are probably most interested in the material on applications to appear in Part III, we will try to complete and publish some chapters in Parts II and III just as soon as adequate preparation has been made in the earlier chapters of Part I. However, because our approach to radiometry differs so much from the traditional treatment, we feel that unnecessary confusion and misunderstanding can be avoided if at least the first nine chapters of Part I are published first and so are available to readers of later chapters.

Finally, we invite the reader to submit comments, criticisms, and suggestions for improving future chapters in this Manual. In particular, we welcome illustrative examples and problems from as widely different areas of application as possible.

As previously stated, we are indebted to a great many individuals for invaluable "feedback" that has helped us to put this text together more effectively. Notable are the inputs and encouragement from the Council on Optical Radiation Measurements (CORM), especially the

CORM Coordinators, Richard J. Becherer, John Eby, Franc Grum, Alton R. Karoli, Edward S. Steeb, and Robert B. Watson, and the Editor of *Electro-Optical Systems Design*, Robert D. Compton. In addition, for editorial assistance, we are grateful to Donald A. McSparron, Joseph C. Richmond, and John B. Shumaker, and particularly to Albert T. Hattenburg.

We are especially grateful to Mrs. Betty Castle for the skillful and conscientious effort that produced the excellent typing of this difficult text. We also want to thank Henry J. Zoranski for his capable help with the figures.

Fred E. Nicodemus, Editor

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March 1976

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SELF-STUDY MANUAL on OPTICAL RADIATION MEASUREMENTS

Part I. Concepts

This is the second in a series of Technical Notes (910-) entitled "Self-Study Manual on Optical Radiation Measurements." It contains the fourth and fifth chapters of this Manual. Additional chapters will continue to be published, similarly, as they are completed. The Manual is a comprehensive tutorial treatment of the measurement of incoherent radiation that is complete enough for self instruction. Detailed chapter summaries make it also a convenient authoritative reference source.

The following radiometric quantities are defined and discussed in Chapter 4: radiant energy, radiant exposure, radiant fluence, radiant density, radiant intensity, radiant flux (surface) density, irradiance, radiant exitance, radiant fluence rate, radiant sterisent, as well as spectral radiant energy, and the other corresponding spectral quantities. In particular, each quantity is related to the quantities previously introduced in Chapters 1-3: radiant flux or power, radiance, and the corresponding spectral quantities. Important interrelationships between the different quantities are also presented, particularly where they help to clarify their significance and the distinctions between them. Treatment of one of the most important interrelationships, the inverse-square law, also covers commonly used approximations involving actual (extended) sources and receivers.

The measurement equation, central to our approach to all of radiometry, is introduced in Chapter 5 through three illustrative measurement problems: a spectral-radiance-comparison measurement; a spectral-irradiance measurement near a large source; and an irradiance measurement with a wide-band radiometer. Normalization of spectrally broad-band measurements is briefly discussed. A general discussion of the measurement equation summarizes and enlarges on the points brought out by the examples. A set of orderly steps for solving the measurement equation is presented and the limitations of the measurement equation developed in this chapter are summarized.

Key Words: Measurement equation; optical radiation measurement; photometry; radiometry; spectroradiometry.

Chapter 4. More on the Distribution of Optical Radiation with Respect to Position and Direction

by Fred E. Nicodemus

<u>In this CHAPTER</u>. We introduce all of the remaining radiometric quantities listed in table 4-1 (a slightly revised version of table A1-3 in Appendix 1 [5]). In Chapter 2 [5], we concentrated only on the concept of radiance as the basic spatial distribution of radiant flux with respect to both position and direction and in Chapter 3 [5] we similarly considered only spectral radiance. Now we treat all of the other spatial distributions of flux, and of energy (time-integrated flux). Included is radiant sterisent, a quantity for characterizing distributed volume sources. In every case the quantity is defined in terms of radiance or is explicitly related to the radiance. Interrelationships between the different quantities are also given to clarify their meaning and significance and to help to distinguish between them. A particularly important interrelationship is the <u>inverse-square law</u> which gives the irradiance element at a given distance from a source element

(point) of known radiant intensity. The problems involved in the use of common approximations to the inverse-square law, applied to actual (extended) sources and receivers, are discussed.

In Chapter 1 (on p. 7 of [5]), we said, "It cannot be too strongly emphasized - - - that everything said here about the fundamentals of radiometry, even though stated in terms of watts, applies equally to *all* forms of optical radiation measurements. For example, the measurement of illumination, for application to vision needs, involves all of the fundamentals, not just those discussed in the chapter on photometry." We can now be more explicit. Compare tables 4-1, 4-2, and 4-3 (previously tables A1-3, -4, and -6 in Appendix 1 [5]). Note that in both table 4-2 and 4-3 there is an entry corresponding to each entry in table 4-1. The main difference is apparent in the units or unit-dimensions. Wherever watts [W] appear in table 4-1, they are replaced by lumens [lm] in table 4-2 and by quanta per second [$q \cdot s^{-1}$] in table 4-3. In addition, table 4-2 gives some equivalent units that are also commonly used for the photometric quantities. For lack of space and *because they are exactly the same*, the defining relations in the third column of table 4-1 are not repeated in the other two tables. The symbols, too, are the same except that, when it is necessary to distinguish between them, the subscript e may be used for the radiometric quantities, v for the photometric quantities, and p for the photon-flux quantities, as indicated by the alternate symbols in the second column of each table. Ordinarily, when the context makes it clear which quantities are being used, the subscripts may be omitted. Also, when, as in this chapter, we want to emphasize the common geometrical relationships that apply equally to all, the quantity symbols are also used without subscripts. We will present the material in this chapter in terms of the radiometric quantities, with unit-dimensions in terms of watts, but *all of the geometrical relationships are equally applicable to photometric and photon-flux quantities.* It is only necessary to replace each term, quantity symbol, and unit-dimension with the corresponding one from the appropriate table to show this explicitly in each case.

Many readers may use only a few of the quantities in these tables and those of interest to them may change from time to time. Accordingly, they may prefer to turn directly to Chapter 5, where we first introduce the measurement equation and deal with the concepts relating directly to the measurement of some radiometric quantities. Then, as they encounter new quantities here or in their reading elsewhere, they can return to the pertinent parts of this chapter where each one is defined and discussed in some detail. To find the chapter where each quantity is defined, see the numbers in parentheses in the first column of table 4-1.

"SIMPLE" SPATIAL DISTRIBUTIONS (with RESPECT to POSITION or DIRECTION).

Until now, we have analyzed all spatial relations connected with the propagation of optical radiation in terms of the basic spatial-distribution function, namely radiance (as a function of ray position and direction). However, a glance at table 4-1 shows that there are some other simpler spatial distributions in common use with special names. These are distributions with respect to position only (flux per unit area) and with respect to direction

Table 4-1

SI Derived Units for Radiometry

Quantity (Chap. No.)†	Symbol	Definition††	Unit	Unit Symbol
radiant energy (4)	$Q, (Q_e)$	$\int \Phi \cdot dt$	joule	$[J]$
radiant (directed-surface) exposure (4)	$H, (H_e)$	dQ/dA; $\iint L \cdot \cos\theta \cdot d\omega \cdot dt$	joule per square meter	$[J \cdot m^{-2}]$
radiant (omni-directional) fluence‡ (4)	$F, (F_e)$	dQ/da; $\iint L \cdot d\omega \cdot dt$	joule per square meter	$[J \cdot m^{-2}]$
radiant (volume) density (4)	$w, (w_e)$	dQ/dV	joule per cubic meter	$[J \cdot m^{-3}]$
radiant power or flux (1)	$\Phi, (\Phi_e)$	dQ/dt	watt	$[W]$; $([J \cdot s^{-1}])$
radiant intensity (4)	$I, (I_e)$	$d\Phi/d\omega$	watt per steradian	$[W \cdot sr^{-1}]$
radiant flux (directed-surface) density (4)	$W, (W_e)$			
irradiance (4)	$E, (E_e)$	$d\Phi/dA$; $\int L \cdot \cos\theta \cdot d\omega$	watt per square meter	$[W \cdot m^{-2}]$
radiant exitance (4)	$M, (M_e)$			
radiant (omni-directional) fluence rate‡ (4)	$F_t, (F_{e,t})$	$d\Phi/da$; $\int L \cdot d\omega$	watt per square meter	$[W \cdot m^{-2}]$
radiance (2)	$L, (L_e)$	$d^2\Phi/(dA \cdot \cos\theta \cdot d\omega)$; $d^2\Phi/(da \cdot d\omega)$	watt per square meter and steradian	$[W \cdot m^{-2} \cdot sr^{-1}]$
radiant sterisent (4)	$L^*, (L^*_e)$	dL_g/ds; dI_g/dV	watt per cubic meter and steradian	$[W \cdot m^{-3} \cdot sr^{-1}]$
- -				
spectral radiant energy (4)	$Q_\lambda, (Q_{e,\lambda})$	$dQ/d\lambda$	joule per nanometer	$[J \cdot nm^{-1}]$
spectral radiance (3)	$L_\lambda, (L_{e,\lambda})$	$dL/d\lambda$	watt per square meter, steradian, and nanometer	$[W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}]$

[other spectral quantities are similarly treated (4)]

†All of these quantities are defined and discussed in Chapters 1, 2, 3, and 4, as listed in the first column.

††All symbols are defined and used elsewhere in this Manual; it is important to distinguish: dA = element of (directed) surface; da = cross-sectional area of spherical element; dL_g = element of generated (emitted or scattered into ray) radiance; dI_g = element of generated radiant intensity; ds = element of distance along ray; dV = element of volume; and dt = element of time.

‡In September 1977 at Berlin, the CIE Technical Committee TC 1.2 on Photometry and Radiometry adopted a number of recommendations for additions and changes in the next edition of the International Lighting Vocabulary [6] that is currently in preparation. Among those recommendations, the terms "spherical exposure" and "spherical irradiance" were given as the preferred terms for what we have called, respectively, "fluence" and "fluence rate", although the latter were also recognized as acceptable alternates, widely used in photobiology. Since our text, using the photobiology terms, is already in press, we have decided not to change it except for the addition of this footnote.

NOTE: This is a corrected version of table A1-3 in Appendix 1 [5].

Table 4-2

SI Derived Units for Photometry

Quantity	Symbol	Unit	Unit Symbol
luminous energy	$Q, (Q_v)$	lumen-second; (candela-steradian-second) lumen-second; (talbot)	[lm·s]; ([cd·sr·s]) [lm·s]
luminous (directed-surface) exposure	$H, (H_v)$	lux-second (candela-steradian-second per square meter) lumen-second per square meter	[lx·s] ([cd·sr·s·m^{-2}]) [lm·s·m^{-2}]
luminous (omni-directional) fluence	$F, (F_v)$	lux-second (candela-steradian-second per square meter) lumen-second per square meter	[lx·s] ([cd·sr·s·m^{-2}]) [lm·s·m^{-2}]
luminous flux	$\Phi, (\Phi_v)$	lumen; (candela-steradian) lumen	[lm]; ([cd·sr]) [lm]
luminous intensity	$I, (I_v)$	candela lumen per steradian	[cd] [lm·sr^{-1}]
luminous flux (directed-surface) density	$W, (W_v)$	lumen (candela-steradian) per square meter lumen per square meter	[lm·m^{-2}]; ([cd·sr·m^{-2}]) [lm·m^{-2}]
illuminance (illumination)	$E, (E_v)$	lux; (candela-steradian per square meter) lumen per square meter	[lx]; ([cd·sr·m^{-2}]) [lm·m^{-2}]
luminous exitance	$M, (M_v)$	lumen (candela-steradian) per square meter lumen per square meter	[lm·m^{-2}]; ([cd·sr·m^{-2}]) [lm·m^{-2}]
luminous (omni-directional) fluence rate	$F_t, (F_{v,t})$	lux; (candela-steradian per square meter) lumen per square meter	[lx]; ([cd·sr·m^{-2}]) [lm·m^{-2}]
luminance	$L, (L_v)$	candela per square meter lumen per square meter and steradian	[cd·m^{-2}] [lm·m^{-2}·sr^{-1}]
luminous sterisent	$L^*, (L^*_v)$	candela per cubic meter lumen per cubic meter and steradian	[cd·m^{-3}] [lm·m^{-3}·sr^{-1}]

NOTE: The first entry or entries for each quantity give the SI units, including, in every case, units in terms of the candela [cd] as the base unit. The last entry for each quantity is the same unit in terms of the lumen [lm] or lumen-second [lm·s], that parallels the corresponding radiometric unit in terms of the watt [W] or joule [J], respectively, for the corresponding quantity in table 4-1. The definitions (defining expressions) in that table, and the footnotes there, also apply to the corresponding quantities listed here.

Table 4-3

Units for Photon-Flux Radiometry

Quantity†	Symbol	Unit	Unit Symbol
photon energy	$Q, (Q_p)$	quantum††	[q]
photon exposure	$H, (H_p)$	quantum per square meter	$[q \cdot m^{-2}]$
photon fluence	$F, (F_p)$	" " " "	$[q \cdot m^{-2}]$
photon flux	$\Phi, (\Phi_p)$	quantum per second	$[q \cdot s^{-1}]$
photon-flux intensity	$I, (I_p)$	quantum per second and steradian	$[q \cdot s^{-1} \cdot sr^{-1}]$
photon-flux (surface) density	$W, (W_p)$		
incident photon-flux density	$E, (E_p)$	quantum per second and square meter	$[q \cdot s^{-1} \cdot m^{-2}]$
photon-flux exitance	$M, (M_p)$		
photon-flux fluence rate	$F_t, (F_{p,t})$	quantum per second and square meter	$[q \cdot s^{-1} \cdot m^{-2}]$
photon-flux sterance (radiance)	$L, (L_p)$	quantum per second, square meter, and steradian	$[q \cdot s^{-1} \cdot m^{-2} \cdot sr^{-1}]$
photon-flux sterisent	$L^*, (L^*_p)$	quantum per second, cubic meter, and steradian	$[q \cdot s^{-1} \cdot m^{-3} \cdot sr^{-1}]$

†Definitions (defining expressions) are the same as for corresponding quantities in table 4-1. Also spectral quantities are formed as shown in that table.

††The number of photons or quanta in a beam of radiation is frequently regarded as a pure (dimensionless) number, the ratio between the energy in that beam and the energy ($h\nu$) of an individual photon or quantum. However, that number is certainly a measure of the "amount of radiation" in the beam and it is not just a number, but is a number <u>of</u> a distinctive physical quantity, just as the number of joules is a physical quantity. Accordingly, it is useful to assign the quantum per second $[q \cdot s^{-1}]$ as the unit of photon flux. Then all of the other geometrical quantities and their interrelationships and units parallel exactly those for radiant flux, luminous flux, or any other flux of a physical quantity propagated in rays that obey the laws of geometrical optics.

NOTE: The einstein $[E] = N_A \cdot [q]$ (where N_A is the Avogadro constant, the number of molecules (particles) per mole [mol] of any substance), is widely used as a (much larger) unit of photon flux. (The latest value of the Avogadro constant in NBS Spec. Pub. 398 (Aug. 1974) is given as

$$N_A = (6.022045 \pm 0.000031) \times 10^{23}\ [\text{particles} \cdot \text{mol}^{-1}].)$$

only (flux per unit solid angle), rather than with respect to both at once. Customarily, these "simpler" quantities are presented and discussed first, before taking up the more general treatment of radiance. However, we have deliberately reversed the usual order because it has been our experience that the apparent simplicity can be misleading. Lack of attention to the simultaneous variation of ray-radiance with respect to both position and direction in actual beams of radiation can often lead to error. For consistently accurate measurement results, the sound approach is to look at every situation first in terms of ray-radiances. A simpler distribution function, with respect to position or direction alone, should be used only when it is then clear, from the analysis in terms of ray-radiances, that its use is appropriate and will not introduce any unacceptable approximations or errors. That analysis can often be very brief, but *every* situation should be considered first in this way.

In subsequent sections we'll take up each of these "simple" distributions, in turn, but, here, we'll first review those presentations briefly to put them into perspective. This means that we'll be using some of the terms before they are fully defined and discussed in detail. For that reason, it may be worthwhile to reread this section after going through the rest of the chapter, to more fully appreciate the significance of the intercomparisons between the different quantities.

We take up first the distribution of flux with respect to position only, in the form of flux per unit area as a function of position. Here we find two possibilities. To distinguish them, we refer to one as a "directed-surface distribution" and the other as an "omni-directional-surface distribution." An example of the first would be the quantity measured by a plane detector responding to incident radiation from the hemisphere above it; the second would be that measured by a spherical detector responding equally to incident radiation from all directions over a full sphere surrounding it.

The first, the directed-surface distribution, is called by the CIE-IEC [6] the "radiant flux (surface) density (at a point of a surface)." They also use the terms "irradiance" [for incident radiant flux (surface) density] and "radiant exitance" [for exitent—leaving the surface—radiant flux (surface) density]. However, they have no nomenclature at all for the second, the omni-directional-surface distribution. It is used primarily by photobiologists and photochemists, nuclear physicists and engineers, and others,[1] many of whom refer to it as the "fluence rate." Both kinds of surface-area

[1]Nuclear scientists generally use the term "energy current density" for the directed-surface distribution that we call "radiant flux (surface) density" and they use "flux density" or "energy flux density" for the omni-directional-surface distribution that we call "radiant fluence rate" [7]. However, "fluence rate" is also given as an acceptable alternate for the latter [8,9], so our choice of that term does not create any direct conflict. With respect to the term "flux density", however, there is an unfortunate, but unavoidable, ambiguity in the existing literature that even the unit-dimensions won't

(Footnote continued on page 8)

resolve because both the CIE-IEC [6] "flux (surface) density" and the nuclear [7,8,9] "flux density" (or "fluence rate") are measured in units of propagated quantity per unit time per unit area, e.g. $[J \cdot s^{-1} \cdot m^{-2}]$ or $[W \cdot m^{-2}]$. All we can do is to warn that when a nuclear scientist designates a quantity as a "flux density" he probably means an omnidirectional-surface distribution that we call "fluence rate" rather than the directed-surface distribution that we call "flux (surface) density" and which he will more likely call "current density". The only way we know to verify this, in the absence of explicit definitions, is through relationships that may be given between each quantity and other radiometric quantities. That's one of the important reasons for presenting such interrelationships between the different radiometric quantities in considerable detail in this chapter.

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distribution are measured in units of watts per square meter $[W \cdot m^{-2}]$. Up to this point, and in tables 4-1 and -2, we've used the descriptive phrases "directed-surface" and "omnidirectional-surface" to emphasize the distinction between these two forms of flux distribution with respect to position. Hereafter, and in table 4-3, we use the less cumbersome terms "flux (surface) density" and "fluence rate", without the descriptive phrases, except where they may be introduced for emphasis. The parenthetical term "surface" is retained, however, because the term "density" alone more often refers to a volume concentration.

The radiant flux (surface) density is the flux per unit area through a surface-area element of fixed orientation, from (or to) all (or any) directions within the hemisphere on one side of it. The radiant fluence rate is the flux per unit cross-sectional area incident on a spherical volume element (which has the same cross-sectional surface area in all directions lying within a complete sphere centered on the volume element). The flux (surface) density is the appropriate quantity to relate to a reference surface that intersects a radiation beam, such as the (imaginary) plane surface across each one of a pair of beam-defining apertures. Situations involving the fluence rate are those where radiation is incident throughout a large solid angle (up to a complete sphere) without a beam-defining aperture at the receiver. An example would be a small amount of matter suspended in space where it is exposed to irradiation from all sides, such as a microbe in an irradiated transparent culture medium or a small volume element at a point in the lasing gas of an "optically pumped" laser.

Note that flux (surface) density may refer either to incident radiation (irradiance) or exitent radiation (exitance)—either receiver or source. Fluence rate, on the other hand, involves only incident radiation—a receiver quantity. [Exitent (emitted or scattered) radiation from a volume source, not referred to a reference surface, will be treated later when we take up radiant sterisent.]

The last of these "simple" distributions, the distribution of radiant flux with respect to direction only, in the form of exitent flux per unit solid angle as a function of direction, in contrast to fluence rate, involves only exitent radiation; it is a source quantity. While it is defined for any source, including extended sources as we'll see shortly, it isn't very useful except when the source can be treated as a "point" source; i.e., when the source is far enough away so that the irradiance it produces on an element of receiving surface is inversely proportional to the square of its distance from that receiver element. This directional distribution of radiant flux is called the "radiant intensity (of a source in a given direction)" by the CIE-IEC [6], and they use that term consistently for just the one quantity. Unfortunately, however, there are many others who use the term "intensity" in a variety of ways, resulting in widespread ambiguity and confusion. For example, many physicists use the term "intensity" for the flux (surface) density. Others, particularly astrophysicists and meteorologists, use the term as a synonym of radiance, although many of them add the modifier "specific" so that they speak of the "specific intensity" of flux per unit area and solid angle. There's just no way to avoid this unfortunate situation in the existing literature, so here is one place where it's particularly important to pay close attention to the units (unit-dimensions) given--for any quantity called "intensity." In this Manual, we consistently follow the CIE-IEC [6] and speak of "intensity" only when we mean flux per unit solid angle in, e.g., watts per steradian $[W \cdot sr^{-1}]$.

We also discuss in some detail the considerations governing the valid application of the quantity called radiant intensity, the circumstances under which a real source may be treated usefully as a "point source"; i.e., one where the irradiance produced by it on a distant element of a receiver obeys the inverse-square law. This analysis is in terms of radiance and throughput[1] and it also introduces the engineering "configuration factors" that are closely related to the concepts of throughput and projected solid angle[2]. The relationships between configuration factors and throughputs and projected solid angles are developed in Appendix 3. With these relationships, the extensive tables of configuration factors that are available can be used to conveniently evaluate the throughputs and/or projected solid angles for a great many beam configurations, between sources and receivers, covered by those published tables.

DIRECTED-SURFACE DISTRIBUTIONS: RADIANT FLUX (SURFACE) DENSITY; IRRADIANCE; RADIANT EXITANCE. There are a number of practical situations where the quantity of significance is just the amount of flux per unit area flowing through (into or out of) a surface, regardless of the orientation of the rays along which the energy is propagating. For example, in determining the required size for a black surface to dissipate a certain amount of power (flux) by radiation, an important quantity is the amount of flux per unit area, the number of watts per square meter $[W \cdot m^{-2}]$, that the surface emits. For another example, the flux from a very distant radiation source is almost always uniformly distributed in rays of equal radiance over any small plane receiving-surface area that intersects

[1]See eq. (2.27) [5].

[2]See Appendix 2 [5].

a narrow portion of the beam from that source. Accordingly, the amount of received power or flux is directly proportional to the receiving area; it is the product of that plane area and the incident flux (surface) density or irradiance, which we will now define.

In order to examine the distribution of radiation with respect to position, consider a photocell lying on a bench or desk below two long tubular fluorescent ceiling lamps, as illustrated in figure 4.1. The radiation (light) from the lamps reaches the photocell through a circular hole in a flat opaque baffle, placed on top of the photocell so that its center point P is just above the center of the photocell. Baffles with holes of different areas ΔA [m^2] are used, always centered at P and all small enough so that all of the flux through them reaches the sensitive surface of the photocell. The output from the photocell is then a measure of the flux $\Delta\Phi$ [W] through the hole from both lamps.

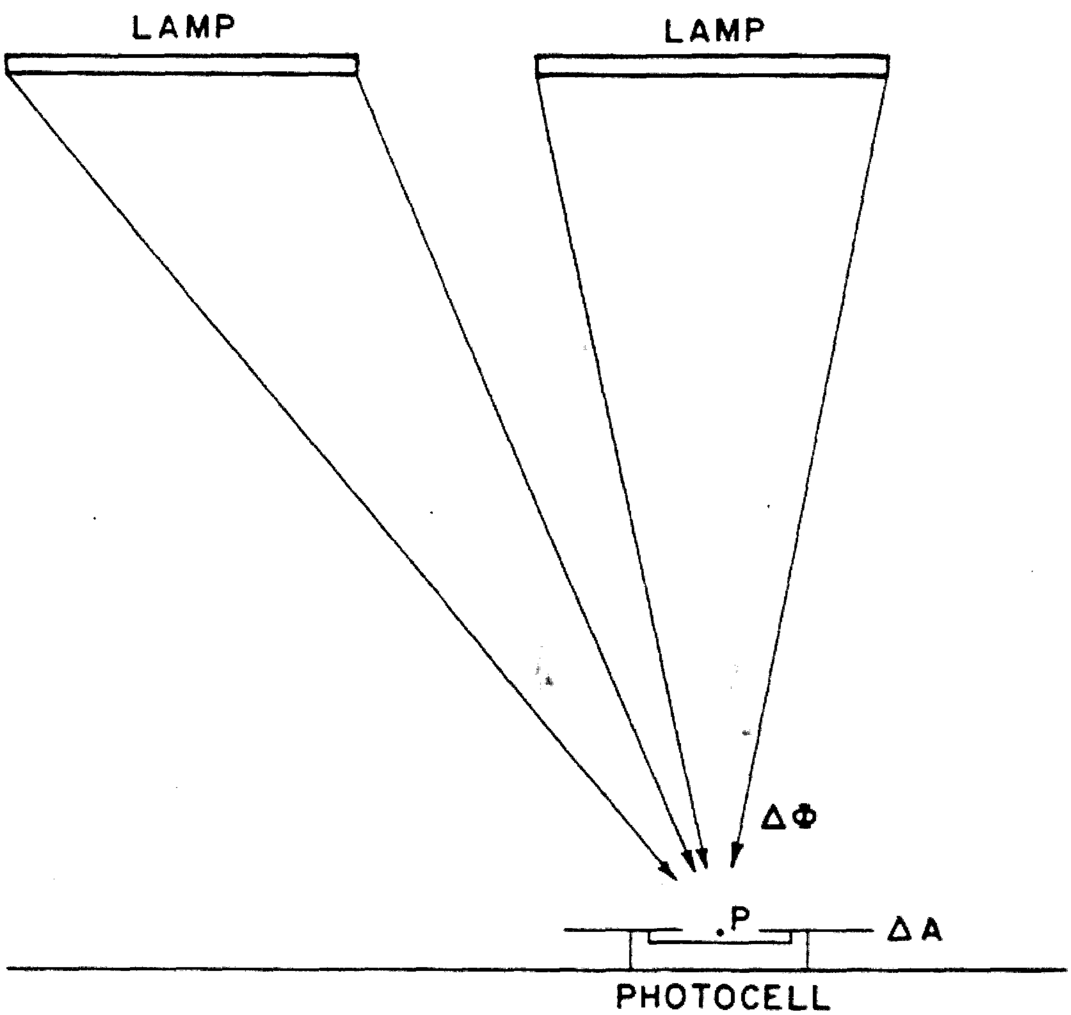

Figure 4.1. Experiment to develop the concept of irradiance.

In fact, we'll assume that the photocell has an ideal response that is the same for a given amount of incident flux $\Delta\Phi$ reaching its sensitive surface at any point and from any direction. As ΔA is reduced, by using smaller and smaller holes in the baffle, the flux $\Delta\Phi$, as measured by the photocell output, also decreases, while the quotient $\Delta\Phi/\Delta A$ approaches a constant. That limiting value, toward which the quotient converges as the area ΔA becomes vanishingly small, can also be described as the quotient of the element of flux $d\Phi(x,y)$ through a surface element $dA(x,y)$ containing the point x,y by the area dA of the element, and is called the flux (surface) density (at a point of a surface)[1]

$$W(x,y) \equiv \lim_{\Delta A \to 0} \frac{\Delta\Phi(x,y)}{\Delta A} \equiv \frac{d\Phi(x,y)}{dA} \; [\mathrm{W \cdot m^{-2}}] \tag{4.1}$$

The orientation of the rays passing through dA need not be specified. They may be concentrated in a small solid angle, as part of a well-collimated beam (e.g., that from a very distant source, such as a star) or, at the other extreme, they may be spread more or less evenly over the complete hemisphere of directions above dA in a widely diffuse field of radiation. In the more general case, if dA is an element of a curved surface, the hemisphere is that above the tangent plane that contains dA.

In order to cover all possibilities when ray directions are confined to the hemisphere on one side, only, of a surface element, we also take account of the sense in which propagation occurs along a ray. It is incident when it is toward the surface; exitent when it is away from the surface. As we'll see when we take up thermal radiation, we are sometimes concerned with simultaneous propagation in both senses, incident and exitent, along each ray direction, as well as with the net energy transfer given by the difference between them. It is this net value that is measured by most detectors. This can be a particularly important consideration in heat-transfer configurations near thermal equilibrium where each of two surfaces at nearly the same temperature may emit substantially toward the other but where there is little or no net exchange of energy between them.

If the sense of propagation along all rays is toward a surface element dA (from one side of dA), the incident radiant flux (surface) density W is usually designated as the irradiance, with the symbol $E, (E_e)$ [6]. Similarly, for propagation away from dA, the exitent radiant flux (surface) density W leaving the surface (on one side) is usually designated as the radiant exitance, with the symbol $M, (M_e)$ [6].

In previous chapters [5] we pointed out that radiance and spectral radiance, being derivatives, are quotients of vanishingly small quantities that, due to limiting noise levels in all measuring equipment, can never, even in principle, be measured exactly. More seriously, reduction of the quantities involved reaches a point where the basic

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[1]The CIE-IEC [6] has no symbol for this quantity other than the more restricted M or E. We have adopted W, formerly widely used for radiant exitance [now $M, (M_e)$].

assumptions of the geometrical-optics model of ray propagation are no longer valid. The same is true with respect to flux (surface) density, including exitance and irradiance, and with respect to all of the other distributions treated in this chapter. However, our use of calculus, based on the assumption of an underlying continuous distribution of infinite resolution, provides useful results that agree well with experimental measurements as long as we are careful not to try to apply those results outside the domain of geometrical optics.

In Chapter 2 [5], we defined the radiance at the point x,y on a reference surface in the direction θ,ϕ of an intersecting ray through that point (through the surface element dA at that point—see figure 4.2) as

$$L(x,y,\theta,\phi) \equiv \frac{d^2\Phi(x,y,\theta,\phi)}{dA\cdot\cos\theta\cdot d\omega} \; [W\cdot m^{-2}\cdot sr^{-1}] \tag{2.14}$$

From this defining equation for radiance, we were also able to write the expression for the element of flux associated with a ray of radiance $L(x,y,\theta,\phi)$ as

$$d\Phi(x,y,\theta,\phi) = L(x,y,\theta,\phi)\cdot\cos\theta\cdot d\omega\cdot dA \tag{2.24}$$

$$= L(x,y,\theta,\phi)\cdot d\Omega\cdot dA \; [W]. \tag{4.2}$$

Accordingly, the total flux through the surface element dA at the point x,y is

$$d\Phi(x,y) = dA\cdot\int_{\omega} L(x,y,\theta,\phi)\cdot d\Omega = dA\cdot\int_{\omega} L(x,y,\theta,\phi)\cdot\cos\theta\cdot d\omega$$

$$= dA\cdot\int_{\phi}\int_{\theta} L(x,y,\theta,\phi)\cdot\cos\theta\cdot\sin\theta\cdot d\theta\cdot d\phi \; [W], \tag{4.3}$$

where the limits of integration for θ (in the range $0 - \pi/2$ [rad]) and ϕ (in the range $0 - 2\pi$ [rad]) include all rays of the beam of interest that pass through dA at x,y. The flux (surface) density (irradiance or radiant exitance, depending on whether propagation is toward or away from dA on the side in question) through the surface element dA at the point x,y is then

$$W(x,y) \equiv d\Phi(x,y)/dA \tag{4.1}$$

$$= \int_{\omega} L(x,y,\theta,\phi)\cdot d\Omega = \int_{\omega} L(x,y,\theta,\phi)\cdot\cos\theta\cdot d\omega$$

$$= \int_{\phi}\int_{\theta} L(x,y,\theta,\phi)\cdot\cos\theta\cdot\sin\theta\cdot d\theta\cdot d\phi \; [W\cdot m^{-2}] \tag{4.4}$$

where, again, the limits of integration include all of those rays of the beam that pass through dA at the point x,y. In particular, if the beam fills the entire hemisphere above the element dA (above the tangent plane containing dA), this becomes

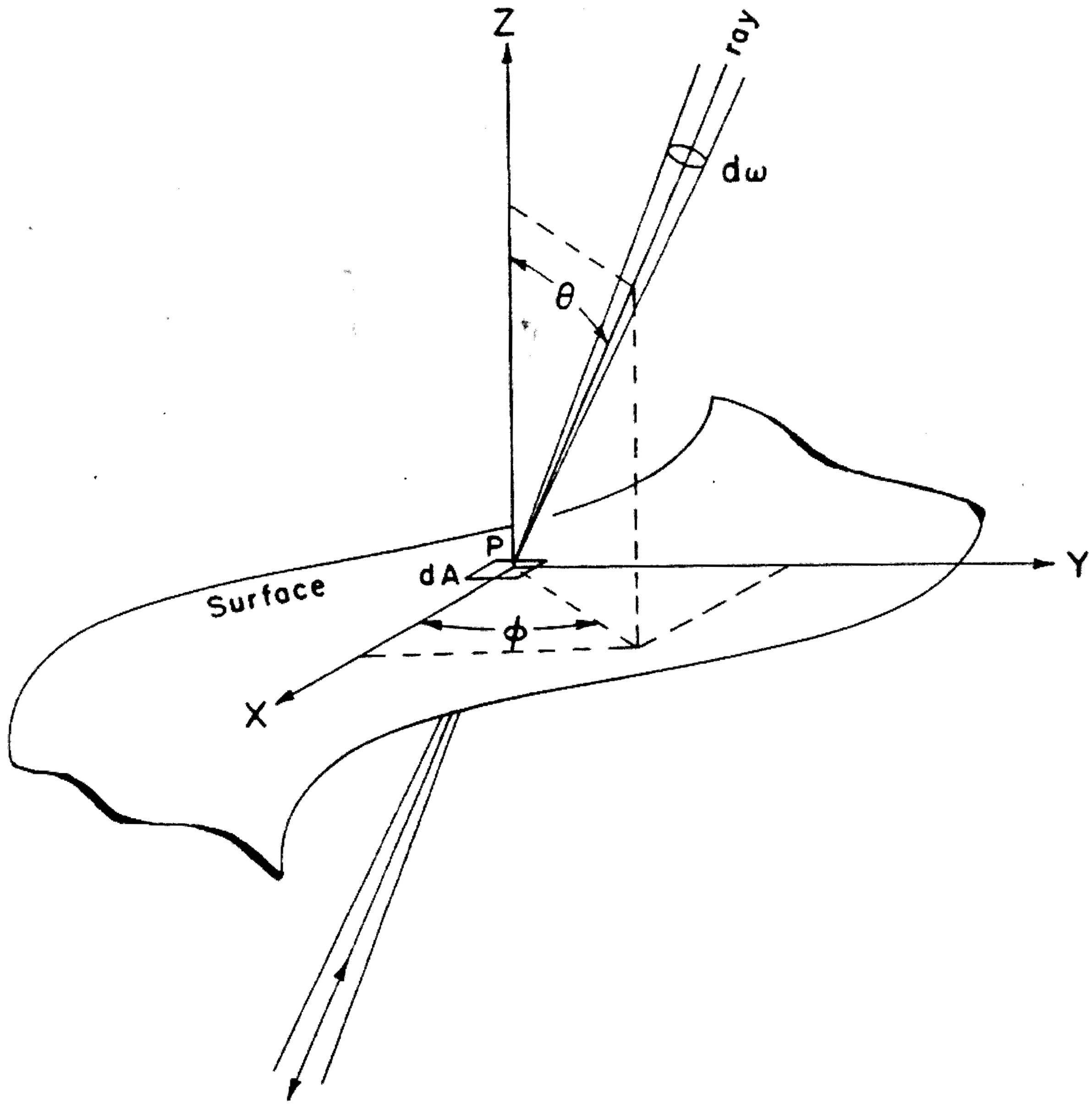

Figure 4.2. Geometry of ray-surface intersection (for the definition of radiance). (This figure previously appeared as figure 2.7 [5].)

$$W(x,y) = \int_h L(x,y,\theta,\phi)\cdot d\Omega = \int_{2\pi} L(x,y,\theta,\phi)\cdot\cos\theta\cdot d\omega$$

$$= \int_o^{2\pi}\int_o^{\pi/2} L(x,y,\theta,\phi)\cdot\cos\theta\cdot\sin\theta\cdot d\theta\cdot d\phi \ [W\cdot m^{-2}]. \qquad (4.4a)$$

Here, $\int_h$ denotes integration over a full hemisphere of 2π [sr] with respect to solid angle. Of course, $E(x,y)$ (irradiance) or $M(x,y)$ (exitance) can always be substituted for $W(x,y)$ {flux (surface) density}, in eqs. (4.4) and (4.4a) or elsewhere, whenever it is appropriate to be more explicit about whether propagation is respectively toward or away from the surface element dA.

Although, as we have pointed out, flux (surface) density (including irradiance and exitance) is not a directional quantity, it is useful to recognize the directionality of the *element* of flux (surface) density by relating it to the flux element (radiance times throughput element) associated with a single ray in the direction θ,ϕ through the surface-area element dA at the point x,y (see figure 4.2):

$$dW(x,y,\theta,\phi) \equiv d\Phi(x,y,\theta,\phi)/dA$$
$$= L(x,y,\theta,\phi)\cdot d\Theta/dA \ [W\cdot m^{-2}], \qquad (4.5)$$

where the throughput element {eq. (2.28) [5]} is

$$d\Theta \equiv dA\cdot d\Omega \equiv dA\cdot\cos\theta\cdot d\omega$$
$$\equiv dA\cdot\cos\theta\cdot\sin\theta\cdot d\theta\cdot d\phi \ [m^2\cdot sr] \qquad (4.6)$$

so that

$$dW(x,y,\theta,\phi) = L(x,y,\theta,\phi)\cdot d\Omega \ [W\cdot m^{-2}]. \qquad (4.7)$$

Again, of course, $dE(x,y,\theta,\phi)$ or $dM(x,y,\theta,\phi)$ may replace $dW(x,y,\theta,\phi)$ to designate, explicitly, the sense of propagation (toward or away from dA) along the ray in the direction θ,ϕ through dA at the point x,y. This also provides an alternate derivation of eq. (4.4), the last two lines of which follow directly from eq. (4.7), since

$$W(x,y) = \int_\omega dW(x,y,\theta,\phi) \ [W\cdot m^{-2}]. \qquad (4.8)$$

FLUX (SURFACE) DENSITY in ISO-RADIANCE BEAMS. The relationship between radiance and flux (surface) density across any intersecting reference surface, is greatly simplified for an iso-radiance[1] beam in which the value of radiance is the same for all rays. Then L is a constant that may be brought outside the integral in eq. (4.4) and the flux (surface) density through a surface element dA at any point x,y becomes

$$W = L \cdot \Omega \ [W \cdot m^{-2}]. \tag{4.9}$$

It is the product of the constant radiance L and the *projected* solid angle $\Omega \equiv \int_\omega \cos\theta \cdot d\omega$ {eq. (2.31) [5]}, where ω is the solid angle filled by the rays of the beam that intersect at the point x,y where the quantity is evaluated. In particular, when all rays into, or from, the full hemisphere above an iso-radiance surface element or plane surface are included in the beam,

$$W = \pi \cdot L \ [W \cdot m^{-2}], \tag{4.10}$$

since the projected solid angle of a full hemisphere is π [sr] {see eq. (A2-15) in Appendix 2 [5]}.

Iso-radiance beams and their properties are very important in radiometry (including photometry). It is obvious from the foregoing that analysis is greatly simplified when iso-radiance conditions exist. Even when all rays are not of exactly the same radiance but are only approximately so, the use of an average value as though it were everywhere the same provides approximate results that are useful for many purposes. However, this has led to such widespread treatment of almost all problems in terms of source-surface radiant exitance and receiver irradiance, particularly in the fields of heat-transfer and illumination engineering, that the directional distribution of radiance is too easily overlooked in situations where it is, in fact, significant. Accordingly, we reemphasize that the sound approach is always to analyze a situation first in terms of the radiance of individual rays and its spatial distribution and to employ one of the "simpler" distributions, in terms of position or direction alone, only after that analysis has shown clearly that this can be done without unacceptable approximation or error.

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[1]Iso-radiance, rather than lambertian, because, strictly, a lambertian surface is one that is isotropic (the same radiance in all directions--at least within the beam of interest) at each point, although it might be non-uniform (varying from point to point). If it is non-uniform, a second surface cutting the beam at a distance from the first will no longer be lambertian. The rays crossing at any one point of the second surface will come from different points of the original surface, so the radiance of a point on the second surface is no longer isotropic. An iso-radiance surface or beam is both uniform and labertian; it has the same radiance everywhere and in all directions so that every surface cutting the beam anywhere is lambertian (within the beam).

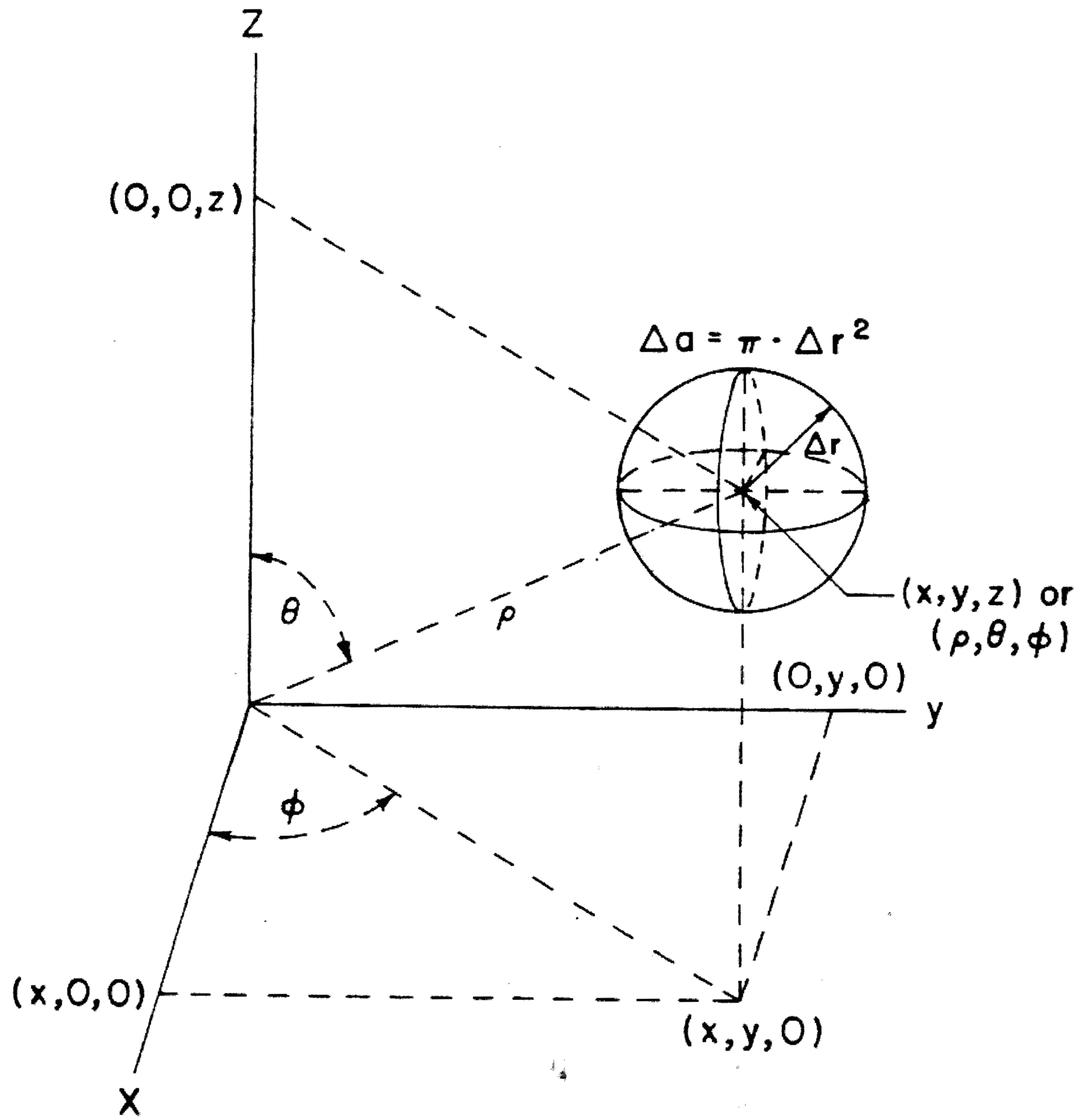

Figure 4.3. A small sphere of radius Δr and cross section $\Delta a = \pi \cdot (\Delta r)^2$, centered at the point x, y, z (or ρ, θ, ϕ).

OMNI-DIRECTIONAL-SURFACE DISTRIBUTION: FLUENCE RATE. The directed-surface distributions, defined by eq. (4.1), are seen, by eq. (4.4), to depend on the orientation of the reference-surface element dA. Accordingly, they are not scalar field quantities, since they do not have a unique value at each point. The omni-directional-surface distribution, on the other hand, involves no reference surface; it does have a unique value at each point in space, making it a scalar field quantity. The fluence rate, as we call it, is defined with reference to the cross-sectional area da of a spherical volume element (which is the same in all directions) rather than the area dA of a surface element of fixed orientation. Sometimes it is called, instead, the "scalar irradiance."

In order to analyze the concept of omni-directional-surface distribution or fluence rate by means of a "thought experiment," consider a small sphere of radius Δr with its center at the point x,y,z in space, as shown in figure 4.3. The area of the circular plane surface through x,y,z perpendicular to any incident ray, i.e., the area of the circular cross-section of the sphere, is then

$$a = \pi \cdot (\Delta r)^2 \ [m^2]. \tag{4.11}$$

We also postulate that, in some way, we can determine the total flux $\Delta\Phi$ [W] incident on the sphere from all directions. (This might be approximated by replacing the sphere with an ideal black spherical receiver and making calorimetric measurements based on its temperature history, etc.) Then, if we keep the center of the sphere fixed at x,y,z and reduce the cross-sectional area Δa by reducing the radius Δr, the fluence rate at the point x,y,z is defined as the limit, as Δa approaches zero, of the quotient $\Delta\Phi/\Delta a$:

$$F_t(x,y,z) \equiv \lim_{\Delta a \to 0} \frac{\Delta\Phi(x,y,z)}{\Delta a} \equiv \frac{d\Phi(x,y,z)}{da} \ [W \cdot m^{-2}], \tag{4.12}$$

where

$F_t(x,y,z)$ $[W \cdot m^{-2}]$ is the fluence rate at the point x,y,z;[1] and

$d\Phi(x,y,z)$ [W] is the flux or power incident on a spherical volume element, of cross-section da $[m^2]$, centered at the point x,y,z.

No limitation is placed on the direction of arrival of the incident radiant flux $d\Phi$.

[1]Neither the CIE-IEC [6] nor the ANSI [10,11] has either a term or a symbol for this quantity, or for the related quantity, fluence. After choosing the symbol F for fluence [12] which we discuss later in this chapter, we preferred to avoid an additional symbol, as used elsewhere [8], for fluence rate by adopting F_t [$\equiv dF/dt$] since, as we will see presently, the fluence rate is the time derivative of the fluence at the same point. In this Manual, we employ the notation $X_p \equiv dX/dp$ for any radiometric quantity X and the following radiation parameters p: position, direction, any spectral parameter (wavelength, wave number, frequency ν, etc.), or time or frequency $f \ll \nu$.

It may be incident from any direction which, unless explicitly limited in a particular case, includes the entire sphere of directions surrounding the point x,y,z, a full 4π-[sr] solid angle.

Next, let's consider the contribution to the fluence rate, defined in eq. (4.12), by the incident element of flux $d\Phi(x,y,z,\theta,\phi)$ associated with a single ray, of radiance $L(x,y,z,\theta,\phi)$ through the point x,y,z from the direction θ,ϕ. That element of flux is given by {see eq. (2.24) [5]}

$$d\Phi(x,y,z,\theta,\phi) = L(x,y,z,\theta,\phi)\cdot\cos\theta_a\cdot d\omega\cdot da \ [W], \tag{4.13}$$

where θ_a is the angle between the ray and the normal to da. But da is the circular intersection of the spherical volume element with a plane through x,y,z perpendicular to the ray, so that $\theta_a = 0$ and $\cos\theta_a = 1$ for every incident ray and eq. (4.13) becomes

$$d\Phi(x,y,z,\theta,\phi) = L(x,y,z,\theta,\phi)\cdot d\omega\cdot da \ [W]. \tag{4.14}$$

The corresponding element of fluence rate is then

$$\begin{aligned} dF_t(x,y,z,\theta,\phi) &\equiv d\Phi(x,y,z,\theta,\phi)/da \\ &= L(x,y,z,\theta,\phi)\cdot d\omega \ [W\cdot m^{-2}], \end{aligned} \tag{4.15}$$

and the total fluence rate at the point x,y,z is the integral over all such elements of incident fluence rate:

$$\begin{aligned} F_t(x,y,z) &= \int_\omega dF_t(x,y,z,\theta,\phi) \\ &= \int_\omega L(x,y,z,\theta,\phi)\cdot d\omega \\ &= \int_\phi\int_\theta L(x,y,z,\theta,\phi)\cdot\sin\theta\cdot d\theta\cdot d\phi \ [W\cdot m^{-2}], \end{aligned} \tag{4.16}$$

where the limits of integration for θ and ϕ include all rays incident on the elementary spherical volume at x,y,z. In particular, if the incident radiation comes from all surrounding directions, the solid angle ω is a full sphere of 4π [sr] and this becomes

$$\begin{aligned} F_t(x,y,z) &= \int_{\text{sphere}} dF_t(x,y,z,\theta,\phi) \\ &= \int_{4\pi} L(x,y,z,\theta,\phi)\cdot d\omega \\ &= \int_o^{2\pi}\int_o^{\pi} L(x,y,z,\theta,\phi)\cdot\sin\theta\cdot d\theta\cdot d\phi \ [W\cdot m^{-2}]. \end{aligned} \tag{4.16a}$$

As was stated earlier, this is a scalar field quantity, since it has a unique value at each point x,y,z in space. By contrast, the incident radiant flux (surface) density or irradiance is, from eq. (4.4a),

$$W(x,y) = \int_h L(x,y,\theta,\phi)\cdot d\Omega = \int_{2\pi} L(x,y,\theta,\phi)\cdot\cos\theta\cdot d\omega \ [W\cdot m^{-2}]. \qquad (4.17)$$

The latter, as we have said, does not have a unique value at a point x,y,z in space. Instead, the value of $W(x,y)$ in eq. (4.17) depends on the orientation of the reference surface at the point x,y; i.e., on the direction of the normal to the element dA at x,y which determines the value of θ, and hence of $\cos\theta$, for each incident ray.[1]

The integrals in eqs. (4,16a) and (4.17) differ in two important respects: (1) the first does not contain the obliquity factor $\cos\theta$, which appears as a weighting factor that varies with surface orientation in the second; (2) the first integral is taken over a solid angle that may extend to the full sphere of 4π [sr] while the second may extend, at maximum, only to the hemisphere of 2π [sr] on just one side of the reference-surface element (of the tangent plane containing the element dA).

Conversely, we can write expressions for radiance, in terms of radiant flux (surface) density and in terms of fluence rate, from eqs. (4.7) and (4.15), respectively. They are

$$L(x,y,\theta,\phi) = \frac{dW(x,y,\theta,\phi)}{d\Omega} = \frac{dW(x,y,\theta,\phi)}{\cos\theta\cdot d\omega} \ [W\cdot m^{-2}\cdot sr^{-1}] \qquad (4.18)$$

and

$$L(x,y,z,\theta,\phi) = \frac{dF_t(x,y,z,\theta,\phi)}{d\omega} \ [W\cdot m^{-2}\cdot sr^{-1}]. \qquad (4.19)$$

Note, particularly, that eq. (4.19) is an expression for the radiance in a given direction at a point in space that does not involve any reference surface. While this is appealing to the theoretician, we don't feel that it is of strong practical significance in connection with optical radiation measurements since such measurements almost invariably involve an instrument with a defined beam, or throughput, involving a reference surface formed by the (imaginary) plane across the receiving aperture or entrance pupil (and the solid angle subtended at each point across that aperture or pupil by the entrance window, the field

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[1]Note that, in the rare cases where we directly compare the incident flux (surface) density $W(x,y)$ or irradiance $E(x,y)$ and the fluence rate $F_t(x,y,z)$ at the same point, the coordinates x,y in eq. (4.17) are position coordinates on the reference surface while x,y,z in eq. (4.16) are coordinates in space. The x,y coordinates of the common point can be the same in both equations only if the reference surface is a plane surface. Then, by appropriate transformation (translation and rotation) the origins and axes of both coordinate systems can be brought into coincidence.

solid angle).[1] Accordingly, the appropriate definition and treatment of radiance, for our purposes, is in terms of a reference surface, as in eq. (2.14) [5] {repeated just before eq. (4.2), above}.

On the other hand, there certainly are occasions when a distribution of incident radiant flux with respect to position is more appropriately or meaningfully given in terms of fluence rate rather than irradiance [incident radiant flux (surface) density]. Accordingly, we need to clearly establish the relationship between these two quantities, both given in $[\mathrm{W \cdot m^{-2}}]$. We substitute E for W in eq. (4.17), since we are concerned only with incident radiation, and compare it directly with eq. (4.16). In making this comparison, we recognize that the ray-direction coordinates θ,ϕ may be referred to arbitrarily oriented fixed axes but that $\cos\theta$ in eq. (4.17) is the cosine of the angle between the ray direction θ,ϕ and the normal to the receiving surface element dA_r upon which it is incident. Accordingly, we'll designate the polar angle from the normal to dA_r as θ_r, to distinguish it from the direction coordinate θ referred to the fixed axes. Then it is clear that the element of fluence rate at a point x,y on some reference surface is

$$dF_t(x,y,\theta,\phi) \equiv dE(x,y,\theta,\phi)/\cos\theta_r \equiv dE_n(x,y,\theta,\phi) \ [\mathrm{W \cdot m^{-2}}], \tag{4.20}$$

where dE_n is the <u>normal irradiance</u> and is equal to the irradiance element dE only when $\cos\theta_r = 1$, i.e., when $\theta_r = 0$ (when the ray direction θ,ϕ coincides with the normal to dA_r). Thus the element of fluence rate at a point is equal to the normal irradiance at that point, defined as the irradiance on a surface element at that point divided by the cosine of the angle between the incident ray and the normal to the surface element. Only the *elements* of fluence rate and irradiance, associated with individual rays (elements of throughput) can be directly related in this way. The integrated total fluence rate and irradiance at a point cannot be directly converted from one to the other. If values for both are needed, they are most easily obtained from the incident radiance distribution and eqs. (4.16a) and (4.17). But this seems unlikely; ordinarily only one or the other will be useful or significant.

Discussion of illustrative examples involving radiant flux (surface) density and radiant fluence rate is deferred until the concept of radiant intensity has been presented. Then the examples will help to clarify all three of these concepts and their interrelationships.

<u>DIRECTIONAL DISTRIBUTION: RADIANT INTENSITY—TRADITIONAL APPROACH</u>.

Consider the beam of radiation flowing from all points on the surface of an extended source S to a small receiving surface of area ΔA, as shown in figure 4.4. Designate the magnitude of the total flux, reaching the receiver from S, as $\Delta\Phi$. Let the solid angle subtended at any point P on the source surface by the receiver be designated as $\Delta\omega$.

[1] The theory of stops (pupils and windows) will be found in any standard text on geometrical optics [13,14].

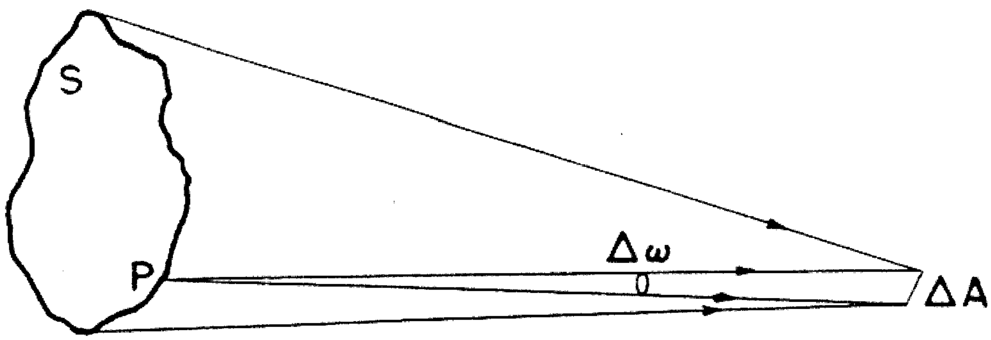

Figure 4.4. Experiment to develop the concept of <u>(radiant) intensity</u> -- traditional approach.

If the receiver is moved arbitrarily far from the source, both $\Delta\Phi$ and $\Delta\omega$ decrease to the vanishing point, but their quotient $\Delta\Phi/\Delta\omega$ approaches a constant value called the <u>radiant intensity</u> I of the source in the direction of the receiver:

$$I \equiv \frac{d\Phi}{d\omega} \ [\mathrm{W \cdot sr^{-1}}]. \tag{4.21}$$

Also, as the receiver moves away, the small solid angle that it subtends at every other point on the source surface becomes more nearly equal to and more nearly parallel to $\Delta\omega$. Ultimately, the rays from all points of S to ΔA "at infinity" can be thought of as forming a set of parallel solid-angle elements $d\omega$, each containing a ray from that source point in the direction of ΔA, as depicted in figure 4.5.

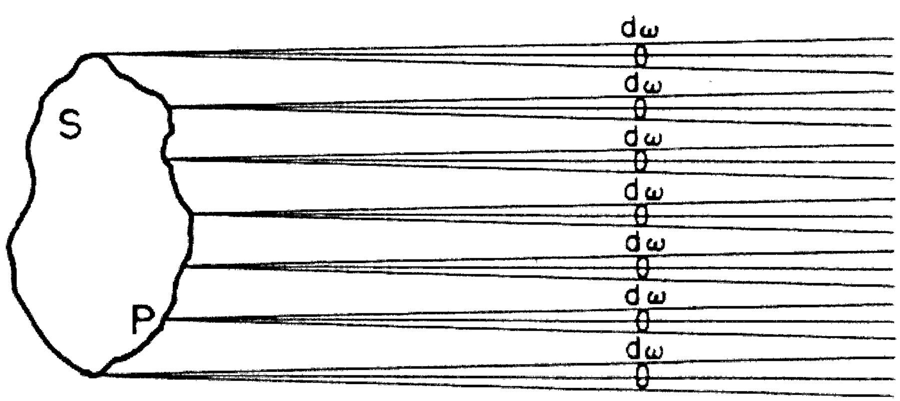

Figure 4.5. Beam from the source of figure 4.4 to a receiver "at infinity."

If L is the radiance, in the direction of the receiver ΔA, of an element of source surface dA_s, then, by eq. (2.15 [5], the element of flux propagated from dA_s to ΔA is $L \cdot dA_s \cdot \cos\theta \cdot d\omega$, where θ is the angle between the normal to dA_s and the ray from dA_s to ΔA. Then the flux in the solid-angle element $d\omega$, propagated from all of S to ΔA, is given by the integral

$$d\Phi = \int_S L \cdot dA_s \cdot \cos\theta \cdot d\omega = d\omega \cdot \int_S L \cdot \cos\theta \cdot dA_s, \tag{4.22}$$

so that

$$I \equiv \frac{d\Phi}{d\omega} = \int_S L \cdot \cos\theta \cdot dA_s \; [\mathrm{W \cdot sr^{-1}}]. \tag{4.23}$$

This gives us an alternative expression for the radiant intensity of an extended source in a given direction in terms of the radiance of each point (surface element) of the source in the given direction.

Measurements of the incident flux, per unit solid angle subtended at the source, reaching a detector from a source at a finite distance are only approximations to the defined quantity given by eq. (4.21) or (4.23)

$$I \approx \frac{\Delta\Phi}{\Delta\omega} \; [\mathrm{W \cdot sr^{-1}}]. \tag{4.24}$$

We'll examine the factors that contribute to the degree of approximation after we've defined radiant intensity a bit more carefully and have established its relationships to other radiometric quantities in a somewhat different manner and in greater detail.

The definition or description of the concept of radiant intensity seems to give rise to more problems than that of any other radiometric quantity. Some readers are puzzled by the fact that we have related the flux $\Delta\Phi$, coming from all parts of the source in figure 4.4 (not just that flowing in the solid angle $\Delta\omega$ from the particular point P shown in the illustration), to the more limited solid angle $\Delta\omega$ in the quotient that converges to the limiting value of eq. (4.21) and that appears in eq. (4.24). The point P in figure 4.4 is *any* point on the emitting surface of the source. There is (not shown) a similar solid angle, approximately equal to $\Delta\omega$, subtended at every other source point from which source radiation reaches the receiver. We use the value $\Delta\omega$, then, as a representative approximation to all of these solid angles, subtended at all points of the source. As the distance between source and receiver increases, the approximation improves until, in the limit, as suggested by figure 4.5, all of the elementary solid angles become parallel and equal to $d\omega$. In a very large number of cases, the approximation becomes practically satisfactory (within useful limits of accuracy) within distances available in the laboratory. As we'll see, we soon reach a point, with most sources, where the flux $\Delta\Phi$ and the solid angle $\Delta\omega$ both become inversely proportional to the square of the distance between convenient points on both source and receiver (the size of which may also be appreciable), so the quotient becomes and remains constant as that distance is further increased. Then we have a useful value of source radiant intensity that characterizes the source emission in the given direction. It is often overlooked, however, that this depends on the fact that the radiance of a point on the surface of a source usually doesn't change abruptly with a small change in direction. We'll examine this point in more detail later, in the text accompanying figure 4.12, below.

DIRECTIONAL DISTRIBUTION: RADIANT INTENSITY. For a more detailed treatment of radiant intensity, consider again the experiment, first introduced in Chapter 2 [5], with an iso-radiance visible source, two beam-forming apertures in opaque screens, and a photocell, arranged as illustrated in figures 4.6 and 4.7(a) (figures 2.2 and 2.3 of Chapter 2 [5]). A uniform, clear propagation path is assumed, with no variation in refractive index and no attenuation (by absorption or scattering). Previously, we found that, as we varied the areas ΔA_1 and ΔA_2, and the respective tilts θ_1 and θ_2, of the apertures as well as the distance D between them, the radiance L, was given by eq. (2.4). That equation is repeated here, but as an approximation

$$L \approx \frac{\Delta\Phi \cdot D^2}{\Delta A_1 \cdot \cos\theta_1 \cdot \Delta A_2 \cdot \cos\theta_2} \tag{4.25}$$

because we also found, subsequently, that the exact expressions for L were given in eqs. (2.5) and (2.13) as

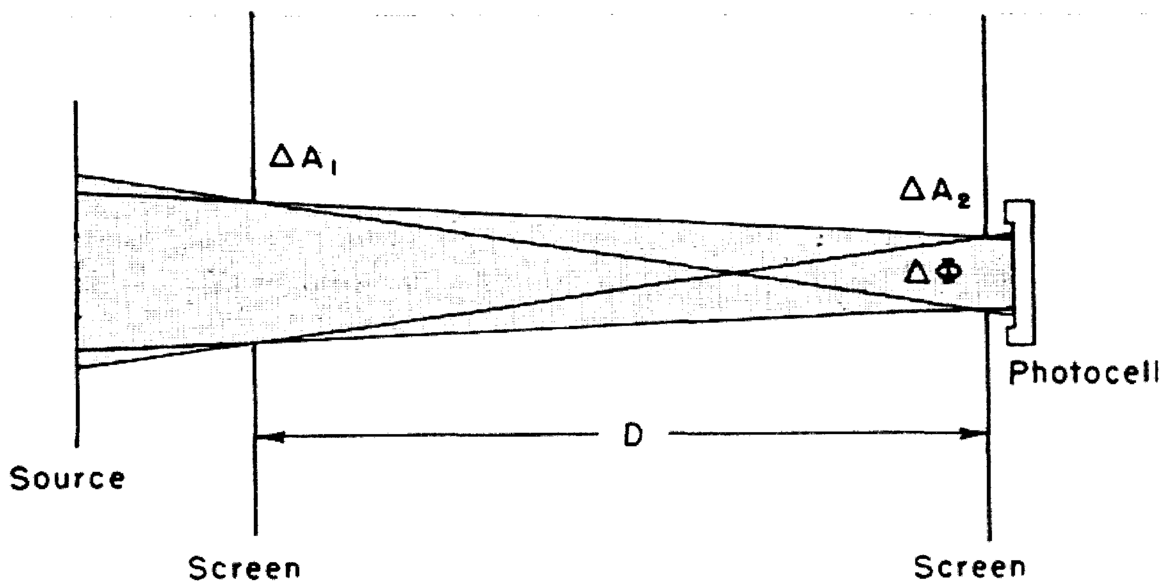

Figure 4.6. Experiment to develop the concept of radiance.
(This figure appeared previously as figure 2.2 [5].)

$$L = \lim_{\substack{\Delta A_1 \to 0 \\ \Delta A_2 \to 0}} \frac{\Delta\Phi}{\Delta A_1 \cdot \cos\theta_1 \cdot (\Delta A_2 \cdot \cos\theta_2 / D^2)}$$

$$= \lim_{\substack{\Delta A_1 \to 0 \\ \Delta\omega_{12} \to 0}} \frac{\Delta\Phi}{\Delta A_1 \cdot \cos\theta_1 \cdot \Delta\omega_{12}} = \frac{d^2\Phi}{dA_1 \cdot \cos\theta_1 \cdot d\omega_{12}} \; [\mathrm{W \cdot m^{-2} \cdot sr^{-1}}]. \tag{4.26}$$

These equations define the radiance L that is geometrically invariant along any ray in a lossless, isotropic (iso-index-of-refraction) medium. Here, the quantity $d\omega_{12} \equiv dA_2 \cdot \cos\theta_2 / D^2$ is the element of solid angle subtended by dA_2 at dA_1 {see eq. (A2-13), Appendix 2 [5]}.

The first aperture screen and the source, taken together, are equivalent to an iso-radiance source, of area ΔA_1 filling the aperture in the screen, because every ray of the beam, through every point across that aperture, has the same radiance L. Whenever it will simplify our discussion, then, we may replace the source-screen combination by a bare iso-radiance plane source of radiance L, of area $\Delta A_s = \Delta A_1$ tilted at an angle $\theta_s = \theta_1$, at that same location. Similarly, the combination of the second aperture screen and the photocell, taken together, is equivalent to, and may be replaced by, a bare plane receiver-detector of area $\Delta A_r = \Delta A_2$, tilted at an angle $\theta_r = \theta_2$, centered at a distance D from the center of the source, and with uniform isotropic response (equal response to flux incident at any point and from any direction within the beam). These equivalent configurations, that may be interchanged at will without altering the propagation of radiant energy in the beam, are illustrated in figures 4.7(a) and (b).

In addition to rewriting eq. (4.26) in terms of the quantities in figure 4.7(b), it will be convenient to rearrange it somewhat to examine what happens as we go to the limits in different ways.

$$L = \lim_{\substack{\Delta A_s \to 0 \\ \Delta\omega_{sr} \to 0}} \frac{\Delta\Phi}{\Delta A_s \cdot \cos\theta_s \cdot \Delta\omega_{sr}}$$

$$= \lim_{\substack{\Delta A_s \to 0 \\ D \to \infty}} \frac{\Delta\Phi}{\Delta A_s \cdot \cos\theta_s \cdot (\Delta A_r \cdot \cos\theta_r / D^2)} \; [\mathrm{W \cdot m^{-2} \cdot sr^{-1}}], \quad \text{or} \tag{4.27}$$

$$= \lim_{\substack{\Delta A_s \to 0 \\ \Delta A_r \to 0}} \frac{\Delta\Phi}{\Delta A_s \cdot \cos\theta_s \cdot (\Delta A_r \cdot \cos\theta_r / D^2)} \; [\mathrm{W \cdot m^{-2} \cdot sr^{-1}}]. \tag{4.28}$$

Note that the solid angle $\Delta\omega_{sr}$ (subtended at ΔA_s by ΔA_r) may be reduced either by increasing D, as in eq. (4.27), or by reducing the receiver area ΔA_r, as in eq. (4.28), or, of course, by doing both. We examine the results of doing one or the other alone, first looking at the effects of increasing the separation distance D as in eq. (4.27)

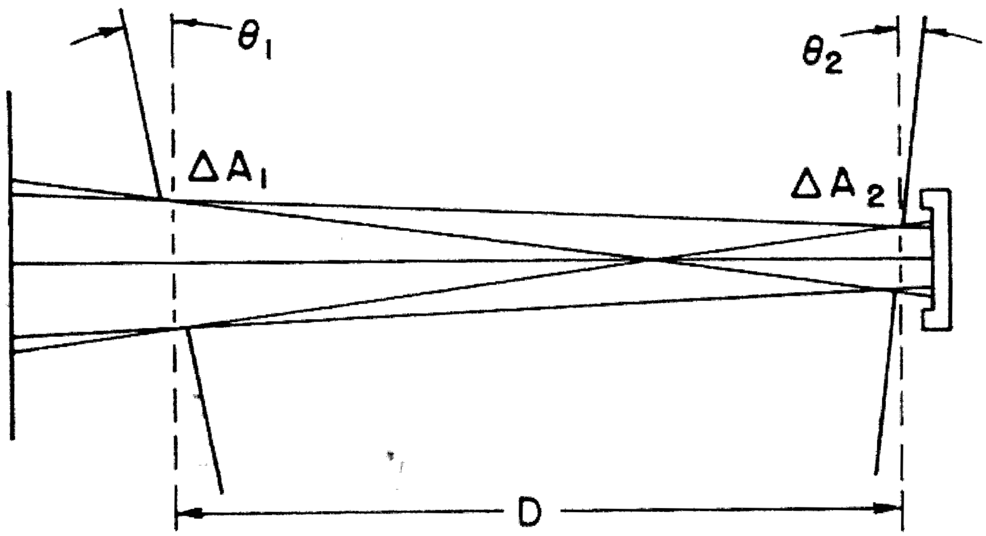

Figure 4.7(a). Tilted Apertures.
(This figure appeared previously as figure 2.3 [5].)

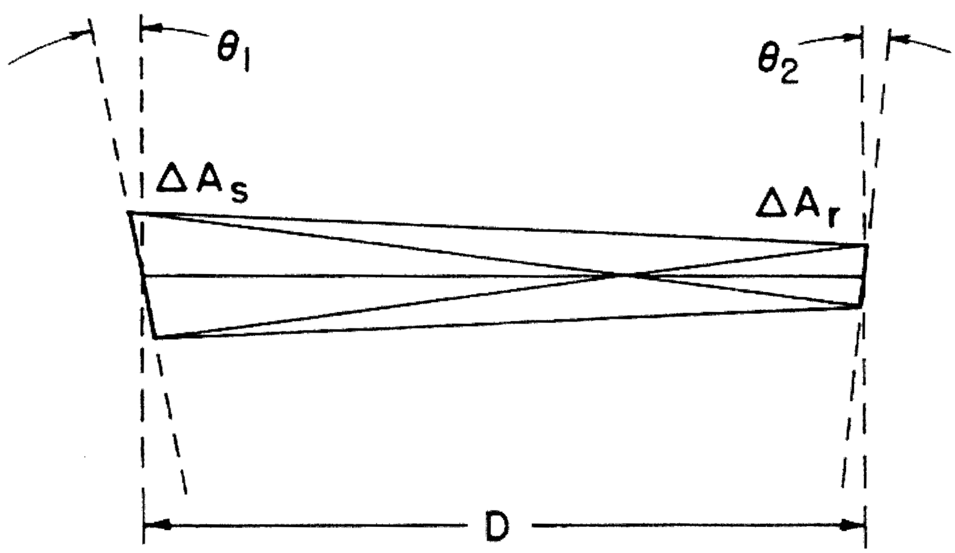

Figure 4.7(b). Bare source and detector equivalent to figure 4.7(a).

while holding the source area ΔA_s fixed. Then, holding D constant, we'll examine the effects of independently varying ΔA_s. A similar analysis of eq. (4.28) will follow in the next section.

As the separation D is increased, both the flux $\Delta\Phi$ and the solid angle $\Delta\omega_{sr}$ are reduced. However, if the small source ΔA_s is strong enough and the small detector ΔA_r is sufficiently sensitive, a point is soon reached where the quotient $\Delta\Phi/\Delta\omega_{sr}$ becomes and remains constant (within achievable measurement accuracy) with further increase of D until there is no longer enough flux $\Delta\Phi$ to produce a measurable response. The limiting value of the quotient $\Delta\Phi/\Delta\omega_{sr}$ is called the radiant intensity ΔI of the source ΔA_s in the direction of the detector (in a direction making an angle θ_s with the normal to the plane source surface):

$$\Delta I = \lim_{D \to \infty} \frac{\Delta\Phi}{\Delta A_r \cdot \cos\theta_r / D^2} = \frac{d\Phi}{d\omega_{sr}} \; [W \cdot sr^{-1}]. \tag{4.29}$$

This can be repeated for different values of source area ΔA_s with similar results except that, as ΔA_s is reduced, ΔI is correspondingly reduced. The maximum distance D where there is still enough flux $\Delta\Phi$ to produce a measurable response is also reduced with decreasing ΔA_s. Accordingly, we can substitute from eq. (4.29) in eq. (4.27) to obtain

$$L = \lim_{\Delta A_s \to 0} \frac{\Delta I}{\Delta A_s \cdot \cos\theta_s} = \frac{dI}{dA_s \cdot \cos\theta_s} \; [W \cdot m^{-2} \cdot sr^{-1}], \tag{4.30}$$

from which we have

$$dI = L \cdot \cos\theta_s \cdot dA_s \; [W \cdot sr^{-1}]. \tag{4.31}$$

Up to this point, for simplicity, we have postulated an iso-radiance source. More generally, however, the limiting values of radiant intensity can be obtained in exactly the same way even when the source radiance varies with position and/or direction. It is only necessary to go to smaller areas and solid angles to obtain the constant values, corresponding to the limiting values that are the radiant intensities. Accordingly, we will restate eq. (4.31) more completely and explicitly: The element of radiant intensity $dI(x,y,\theta,\phi)$, in the direction θ,ϕ, of a source-surface element dA_s of radiance $L(x,y,\theta,\phi)$ at the point x,y and in that direction, is given by

$$dI(x,y,\theta,\phi) = L(x,y,\theta,\phi) \cdot \cos\theta_s \cdot dA_s \; [W \cdot sr^{-1}]. \tag{4.32}$$

Then, by integrating both sides of eq. (4.32), we obtain an alternative defining equation for the radiant intensity of an extended source of radiating (emitting and/or reflecting or scattering) area A_s:

$$I(\theta,\phi) = \int_{A_s} dI(x,y,\theta,\phi)$$
$$= \int_{A_s} L(x,y,\theta,\phi)\cdot\cos\theta_s\cdot dA_s \ [W\cdot sr^{-1}]. \qquad (4.33)$$

Note that, in these equations, the angle θ_s (between a ray and the normal to dA_s) will be equal to the ray-direction-coordinate angle θ only when the element dA_s is oriented with its normal parallel to the polar axis of the direction coordinates. Generally, for a non-planar reference surface, the angle θ_s for parallel rays will vary from point to point across that reference surface, as illustrated in figure 4.8.

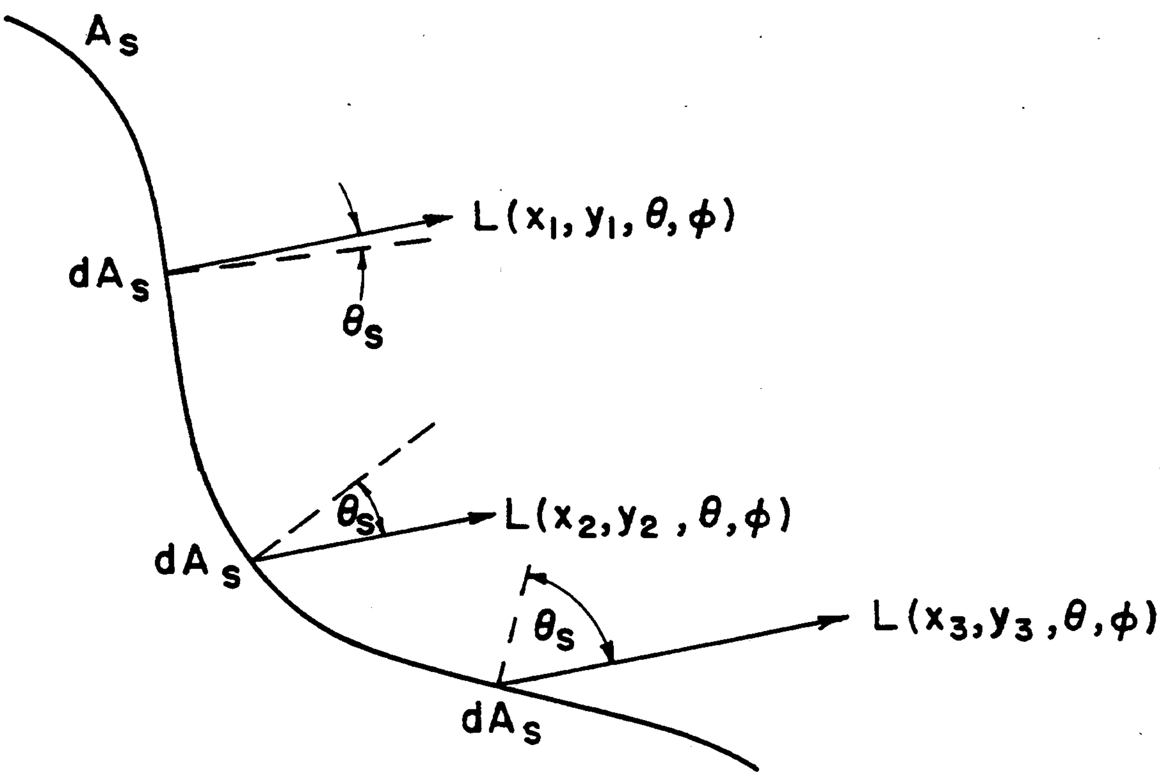

Figure 4.8. Illustration to clarify the designations θ and θ_s. Given a fixed direction θ,ϕ and a curved reference surface, the angle θ_s between the direction θ,ϕ and the normal to a surface element dA_s varies from point to point across the curved surface.

POINT SOURCES and RECEIVERS and the INVERSE-SQUARE LAW. Although eq. (4.33) explicitly defines a quantity called the "radiant intensity" for any source of extended area A_s, it turns out that this quantity is useful, in general, only when the source can be regarded or treated as a "point source." The classic example of what is meant by "treated as a 'point source'" is a star, even the nearest of which, in spite of being, in many cases, incredibly large in comparison with the earth, or even the sun, is so far distant that our telescopes can't distinguish or "resolve" different points on its surface. Accordingly, for all practical purposes, the rays reaching the earth from all parts of the emitting surface of a star are those that travel from it in parallel lines in a single direction {the direction θ,ϕ in eq. (4.33)}. In a great many cases, when shorter distances are involved, a criterion that determines whether or not we can treat an extended source as a point source, with a useful value of radiant intensity, is whether or not the irradiance or radiant fluence rate at a point of interest, due to the source in question, obeys the inverse-square law within a given tolerance. Accordingly, we next want to look more closely at the relationship between radiant intensity and irradiance or radiant fluence rate known as the "inverse-square law."

We can obtain the inverse-square law by going back to

$$L = \lim_{\substack{\Delta A_s \to 0 \\ \Delta A_r \to 0}} \frac{\Delta\Phi}{\Delta A_s \cdot \cos\theta_s \cdot (\Delta A_r \cdot \cos\theta_r / D^2)} \ [\mathrm{W \cdot m^{-2} \cdot sr^{-1}}] \tag{4.28}$$

to see what happens as we first reduce ΔA_r while holding ΔA_s constant. As ΔA_r becomes smaller, the flux $\Delta\Phi$ reaching the detector from the source is correspondingly reduced, but the quotient $\Delta\Phi/\Delta A_r$ becomes and remains constant, and we recognize [from eq. (4.1)] that, in the limit, this quotient is the incident radiant flux (surface) density or irradiance ΔE. We designate it as ΔE, rather than just E, because it is also dependent on ΔA_s. Accordingly, eq. (4.28) becomes

$$L = \lim_{\Delta A_s \to 0} \frac{\Delta E \cdot D^2}{\Delta A_s \cdot \cos\theta_s \cdot \cos\theta_r} = \frac{dE \cdot D^2}{dA_s \cdot \cos\theta_s \cdot \cos\theta_r} \ [\mathrm{W \cdot m^{-2} \cdot sr^{-1}}]. \tag{4.34}$$

When this is rearranged and compared with eq. (4.31), we have

$$L \cdot dA_s \cdot \cos\theta_s = dI = \frac{dE \cdot D^2}{\cos\theta_r} \ [\mathrm{W \cdot sr^{-1}}] \tag{4.35}$$

from which we obtain the inverse-square law, giving the element of normal irradiance dE_n or radiant fluence rate dF_t [see eq. (4.20)] at a distance D from the source element dA_s in a direction in which its intensity is dI:

$$dE_n = dF_t = dE/\cos\theta_r = dI/D^2 = L \cdot \cos\theta_s \cdot dA_s / D^2 \ [\mathrm{W \cdot m^{-2}}], \tag{4.36}$$

or, more explicitly,

$$dE_n(x_r,y_r,z_r,\theta,\phi) = dF_t(x_r,y_r,z_r,\theta,\phi) = dE(x_r,y_r,z_r,\theta,\phi)/\cos\theta_r$$

$$= dI(x_s,y_s,z_s,\theta,\phi)/D^2$$

$$= L(x_s,y_s,z_s,\theta,\phi)\cdot\cos\theta_s\cdot dA_s/D^2 \ [\mathrm{W\cdot m^{-2}}], \qquad (4.36a)$$

where (see figure 4.9)

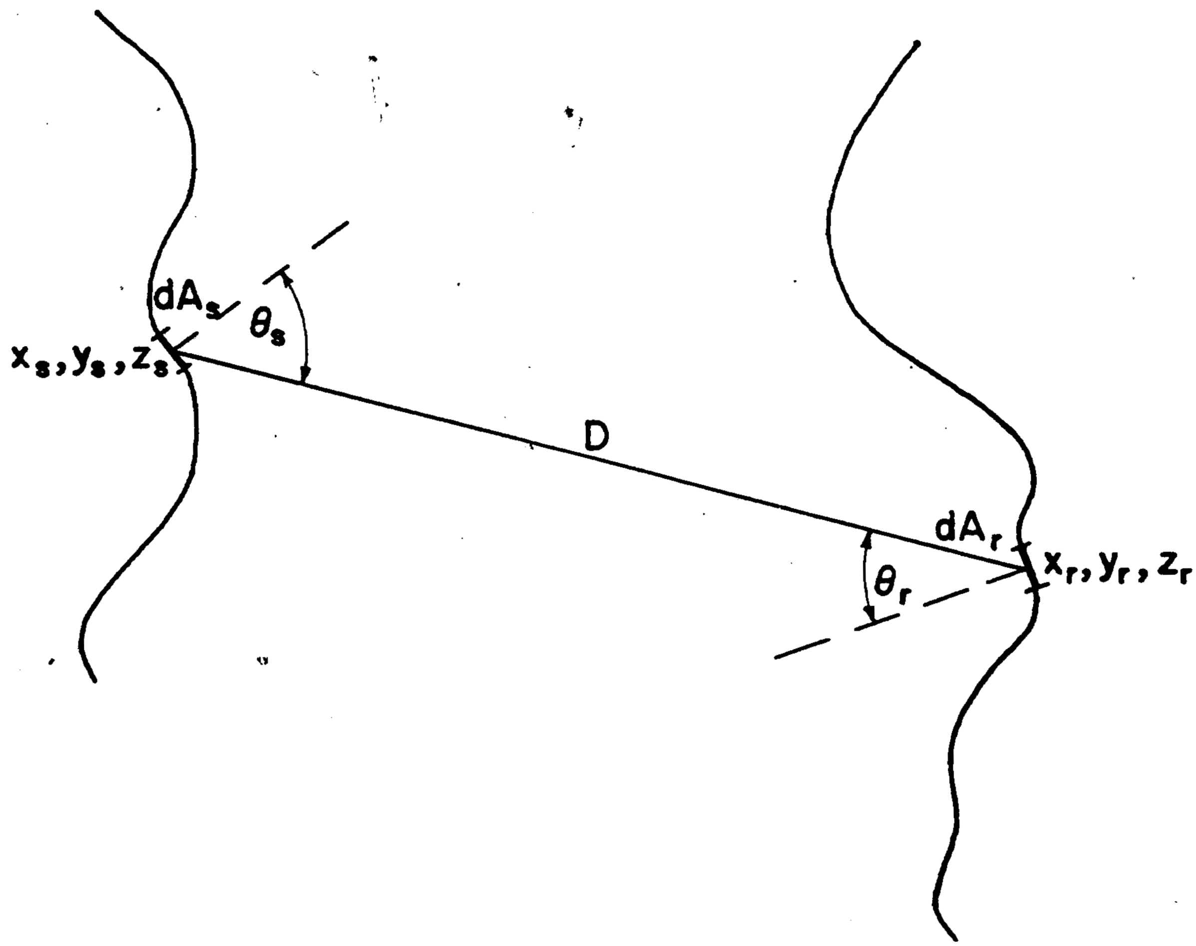

Figure 4.9. A section through source and receiver surfaces of arbitrary shape. A source-surface element dA_s at x_s,y_s,z_s and a receiver-surface element dA_r at x_r,y_r,z_r are separated by a distance D. For purposes of illustration, the normals to the two elements are shown as lying in the same plane although they are not actually so constrained.

$$D^2 = (x_r - x_s)^2 + (y_r - y_s)^2 + (z_r - z_s)^2 \ [m^2]$$

is the square of the (slant) distance between the source element at x_s, y_s, z_s and the receiver element at x_r, y_r, z_r,

$$\theta = \tan^{-1} \{[(x_r - x_s)^2 + (y_r - y_s)^2]^{1/2}/(z_r - z_s)\} \ [rad],$$

$$\phi = \tan^{-1} \{(y_r - y_s)/(x_r - x_s)\} \ [rad],$$

θ_s [rad] is the angle between the direction θ, ϕ and the normal to the element of source emitting surface dA_s at x_s, y_s, z_s, and

θ_r [rad] is the angle between the direction θ, ϕ and the normal to the element of receiving surface dA_r at x_r, y_r, z_r.

The inverse-square law is a relationship for a single ray or throughput element through a source-surface element dA_s, of intensity $dI = L \cdot \cos\theta_s \cdot dA_s$, in the direction θ, ϕ from the point x_s, y_s, z_s. It gives the normal irradiance dE_n [$= dE/\cos\theta_r$] or radiant fluence rate dF_t at a distance D in the direction θ, ϕ from that source element. Integrals based on these relationships can successfully account for the observed values of radiation flux in beams involving extended source and receiver surfaces. Nevertheless, although a "point source" is often specified, it is not unusual to find less precise statements to the effect that the flux received from a distant source varies inversely with the square of that distance. Such a statement can only be approximately true, in general, when applied to real (extended) sources and receivers. The inverse-square law [eq. (4.36)], as we have seen, applies exactly to each source-element—receiver-element pair and the direct or slant distance between them. That distance, however, will, in general, be different, perhaps only slightly different but still different, for different pairs. Thus, an overall inverse-square relationship based on just a single (average or nominal) source-receiver distance can, in general, only be an approximation. Next, we'll take a closer look at such approximations to the inverse-square law and try to evaluate the extent of their usefulness.

INVERSE-SQUARE-LAW APPROXIMATIONS for EXTENDED SOURCES and RECEIVERS. There are three main considerations that, together, determine when, in a given measurement configuration, it is useful to treat a source as a point source, characterized by its radiant intensity {eq. (4.33)}, or as an extended source, characterized by its (average) radiance:

(1) whether or not the source fills the entrance window[1] or angular field of view of the radiometer;[2]

(2) the size of the maximum transverse dimensions of the source, and of the instrument receiving aperture or entrance pupil,[1] relative to the distance D between them; and

(3) the directional distribution of radiance from each surface element dA_s across the radiating surface of the source A_s, and particularly the effects of obscuration (when an opaque portion of an irregularly shaped source may block rays from other parts of the emitting surface).

So far, we have considered only fairly regular well-behaved sources, most of them plane surfaces or equivalent to such a plane emitting surface, but when we start dealing with real sources, such as the very common tungsten lamps with coiled filaments, severe problems can arise due to the fact that parts of the emitting filament lie behind other opaque portions of the coil. We'll discuss this more a bit later. In addition, it should be noted that the problem can also be complicated by directional variations in instrument response. But we'll leave it with just that caution and assume now that we have good instruments that respond equally to incident flux from any direction within the angular field, keeping in mind that things may be quite different if this important assumption is not valid.

Although the three considerations are listed above in the order in which they are usually checked, we'll start with the second, the transverse dimensions of source and/or receiver in relation to their nominal separation distance D. The geometrical relations involved can be seen more clearly if we simplify our experimental configuration to one often employed in photometry, observing a small source with a small detector on an optical bench where the distance between them is easily changed and easily measured. In figure 4.10 we see an iso-radiance source consisting of a tungsten lamp in an opaque housing with an opal-glass diffuser across one side to produce an iso-radiance radiating surface. An opaque screen with a small circular hole of area ΔA_s, perpendicular to the bench so that $\theta_s = 0$ $(\cos\theta_s = 1)$, limits the beam of rays from source to receiver, establishing the

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[1]See any standard text on geometrical optics [13,14]. A future chapter is planned dealing with stops, pupils, windows, and baffles and their role in radiometry.

[2]In a scanning system, we are concerned with filling the instantaneous entrance window or angular field of view; in an imaging system we are concerned with filling the instantaneous entrance window or angular field of view of a resolution element.

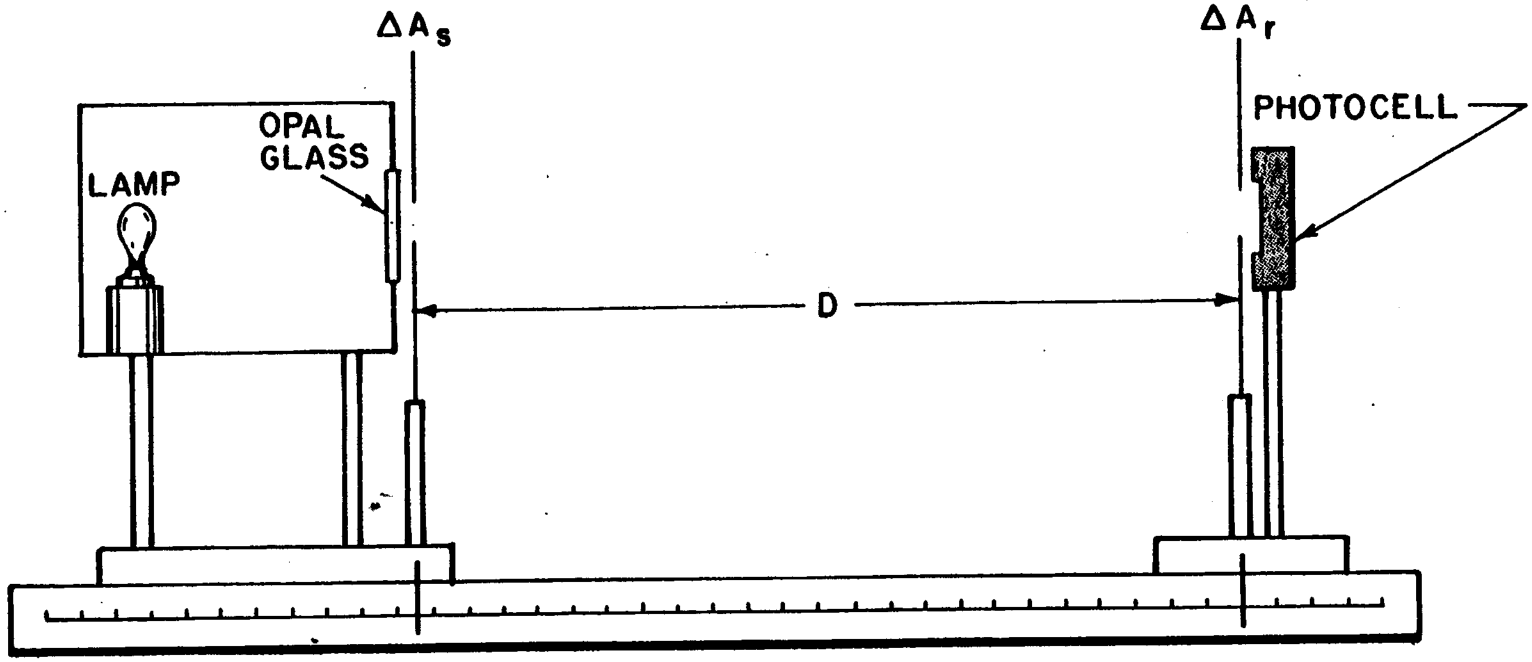

Figure 4.10. Experiment to develop the range of usefulness of the concept of extended-source <u>(radiant) intensity</u> and of practical approximations to the <u>inverse-square law</u> of irradiation.

position, orientation, and area of the effective source ΔA_s, as before in figure 4.7(b). All of these source components are carried, and move together as a unit, on a singl optical-bench carriage with an index or cursor mark aligned with the vertical plane of the screen (the plane of the source ΔA_s). A second carriage carries the receiver components, which consist of another vertical screen with a small aperture of area ΔA_r ($\theta_r = 0$; $\cos\theta_r = 1$), followed by a photocell with a sensitive area large enough to be sure to accept all of the rays of the beam through this second aperture. Again, the cursor mark or index on the receiver carriage is aligned with the vertical plane of the aperture screen so the distance D between the source ΔA_s and the receiver ΔA_r is easily measured by the difference between the cursor-index readings for the two carriages on the horizontal scale of the optical bench.

The value of the flux $\delta\Phi$ in the beam is measured by the photocell. Its approximate value in terms of beam parameters is given by

$$\delta\Phi \approx (L\cdot\Delta A_s/D^2)\cdot\Delta A_r \approx (\Delta I_s/D^2)\cdot\Delta A_r \approx \Delta E_{rs}\cdot\Delta A_r \text{ [W]}, \tag{4.37}$$

since, from eq. (4.31), a plane iso-radiance (L = a constant) source of area ΔA_s has a radiant intensity $\Delta I_s \approx L\cdot\cos\theta_s\cdot\Delta A_s$ and since $\cos\theta_s = \cos\theta_r = 1$. For a still larger source and receiver, of area A_s and A_r, respectively, this same approximation exists and we want, now, to determine the degree of approximation involved in

$$\Phi \approx E\cdot A_r \approx (I/D^2)\cdot A_r \approx L\cdot(A_s\cdot A_r/D^2) \text{ [W]}, \tag{4.38}$$

or, putting it another way, we want to evaluate the discrepancy

$$\Delta\Phi \equiv \Phi - (L\cdot A_s\cdot A_r/D^2) \text{ [W]} \tag{4.39}$$

in order to assess the percentage or relative inaccuracy $\Delta\Phi/\Phi$ as a function of the transverse dimensions of A_s and A_r relative to D.

In order to evaluate eq. (4.39), we need an exact expression for the flux Φ in the beam between the extended source and the extended receiver. But we already have such an expression in eq. (2.12) [5]:

$$\Phi = L\cdot\int_{A_r}\int_{A_s} D_s^{-2}\cdot\cos\theta_s\cdot\cos\theta_r\cdot dA_s\cdot dA_r = L\cdot\Theta \text{ [W]}, \tag{4.40}$$

where D_s is the slant distance between each pair of surface elements dA_s and dA_r, *not* the nominal separation or the perpendicular distance D between the vertical planes of A_s and A_r, as read on the optical bench. The throughput Θ between two parallel coaxial circular plane areas A_s and A_r separated by a distance D along the common axis through their centers is available in Appendix 3. If the radius of A_s is r_s, and of A_r is r_r, we can write [see eq. (A3-15)]

$$\Theta = (\pi^2/2)\cdot\{(r_s^2 + r_r^2 + D^2) - [(r_s^2 + r_r^2 + D^2)^2 - 4r_s^2\cdot r_r^2]^{\frac{1}{2}}\} \text{ [m}^2\cdot\text{sr]}. \tag{4.41}$$

We can also combine eqs. (4.39) and (4.40) to simplify the expression for the relative discrepancy $\Delta\Phi/\Phi$.

$$\Delta\Phi = L\cdot\Theta - L\cdot(A_s\cdot A_r/D^2) = L\cdot\Delta\Theta \text{ [W]}, \tag{4.42}$$

where

$$\Delta\Theta \equiv \Theta - (A_s\cdot A_r/D^2) \text{ [m}^2\cdot\text{sr]} \tag{4.43}$$

from which it is clear that

$$\Delta\Phi/\Phi = \Delta\Theta/\Theta \text{ [dimensionless]} \tag{4.44}$$

is the relative discrepancy.

There is a common "rule of thumb" in radiometry (photometry) that application of the approximate inverse-square relationship

$$\Theta \approx A_s\cdot A_r/D^2 \text{ [m}^2\cdot\text{sr]} \tag{4.45}$$

will be adequate for accuracies of about 99 per cent when the separation D is at least 10 times the maximum transverse dimension of A_s or A_r. For example, in the present case, where $A_s = \pi\cdot r_s^2$ and $A_r = \pi\cdot r_r^2$, those transverse dimensions are $2r_s$ and $2r_r$, respectively, so we want to evaluate the inaccuracy $\Delta\Theta$ when $D \geq 10\cdot(2r_s)$ and $D \geq 10\cdot(2r_r)$. To make it simple, let $r_s = r_r$ and $D = 20\ r_s = 20\ r_r$, and solve eqs. (4.43) and (4.41) to obtain the relative inaccuracy $\Delta\Theta/\Theta$. The result is just under 0.005 or 0.5%. But, to get this, we have assumed ideal conditions, an iso-radiance source and a receiver-detector that responds uniformly and isotropically to flux incident at any point across A_r from any direction within the beam from A_s.

For a somewhat better idea of the applicability of this "rule of thumb" to real configurations, consider the extreme possibility that the coaxial-parallel-disc source and receiver are both so non-uniform that all of the radiation emitted isotropically by the source disc actually comes only from a small region about a point S at the edge of the disc while all of the non-uniform isotropic responsivity of the receiver is similarly concentrated in a small sensitive region about a point R at the edge of that disc, diametrically opposite to point S on the source disc, as depicted in figure 4.11. The actual slant distance separating these points is then $D_s = 2\{r^2 + (D^2/4)\}^{1/2}$, where $D = 20r$. Then the relative or fractional error resulting from the use of D rather than D_s in the inverse-square relationship is $(D_s^2 - D^2)/D^2 = 0.01$ or 1%. Thus, the "rule of thumb" still holds for an estimated accuracy of 99%, even with such extreme non-uniformity as long as both source and receiver are still isotropic at each point of A_s and A_r.

This brings us to the third consideration, the directional distribution of radiation [and of receiver-detector response]. Although the isotropic assumption is not realistic over a full hemisphere for most surfaces, it is often a good approximation over a fairly wide range of directions about the normal. Accordingly, our "rule of thumb" approach is a

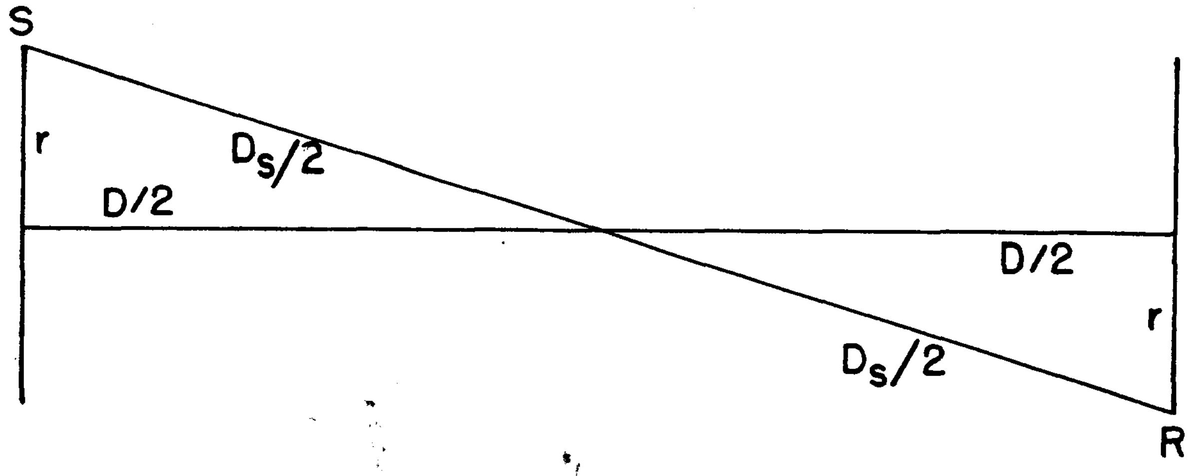

Figure 4.11. Extremely non-uniform isotropic source and receiver to assess a "worst-case" approximation to the inverse-square law.

Source and receiver are equal, parallel, co-axial, circular discs of radius r separated by distance D. The slant distance D_s between extreme points S and R is seen to be

$$D_s = (4r^2 + D^2)^{\frac{1}{2}}.$$

Evaluated for $D = 20r$, $D_s^2 = 1.01 \cdot D^2$.

reasonable one for most sources and receivers with reasonably flat emitting and receiving surfaces that are oriented perpendicular to the axis between their centers, as in the illustrative experimental configurations we've been discussing. We've also seen, in figure 2.10 of Chapter 2 [5], that, with truly iso-radiance emitting surfaces, the shape is immaterial as long as it subtends the same solid angle at the receiver. However, with real emitting surfaces the radiance may change when the direction to a receiver from an off-axis point changes slightly as the receiver moves farther away. For example, if the source in figure 4.11 is not isotropic, we can't base our conclusion on just the geometrical relations but must take into account that the radiance L of the rays from S to R is also a function of the separation distance D. The situation becomes even worse with sources of such irregular shape that rays from some parts of the emitting surface can be blocked in some directions by other parts of the source itself while other rays in directions very close to them are not blocked, thus producing violent variations in the directional distribution of source radiance (see figure 4.12). Not only does this alter the directional distribution of radiance; it may also change the effective emitting area A_s as a function of direction. With the regularly periodic structure of a helical lamp filament (see figure 4.12), one would expect that from some distant observation points, most of the inner sides of the far loops of the coil would be hidden behind the near loops while a small rotation

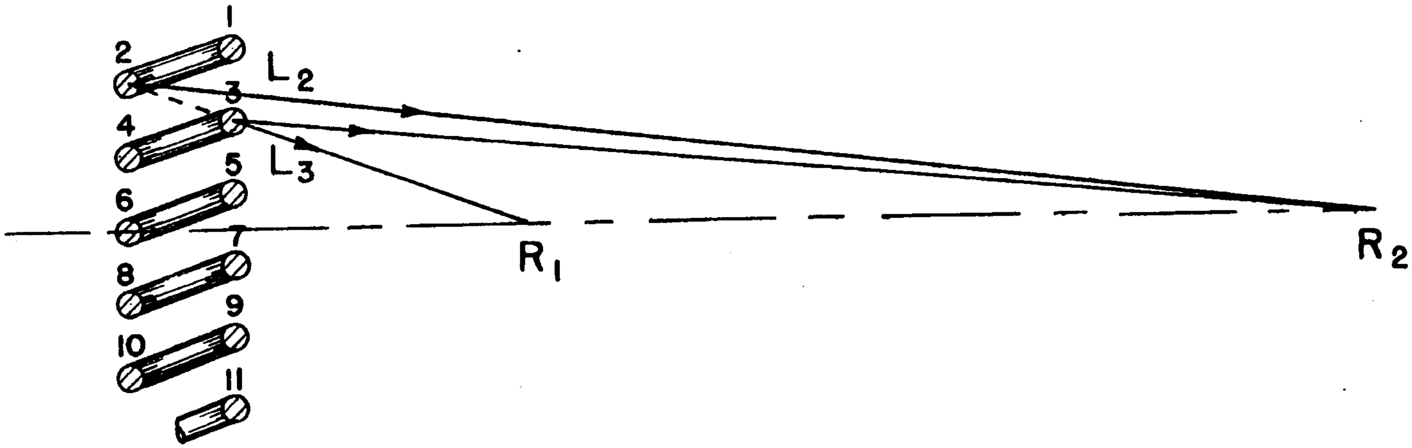

Figure 4.12. Illustration showing how obscuration invalidates the isotropic assumption implicit in the common approximation to the inverse-square law.

Circles 1 to 11 represent sections through successive turns of a helical-coil lamp filament. Turn 2 is obscured completely by turn 3 when viewed from axis point R_1 so the source radiance in that direction is just L_3, the radiance of the cooler outer surface of the coil. From R_2, however, both turn 3 of radiance L_3 and, adjacent to it, turn 2 of radiance L_2 are seen, and $L_2 > L_3$ because the inner surfaces not only emit but also reflect incident radiation from adjacent coils.

of the lamp would bring them all into view in the spaces between the near loops; and that this might be repeated as the lamp is rotated further. Just such a periodic variation (of about one per cent) in received flux with lamp rotation has been observed (see figure 18 and accompanying text in [15]). This situation is too complex for a precise analysis except to point out that, for such sources, there may not be a sufficiently accurate inverse-square relationship such as eq. (4.38), even though the inverse-square law of eq. (4.36a) is still perfectly valid between every pair of source- and receiver-surface elements. The *overall radiant intensity of the entire source,* in such cases, may be too variable to be a useful quantity by which to characterize the lamp to better than a few per cent.

Finally, to return to the first criterion, we look briefly at the relationship between a source and the angular field of view of an instrument (the solid angle ω subtended at the entrance pupil by the entrance window—see figure 4.13).[1] Our discussion of the experimental situation depicted in figure 4.6 (figure 2.2 of [5]), has established that the flux $\Delta\Phi$ through the two beam-defining apertures is the same, regardless of the position and orientation of the source, as long as all rays of the beam originate on an iso-radiance surface. Since the entrance window, of area A_E, and entrance pupil, of area A_r, of an optical system (see figure 4.13) are equivalent to just such a pair of beam-defining apertures for the rays that will pass through that optical system, *an iso-radiance source that fills the entrance window or field of view (as well as the entrance pupil) with rays of the same radiance L will produce the same flux through the optics regardless of its position and orientation.* In fact, we've already used the exact expression for the flux in such a beam in eq. (4.40):

$$\Phi = L\cdot\int_{A_E}\int_{A_r} D_s^{-2}\cdot\cos\theta_E\cdot\cos\theta_r\cdot dA_r\cdot dA_E = L\cdot\Theta \text{ [W]}. \qquad (4.40a)$$

The throughput Θ between entrance window and entrance pupil is a constant of the instrument. It is the same as the throughput between the aperture stop (same as entrance pupil in this oversimplified case) and the field stop. Accordingly, *if the flux responsivity* $R_\Phi(x,y,\theta,\phi)$ *of the instrument is* <u>*not*</u> *a variable function of position and direction* but has the same constant value R_Φ for every ray of the incident beam,[2] the instrument output can be treated as a measurement of the incident radiance $L = \Phi/\Theta$, as well as of the flux Φ in that beam. If the source is not an iso-radiance source so that the radiance cannot be taken outside the integral in eq. (4.40a), that equation can be written with the variable radiance inside the integral as $\Phi = \int L\cdot d\Theta$ and the quantity Φ/Θ then gives us an average value of the incident radiance, thus

[1]See any standard text on geometrical optics [13,14]. A future chapter is planned dealing with stops, pupils, windows, and baffles and their role in radiometry.

[2]The effects of a variable $R_\Phi(x,y,\theta,\phi)$, when this important assumption is not satisfied, will be covered in Chapter 5 on the measurement equation.

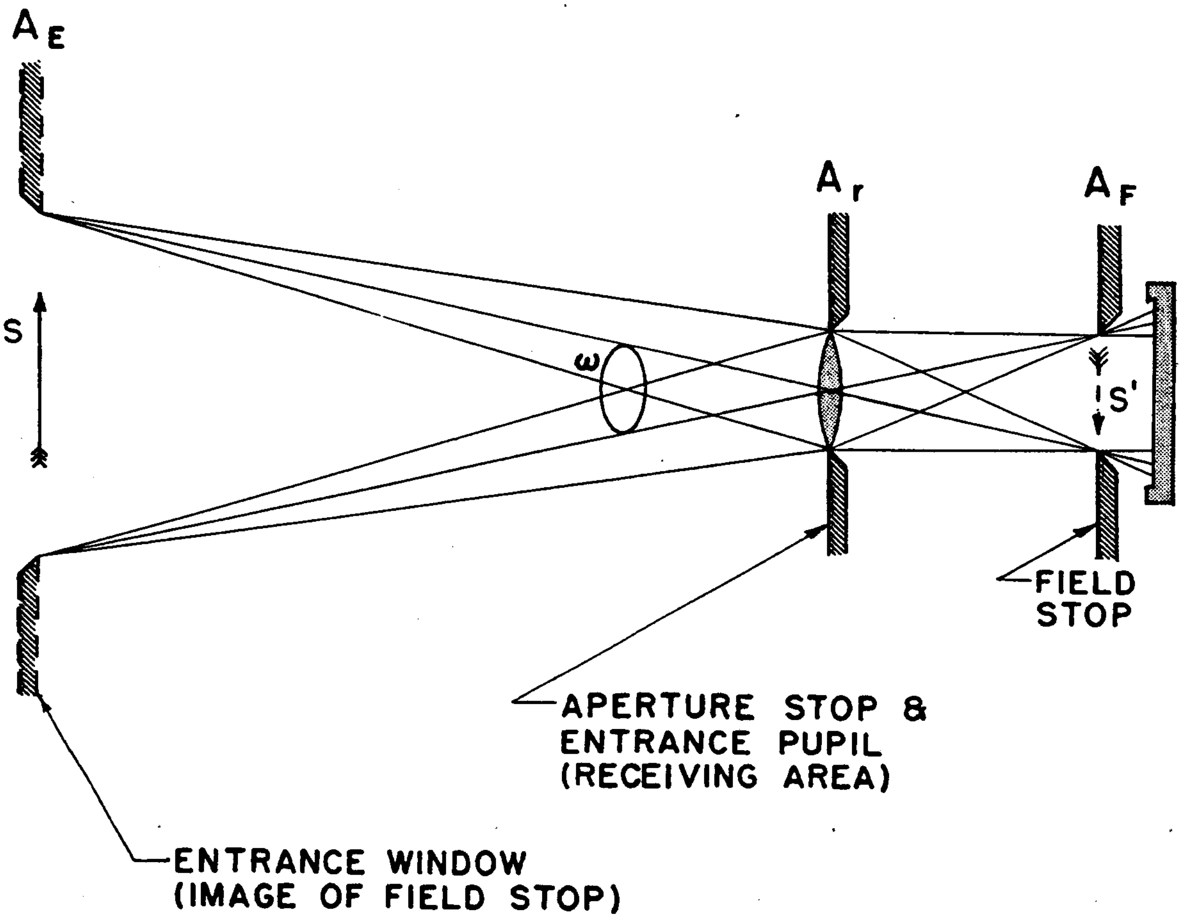

Figure 4.13. A very simple radiometer with a single lens, showing the entrance window (image of field stop in object space) and the entrance pupil (image of aperture stop in object space--here coincident with the aperture stop).

S -- source

S' -- source image

$$\Phi/\Theta = (1/\Theta)\cdot\int L\cdot d\Theta = \bar{L}\ [W\cdot m^{-2}\cdot sr^{-1}]. \tag{4.46}$$

The foregoing applies only when radiation from an extended source completely fills the entrance window or field of view (and the entrance pupil) and, in the case of the non-isoradiance source, the average radiance of eq. (4.46) is averaged only over that portion of the source that lies within the entrance window, only that portion that contributes rays to the beam accepted by the instrument throughput. On the other hand, when a source fills only a portion of the entrance window, as illustrated in figure 4.13, so that rays from each point of the entire exposed portion of the emitting surface reach the full entrance pupil and are included in the measured beam, the instrument output, as a measure of the flux Φ' in that beam (*again assuming constant flux responsivity* R_Φ that is the same for every ray of the incident beam),[1] when divided by the area A_r of the entrance pupil (also a constant of the instrument), is a measure of the (average) irradiance $E = \Phi/A_r\ [W\cdot m^{-2}]$ at the entrance pupil. Then if, in addition, the size, distance, and configuration of the source are such that, as discussed earlier with respect to the second and third considerations, it has a meaningful radiant intensity I at that distance (from source to entrance pupil), producing an irradiance E that is inversely proportional to the square of that distance, the instrument output (*again assuming constant* R_Φ)[1] may also be treated as a measure of the apparent source intensity at the given distance D. In general, this apparent radiant intensity will be equal to the actual source intensity only if there is no attenuation (loss by absorption or scattering) along the optical path between source and measuring instrument. The apparent radiant intensity is given by

$$I' \equiv \tau^*\cdot I = D^2\cdot\Phi/A_r\ [W\cdot sr^{-1}], \tag{4.47}$$

where

I' $[W\cdot sr^{-1}]$ is the apparent radiant intensity of the source as viewed from the instrument at a distance of D [m],

I $[W\cdot sr^{-1}]$ is the radiant intensity of the source as given by eq. (4.33), and

τ^* is the propagance, as defined by eq. (2.38) [5], of the optical path, of length D [m], between source and instrument (entrance pupil or receiving aperture) where the measured flux is Φ [W] (R_Φ = *const. is the same for all rays*[1]) through an aperture of area A_r $[m^2]$.

For example, the source quantity (radiance or radiant intensity) measured by the flux reaching an ideal photocell, *with uniform and isotropic responsivity* R_Φ *for all incident*

[1]The effects of a variable $R_\Phi(x,y,\theta,\phi)$, when this important assumption is not satisfied, will be covered in Chapter 5 on the measurement equation.

rays,[1] positioned to receive the entire beam through a telescope that is focused on the moon, will depend on the magnification used. At low magnification, where the entire moon is seen through the telescope, the output is a measure of the irradiance at the telescope objective and, in combination with the distance to the moon, a measure of the moon's apparent radiant intensity. On the other hand, at higher magnification, where only a portion of the moon's surface is seen in the field, the output is a measure of the average apparent radiance of just that portion of the moon's surface. (Again we use the word "apparent" to include the effects of attenuation; the apparent radiance is $L' \equiv \tau^{*} \cdot L$.)

One further consideration that may occasionally arise is the effect of the longitudinal extent of a source toward, or away from, the receiver (we have so far considered only the transverse dimensions). It is clear from the discussion accompanying figure 2.10 [5] that the irradiance produced at a distant point by an iso-radiance source depends only on the solid angle subtended at that point by its periphery or its extremities (as "seen" from the receiver) and is otherwise independent of its size, position, and configuration within that solid angle (strictly, it is proportional to the projected solid angle $\Omega \equiv \int \cos\theta \cdot d\omega$). Hence, longitudinal effects do not occur for iso-radiance sources. For non-uniform sources, we can obtain an indication of the effects of longitudinal extent on approximations to the inverse-square law by examining the configuration in figure 4.14. Two "point-source" elements of an extended volume source (see next section) each of radiant intensity dI [$W \cdot sr^{-1}$], are at distances of $(1+\delta) \cdot D$ and $(1-\delta) \cdot D$ from an observation point 0, where the resulting element of normal irradiance or radiant fluence rate is

$$dE_n = dF_t = \frac{dI}{(D + \delta \cdot D)^2} + \frac{dI}{(D - \delta \cdot D)^2} \approx \frac{2dI}{D^2} \; [W \cdot m^{-2}]. \tag{4.48}$$

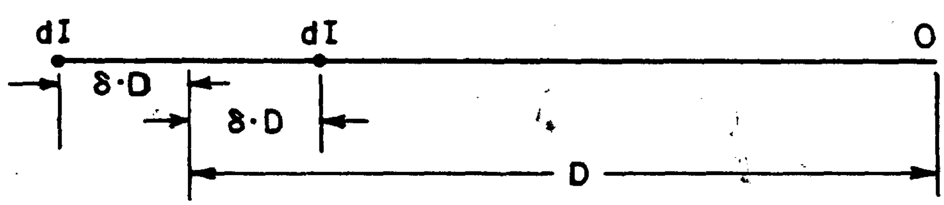

Figure 4.14. Source-point configuration for assessing the effects of longitudinal extent on approximations to the inverse-square law.

[1]The effects of a variable $R_{\Phi}(x,y,\theta,\phi)$, when this important assumption is not satisfied, will be covered in Chapter 5 on the measurement equation.

The approximation involved is, clearly,

$$\frac{1}{(1+\delta)^2} + \frac{1}{(1-\delta)^2} \approx 2. \qquad (4.48a)$$

It is easily shown that the approximation will be within 1% if $\delta \le 0.06$. The approximation improves if more source points are distributed over the intervening space.

VOLUME-SOURCE DISTRIBUTION: RADIANT STERISENT. Up to this point, we have considered only sources that have emitting surfaces or, at least, only those where it is feasible to refer the exitent radiance to some convenient reference surface. However, there are also distributed sources where radiation is emitted, or incident radiation is scattered, from points throughout a volume where it may not be feasible to establish the source-radiance distribution directly with respect to any reference surface. For example, in studying the night-sky airglow,[1] the radiance, or spectral radiance, due to the emitting layer is observed from the ground where the radiation coming from altitudes of about 50 [km] or more seems to come from an emitting surface having the observed radiance. However, the radiation is actually coming from excited atmospheric molecules distributed throughout an emitting region that may extend vertically over an altitude range measured in kilometers. Theoreticians concerned with the details of the emission processes need to relate the observed sky radiance to the volume distribution of source emission, so we need a radiometric quantity to characterize that distribution.

A more familiar example is the daytime sky where the radiance (actually, the corresponding luminance--see table 4-2) that we see consists of scattered sunlight reaching us from points distributed throughout a huge volume of air. When the sky is clear and blue, the scattering points are primarily the air molecules themselves, as well as some dust and aerosol particles; when the sky is hazy, misty, or "smoggy", there are many more and larger scattering particles of various aerosols and atmospheric contaminants. We're not concerned, however, with the detailed mechanisms, only with the fact that the source of radiation is widely distributed throughout an extended volume.

The International Lighting Vocabulary (CIE-IEC) [6] lists no radiometric quantity that is a volume-source distribution. We have adopted the term radiant sterisent and the symbol L^*, proposed by Jones [16,17] and by a Nomenclature Committee of the Optical Society of America [18], for the "generated" (emitted and/or scattered) radiance per unit path length along a ray, or the equivalent "generated" radiant intensity per unit volume, by which such a distributed source can be characterized. This concept will now be developed.

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[1]Nocturnal airglow is the emission of light from extremely high-altitude atmospheric regions where gases at very low pressures and densities have been excited by incident solar radiation during the daytime so that they luminesce faintly over the whole sky at night at certain characteristic wavelengths.

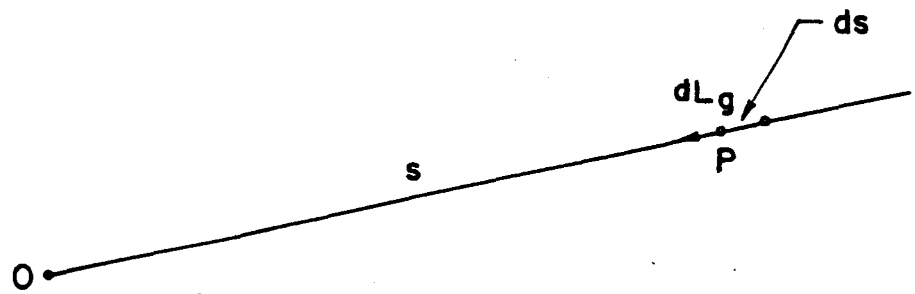

Figure 4.15. A ray path through an extended volume of radiating (emitting and/or scattering) medium.

Consider a region in which radiation is being generated (emitted and/or scattered) at all points throughout an extended volume. Let the point O in figure 4.15 be a point of observation or measurement, and consider the incident radiance at O from a single direction (along a single ray). Let the distance along that ray from O to any point P on the ray be s [m]. If contributions to the incident radiance at O are generated (emitted and/or scattered) along the ray path at the rate of $L^*(s)$ units of radiance per meter of path length $[W \cdot m^{-2} \cdot sr^{-1} \cdot m^{-1}]$, the radiance of an element of path length ds at P in the direction of the ray to O is $dL_g = L^*(s) \cdot ds \ [W \cdot m^{-2} \cdot sr^{-1}]$. If there is no attenuation (and if the intervening medium is uniform and isotropic—iso-refractive-index), this element of path ds will then contribute an amount $dL = dL_g$ to the observed or measured radiance incident from that direction (along the given ray from P) at O, because of the geometrical invariance of radiance—see Chapter 2 [5]. However, if there is attenuation over the intervening path of s [m] from P to O, so that the propagance[1] is $\tau^*(s)$, the incident radiance at point O from just the element ds at P is

$$dL = \tau^*(s) \cdot dL_g = \tau^*(s) \cdot L^*(s) \cdot ds \ [W \cdot m^{-2} \cdot sr^{-1}]. \tag{4.49}$$

The observed or measured incident radiance at O, from the direction of the ray through P, is then the integral of all such contributions from the entire path, or

$$L = \int_0^\infty L^*(s) \cdot \tau^*(s) \cdot ds \ [W \cdot m^{-2} \cdot sr^{-1}]. \tag{4.50}$$

Accordingly, referring to any point P as the point x,y,z and designating the ray direction at P(x,y,z) by the direction coordinates θ,ϕ, for greater generality, the volume distribution of generated radiance, as a function of position and direction throughout any such distributed source, is characterized by the <u>radiant sterisent</u> (in a given direction at a point)

$$L^*(x,y,z,\theta,\phi) \equiv \frac{dL_g(x,y,z,\theta,\phi)}{ds} \ [W \cdot m^{-3} \cdot sr^{-1}], \tag{4.51}$$

[1]See Chapter 2 [5] for these relations involving propagance τ^*.

where

$L^*(x,y,z,\theta,\phi)$ [$W\cdot m^{-3}\cdot sr^{-1}$] is the radiant sterisent at the point x,y,z in the direction θ,ϕ;

$dL_g(x,y,z,\theta,\phi)$ [$W\cdot m^{-2}\cdot sr^{-1}$] is the generated (emitted and/or scattered) radiance of the medium along the path element ds, in the direction θ,ϕ, at the point x,y,z; and

$ds = (dx^2 + dy^2 + dz^2)^{\frac{1}{2}}$ [m] is an element of path length along the given ray in the direction θ,ϕ at the point x,y,z so that $\theta = \cos^{-1}(dz/ds)$ and $\phi = \tan^{-1}(dy/dx)$.[1]

From the unit-dimensions, [$W\cdot m^{-3}\cdot sr^{-1}$], it appears that radiant sterisent is radiant intensity [$W\cdot sr^{-1}$] per unit volume [m^{-3}], and this is, indeed the case, as is easily demonstrated. Consider a cylindrical element of volume dV, along the path element ds of figure 4.15, with parallel end faces, each of area dA and tilted at an angle θ_s between the normal to dA and the ray path along the element ds, as illustrated in figure 4.16. The volume element $dV = ds\cdot\cos\theta_s\cdot dA$ is indefinitely small ($\Delta s\to 0$ and $\Delta A\to 0$)

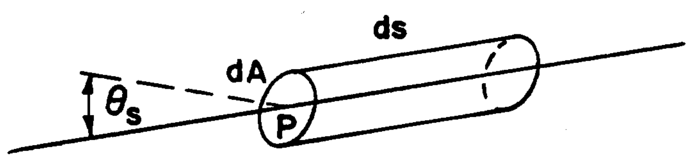

Figure 4.16. An elementary volume along the ray path at P in figure 4.15.

so that the generation (emission and/or scattering) of radiation is uniform throughout dV and the radiance dL_g is the same over all parts of the end-face element dA. Then, from eq. (4.32), the element of radiant intensity of the end face dA in the direction θ,ϕ is

$$dI_g = dL_g\cdot\cos\theta_s\cdot dA \;[W\cdot sr^{-1}], \tag{4.52}$$

so that

$$dL_g = dI_g/(\cos\theta_s\cdot dA). \tag{4.53}$$

[1]The relationships between spherical and rectangular coordinates are summarized in Appendix 2 [5] for those who may wish to refresh their memories on that topic.

Then, from eqs. (4.51) and (4.53), we can write

$$L^* \equiv \frac{dL_g}{ds} \equiv \frac{dI_g}{ds\cdot\cos\theta_s\cdot dA} \equiv \frac{dI_g}{dV}\ [W\cdot m^{-3}\cdot sr^{-1}], \tag{4.54}$$

or, more explicitly

$$L^*(x,y,z,\theta,\phi) \equiv \frac{dI_g(x,y,z,\theta,\phi)}{dV}\ [W\cdot m^{-3}\cdot sr^{-1}], \tag{4.55}$$

where

$dI_g(x,y,z,\theta,\phi)$ $[W\cdot sr^{-1}]$ is the generated radiant intensity of the volume element $dV = ds\cdot\cos\theta_s\cdot dA$ $[m^{-3}]$ at the point x,y,z in the direction θ,ϕ, where

θ,ϕ [rad], as before, designates the direction of the ray (and of its path-length element ds) at the point x,y,z.

In developing the concept of radiant sterisent and its use in the radiative-transfer relation of eq. (4.50), we have integrated along the entire path, from the point 0 to infinity, without considering that there might be something other than the distributed-source medium along that path. Now, we want to look at the case where the distributed-source medium is found only along a finite path extending between the observation or measurement point 0 and an opaque source. Let the opaque surface have a radiance L_1 in the direction of point 0 at a distance s_1 from 0 along the ray path. The propagance, again, is given by $\tau^*(s)$ for $0 \le s \le s_1$, but now, because the path is blocked by the opaque surface at $s = s_1$, $\tau^*(s) = 0$ for all $s > s_1$. Then, if $L^*(s)$ is still the sterisent along the ray path in the intervening medium, the observed radiance from the direction of the given ray incident at point 0 is now given by

$$L = L_1\cdot\tau^*(s_1) + \int_0^{s_1} L^*(s)\cdot\tau^*(s)\cdot ds\ [W\cdot m^{-2}\cdot sr^{-1}]. \tag{4.56}$$

The integral vanishes for all values of s greater than s_1 because the propagance $\tau^*(s)$ goes to zero, so it is immaterial whether the upper limit is shown as s_1 or as infinity.

While equations like eq. (4.56) and the approach used in setting it up are most likely to be used in any applications to evaluate the radiometric quantities involved, some readers may be interested in the possibility for making the sterisent a more general distribution function that can account for discrete as well as continuously distributed sources. This is conceptually appealing because eq. (4.50) then becomes a very general equation of radiative transfer. It is accomplished by introducing the discontinuous distribution function known as the Dirac delta-function [19]. For example, in the situation covered by eq. (4.56), let the sterisent be, instead,

$$L^*(s) \;=\; L_1 \cdot \delta(s - s_1) + L_m^*(s) \;\; [W \cdot m^{-3} \cdot sr^{-1}] \qquad (4.57)$$

where

$L_m^*(s)$ $[W \cdot m^{-3} \cdot sr^{-1}]$ is the sterisent of the intervening medium; and

$\delta(s - s_1)$ is a Dirac delta-function that satisfies the following defining relations [19]:

$\delta(u) = 0$ for $u \neq 0$,

$\int \delta(u) \cdot du = 1$, and

$\int f(u) \cdot \delta(u-a) \cdot du = f(a)$,

when the integration is carried out, in each case, over any range of the variable that includes the zero of the argument of the δ-function.

It is easily verified that, following the above rules for the δ-function, substitution of eq. (4.57) into eq. (4.50) leads to the same result as eq. (4.56). While it is appealing and conceptually helpful to thus treat eq. (4.50) as a very general equation of radiative transfer, any attempt to apply it to an actual situation can rapidly develop considerable complexity.

Since, as we have shown, the radiant sterisent is the radiant intensity per unit volume, the radiant intensity of an extended volume source in which there is no significant attenuation is just the volume integral of the sterisent

$$I(\theta,\phi) \;=\; \int_V L^*(x,y,z,\theta,\phi) \cdot dV \;\; [W \cdot sr^{-1}], \qquad (4.58)$$

where the element of volume $dV \equiv dx \cdot dy \cdot dz$ $[m^3]$. In the presence of appreciable attenuation, on the other hand, it is necessary to first establish ray-radiances at an appropriate reference surface by eq. (4.50) and then to compute the source intensity for the radiance distribution at that reference surface by eq. (4.33). Whether this is useful or not in either case depends, of course, on the size-distance relationships already discussed with respect to the intensity of extended sources and the approximations to the inverse-square law.

ENERGY (TIME-INTEGRATED FLUX) DISTRIBUTIONS. As stated in Chapter 1 [5], we're mainly interested, in this Manual, in the measurement of radiant flux or power in watts [W]. However, there are also many occasions when the effects of interest produced by a radiation beam are related, instead, to the time-integrated flux, i.e., the energy, in joules [J]. A very common example is that of photographic exposure, including that from the pulsed output of various photoflash illumination devices. The incident flux reaching a particular area of a photographic film may vary greatly with time due to the action of a camera shutter and/or the pulsed-light output of a flash lamp but, within rather wide limits of so-called "reciprocity" (between time and irradiance), it is the time integral of the incident flux, i.e., the incident energy, that determines the density of the resulting photographic image—along with other factors, such as temperature and "development" (chemical processing). The radiation "dose" in various photobiological or medical applications, such as photosynthesis in plants exposed to sunlight or the erythemal or "sunburn" effect of ultraviolet radiation on the human skin, also depends primarily on the total energy involved. Accordingly, we take a brief look at radiometric quantities that are spatial distributions of radiant energy, rather than radiant flux. Such quantities include radiant (directed-surface) exposure H [$J \cdot m^{-2}$], radiant (omni-directional) fluence F [$J \cdot m^{-2}$], and radiant (volume) density w [$J \cdot m^{-3}$].

DIRECTED-SURFACE DISTRIBUTION of INCIDENT RADIANT ENERGY: RADIANT EXPOSURE.

Consider a time-varying beam of radiation, incident on a given surface. If the total energy, incident on a small area of that surface ΔA [m^2] about the point x,y in a time interval from t_1 to t_2 is ΔQ[J], we define the radiant exposure (at that point and in that time interval) as

$$H \equiv \lim_{\Delta A \to 0} \frac{\Delta Q}{\Delta A} \equiv \frac{dQ}{dA} \; [J \cdot m^{-2}]. \tag{4.59}$$

Since the beam is varying with time, the radiant flux element $d\Phi(t)$ incident on each area element dA is a function of time, making the irradiance at any point x,y also a function of time

$$E(t) \equiv \frac{d\Phi(t)}{dA} \; [W \cdot m^{-2}]. \tag{4.60}$$

In terms of the time-varying incident flux $d\Phi(t)$, the total incident energy in the interval from t_1 to t_2 is then

$$dQ = \int_{t_1}^{t_2} d\Phi(t) \cdot dt \; [J], \tag{4.61}$$

so that

$$H \equiv \frac{dQ}{dA} = \int_{t_1}^{t_2} \frac{d\Phi(t)}{dA} \cdot dt = \int_{t_1}^{t_2} E(t) \cdot dt \; [J]. \tag{4.62}$$

The radiant exposure is, thus, the time integral of the irradiance [the incident radiant flux (surface) density]. In fact, each of the relations previously given between irradiance [incident radiant flux (surface) density] and another radiometric quantity transforms directly to that for radiant exposure by expressing the other quantity as a function of time and integrating it over the appropriate time interval. For example, eq. (4.7) becomes

$$dH(x,y,\theta,\phi) \;=\; d\Omega \cdot \int_{t_1}^{t_2} L(x,y,\theta,\phi,t)\cdot dt \;[\mathrm{J}\cdot\mathrm{m}^{-2}]. \tag{4.63}$$

Note, however, that, in unusual situations where there is interaction between the time dependence and the effects of other radiation parameters, as in some scanning instruments, such transformations cannot be made so simply.

OMNI-DIRECTIONAL-SURFACE DISTRIBUTION of INCIDENT RADIANT ENERGY: RADIANT FLUENCE. Radiant fluence, like radiant fluence rate, is defined in terms of the radiation incident on a small sphere of radius Δr and cross-sectional area $\Delta a = \pi\cdot(\Delta r)^2$ [m^2] [eq. (4.11)]. The total energy incident on the sphere from all directions in a given time period of interest, from t_1 to t_2, is ΔQ [J]. Then, if we keep the center of the sphere fixed at the point x,y,z (figure 4.3) while we reduce the cross-sectional area Δa by reducing the radius Δr, the radiant fluence at the point x,y,z is defined as the limit, as Δa approaches zero, of the quotient $\Delta Q/\Delta a$ (since the CIE-IEC [6] have no symbol, we choose F, used by photobiologists [12]):

$$F(x,y,z) \;\equiv\; \lim_{\Delta a \to 0} \frac{\Delta Q}{\Delta a} \;\equiv\; \frac{dQ}{da} \;[\mathrm{J}\cdot\mathrm{m}^{-2}]. \tag{4.64}$$

If the element of flux incident from all directions at the point x,y,z is varying with time $d\Phi(t)$, the total incident energy in the interval from t_1 to t_2 is then given by eq. (4.61)

$$dQ \;=\; \int_{t_1}^{t_2} d\Phi(t)\cdot dt \;[\mathrm{J}]. \tag{4.61}$$

Then, from eqs. (4.64), (4.61), and (4.12),

$$F \;\equiv\; \frac{dQ}{da} \;=\; \int_{t_1}^{t_2} \frac{d\Phi(t)}{da}\cdot dt \;=\; \int_{t_1}^{t_2} F_t(t)\cdot dt \;[\mathrm{J}\cdot\mathrm{m}^{-2}]. \tag{4.65}$$

Thus, as might be expected, radiant fluence is the time integral of the radiant fluence rate at the same point. And, again, each of the relations previously given between radiant fluence rate and another radiometric quantity transforms directly to that for radiant fluence by expressing the other quantity as a function of time and integrating it over the appropriate time interval (except when there is interaction between the time dependence and the effects of other radiation parameters). In particular, from eq. (4.20), we can write the relationship between elements of radiant exposure and radiant fluence at a point x,y on some reference surface as

$$dF(x,y,\theta,\phi) \;\equiv\; dH(x,y,\theta,\phi)/\cos\theta_r \;\equiv\; dH_n(x,y,\theta,\phi) \;[\mathrm{J}\cdot\mathrm{m}^{-2}], \tag{4.66}$$

where dH_n is an element of normal exposure and is equal to the exposure element dH only when $\cos\theta_r = 1$, i.e., when $\theta_r = 0$ (when the ray direction θ,ϕ coincides with the normal to the surface element dA_r). Thus the element of radiant fluence at a point is equal to the normal exposure at that point, defined as the exposure on a surface element that is normal to the ray at that point. Only elements of fluence and exposure, associated with individual rays (elements of throughput) can be directly related in this way. The integrated total fluence and exposure at a point cannot be directly converted from one to the other. If values for both are needed, they are most easily obtained from the incident radiance distribution and the counterparts of eqs. (4.16a) and (4.17). But this seems unlikely; ordinarily only one or the other will be useful or significant.

Note that eq. (4.65) is consistent with

$$F_t(x,y,z) = \frac{dF(x,y,z)}{dt} \; [\mathrm{W \cdot m^{-2}}], \tag{4.67}$$

justifying our use of the notation F_t (which means $\equiv dF/dt$)[1] for fluence rate.

<u>VOLUME DISTRIBUTION of RADIANT ENERGY: RADIANT (VOLUME) DENSITY</u>. The volume distribution of radiant energy is not included in the CIE-IEC nomenclature [6]. However, there are some occasions when it is used in radiometry, and it is included in the ANSI nomenclature [11], so we include it here for completeness.

The <u>radiant (volume) density</u> at a point x,y,z is defined as the quotient of the radiant energy contained in a small volume about the given point by the magnitude of the volume, as that volume is made indefinitely small:

$$w(x,y,z) \equiv \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} \equiv \frac{dQ}{dV} \; [\mathrm{J \cdot m^{-3}}]. \tag{4.68}$$

So stated, the concept is clear and appears quite simple. However, when we try to relate it to other radiometric quantities, we must take into account the fact that radiant energy is in constant motion, propagating at high speeds that, in a vacuum, reach the well-known value $c \approx 3 \times 10^8 \; [\mathrm{m \cdot s^{-1}}]$ {= (2.997 924 58 ± 0.000.000.012) × $10^8 \; [\mathrm{m \cdot s^{-1}}]$}. The problem is to evaluate the instantaneous radiant energy density or the average density over a very short time period.

In order to do this, consider a ray incident at the point x,y,z from the direction θ,ϕ (see figure 4.17). If the radiance is $L(x,y,z,\theta,\phi)$, the element of fluence rate and the flux element incident on an elementary sphere of cross section da centered at x,y,z are given by eqs. (4.15) and (4.14), respectively:

$$dF_t(x,y,z,\theta,\phi) = L(x,y,z,\theta,\phi) \cdot d\omega \; [\mathrm{W \cdot m^{-2}}] \tag{4.15}$$

and

$$d\Phi(x,y,z,\theta,\phi) = L(x,y,z,\theta,\phi) \cdot d\omega \cdot da \; [\mathrm{W}]. \tag{4.14}$$

[1]In this Manual $X_p \equiv dX/dp$ for any radiometric quantity X and the following radiation parameters p: position, direction, a spectral parameter (wavelength, wave number, frequency ν, etc.), or time or frequency $f \ll \nu$.

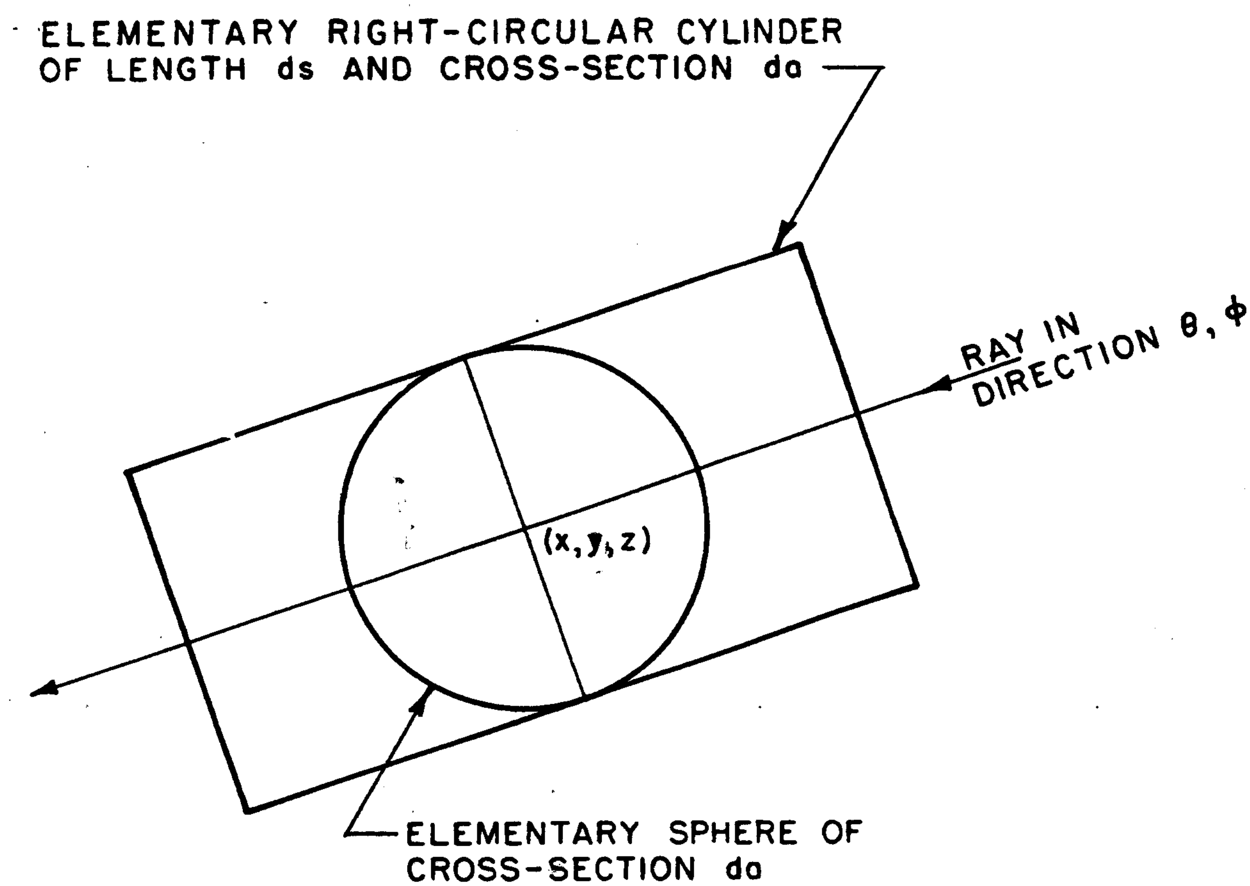

Figure 4.17. A sectional view showing a ray through the point x,y,z and along the axis of a right-circular cylinder volume element tangent to an inscribed spherical volume element (see, also figure 4.3).

Figure 4.17 also shows a right circular cylinder, of length ds along the ray as its axis, centered at x,y,z, and with cross-sectional area da, so that it is tangent to the inscribed sphere, also of cross section da and centered at x,y,z. The volume of this cylindrical element is

$$dV = da \cdot ds \ [m^3], \tag{4.69}$$

and we denote the amount of radiant energy contained in that volume at any instant as dQ [J]. In terms of the flux element $d\Phi$, the energy passing through a transverse element da at any point on the ray in an elementary time interval dt is $dQ = d\Phi \cdot dt$ [J]. But the time required for all of the energy in the cylindrical element at a given instant to pass out through one end is just $dt = ds/v = n \cdot ds/c$. This is the time for the energy at one end, flowing at the velocity $v = c/n$ $[m \cdot s^{-1}]$, to flow just the distance ds to the opposite end. Here, $n \equiv c/v$ is the index of refraction of the medium. Accordingly, the energy in the element at any instant is given by

$$dQ = d\Phi \cdot n \cdot ds/c \ [J]. \tag{4.70}$$

Now, we can combine eqs. (4.68), (4.14), (4.69), and (4.70) to obtain the element of radiant (volume) density due to this elementary beam along the single ray as[1]

$$\begin{aligned} dw(x,y,z,\theta,\phi) &\equiv \frac{dQ}{dV} = \frac{n \cdot d\Phi}{c \cdot da} \\ &= (n/c) \cdot L(x,y,z,\theta,\phi) \cdot d\omega \ [J \cdot m^{-3}]. \end{aligned} \tag{4.71}$$

But, from eq. (4.15), this is equivalent to[1]

$$dw(x,y,z,\theta,\phi) = (n/c) \cdot dF_t(x,y,z,\theta,\phi) \ [J \cdot m^{-3}]. \tag{4.72}$$

The same is true for the elements associated with all other rays through x,y,z (at a given time) from all directions θ,ϕ, so, integrating both sides of eq. (4.72) with respect to solid angle (direction) $d\omega \equiv \sin\theta \cdot d\theta \cdot d\phi$, we can write[1]

$$\begin{aligned} \int_\omega dw(x,y,z,\theta,\phi) &= (n/c) \cdot \int_\omega dF_t(x,y,z,\theta,\phi) \\ w(x,y,z) &= (n/c) \cdot F_t(x,y,z) \ [J \cdot m^{-3}]; \end{aligned} \tag{4.73}$$

the radiant (volume) density at a point (at a given time) is equal to n/c times the fluence rate at that point (at the given time). Also, from eq. (4.16a), we can relate this to the incident radiance by

$$w(x,y,z) = (n/c) \cdot \int_{4\pi} (L(x,y,z,\theta,\phi) \cdot d\omega \ [J \cdot m^{-3}]. \tag{4.74}$$

Note, particularly, that all quantities in this equation are evaluated at the point x,y,z, including the radiance L. If the radiance has a value L_1 at some other point x_1,y_1,z_1 along the ray, where the ray direction is θ_1,ϕ_1 and the refractive index is n_1, then, by eq. (2.23) [5],

$$L(x,y,z,\theta,\phi) = (n/n_1)^2 \cdot L_1(x_1,y_1,z_1,\theta_1,\phi_1) \ [W \cdot m^{-2} \cdot sr^{-1}]$$

and, furthermore, this value must be multiplied or divided (depending on the direction of propagation) by the propagance τ^* of the ray path between x,y,z and x_1,y_1,z_1, as in eq. (2.40) or (2.39) [5].

---

[1]Note that in a dispersive medium, where $n = n(\lambda)$, these equations must be written for the spectral quantities; for example,

$$dw_\lambda(x,y,z,\theta,\phi,\lambda) = [n(\lambda)/c] \cdot L_\lambda(x,y,z,\theta,\phi,\lambda) \cdot d\omega \ [J \cdot m^{-3} \cdot nm^{-1}], \tag{4.71a}$$

and

$$w_\lambda(x,y,z,\lambda) = [n(\lambda)/c] \cdot F_{t,\lambda}(x,y,z,\lambda) \ [J \cdot m^{-3} \cdot nm^{-1}]. \tag{4.73a}$$

SPECTRAL RADIOMETRIC QUANTITIES. Although this chapter is devoted to spatial distributions, it should be noted that all of the spatial radiometric quantities discussed here are transformed to the corresponding spectral quantities in just the same way that radiance was transformed to spectral radiance in Chapter 3 [5].

In general, if X is any radiometric quantity, including L, L^*, F_t, M, E, W, I, Φ, w, F, H, or Q, and if μ is a spectral parameter or variable, including wavelength λ, frequency ν, or wave number σ (see footnote at beginning of Chapter 3 [5]), then the corresponding spectral radiometric quantity is

$$X_\mu \equiv dX/d\mu \; [X\cdot\mu^{-1}]. \tag{4.75}$$

For example: spectral radiant energy, in terms of wavelength, is $Q_\lambda \equiv dQ/d\lambda$ $[J\cdot nm^{-1}]$; spectral radiant sterisent, in terms of wave number, is $L^*_\sigma \equiv dL^*/d\sigma$ $[W\cdot m^{-3}\cdot sr^{-1}\cdot(cm^{-1})^{-1}]$ or $[W\cdot m^{-3}\cdot sr^{-1}\cdot cm]$ (i.e., in watts per cubic meter, steradian, and reciprocal centimeter); and spectral radiant density in terms of frequency, is $w_\nu \equiv dw/d\nu$ $[J\cdot m^{-3}\cdot THz^{-1}]$.† These examples, like almost all of our discussion, are in terms of energy units, [J] and [W]. One should keep in mind that they apply equally to any other form of flux that is propagated in accordance with geometrical (ray) optics. For example, spectral photon-flux exitance, in terms of wavelength, is (see table 4.3) $M_{p,\lambda} \equiv dM_p/d\lambda$ $[q\cdot s^{-1}\cdot m^{-2}\cdot nm^{-1}]$. Spectral quantities could also be formed in the same way from the luminous-flux or photometric quantities of table 4.2, but we're not aware of any useful application for such quantities.

SUMMARY of CHAPTER 4. All of the radiometric quantities in table 4-1 have now been introduced, with definitions and discussions of their significance and their interrelations with other radiometric quantities, particularly radiant flux, radiant energy, radiance, or spectral radiance. The "simple" spatial distributions of radiant flux include,

(1) the directed-surface distributions:
- radiant flux (surface) density W, including
  - irradiance E, and
  - radiant exitance M;

(2) the omni-directional-surface distribution:
- radiant fluence rate F_t; and

(3) the directional distribution:
- radiant intensity I.

Important interrelationships include the one between radiance and flux (surface) density in iso-radiance beams and the inverse-square law that relates point-source radiant intensity to the resulting irradiance as a function of distance, as well as approximations to the inverse-square law involving extended sources and receivers. Also presented is the

†One teraherz $1[THz] = 10^{12}[Hz]$.

volume-source distribution radiant sterisent L^* and its use in a radiative-transfer equation. In addition, there are energy (time-integrated flux) distributions corresponding to all of the flux distributions. Those given special names and symbols, and listed in table 4-1, include

(1) the directed-surface distribution of incident radiant energy: radiant exposure H;

(2) the omni-directional-surface distribution of incident radiant energy: radiant fluence F; and

(3) the volume distribution of radiant energy: radiant (volume) density w.

Finally, there is a spectral radiometric quantity X_μ corresponding to each radiometric quantity X and spectral parameter μ.

With respect to the spatial parameters, a radiometric measurement situation should *always* be analyzed, first, in terms of the distribution of ray-radiance with respect to both position (across a reference surface) and direction (through each point of that surface) to account for the full range of possibilities (see Chapter 2 [5]). If it is then found that the assumption of uniformity, or the use of an average value, with respect to part of that distribution will not produce unacceptable error, a "simpler" distribution of radiant flux with respect to position (area) or direction (solid angle) alone can be used.

The flux per unit area, as a function of position, is given in two ways: The first, the directed-surface distribution, is called the radiant flux (surface) density (at a point of a surface)

$$W(x,y,) \equiv d\Phi(x,y)/dA \ [W{\cdot}m^{-2}], \tag{4.1}$$

where the directional distribution of the flux $d\Phi(x,y)$ [W] through the surface element dA [m^2] at the point x,y need not be specified. This quantity is related to the radiance distribution by

$$\begin{aligned} W(x,y) &= \int_\omega L(x,y,\theta,\phi){\cdot}d\Omega = \int_\omega L(x,y,\theta,\phi){\cdot}\cos\theta{\cdot}d\omega \\ &= \int_\phi \int_\theta L(x,y,\theta,\phi){\cdot}\cos\theta{\cdot}\sin\theta{\cdot}d\theta{\cdot}d\phi \ [W{\cdot}m^{-2}], \end{aligned} \tag{4.4}$$

where the intersecting rays at the point x,y fill a solid angle $\omega \le 2\pi$ [sr] that may include the entire hemisphere on one side of the element dA (of the tangent plane containing that element, if the reference surface is curved). For an incident beam, $W(x,y) = E(x,y)$ is called the irradiance; for an exitent beam, $W(x,y) = M(x,y)$ is the radiant exitance.

The second form of flux per unit area, the omni-directional-surface distribution, is called the radiant fluence rate (at a point in space)

$$F_t(x,y,z) \equiv d\Phi(x,y,z)/da \ [W\cdot m^{-2}], \tag{4.12}$$

where, again, the directional distribution need not be specified for the flux $d\Phi(x,y,z)$ [W] incident on a spherical volume element, of cross section da $[m^2]$ and centered at the point x,y,z. This quantity is related to the incident radiance distribution by

$$F_t(x,y,z) = \int_\omega L(x,y,z,\theta,\phi)\cdot d\omega$$

$$= \int_\phi \int_\theta L(x,y,z,\theta,\phi)\cdot \sin\theta\cdot d\theta\cdot d\phi \ [W\cdot m^{-2}], \tag{4.16}$$

where the intersecting rays at the point*, x,y,z fill a solid angle $\omega \le 4\pi$ [sr] that may include the entire sphere of directions surrounding the point x,y,z.

Equations (4.4) and (4.16) can be turned around to obtain the following expressions for radiance in terms of radiant flux (surface) density and of radiant fluence rate, respectively:

$$L(x,y,\theta,\phi) = \frac{dW(x,y,\theta,\phi)}{d\Omega} = \frac{dW(x,y,\theta,\phi)}{\cos\theta\cdot d\omega} \ [W\cdot m^{-2}\cdot sr^{-1}] \tag{4.18}$$

and

$$L(x,y,z,\theta,\phi) = \frac{dF_t(w,y,z,\theta,\phi)}{d\omega} \ [W\cdot m^{-2}\cdot sr^{-1}]. \tag{4.19}$$

The third of the "simple" distributions, the purely directional distribution of radiant flux, is the radiant intensity (in a direction)

$$I(\theta,\phi) \equiv d\Phi(\theta,\phi)/d\omega \ [W\cdot sr^{-1}] \tag{4.21a}$$

$$= \int_{A_s} L(x,y,\theta,\phi)\cdot \cos\theta_s\cdot dA_s \ [W\cdot sr^{-1}]. \tag{4.33}$$

Although there is a value of radiant intensity for any source, by eq. (4.33), that quantity is not particularly useful except to characterize a source in a configuration where it may be treated as a "point source"; i.e., at distances where the irradiance that it produces varies inversely as the square of that distance. The inverse square law applies, strictly, only to elementary or "point" sources and receivers. The normal irradiance dE_n $[\equiv dE/\cos\theta_r]$ on a receiver element dA_r at the point x_r,y_r,z_r, or the radiant fluence rate dF_t incident on an elementary receiver volume of cross section da at the point x_r,y_r,z_r, at a distance D from a source element dA_s at the point x_s,y_s,z_s of radiant intensity dI in the direction θ,ϕ of the receiver at x_r,y_r,z_r, at an angle θ_s from the normal to dA_s, is given by

$$dE_n = dF_t = dE/\cos\theta_r = dI/D^2 = L\cdot\cos\theta_s\cdot dA_s/D^2 \ [W\cdot m^{-2}]. \tag{4.36}$$

More explicitly,

$$dE_n(x_r,y_r,z_r,\theta,\phi) = dF_t(x_r,y_r,z_r,\theta,\phi) = dE(x_r,y_r,z_r,\theta,\phi)/\cos\theta_r$$

$$= dI(x_s,y_s,z_s,\theta,\phi)/D^2$$

$$= L(x_s,y_s,z_s,\theta,\phi)\cdot\cos\theta_s\cdot dA_s/D^2 \ [\mathrm{W\cdot m^{-2}}]. \tag{4.36a}$$

When relations of this form are written for sources and receivers of finite dimensions, the distance D can only be a nominal or average distance because the slant distance D_s varies for different pairs of source and receiver elements, so the relations are only approximations to the inverse-square law. As an example, a simple case of parallel, plane, circular, co-axial, iso-radiance source and ideal receiver-detector, of area A_s and A_r, respectively, separated by a distance D along the axis between their centers, is examined. The incident flux at the receiver is

$$\Phi \approx E\cdot A_r \approx (I/D^2)\cdot A_r \approx L\cdot(A_s\cdot A_r/D^2) \ [\mathrm{W}], \tag{4.38}$$

where L is the source radiance, I is its radiant intensity, and E is the irradiance at the center of the detector-receiver. The discrepancy between the actual and inverse-square-approximation values of flux is

$$\Delta\Phi \equiv \Phi - L\cdot(A_s\cdot A_r/D^2) \ [\mathrm{W}]. \tag{4.39}$$

The relative error of the approximation to the inverse-square law is $\Delta\Phi/\Phi$ or, in terms of throughput (dividing through by L), $\Delta\Theta/\Theta$. For the conditions stated, it is shown that this error is less than one per cent when the maximum transverse dimension of source or receiver is no more than one tenth of the distance D between them. In fact, the use of D, the distance between centers, rather than the slant distance D_s between a pair of single points on source and receiver, does not introduce an error exceeding one per cent in the inverse-square relationship within these size limits, even for extreme points at opposite edges of source and receiver (see figure 4.11.) A brief examination shows effects of about the same magnitude due to the longitudinal dimensions of a source. The irradiance from a uniform iso-radiance source, however, depends only on the projected solid angle that it subtends (see figure 2.10 [5]).

Altogether, three main considerations determine when a source may be usefully treated as a point source, characterized by its radiant intensity, in making measurements:

(1) the size of the source relative to filling the entrance window or angular field of view of the measuring instrument (radiometer),

(2) the size of the maximum transverse dimensions of source and receiver (instrument receiving aperture or entrance pupil) relative to the distance separating them, and

(3) the directional distribution of radiance from each source-surface element and the effects of obscuration of portions of that surface.

These three considerations are discussed in some detail.

The volume-source distribution is defined as the <u>radiant sterisent</u> (in a direction at a point)

$$L^* \equiv dL_g/ds \equiv dI_g/dV \ [W \cdot m^{-3} \cdot sr^{-1}]. \tag{4.54}$$

More explicitly, the radiant sterisent at the point x,y,z in the direction θ,ϕ is

$$L^*(x,y,z,\theta,\phi) \equiv dL_g(x,y,z,\theta,\phi)/ds \ [W \cdot m^{-3} \cdot sr^{-1}], \tag{4.51}$$

where $dL_g(x,y,z,\theta,\phi)$ $[W \cdot m^{-2} \cdot sr^{-1}]$ is the generated (emitted and/or scattered) radiance of the medium in the path element ds at the point x,y,z and in the direction θ,ϕ (along a given ray in that direction). Alternatively,

$$L^*(x,y,z,\theta,\phi) \equiv dI_g(x,y,z,\theta,\phi)/dV \ [W \cdot m^{-3} \cdot sr^{-1}], \tag{4.55}$$

where $dI_g(x,y,z,\theta,\phi)$ $[W \cdot sr^{-1}]$ is the generated radiant intensity of the volume element $dV = ds \cdot \cos\theta_s \cdot dA$ $[m^3]$ at the point x,y,z in the direction θ,ϕ. Thus sterisent is the generated radiance per unit path length or the generated radiant intensity per unit volume. Its significance is most clearly apparent in the radiative-transfer equation for the observed radiance at a point from a given direction

$$L = \int_0^\infty L^*(s) \cdot \tau^*(s) \cdot ds \ [W \cdot m^{-2} \cdot sr^{-1}], \tag{4.50}$$

where $L^*(s)$ is the sterisent in the direction of the observation point at a distance s from that point along the ray path in the given direction and $\tau^*(s)$ is the propagance over the intervening path. This is a general relation of very wide application if the distribution of generated radiance, the sterisent, is broadened to include discontinuities, expressed by using Dirac delta-functions, as well as continuous distribution functions.

The radiant intensity of an extended volume source with negligible internal attenuation is given by

$$I(\theta,\phi) = \int_V L^*(x,y,z,\theta,\phi) \cdot dV \ [W \cdot sr^{-1}]. \tag{4.58}$$

However, in the presence of substantial attenuation within the source medium, it is necessary to first establish ray-radiance at an appropriate reference surface by eq. (4.50) and then to compute the source intensity from the radiance distribution at that reference surface by eq. (4.33). Whether this is useful or not, in either case, depends, of course, on the size-distance relationships already discussed with respect to the intensity of extended sources and approximations to the inverse-square law.

There are energy (time-integrated flux) distributions, corresponding to all of the spatial distributions of radiant flux but only the more commonly used have been given special names and symbols. In many cases it is the total energy, the time integral of flux, in a given time interval that is of most interest and significance.

The directed-surface distribution of incident radiant energy is the radiant exposure (at a point of a surface)

$$H(x,y) \equiv dQ(x,y)/dA = \int_{t_1}^{t_2} E(x,y,t)\cdot dt \ [J\cdot m^{-2}]. \tag{4.62a}$$

It is the time integral of the irradiance at the given point.

The omni-directional-surface distribution of incident radiant energy is the radiant fluence (at a point)

$$F(x,y,z) \equiv dQ(x,y,z)/da \ [J\cdot m^{-2}] \tag{4.64a}$$

$$= \int_{t_1}^{t_2} F_t(x,y,z,t)\cdot dt \ [J\cdot m^{-2}]. \tag{4.65a}$$

It is the time integral of the radiant fluence rate at the given point.

The volume distribution of radiant energy is the radiant (volume) density (at a point)

$$w(x,y,z) \equiv dQ(x,y,z)/dV \ [J\cdot m^{-3}] \tag{4.68a}$$

$$= (n/c)\cdot F_t(x,y,z) \ [J\cdot m^{-3}] \tag{4.73}$$

$$= (n/c)\cdot \int_{4\pi} L(x,y,z,\theta,\phi)\cdot d\omega \ [J\cdot m^{-3}]. \tag{4.74}$$

It is equal to n/c times the fluence rate at the given point (and at a given time), where n is the index of refraction of the medium and c is the vacuum velocity of electromagnetic radiation. (In a dispersive medium, similar relations exist but only between the spectral quantities at each wavelength.)

Finally, spectral radiometric quantities are formed, corresponding to each spatial radiometric quantity X, just by taking the derivative with respect to the appropriate spectral (variable) parameter μ:

$$X_\mu \equiv dX/d\mu. \tag{4.75}$$

The reader is also reminded that all of the foregoing quantities in terms of energy [J] and energy flux or power [W], including the spectral quantities, have their counterparts in terms of any other form of flux that is propagated in rays that obey the laws of geometrical optics. The photometric quantities, in terms of flux in lumens [lm], corresponding to the radiometric quantities of table 4.1 are listed in table 4.2. The photon-flux quantities, in terms of flux in quanta per second $[q \cdot s^{-1}]$, are similarly listed in table 4.3.

Chapter 5. An Introduction to the Measurement Equation

by Henry J. Kostkowski and Fred E. Nicodemus

In this CHAPTER. The measurement equation, which is central to our approach to all of radiometry (optical radiation measurements), is introduced by deriving simple measurement equations for three illustrative measurement problems. First, however, there is a brief presentation and discussion of responsivity, the output signal of a radiometer per unit input of incident radiation, an essential factor in the measurement equation. The measurement equation relates the instrument output resulting from a measurement to the distribution of incident spectral radiance in terms of all of the radiation parameters as well as significant environmental and instrumental parameters. By appropriate substitution, the output signal is similarly related to the radiometric quantity to be measured, if it is other than the incident spectral-radiance distribution. The examples here deal only with the radiation parameters of position, direction, and spectrum (wavelength). The radiation parameters of polarization and of time or frequency of scintillation or modulation and the instrumental and environmental parameters have not yet been treated in this Manual. The three measurement problems are: (1) transferring the spectral-radiance calibration of a tungsten-ribbon lamp, (2) determining the spectral irradiance near a large source, and (3) determining irradiance with a broad-band (broad-spectral-band) radiometer. The last problem also leads into a brief discussion of normalization. A general discussion summarizes and enlarges on the important features and properties of the measurement equation brought out by the illustrative problems. A set of orderly steps for approaching the solution of the measurement equation is presented. The limitations of the measurement equation developed in this chapter are summarized.

RESPONSIVITY. The response of an instrument used for optical radiation measurements, for example a radiometer or a photometer, is an output signal S that may have many different forms. It may be a voltage, a current, a photographic-film density, or just the deflection of a meter needle or the displacement of a line on a recorder strip chart. Each of these "signals" is measured in different units so, for general statements that apply to all of them, we just designate them as units of output signal[1] [S] (we use the upper-case "S" to distinguish them from the unit of time, the second [s]).

In order to interpret or evaluate this output signal as a measure of the incident radiation that produces it, we need the relationship between it and that incident radiation. That relationship is stated mathematically by the function called the instrument responsivity,

[1]This may, variously, be called the instrument "response" or "output". The nomenclature is not well standardized. However, since most such outputs are electrical "signals", we have taken the term and symbol from the International Electrotechnical Commission (IEC) [20] that are readily compatible with treatments of electrical "signal" processing and display. For those who may be interested, this choice of nomenclature is discussed in more detail in Appendix 4.

the output signal per unit input of incident radiation [17]. Most instruments respond to the time derivative of the incident radiant energy, i.e., to the radiant flux[1] Φ [W]. Accordingly, we will be concerned, mostly, with the flux responsivity

$$R_\Phi \equiv dS/d\Phi \ [S\cdot W^{-1}], \tag{5.1}$$

the units of output signal per watt of incident radiant flux. It follows that the element of output signal produced by an element of incident radiant flux is given by

$$dS = R_\Phi \cdot d\Phi \ [S]. \tag{5.2}$$

On the other hand, when "reducing" measurement data, some people prefer, instead, to use

$$d\Phi = dS/R_\Phi = K_\Phi \cdot dS \ [W], \tag{5.3}$$

where $K_\Phi \equiv 1/R_\Phi$ $[W\cdot S^{-1}]$ is sometimes called the calibration "constant" although, like the responsivity, as we'll see, it is often quite variable.

With many radiometers the output signal, and hence, the responsivity, varies considerably with the position and direction of incoming rays at the receiving aperture. It is easy to demonstrate this fact by moving the narrow beam of a small, intense, well-collimated source in front of the instrument. And we've already seen in figure 3.1 (Chapter 3 [5]) that responsivity can also be a highly variable function of wavelength. In general, then, responsivity [eq. (5.1)] and the element of output signal [eq. (5.2)] must be written as functions of both the spatial and spectral radiation parameters:

$$R_\Phi(x,y,\theta,\phi,\lambda) \equiv dS(x,y,\theta,\phi,\lambda)/d\Phi(x,y,\theta,\phi,\lambda), \tag{5.1a}$$

and

$$\begin{aligned} dS(x,y,\theta,\phi,\lambda) &= R_\Phi(x,y,\theta,\phi,\lambda)\cdot d\Phi(x,y,\theta,\phi,\lambda) \\ &= R_\Phi(x,y,\theta,\phi,\lambda)\cdot L_\lambda(x,y,\theta,\phi,\lambda)\cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S]. \end{aligned} \tag{5.2a}$$

The instrument responsivity is, in general, determined by (1) the propagance of optical elements and paths internal to the instrument, (2) the responsivity of the detector or transducer element that transforms the incident radiant flux into some other form of "signal", and (3) the signal processing by components that modify (amplify or filter) the signal from the detector-transducer to produce the final instrument output signal, including the final signal display or recording components. The effects of all of these

[1]Others (e.g., photographic films and instruments for measuring light pulses) respond to the energy Q [J], the time integral of the flux. For them, the energy responsivity is similarly defined as $R_Q \equiv dS/dQ$ $[S\cdot J^{-1}]$. (The radiometric quantities and units are listed in table 4-1; see also Appendix 1 [5].)

factors and their interactions are combined in the overall flux responsivity R_Φ.

Another consideration that must not be overlooked in real situations is that of linearity. A radiation detector is <u>linear</u> if its flux responsivity is the same for any magnitude of flux incident on the detector as long as the flux distribution in position, direction, and wavelength (and time and polarization) is not changed. Fortunately, many radiation detectors are quite linear over wide ranges and, often, they are used only within these linear ranges. On the other hand, some instruments are deliberately designed to have non-linear outputs. For example, a logarithmic output is sometimes used in applications calling for relatively rapid measurements of widely differing amounts of flux. In this chapter, we assume linearity, i.e., that the responsivity (the spectral-directional-positional flux responsivity[1]) is not a function of the level of input radiation nor, hence, of the output-signal magnitude. This simplifies the discussion of other aspects of the measurement situation, and it will still be adequate for a great many applications.

We again remind the reader that, for the present, we are ignoring dependence on time and polarization. We also call attention to the fact that responsivity may sometimes show hysteresis, where its value is a function of time and depends on the history of the instrument's exposure to radiation. This, too, we'll ignore for the present.

<u>EXAMPLES of the MEASUREMENT EQUATION</u>. We have now developed enough of the fundamentals so that we can introduce the measurement equation. This important equation relates the output signal from a radiometric instrument or measuring device to the distribution of radiation incident on it. The first step in solving this equation is to modify it so that it contains the radiometric quantity to be measured. The equation and its solution depend greatly on all of the relevant physical factors that we'll refer to, collectively, as the <u>measurement configuration</u> and on the quantity to be measured. The measurement configuration always includes one or more radiation sources, the measuring instrumentation, and the propagation path(s) for the radiation beam(s) between them. Accordingly, rather than to attempt a general discussion that will cover all of the complex possibilities, it will be

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[1]Note that these parameter modifiers denote only a functional dependence in the case of responsivity, not a derivative (concentration or distribution) as with a radiometric quantity. For example, spectral radiance is

$$L_\lambda \equiv dL/d\lambda \ [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}]$$

with different unit-dimensions than those of radiance L $[\mathrm{W \cdot m^{-2} \cdot sr^{-1}}]$; but spectral flux responsivity is not a derivative with respect to wavelength, it is just the flux responsivity as a function of wavelength, with no change in unit-dimensions:

$$R_\Phi(\lambda) \equiv dS/d\Phi|_\lambda \equiv (dS/d\lambda)/(d\Phi/d\lambda)|_\lambda \equiv S_\lambda/\Phi_\lambda|_\lambda \ [\mathrm{S \cdot W^{-1}}].$$

The effect of the modifier "spectral" is only to emphasize the functional dependence on the spectral parameter (in this case, wavelength).

simpler to start by setting up and discussing the measurement equations for three simple illustrative problems: (1) transferring the spectral-radiance calibration of a tungsten-ribbon lamp, (2) determining the spectral irradiance near a large source, and (3) determining irradiance with a broad-band (broad-spectral-band) radiometer.

Problem 1. Transferring the spectral-radiance calibration of a tungsten-ribbon lamp.

This problem commonly arises when one has obtained a spectral-radiance-standard lamp (usually a ribbon-filament tungsten lamp) from a national or commercial standards laboratory and wants to calibrate another similar lamp against this standard for use as a working standard, to conserve the limited working life of the reference-standard lamp. Such a comparison calibration measurement is usually performed by observing the two ribbon-filament lamps in succession with a spectroradiometer.

A spectroradiometer typically used for such comparison measurements consists of (see figure 5.1)

(1) focusing optics (a mirror system is usually used, the lens (O) is shown for simplicity of illustration) that image the desired portion (W) of the source emitting surface onto the entrance slit (S_1) of

(2) a monochromator (M), that selects and transmits a narrow wavelength band of the received source radiation to

(3) a detector (P), that produces an output, usually an electrical signal current or voltage, proportional to the incident spectral radiant flux, which is then fed to

(4) an electronic amplifier (E), the output of which is displayed or recorded in the desired final form by

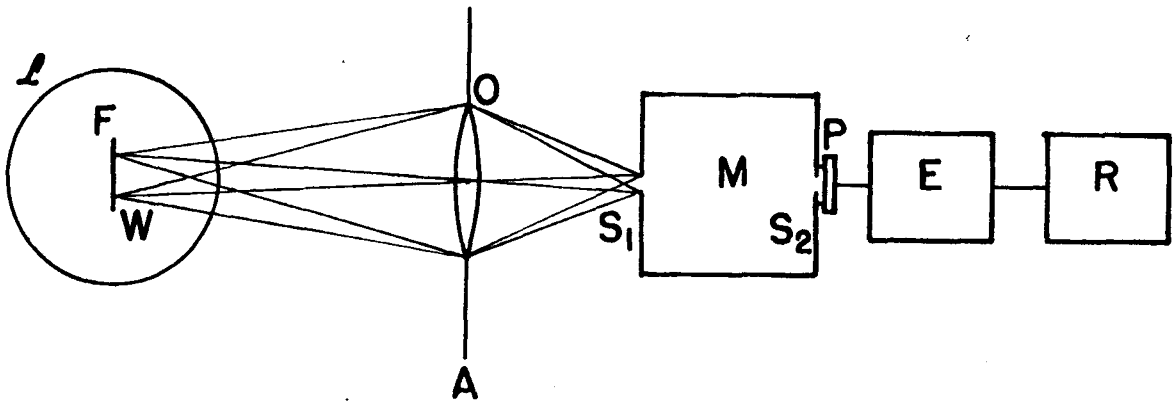
(5) an a propriate readout device (R).

Since we're concerned primarily with the basic concepts here in Part I, we'll postpone any discussion of the details of these devices and components until Part II—Instrumentation and Part III--Applications, and will confine our attention now to just their main functions.

Our first concern is to determine just how the instrumentation selects or determines the incident beam to which it responds in terms of the spectral-ray radiance distribution at its receiving aperture. The three parameters with which we are concerned, for the moment, are position, direction, and spectrum (wavelength):

(a) position (at the receiving aperture) -- Only rays through points within the receiving aperture are accepted and measured; all others are excluded.

(b) direction (at the receiving aperture) -- At each point within the receiving aperture, only the converging rays in the solid angle subtended at that point by a desired target area on the source emitting surface (the ribbon filament) are accepted and measured. Accordingly, the throughput of the system is that of the beam defined by the receiving aperture and the target area in combination. Without examining in detail the further paths of these rays through the monochromator until they reach the detector beyond the exit slit, we recognize that the responsivity may vary from ray to ray by writing it as



ℒ -- tungsten lamp with ribbon filament F

W -- target area on emitting surface of F

O -- external focusing optics (for illustrative simplicity we show a single thin lens)

A -- receiving aperture

M -- monochromator, with entrance slit S_1 and exit slit S_2 (internal details not shown)

P -- photocell, responds to radiation emerging from exit slit S_2

E -- electronics (amplification and signal processing)

R -- readout (recording and/or display of output "signal")

Figure 5.1. Diagramatic horizontal section of measurement configuration for spectral-radiance comparison measurements.

Note: For those interested in or familiar with the theory of stops [13,14], the target area above is the entrance window of the entire optical system and the receiving aperture is the entrance pupil. We plan to treat apertures, stops, pupils, windows, and baffles and their roles in radiometry in a future chapter.

$R(x,y,\theta,\phi)$, as in the preceding section.

(c) spectrum (wavelength, at the receiving aperture) -- The acceptance and response of the instrument to different wavelengths present in the incident radiation depends on (1) the extent to which those wavelengths appear in the beam emerging from the exit slit of the monochromator and (2) the spectral responsivity of the detector on which that beam is incident. The spectral propagance of any external optics and optical paths usually has a relatively minor effect although filters that substantially limit the spectral pass band are sometimes used along with a monochromator. However, the overall band width $\Delta\lambda$ in which the spectral responsivity of the instrument as a whole is significantly non-zero is usually determined primarily by the spectral characteristics of the monochromator together with the spectral responsivity of the detector. If $\Delta\lambda_m$ is the spectral pass band of the monochromator (and any associated optics and optical paths), $\Delta\lambda \leq \Delta\lambda_m$. They are equal if the detector responds to all wavelengths passed by the monochromator, etc.; if not, the overall spectral band width $\Delta\lambda$ may be less than the monochromator band width $\Delta\lambda_m$.

Without examining its details, it is sufficient for this discussion to assume that the monochromator is an instrument designed to select and pass, to the detector at its exit slit, a desired portion of the spectrum of radiation incident on its entrance slit. That portion is characterized by a wavelength setting λ_o and a spectral bandwidth or pass band $\Delta\lambda_m = \lambda_2 - \lambda_1$. All wavelengths λ reaching the detector, lie in the interval $\lambda_1 \leq \lambda \leq \lambda_2$; all wavelengths $\lambda < \lambda_1$ and $\lambda > \lambda_2$ are blocked and do not appear in the emerging beam at the exit slit. The spectral bandwidth of the monochromator $\Delta\lambda_m$ is primarily controlled by adjusting the widths of the slits, and may also be a function of the setting λ_o.

This rather sketchy description of monochromator operation has assumed some ideal conditions and, for our simple example, we continue to assume that they exist. However, the reader is cautioned that strong incident radiation outside any spectral bandwidth $\Delta\lambda$ may not be reduced completely to zero (completely blocked) by an instrument or component designed for that purpose.[1] A monochromator is no exception. Real monochromators, because of internal scattering, always have some scattered radiation of unwanted wavelengths present in the exit beam, although with good designs this can be kept extremely low. However, when attempting to measure weak radiation of given wavelengths in the presence of strong radiation of other wavelengths, the scattered radiation can become a significant factor. This is particularly true when the wavelength separation is small. A good example is the difficult problem of measuring the very weak terrestrial solar ultraviolet irradiation at about 290 [nm], that is passed by the nearly opaque layer of atmospheric ozone, when this is done in the presence of significantly stronger radiation at slightly longer wavelengths to which the ozone (and the rest of the atmosphere) is highly transparent.

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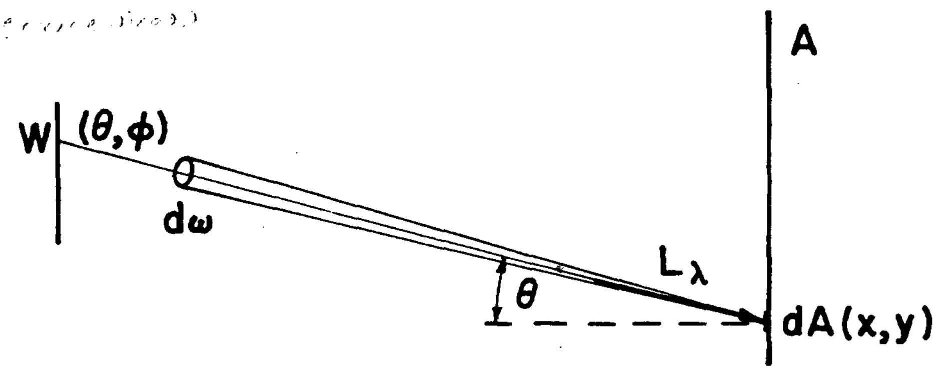
[1]This caveat applies generally to filtering and blocking with respect to <u>any</u> parameter, not just the spectral parameter.

Here the level changes by about six orders of magnitude within a wavelength interval of 30 [nm].

Now we're ready to set up the measurement equation for the incident distribution of radiation at the receiving aperture of our instrument. Figure 5.2 shows a single ray from a point within the target area at the source to the area element dA at the point x,y of the receiving aperture. It is incident there within the solid-angle element $d\omega$ from the direction θ,ϕ. From eq. (3.10 [5]), the element of radiant flux $d\Phi$ incident on the receiving aperture along this ray of incident spectral radiance $L_\lambda(x,y,\theta,\phi,\lambda)$ is given by

$$d\Phi(x,y,\theta,\phi,\lambda) = L_\lambda(x,y,\theta,\phi,\lambda)\cdot dA\cdot\cos\theta\cdot d\omega\cdot d\lambda \text{ [W]}. \tag{3.10}$$



W — target area (on lamp filament)

A — surface (imaginary) area across receiving aperture

$L_\lambda(x,y,\theta,\phi,\lambda)$ -- incident spectral radiance at area element dA(x,y) of A within solid-angle element $d\omega$ from direction θ,ϕ and in wavelength element $d\lambda$ at wavelength λ.

Figure 5.2. Incident ray from lamp filament to receiving aperture.

We can think of such an element of flux arriving at the receiving aperture along each of the rays that connect every point of the target area with all points across the receiving aperture. Each element of incident flux produces a corresponding element of output signal

$$dS(x,y,\theta,\phi,\lambda,\lambda_o) = R_\Phi \cdot d\Phi$$

$$= R_\Phi(x,y,\theta,\phi,\lambda,\lambda_o) \cdot L_\lambda(x,y,\theta,\phi,\lambda) \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S]. \qquad (5.4)$$

where R_Φ is the flux responsivity of the spectroradiometer, when set for a wavelength λ_o, for this *particular* flux element (for the ray from the direction θ,ϕ) at the point x,y of the receiving aperture and at wavelength λ. The total output signal S of the spectroradiometer is then the sum (integral) of all such signal elements dS for all rays contained in the optical beam accepted by the measuring instrument. Included are all points across the receiving aperture and, at each point, the directions of all rays from every point of the target area. In addition, the integration must be carried out over all wavelengths for which the responsivity is not zero. Thus the total signal is given by

$$S(A,\omega,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda} \int_A \int_\omega R_\Phi \cdot L_\lambda \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S], \qquad (5.5)$$

where

- ω [sr] is the target solid angle, the solid angle enclosed by the extreme rays from the target area that converge at a point x,y of the receiving aperture [accordingly, ω can be, and usually is, a function of the position x,y and so could be written more explicitly as $\omega(x,y)$],

- $\Delta\lambda$ [nm] is the wavelength interval over which the instrument spectral flux responsivity is significantly non-zero, i.e., where $R_\Phi(\lambda_o,\lambda) \neq 0$ when the instrument is set on wavelength λ_o.

This equation [eq. (5.5)] is the measurement equation for this problem. It relates the output signal of the measuring instrument to the spectral-radiance distribution incident at the receiving aperture of the instrument.

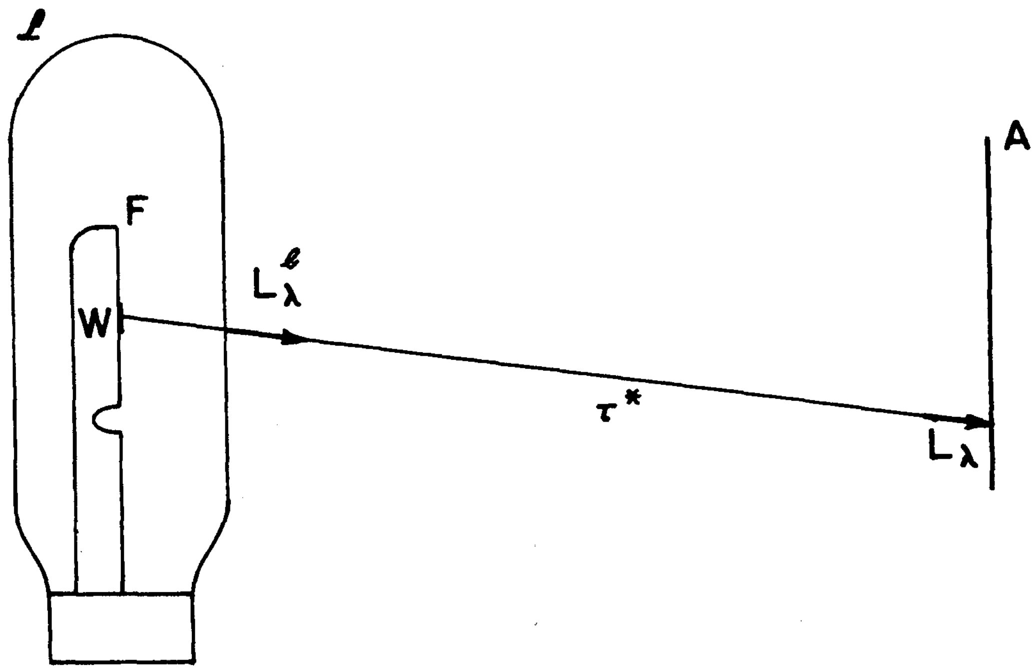
This measurement equation is similar to eq. (3.11) [5], which gives the total flux in a beam at its intersection with a reference surface as

$$\Phi = \int_A \int_\omega \int_{\Delta\lambda} L_\lambda(x,y,\theta,\phi,\lambda) \cdot d\lambda \cdot \cos\theta \cdot d\omega \cdot dA \ [W]. \qquad (3.11)$$

The difference is that, in the measurement equation, each element of flux is "weighted" by the corresponding value of the responsivity. In other words, we can regard the total output signal S as the responsivity-weighted flux in the beam received by the instrument. For the measurement equation, the reference surface is the instrument receiving aperture.

The first step in solving the measurement equation for a particular problem is to introduce into that equation the radiometric quantity one is interested in measuring. In this problem, the quantity to be measured is the spectral radiance, but it is the spectral radiance L_λ^ℓ at the lamp[1] rather than the spectral radiance L_λ incident at the receiving aperture of the spectroradiometer (see figure 5.3). If we assume that the index of refraction (that of air) is the same at both these points, eq. (3.14) [5] gives their relationship as

$$L_\lambda(x,y,\theta,\phi,\lambda) = L_\lambda^\ell(x,y,\theta,\phi,\lambda)\cdot\tau^*(x,y,\theta,\phi,\lambda)\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}], \tag{5.6}$$



W -- target area (on ribbon filament F of lamp $\mathcal{L}$)
A -- surface (imaginary) across receiving aperture
L_λ^ℓ -- spectral radiance of ray just outside of lamp envelope
L_λ -- incident spectral radiance of same ray at receiving aperture after traversing path of propagance τ^*.

Figure 5.3. Spectral radiances at points along a single ray from target area to receiving aperture.

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[1] Because of the way in which radiance standards are used, the spectral radiance is specified at the outer surface of the lamp envelope. This value of radiance is a property of the lamp (and the operating conditions -- voltage or current, etc.) regardless of the propagance between it and a remote instrument.

where

$\tau^*(x,y,\theta,\phi,\lambda)$ is the spectral propagance over the intervening path along the specified ray (between the lamp envelope and the receiving aperture of the instrument).

Thus, the equation, as finally modified to contain the radiometric quantity L_λ^ℓ, that we're actually interested in measuring in this problem, is

$$S(A,\omega,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda}\int_A\int_\omega R_\Phi\cdot\tau^*\cdot L_\lambda^\ell\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda\ [S]. \tag{5.7}$$

Now we want to compare L_λ^ℓ to the spectral radiance L_λ^s of a standard lamp. The corresponding equation for a similar observation with a standard lamp is

$$S^s(A,\omega^s,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda}\int_A\int_{\omega^s} R_\Phi\cdot\tau^{*s}\cdot L_\lambda^s\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda\ [S], \tag{5.8}$$

where, by the use of the superscript s, we have allowed for the possibility that the target area and its subtended solid angle, the target solid angle ω^s, as well as the propagance τ^{*s}, associated with the measurement of the standard lamp may be different. On the other hand, we don't need the superscript s on the remaining quantities since the receiving-aperture area A, and the wavelength setting λ_o and pass band $\Delta\lambda$ and, hence, the spectral responsivity $R_\Phi(\lambda_o,\lambda)$ of the spectroradiometer are all kept the same for both measurements [eqs. (5.7) and (5.8)].

We can easily solve eqs. (5.7) and (5.8) for the spectral radiance L_λ^ℓ of the uncalibrated lamp, in terms of the spectral radiance L_λ^s of the standard lamp and of the two observed output signals S and S^s, if, for each lamp, the value of spectral radiance is the same everywhere in the beam and at all wavelengths in the interval $\Delta\lambda$. When that is true, we can bring L_λ^ℓ and L_λ^s outside the integrals in each case, resulting in

$$S = L_\lambda^\ell\cdot\int_{\Delta\lambda}\int_A\int_\omega R_\Phi\cdot\tau^*\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda\ [S] \tag{5.9}$$

and

$$S^s = L_\lambda^s\cdot\int_{\Delta\lambda}\int_A\int_{\omega^s} R_\Phi\cdot\tau^{*s}\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda\ [S]. \tag{5.10}$$

By positioning the standard lamp filament in exactly the same location as that of the uncalibrated lamp when it was measured and leaving the focusing adjustment and position of the spectroradiometer unchanged, the corresponding target (solid) angles ω and ω^s and propagances τ^* and τ^{*s} will also be, respectively, the same. In other words, by making the measurements under identical conditions, the integrals in eqs. (5.9) and (5.10) are identical and dividing one of these equations by the other gives

$$L_\lambda^\ell = (S/S^s)\cdot L_\lambda^s\ [\mathrm{W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}}]. \tag{5.11}$$

This simple relationship is often used in making a comparison calibration of two such lamps. By deriving it, in this way, from the full measurement equation (at least in terms of the parameters of position, direction, and spectrum), we can see that it has a solid basis and, in addition, we are explicitly aware of all of the assumptions or conditions that must be satisfied for it to be valid.

The requirement, in deriving eq. (5.11), for constancy of spectral radiance throughout the beam (A and ω) and at all wavelengths in the interval $\Delta\lambda$ for these ribbon-filament lamps is never actually realized. There are temperature and emissivity gradients on the tungsten surface that produce variations in spectral radiance relative to both position and direction. In addition, the spectral radiance for tungsten varies significantly with wavelength. The variations relative to position and direction may be minimized by making A and ω sufficiently small so that the range of variation is about one per cent or less. Then, in the measurement equation, eq. (5.7),

$$S(A,\omega,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda} \bar{L}_\lambda^\ell \cdot \left(\int_A \int_\omega R_\Phi \cdot \tau^* \cdot \cos\theta \cdot d\omega \cdot dA\right) \cdot d\lambda \ [S] \tag{5.12}$$

$$= \int_{\Delta\lambda} \bar{L}_\lambda^\ell(\lambda,A,\omega) \cdot R_L^\ell(\lambda_o,\lambda,A,\omega) \cdot d\lambda \ [S], \tag{5.13}$$

where

$$\bar{L}_\lambda^\ell(\lambda,A,\omega) \equiv \frac{\int_A \int_\omega L_\lambda^\ell \cdot R_\Phi \cdot \tau^* \cdot \cos\theta \cdot d\omega \cdot dA}{\int_A \int_\omega R_\Phi \cdot \tau^* \cdot \cos\theta \cdot d\omega \cdot dA} \ [W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}] \tag{5.14}$$

and

$$R_L^\ell(\lambda_o,\lambda,A,\omega) \equiv \int_A \int_\omega R_\Phi \cdot \tau^* \cdot \cos\theta \cdot d\omega \cdot dA \ [S \cdot W^{-1} \cdot m^2 \cdot sr]. \tag{5.15}$$

$\bar{L}_\lambda^\ell(\lambda,A,\omega)$ is the weighted average of the lamp spectral radiance L_λ^ℓ over A and ω, weighted by $R_\Phi \cdot \tau^* \cdot \cos\theta$, and the lamp-spectral-radiance responsivity $R_L^\ell(\lambda_o,\lambda,A,\omega)$ is the integral over A and ω of this weighting function.[1] A similar set of equations exists for the average spectral radiance of the standard lamp $\bar{L}_\lambda^s(\lambda,A,\omega)$ $[W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}]$.

The variation with respect to wavelength can *not* be treated in exactly the same way because the flux responsivity R_Φ and, therefore, the radiance responsivity R_L both vary all the way from zero to their respective peak values in the wavelength interval $\Delta\lambda$.

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[1]Note that this is the radiance responsivity to the total beam; $R_L \equiv dS/dL$ where dS is an elementary change in the output signal and dL is an elementary change in the value of radiance of an incident iso-radiance beam that produces the signal element dS.

However, lamps such as these are usually operated so that their relative spectral distributions are virtually the same. In this case,

$$\bar{L}^{\ell}_{\lambda}(\lambda,A,\omega) \equiv K_1 \cdot \bar{\ell}^{\ell}_{\lambda}(\lambda,A,\omega) \tag{5.16}$$

and

$$\bar{L}^{s}_{\lambda}(\lambda,A,\omega) \equiv K_2 \cdot \bar{\ell}^{s}_{\lambda}(\lambda,A,\omega) \approx K_2 \cdot \bar{\ell}^{\ell}(\lambda \cdot A,\omega), \tag{5.17}$$

where $\bar{\ell}^{\ell}_{\lambda}$ and $\bar{\ell}^{s}_{\lambda}$ are the relative average spectral radiances (averaged over A and ω and adjusted to have the same maximum value) of the two lamps and K_1 and K_2 are constants. The next step is to substitute eq. (5.16) into eq. (5.13) and eq. (5.17) into the standard-lamp equivalent of eq. (5.13) and take the quotient of the resulting expressions to obtain

$$\frac{S(A,\omega,\Delta\lambda,\lambda_o)}{S^s(A,\omega,\Delta\lambda,\lambda_o)} = \frac{K_1 \cdot \int_{\Delta\lambda} \bar{\ell}^{\ell}_{\lambda} \cdot R^{\ell}_{L} \cdot d\lambda}{K_2 \cdot \int_{\Delta\lambda} \bar{\ell}^{\ell}_{\lambda} \cdot R^{\ell}_{L} \cdot d\lambda} = \frac{K_1}{K_2} . \tag{5.18}$$

But, from eqs. (5.16) and (5.17),

$$K_1/K_2 = \bar{L}^{\ell}_{\lambda} \, / \, \bar{L}^{s}_{\lambda} . \tag{5.19}$$

Accordingly, combining eqs. (5.18) and (5.19), we have

$$\bar{L}^{\ell}_{\lambda} = (S/S^s) \cdot \bar{L}^{s}_{\lambda} \; [W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}]. \tag{5.20}$$

Equation (5.20) is the same as eq. (5.11), except that here the two spectral radiances, unknown and standard, are averages over A and ω, as defined in eq. (5.14).

The primary objective of the treatment just presented is to introduce the measurement-equation approach. Discussion of further details of actual applications of eq. (5.11) or (5.20) will be postponed for later chapters. However, we would like to add that, by limiting the target-area dimensions used in such comparison calibrations to one millimeter or less, aperture angles (subtended by the instrument receiving aperture at the lamp source) to about 0.01 steradian, and spectral pass band to about 2.5 nanometers, the uncertainties in such comparisons have been limited at NBS to much less than 1%. It should also be noted that, when average-spectral-radiance values are used in a maximum-accuracy calibration, they apply only to the particular beam from the particular target area (W) observed. Hence, means must be provided for accurately identifying a desired portion of the lamp filament and a specific solid angle to insure identical beam geometry.

Problem 2. Determining the spectral irradiance near a large source.

This second measurement problem is illustrated in figure 5.4. We want to measure the spectral irradiance produced by a horizontal fluorescent lamp at a point x_o, y_o on a horizontal plane surface beneath it, as shown. The vertical distance z_ℓ from the lamp to the horizontal plane of x_o, y_o is approximately equal to the length of the fluorescent tube, so the solid angle ω^ℓ subtended by the lamp at that point is much greater than the solid angle ω^s subtended there by a standard of spectral irradiance (e.g., a small calibrated tungsten-halogen lamp) at the distance for which it is calibrated (usually 0.5 [m]; see figure 5.5). As we'll see presently, we don't need the exact values of these solid angles so these approximate statements will suffice for the description and analysis of the measurement configuration.

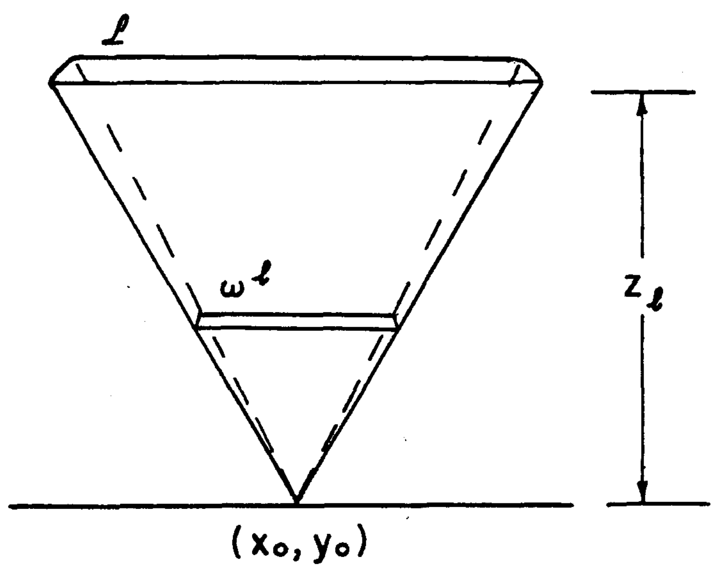


Figure 5.4. Configuration for measuring spectral irradiance at x_o, y_o from fluorescent lamp $\mathcal{L}$.

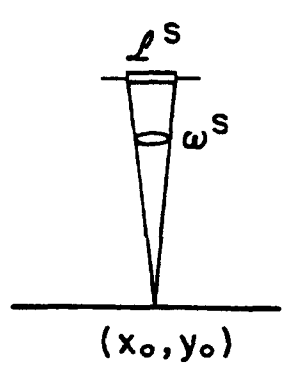


Figure 5.5. Configuration for spectral-irradiance calibration measurement with standard lamp $\mathcal{L}^s$.

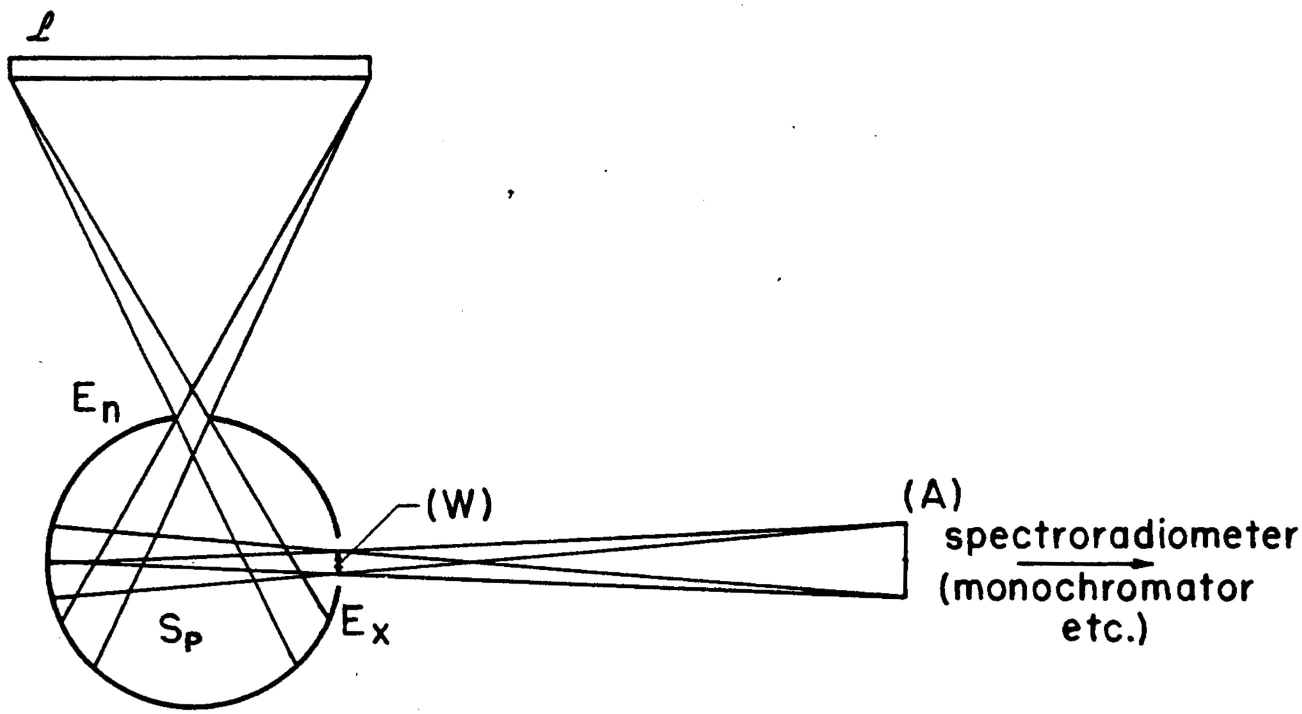
An apparatus commonly used at NBS to make such measurements of spectral irradiance is shown in figure 5.6. It consists of a spectroradiometer and a small averaging sphere.[1] The spectroradiometer is just like the one in figure 5.1, used to compare or measure spectral radiance in problem 1. The averaging sphere is a hollow sphere, typically a few centimeters in diameter, with uniform diffusely reflecting inner walls and two ports, as shown. Radiation incident through the entrance port strikes a portion of the inner wall from which no rays can be reflected *directly* into the beam to the spectroradiometer. The spectroradiometer is focused, by its external optics, with its target area (W) entirely within the exit port of the sphere, as shown in figure 5.6. The function of the averaging sphere is to produce isotropic spectral-flux responsivity over a very wide solid angle of incidence. The multiple internal diffuse reflections, each scattering the incident radiation to all parts of the sphere wall, from which it is again so redistributed, produce uniform irradiation of the wall opposite the exit port, regardless of the direction of the entering rays through the entrance port, within very wide limits. Accordingly, the reflected rays from that wall toward the spectroradiometer through the exit port all have the same value of spectral radiance, and that value is proportional to the spectral flux in the beam incident through the entrance port. We have described the averaging sphere as having ideal properties. With a real sphere, corrections may be required for the highest accuracy, but we won't be concerned with them in this chapter.

The measurement is made with the entrance port of the averaging sphere in a horizontal plane and with the center of the entrance port at x_o, y_o. That entrance port is the receiving aperture for the entire instrument, limiting the incident beam to be measured. Again, we are concerned only with the three radiation parameters -- position, direction, and spectrum (wavelength) -- for the incident rays at that receiving aperture:

(a) position (at the receiving aperture of the entire instrument) -- By its uniform diffusion of the incident radiation, the averaging sphere insures the same full, uniform irradiation of both the target area (W) and the receiving aperture (A) *of the spectroradiometer* (see figure 5.6) by all rays entering the entrance port of the sphere, regardless of position there. This makes the overall flux responsivity of the system R_Φ independent of position x,y within the entrance port (receiving aperture of entire instrument). Also, the limits of the beam are the boundaries of the entrance port.

(b) direction (at the receiving aperture of the entire instrument) -- In the same way, the overall flux responsivity R_Φ is independent of incident-ray direction θ,ϕ at the receiving aperture (entrance port of sphere) over wide limits, at least to the solid angle subtended there by the fluorescent lamp. The rays of the desired beam from the source are limited by the dimensions of the source, not by the measuring instrument which can usually accept rays over a wider solid angle. Accordingly, it is necessary to make sure that there are no other sources present from which a significant amount of additional incident radiation might produce an erroneous measurement. When this is not possible, as at infrared wavelengths where all matter at room temperature is emitting thermal radiation

[1]Sometimes called an integrating sphere.



ℒ — fluorescent lamp

S_p — averaging sphere (not drawn to scale; sphere is usually only a few centimeters in diameter)

E_n — receiving aperture (entrance port of averaging sphere S_p)

E_x — exit port of averaging sphere S_p [contains target area (W) for spectroradiometer (see figure 5.1)]

(A) — receiving aperture of spectroradiometer of figure 5.1

Figure 5.6. Sketch of some essential parts of the configuration for a measurement of spectral irradiance.

in significant amounts, special techniques for cancelling the effects of unwanted "background" radiation are used to obtain the desired measurement results. We won't go into the details of those techniques now, however, In any event, the incident rays of the desired beam are those within the solid angle subtended at each point of the receiving aperture (entrance port of sphere) by the source.

(c) spectrum (wavelength, at the receiving aperture of the entire instrument) -- Ideally, the averaging-sphere-wall coating is selected to have as nearly uniform spectral reflectance (as a function of wavelength) as possible. The wavelength limits, consequently, are those established by the spectral bandwidth $\Delta\lambda$ at the wavelength setting λ_o of the spectroradiometer. Any small spectral non-uniformity of reflectance in the sphere merely affects the overall spectral distribution of responsivity which, thus may be slightly different from that for the spectroradiometer used alone as in problem 1.

Again, as in problem 1, we first write a general expression, in terms of the flux responsivity R_Φ, for the signal element dS produced by an element of flux $d\Phi$ incident on the receiving aperture (entrance port -- see figure 5.7)

$$dS(x,y,\theta,\phi,\lambda,\lambda_o) = R_\Phi \cdot d\Phi$$

$$= R_\Phi(x,y,\theta,\phi,\lambda,\lambda_o) \cdot L_\lambda(x,y,\theta,\phi,\lambda) \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S]. \quad (5.4)$$

And, again, the measurement equation is obtained by integrating eq. (5.4) to obtain the expression for the total output signal

$$S(A,\omega,\Delta\lambda,\lambda_o) = \int_{\Delta\lambda} \int_A \int_\omega R_\Phi \cdot L_\lambda \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S], \quad (5.5)$$

where, now,

ω [sr] is the acceptance solid angle that, at any point x,y in the receiving aperture (entrance port of sphere), includes all converging incident rays to which the instrument responds,

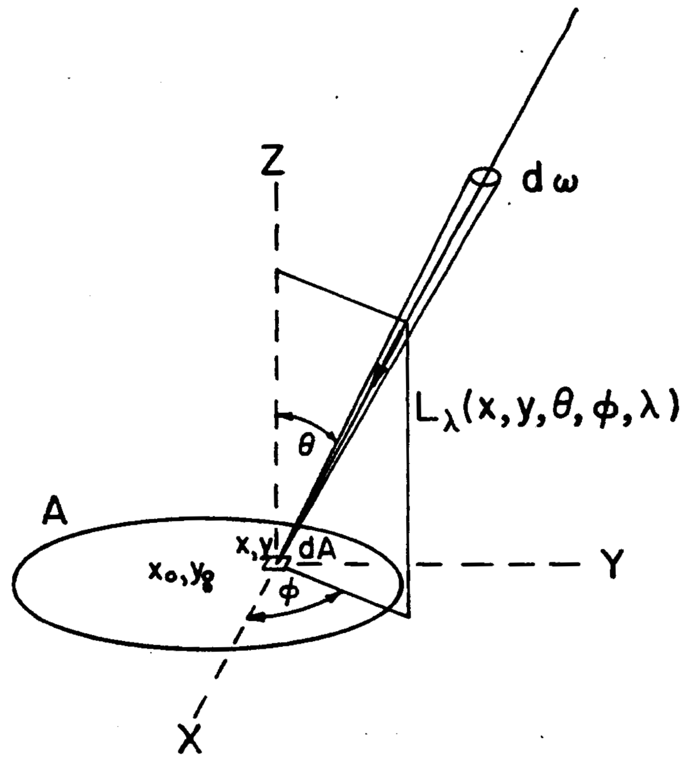
A [m^2] is the area of the receiving aperture (entrance port of sphere), and

$\Delta\lambda$ [nm] is the wavelength pass band of the entire instrument, including all wavelengths for which the spectral flux responsivity is significantly non-zero; i.e., where $R_\Phi(\lambda_o,\lambda) \neq 0$.

The first step in solving this measurement equation is, again, to introduce the radiometric quantity to be measured (in terms of the incident spectral-radiance distribution). The incident spectral irradiance at the point x,y and the wavelength λ [see eq. (4.4) and the last section of Chapter 4 entitled "Spectral Radiometric Quantities"] is given by

$$E_\lambda(x,y,\lambda) \equiv d\Phi_\lambda(x,y,\lambda)/dA$$

$$\equiv \int_\omega L_\lambda(x,y,\theta,\phi,\lambda) \cdot \cos\theta \cdot d\omega \ [W \cdot m^{-2}]. \quad (5.21)$$



A — circular area of receiving aperture (entrance port of averaging sphere)

x_o, y_o — center point of A

x,y — any point within A

$L_\lambda(x,y,\theta,\phi,\lambda)$ — spectral radiance incident at x,y from direction θ,ϕ at wavelength λ

Figure 5.7. Designation of spectral-radiance distribution incident at receiving aperture

In eq. (5.5), if R_Φ were brought outside the *inner* integral, that integral would be identical to that of eq. (5.21) [see eq. (5.22)]. But we have already seen that the effect of the averaging sphere is to make the flux responsivity independent of the position and direction of the incident rays within A and ω. Accordingly, we can write $R_\Phi(x,y,\theta,\phi,\lambda,\lambda_o) = R_\Phi(\lambda_o,\lambda)$ so that eq. (5.5) becomes

$$S(A,\omega,\Delta\lambda,\lambda_o) \;=\; \int_{\Delta\lambda}\int_A \int_\omega L_\lambda(x,y,\theta,\phi,\lambda)\cdot\cos\theta\cdot d\omega \cdot R_\Phi(\lambda_o,\lambda)\cdot dA\cdot d\lambda \;[S], \tag{5.22}$$

and we can replace the inner integral (in large parentheses) with $E_\lambda(x,y,\lambda)$ [eq. (5.21)] to give

$$S(A,\Delta\lambda,\lambda_o) \;=\; \int_{\Delta\lambda}\int_A E_\lambda(x,y,\lambda)\cdot R_\Phi(\lambda_o,\lambda)\cdot dA\cdot d\lambda \;[S]. \tag{5.23}$$

Actually, when it is clear that the fluorescent lamp is the only significant source irradiating the receiving aperture, the incident spectral radiance L_λ at any point x,y in that receiving aperture is zero for all directions outside the solid angle ω^ℓ subtended at that point by the lamp. In that case the integration in eq. (5.22) need be carried out only over ω^ℓ rather than over the full acceptance solid angle ω, so that

$$E^\ell_\lambda(x,y,\lambda) \;=\; \int_{\omega^\ell} L_\lambda(x,y,\theta,\phi,\lambda)\cdot\cos\theta\cdot d\omega \;[\mathrm{W\cdot m^{-2}\cdot nm^{-1}}]. \tag{5.24}$$

Furthermore, if the lamp distance is large enough in relation to the size of the receiving aperture, neither the size of the solid angle ω^ℓ nor the value of the incident spectral radiance L_λ from any point of the lamp will change significantly as a function of position x,y across the receiving aperture. In other words, the spectral irradiance will be constant, and equal to its value at the center x_o,y_o across the entire aperture

$$E^\ell_\lambda(x,y,\lambda) \;=\; E^\ell_\lambda(x_o,y_o,\lambda) \;[\mathrm{W\cdot m^{-2}\cdot nm^{-1}}]. \tag{5.25}$$

If we also assume, for the moment, that the spectral irradiance does not vary significantly with wavelength λ within the pass band $\Delta\lambda$ for any given setting λ_o, we may also write that[1]

$$E^\ell_\lambda(x,y,\lambda) \;=\; E^\ell_\lambda(x_o,y_o,\lambda_o) \;[\mathrm{W\cdot m^{-2}\cdot nm^{-1}}], \tag{5.26}$$

which may be treated as a constant and brought outside the integrals in eq. (5.23), leaving

[1]Tacitly assuming that λ_o lies within $\Delta\lambda$. This may not be true for a nominal instrument "setting" but will hold for the calibrated or "corrected" value corresponding to that "setting".

$$S^{\ell}(A,\Delta\lambda,\lambda_o) = A\cdot\int_{\Delta\lambda} E_{\lambda}^{\ell}(x_o,y_o,\lambda)\cdot R_{\Phi}(\lambda_o,\lambda)\cdot d\lambda$$

$$= E_{\lambda}^{\ell}(x_o,y_o,\lambda_o)\cdot A\cdot\int_{\Delta\lambda} R_{\Phi}(\lambda_o,\lambda)\cdot d\lambda$$

$$= E_{\lambda}^{\ell}(x_o,y_o,\lambda_o)\cdot\overline{R_{\Phi}}(\lambda_o)\cdot A\cdot\Delta\lambda \text{ [S]}. \qquad (5.27)$$

The average responsivity $\overline{R_{\Phi}}(\lambda_o) \equiv (1/\Delta\lambda)\cdot\int_{\Delta\lambda} R_{\Phi}(\lambda_o,\lambda)\cdot d\lambda$ can be eliminated along with A and $\Delta\lambda$, by making a similar measurement of the spectral irradiance produced in the receiving aperture by a tungsten-halogen-lamp standard of spectral irradiance. The corresponding equation for the standard lamp can be derived, step by step, in the same manner as for the fluorescent lamp. The result, corresponding to eq. (5.27), is

$$S^{s}(A,\Delta\lambda,\lambda_o) = E_{\lambda}^{s}(x_o,y_o,\lambda_o)\cdot\overline{R_{\Phi}}(\lambda_o)\cdot A\cdot\Delta\lambda \text{ [S]}. \qquad (5.28)$$

By combining eqs. (5.27) and (5.28), we obtain

$$E_{\lambda}^{\ell}(x_o,y_o,\lambda_o) = (S^{\ell}/S^{s})\cdot E_{\lambda}^{s}(x_o,y_o,\lambda_o) \text{ } [\mathrm{W\cdot m^{-2}\cdot nm^{-1}}], \qquad (5.29)$$

the simplest relation for measuring the spectral irradiance of the fluorescent lamp in terms of that of the standard lamp.

Note that, since we are concerned with irradiance at the reference surface across the entrance port of the sphere, we have not introduced the propagance τ^* as we did in problem 1, where we were measuring L_{λ} at the lamp source. However, if the propagance τ^* over the relatively short path in the laboratory for the standard-lamp measurement should be significantly different from that in the standards laboratory where that lamp was calibrated, this could produce an error in our results.

The most critical condition for this measurement is the constancy of directional responsivity over the solid angle of acceptance ω, required to go from eq. (5.5) to eq. (5.22). Actually, as we can see from eq. (5.24) it is sufficient if the constant directional responsivity holds only over the smaller solid angle ω^{ℓ}, as well as over the still smaller ω^{s} for the corresponding treatment of the standard lamp, providing no other sources are present. Fortunately, as previously mentioned, averaging spheres that perform well in this respect are available. Also, it is not difficult to verify this performance if a sufficiently collimated beam, wide enough to more than fill the entrance port of the averaging sphere, is available. When the angle θ between the collimated-beam direction and the normal to the plane of the entrance port is varied, the instrument output S should vary in direct proportion to $\cos\theta$ (a so-called lambertian responsivity characteristic). This is readily apparent in eq. (5.5) if ω is made small (a small distant source) and all other quantities on the right-hand side of the equation, particularly R_{Φ}, remain constant while θ is varied.

Equation (5.25) was based on two assumptions: (1) that the solid angle ω^{ℓ} (subtended by the lamp) does not change significantly as a function of position x,y across the receiving aperture, and (2) that the incident spectral radiance L_λ from any point of the lamp also does not change significantly with x,y. If we set the height of the lamp z_ℓ above x_o,y_o equal to the length of the lamp, and the receiving-aperture diameter approximately equal to $0.01 \cdot z_\ell$, it is easily shown that the angle subtended by the long dimension of the lamp will change by much less than a tenth of one per cent between the center x_o,y_o and the edge of that aperture. The change is obviously much less for the shorter dimension of the fluorescent lamp and for the smaller tungsten-halogen lamp. On the other hand, the possible variation of lamp spectral radiance L_λ with direction from each point of the lamp may not be so easy to measure or evaluate. However, the constancy of irradiance across the receiving aperture, assumed in eq. (5.25), can be checked directly by placing over it a screen with the smallest possible hole that allows just enough flux to pass to give a measurable output and observing the constancy of that output as the hole is moved about across the receiving aperture. If necessary, the spectroradiometer slits can be widened to increase $\Delta\lambda$ and pass more flux to produce a stronger signal output for this purely geometrical check, since there is no reason to believe that the directional distribution will be significantly different at slightly different wavelengths.

Finally, we have assumed, in going from eq. (5.25) to eq. (5.26) that E_λ is constant with respect to λ within $\Delta\lambda$. For thermal-radiation sources, such as tungsten lamps and blackbody simulators, it is usually possible to make the spectral pass band $\Delta\lambda$ of the spectroradiometer narrow enough so that E_λ does not change by a significant amount within that spectral interval. However, when there are spectral lines, as in the radiation from a fluorescent lamp, or where thermal radiation falls off rapidly, particularly at the short-wavelength end of the spectral curve of thermal emission, there may be large variations in E_λ within the interval $\Delta\lambda$. We'll have to leave much of the treatment of that problem for a later chapter, after the concept of slit function has been developed. For the present, we recognize that when $\Delta\lambda$ is large enough so that there are significant variations in the spectral irradiance $E_\lambda(\lambda)$ over the spectral pass band $\Delta\lambda$, the situation is essentially no different than that of a broad-band radiometer with a non-uniform spectral responsivity, which we take up next in problem 3. Accordingly, all of the discussion in problem 3 (following) is equally applicable here. As will be clear from that discussion, there are always problems when *both* the incident spectral distribution and the spectral responsivity vary significantly across the pass band. With a monochromator there is always large variation of the overall instrument spectral responsivity, about which we will learn more in a later chapter.

Problem 3. Determining irradiance with a broad-band radiometer.

A practical example of this problem is determining the irradiance (incident watts per square meter) between 400 and 500 [nm] produced at the treatment distance by a bank of blue lamps used in the phototherapy of infant jaundice. A schematic diagram of such an experimental setup is shown in figure 5.8. The measurement configuration is similar to that of problem 2 (figure 5.4) except that a filter radiometer that responds to incident radiation in a broader spectral pass band is used in place of the spectroradiometer. We

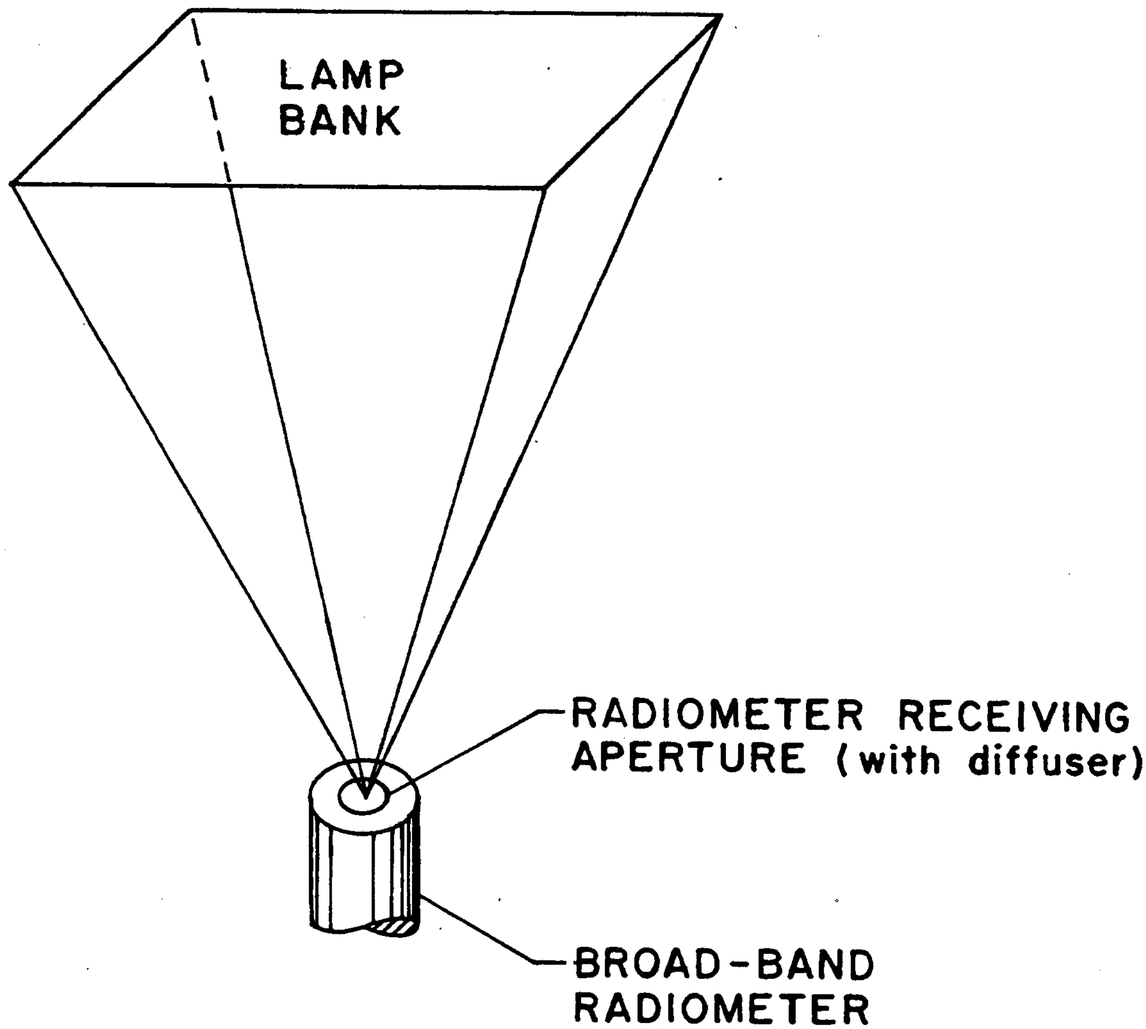


Figure 5.8. Sketch of parts of configuration for measurement of irradiance with broad-band (broad-spectral-band) radiometer.

still need spatially uniform responsivity, so some form of diffuser, such as an averaging sphere, is again used to achieve uniform flux responsivity to all rays of any given wavelength incident on the receiving aperture from a wide range of directions. The wider spectral pass band is now determined primarily by a spectral filter in conjunction with the spectral responsivity of the detector or transducer. The spectral characteristics of other portions of the internal optical path, including the diffuser, may also contribute to the varying overall spectral responsivity.

The measurement equation for this problem is obtained in the same manner as that for problem 2, and is identical to it [eq. (5.5)] except that the spectral pass band $\Delta\lambda$ is larger, now of the order of 100 [nm], and there is no wavelength "setting" λ_o:

$$S(A,\omega,\Delta\lambda) = \int_{\Delta\lambda} \int_{A} \int_{\omega} R_\Phi \cdot L_\lambda \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S], \tag{5.30}$$

where now

ω [sr] is the acceptance solid angle of the instrument at the point x,y within its receiving aperture,

A [m^2] is the area of the receiving aperture, and

$\Delta\lambda$ [nm] is the spectral pass band of the instrument, including all wavelengths λ for which the spectral flux responsivity is significantly non-zero -- $R_\Phi(\lambda) \neq 0$. [This is determined by the combined spectral characteristics of the detector and all internal path(s) and optical elements(s), including the diffuser and, especially any spectral filter(s).]

As in problem 2, it is essential that the flux responsivity R_Φ be constant everywhere within the acceptance solid angle ω, or at least as much of that solid angle as is filled by rays of sufficient radiance to make a significant contribution to the output signal of the instrument. If there are no other sources present, it is the portion of ω filled by rays from the bank of lamps being measured, including reflections and scattering from the fixture in which they are mounted. Then, as before, the responsivity can be taken outside of the inner integral, the integral with respect to solid angle, so that that integral gives us the spectral irradiance. Then the equation becomes [eq. (5.23]

$$S(A,\Delta\lambda) = \int_{\Delta\lambda} \int_{A} E_\lambda(x,y,\lambda) \cdot R_\Phi(\lambda) \cdot dA \cdot d\lambda \ [S]. \tag{5.31}$$

Again, as before, by a suitable choice of the size of the receiving aperture in relation to the working (treatment) distance plus adequate experimental verification, we may treat E_λ as well as R_Φ as constant for all points x,y over the aperture area A, so that

$$S(\Delta\lambda) = A \cdot \int_{\Delta\lambda} R_\Phi(\lambda) \cdot E_\lambda(\lambda) \cdot d\lambda \ [S]. \tag{5.32}$$

Although eq. (5.32) appears to be a relatively simple equation, with only one variable parameter, the wavelength λ, it is a remarkably troublesome one and it represents a situation that arises again and again in radiometry. So we'll take a particularly careful look at it. Note that the quantity that we want to measure in this case is the irradiance

$$E = \int_{400}^{500} E_\lambda(\lambda)\cdot d\lambda \ [W\cdot m^{-2}], \tag{5.33}$$

the integral from 400 to 500 [nm] of the spectral irradiance $E_\lambda(\lambda)$. Unfortunately, however, eq. (5.31) does not give us a measurement of this desired quantity E [of eq. (5.33)] because the integral in eq. (5.32) is a weighted integral of the spectral irradiance, with the spectral flux responsivity $R_\Phi(\lambda)$ as the weighting function, and $\Delta\lambda$ can seldom be made to correspond exactly to the desired wavelength interval, in this case 400-500 [nm].

If we were so fortunate as to have an instrument, with uniform spectral responsivity[1] $R(\lambda) = R$ over its entire pass band $\Delta\lambda = \lambda_2 - \lambda_1$ and zero responsivity $R(\lambda) = 0$ outside of that band [for $\lambda < \lambda_1$ and $\lambda > \lambda_2$], and with the limits exactly matching the desired interval ($\lambda_1 = 400$ [nm] and $\lambda_2 = 500$ [nm]), as depicted by the dashed lines in figure 5.9, there would, of course, be no problem. Then eq. (5.32) simplifies to

$$S(\Delta\lambda) = A\cdot R_\Phi\cdot\int_{400}^{500} E_\lambda(\lambda)\cdot d\lambda = A\cdot R_\Phi\cdot E \ [S], \tag{5.34}$$

and we need only make a similar calibration measurement, with a known standard source substituted for the unknown, to also obtain

$$S^s(\Delta\lambda) = A\cdot R_\Phi\cdot\int_{400}^{500} E^s_\lambda(\lambda)\cdot d\lambda = A\cdot R_\Phi\cdot E^s \ [S]. \tag{5.35}$$

We can then combine eqs. (5.34) and (5.35) to obtain

$$E = (S/S^s)\cdot E^s = (S/S^s)\cdot\int_{400}^{500} E^s_\lambda(\lambda)\cdot d\lambda \ [W\cdot m^{-2}], \tag{5.36}$$

which can be evaluated from the values of $E^s_\lambda(\lambda)$ furnished with the spectral-irradiance standard. But such an ideal instrument response is very difficult, if not impossible, to approximate adequately.

Now let's consider instruments with non-uniform spectral responsivity. In figure 5.9, in addition to the dashed lines showing the ideal "rectangular" responsivity just discussed, the solid curve shows a more realistic spectral-responsivity function for a broad-band

[1]Here R may be flux responsivity R_Φ, irradiance responsivity $R_E = A\cdot R_\Phi$, [see below, eq. (5.49)], etc. It is $R_\Phi(\lambda) = R_\Phi$ that we use in eq. (5.34).

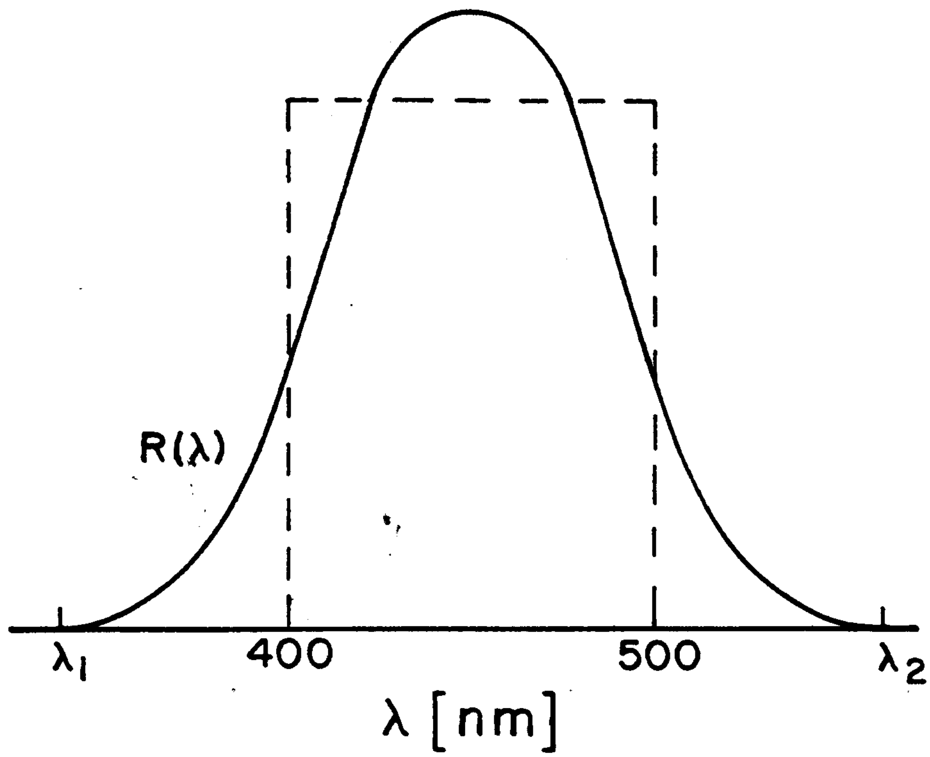


Figure 5.9. Spectral responsivity $R(\lambda)$ of broad-band radiometer. The dashed lines represent the ideal spectral responsivity $R(\lambda)$ desired for problem 3.

radiometer. The responsivity is not constant over all of the pass band and the pass band does not have sharp limits outside of which the responsivity vanishes completely. In such cases, we can still obtain the desired value of measured irradiance E, given by eq. (5.33), if we know *both* the *relative* responsivity $r(\lambda)$ [dimensionless] *and* the *relative* distribution of incident spectral irradiance $e_\lambda(\lambda)$ $[nm^{-1}]$, defined, respectively, by

$$R_\Phi(\lambda) \equiv K_1 \cdot r(\lambda) \; [S \cdot W^{-1}], \tag{5.37}$$

and

$$E_\lambda(\lambda) \equiv K_2 \cdot e_\lambda(\lambda) \; [W \cdot m^{-2} \cdot nm^{-1}], \tag{5.38}$$

where K_1 $[S \cdot W^{-1}]$ and K_2 $[W \cdot m^{-2}]$ are both constants with respect to wavelength.[1] Then, when we substitute from eq. (5.37) and (5.38) into eq. (5.32), we have

[1]We are not, at this point, addressing the question of how to determine either $r(\lambda)$ or $e_\lambda(\lambda)$ (which can be a very difficult problem). All we're concerned about now is the concepts involved when we *do* have at least this much information.

$$S(\Delta\lambda) = A \cdot K_1 \cdot K_2 \cdot \int_{\lambda_1}^{\lambda_2} r(\lambda) \cdot e_\lambda(\lambda) \cdot d\lambda \text{ [S]}, \tag{5.39}$$

where, now, the limits of $\Delta\lambda$ ($\equiv\lambda_2 - \lambda_1$) extend below 400 [nm] and above 500 [nm] to include all wavelengths where $R_\Phi(\lambda) \neq 0$. In figure 5.9 it is evident that, at both limits, the responsivity approaches zero gradually so that exact values for λ_1 and λ_2 are not clearly established. The values assigned to them will depend, in most cases on the noise level of the measuring equipment. In addition, as suggested earlier, this choice also depends on the incident radiation. The integrated contribution of very strong incident radiation at wavelenghts where $R_\Phi(\lambda)$ is supposed to be zero but is actually only very small can be significant while ordinarily, with normal levels of spectral irradiance at these wavelengths, there is no observable contribution. Accordingly, λ_1 must be chosen small enough and λ_2 large enough to include all significantly non-zero values of $R_\Phi(\lambda)$ in the given situation.

For a corresponding measurement of a standard of spectral irradiance, we similarly substitute only from eq. (5.37) into eq. (5.32), giving

$$S^s(\Delta\lambda) = A \cdot K_1 \cdot \int_{\lambda_1}^{\lambda_2} r(\lambda) \cdot E^s_\lambda(\lambda) \cdot d\lambda \text{ [S]}. \tag{5.40}$$

Next, we divide eq. (5.39) by eq. (5.40) to obtain

$$\frac{S}{S^s} = K_2 \cdot \frac{\int_{\Delta\lambda} r(\lambda) \cdot e_\lambda(\lambda) \cdot d\lambda}{\int_{\Delta\lambda} r(\lambda) \cdot E^s_\lambda(\lambda) \cdot d\lambda}, \tag{5.41}$$

from which we can evaluate K_2 as

$$K_2 = \frac{S}{S^s} \cdot \frac{\int_{\Delta\lambda} r(\lambda) \cdot E^s_\lambda(\lambda) \cdot d\lambda}{\int_{\Delta\lambda} r(\lambda) \cdot e_\lambda(\lambda) \cdot d\lambda} \text{ [W}\cdot\text{m}^{-2}\text{]}. \tag{5.42}$$

Finally, combining eq. (5.33), (5.38), and (5.42), the result is

$$E = K_2 \cdot \int_{400}^{500} e_\lambda(\lambda) \cdot d\lambda$$

$$= \frac{S}{S^s} \cdot \frac{\int_{\Delta\lambda} r(\lambda) \cdot E^s_\lambda(\lambda) \cdot d\lambda}{\int_{\Delta\lambda} r(\lambda) \cdot e_\lambda(\lambda) \cdot d\lambda} \cdot \int_{400}^{500} e_\lambda(\lambda) \cdot d\lambda \text{ [W}\cdot\text{m}^{-2}\text{]}, \tag{5.43}$$

where all of the quantities on the right-hand side of the equation are known.

In practice, accurate measurements of irradiance with a broad-band radiometer require the solution of eq. (5.43) because instruments with ideal "rectangular" spectral responsivities are just not available. If the relative spectral responsivity $r(\lambda)$ of the radiometer *and* the relative spectral distribution $e_\lambda(\lambda)$ of the irradiance being measured, which are both required for a solution of eq. (5.43) are not known, one must resort to narrow-band measurements using a suitable spectroradiometer. In this case the spectral irradiance is measured as a function of wavelength and the desired irradiance is obtained by integration over the wavelength interval of interest, as in eq. (5.33).

NORMALIZATION of BROAD-BAND MEASUREMENT RESULTS. A quantity that is occasionally useful and that can be obtained with a broad-band radiometer, when only the relative spectral flux responsivity $r(\lambda)$ of the radiometer is available, is a lower bound for the value of

$$E(\Delta\lambda) = \int_{\Delta\lambda} E_\lambda(\lambda)\cdot d\lambda \ [W\cdot m^{-2}], \tag{5.44}$$

the irradiance over the spectral band $\Delta\lambda = \lambda_2 - \lambda_1$ of the radiometer.[1] This can be done by setting the peak relative responsivity $r(\lambda_p) = 1$, where λ_p is the wavelength at which the spectral responsivity has its maximum or peak value. This is equivalent to setting the constant K_1 in eq. (5.37) equal to that maximum responsivity: $K_1 = R_\Phi(\lambda_p)$. At all other wavelengths $r(\lambda) \le 1$, so we can write

$$\int_{\Delta\lambda} r(\lambda)\cdot E_\lambda(\lambda)\cdot d\lambda \le \int_{\Delta\lambda} E_\lambda(\lambda)\cdot d\lambda. \tag{5.45}$$

This is often referred to as peak normalization and the quantity on the left-hand side of eq. (5.45) is called the peak-normalized value of incident irradiance E_{np}. Accordingly, eq. (5.45) can be combined with eq. (5.44) and rewritten as

$$E_{np}(\Delta\lambda) \le E(\Delta\lambda) \ [W\cdot m^{-2}], \tag{5.45a}$$

where

$$\begin{aligned} E_{np}(\Delta\lambda) &\equiv \int_{\Delta\lambda} r(\lambda)\cdot E_\lambda(\lambda)\cdot d\lambda, \quad \text{for} \quad r(\lambda) \le r(\lambda_p) = 1, \\ &= K_2\cdot\int_{\Delta\lambda} r(\lambda)\cdot e_\lambda(\lambda)\cdot d\lambda. \end{aligned} \tag{5.46}$$

With this last expression, eq. (5.46), it clearly follows from eq. (5.41) that

[1] $E(\Delta\lambda)$ given by eq. (5.44) is equal to E of eq. (5.33) only when $\lambda_1 = 400$ [nm] and $\lambda_2 = 500$ [nm]; i.e., when the instrument has exactly the desired pass band.

$$E_{np}(\Delta\lambda) = \frac{S}{S^s} \cdot \int_{\Delta\lambda} r(\lambda) \cdot E_\lambda^s(\lambda) \cdot d\lambda \ [W \cdot m^{-2}], \tag{5.47}$$

which can be evaluated because all factors on the right-hand side of the equation are known. Thus, our measurement gives a value for the peak-normalized irradiance $E_{np}(\Delta\lambda)$ which, by eq. (5.45a) is a lower bound for the irradiance $E(\Delta\lambda)$. In addition, to the extent that the spectral responsivity $R_\Phi(\lambda)$ approximates the rectangular dashed "curve" in figure 5.9, with limits at 400 and 500 [nm], $E_{np}(\Delta\lambda)$ is an approximation to the lower bound for the desired irradiance E of eq. (5.33).

There are other "normalization" practices where, for various reasons, different choices are made for the value of K_1, which, as we have seen, can be assigned arbitrarily. We won't go into all the details of this rather complex subject here. There is a very general treatment [21] for anyone who wishes to dig more deeply.

When actual values can't be obtained, normalized values usually are reported in order to try to obtain results of some value from such broad-band measurements of incident radiation of unknown spectral distribution, made with instruments of non-uniform spectral responsivity. Unfortunately, the normalization methods used are not always clearly defined or specified, which defeats their purpose. Also, repeated use of terms applied to some of them seems to build up confidence, based mostly on familiarity, that blinds many people to their limitations. There just isn't any way to get around the fact that, without the incident relative spectral distribution, quantities measured in lumens, roentgens, the "effective watts" used by many in the military-infrared community, etc., and other similar quantities, can't be converted directly to the actual quantities in absolute units. Normalized quantities (spectral-responsivity-weighted-integral quantities) can be directly related, in general, only to measurement results from instruments having the same relative spectral responsivity $r(\lambda)$. When they are combined with approximate information about the incident spectral distribution, they can provide correspondingly approximate values or estimates of the true irradiance (or other radiometric quantity). The strong feelings of confidence that are often attached to such normalized units and quantities probably stem from the development of a faculty for making such estimates, based on estimates or assumptions about the incident spectral distribution of which the individual may not even be consciously aware.

We note, also, in passing, that similar considerations arise in connection with non-uniform responsivity with respect to *any* of the radiation parameters, not just the spectral parameter. For example, many instruments do not have a uniform directional responsivity, particularly near the edges of the field of view which is often not very sharply defined. When that is the case, the output signal is again a responsivity-weighted integral of the incident radiance distribution and similar normalization considerations are applicable, since the mathematical relationships are exactly parallel to those in the spectral case just analyzed. This is all covered in the general treatment [21].

RESPONSIVITY CALIBRATIONS. There are many situations where the user of a radiometer or radiometric instrument may not make a calibration measurement or measurements each time he makes a measurement of an unknown radiometric quantity. This may be for a number of reasons that we won't attempt to analyze except to point out that, when substantial periods of time elapse between a measurement and the related calibration, care should be taken to make sure that the instrument stability is adequate to meet the needed measurement accuracy. In any case, when this is done, say, for a broad-band measurement of irradiance, as in problem 3, the calibration measurement [eq. (5.35)] is then treated as a measurement to establish the flux responsivity of the instrument as

$$R_\Phi = S^s/(A \cdot E^s) \ [S \cdot W^{-1}] \tag{5.48}$$

or, even more directly, the irradiance responsivity as

$$R_E = S^s/E^s = A \cdot R_\Phi \ [S \cdot W^{-1} \cdot m^2]. \tag{5.49}$$

This value of responsivity may then be used to evaluate the measured values of irradiance from unknown sources in subsequent measurements as

$$E = S/R_E = S/(A \cdot R_\Phi) \ [W \cdot m^{-2}] \tag{5.50}$$

as long as the instrument stability is adequate to meet the desired accuracies.

GENERAL DISCUSSION of the MEASUREMENT EQUATION and ITS USE. We have introduced the measurement equation by deriving it and obtaining its solution for three rather common radiometric problems. For each problem, the equation turned out to have the same form, namely

$$S = \int_{\Delta\lambda} \int_A \int_\omega R_\Phi \cdot L_\lambda \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [S]. \tag{5.30a}$$ [1]

Actually, this is what one would expect, because the equation is simply a summing up (integrating) of all the elements of radiometer output signal

$$dS = R_\Phi \cdot L_\lambda \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \tag{5.4a}$$

produced by all the elements of radiant flux

$$d\Phi = L_\lambda \cdot \cos\theta \cdot d\omega \cdot dA \cdot d\lambda \ [W] \tag{3.10a}$$

that are incident on the receiving aperture of the radiometer. These statements are quite general; accordingly, eq. (5.30a) is applicable to any optical radiation measurement problem involving incoherent radiation.

Unfortunately, there is no unique general solution to this measurement equation. Even

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[1]This is identical to eq. (5.30) except that functional dependence of S is not shown explicitly.

if the spectral responsivity $R_\Phi(x,y,\theta,\phi,\lambda)$ is completely specified as a function of position x,y (on A), direction θ,ϕ (in ω), and wavelength λ (in $\Delta\lambda$), there are an unlimited number of spectral radiance functions $L_\lambda(x,y,\theta,\phi,\lambda)$ that would produce the same observed output signal S. Moreover, L_λ in eq. (5.30a) can't be replaced by a desired radiometric quantity that is an integral of L_λ (e.g., E_λ, E, or F_t) unless the equation can be modified to contain that integral. This requires that R_Φ and possibly other quantities in the measurement equation be constant with respect to one or more radiation parameters. These difficulties are inherent in the multidimensionality of optical radiation. In principle, it is always *possible* to use a spectroradiometer with small enough $\Delta\lambda$, A, and ω so that L_λ is sufficiently constant over each of those integration intervals that it can be removed from all the integrals. Then measurement of an unknown, and of a known standard, allow one to obtain L_λ. If this is done for all possible combinations of the radiation parameters, any radiometric quantity desired can then be calculated. This, however, is usually an extremely difficult and tedious task and is generally *impractical*.

Accordingly, the practical solution is usually to try to select a measuring instrument and a measurement configuration to satisfy certain conditions, at least within a desired degree of approximation, that will make it possible to modify the measurement equation to include the desired radiometric quantity and to obtain a unique solution. The kinds of conditions we are talking about are illustrated in the three sample problems. They include the constancy or approximate constancy of a radiation quantity or of the responsivity relative to one or more radiation parameters so that these functions can be brought outside one or more integrals, the use of an average to replace one or more of the integrals, and the use of the relative spectral distribution, if known, of the otherwise unknown radiometric quantity being measured. The major advantages of using the measurement equation are that it makes clear that such conditions must be sought and provides insight and a systematic approach towards finding them. In fact, without such an approach, we believe it is highly unlikely that one can make state-of-the-art measurements, or even less accurate measurements, with a meaningful estimate of the uncertainty.

To assist the reader in making use of the measurement equation, we have prepared a list of steps to follow. This is not a comprehensive list, however, because there are two more radiation parameters not yet treated as well as other concepts still to be developed (see next section). In later chapters, the kinds of problems that can be addressed will be extended and alternatives in approach will be provided. In fact, the entire Self-Study Manual can be thought of as a treatment in which concepts and techniques are developed for use in the measurement equation and expertise in its use is developed through discussion and analysis of various applications. Nevertheless, the steps that are listed below are a useful beginning and we recommend that they be followed routinely in applying the measurement equation.

1. Prepare a diagram, approximately to scale, of the entire measurement configuration, showing all pertinent radiation sources, both the source(s) of the radiation beam to be measured and any source(s) of unwanted or background radiation, including objects from which radiation might be scattered or reflected into the measuring instrumentation. Also show the measuring instrumentation in sufficient detail to establish the features required in the following steps.

2. Place the calibration standard that is to be used on this same diagram.

3. Determine the approximate size and position of the receiving aperture A and the acceptance (field) solid angle ω of the instrumentation and show them on the diagram. Show a few typical rays within them as well as the extreme rays that indicate their limits. These values of A and ω, along with $\Delta\lambda$ from the next step and an estimate of R, can be used in the measurement equation to obtain approximations of the expected output signals.

4. Prepare a graph of the relative spectral responsivity of the measuring instrumentation as a function of wavelength, if this information is available,[1] showing the limits λ_1 and λ_2 of the spectral pass band $\Delta\lambda = \lambda_2 - \lambda_1$. On the same graph, plot also, if available, the approximate relative spectral radiance distribution in the incident beam that is to be measured.

Taken together, such a diagram and graph(s) provide a rough picture that is very helpful in visualizing the important interrelationships between the different parts of the measurement configuration when they are more carefully analyzed in later steps. It also provides a basis for preliminary estimates of the adequacy of the instrumentation to make the desired measurement, the appropriateness of the spectral responsivity relative to the wavelength range present in the beam to be measured, the size and configuration of the instrument throughput relative to the geometry of the beam to be measured, etc. If everything appears to be in order, indicating that the proposed measurement is feasible, we can continue with the modification and simplification of the equation.

5. Write down the mathematical relationship (from previous chapters, particularly Chapter 4) between the radiometric quantity to be measured and the spectral radiance incident on the receiving aperture of the radiometer. Determine how the measurement equation must be modified, and whether such a modification can be carried out with sufficient accuracy, so that this relationship can be used to replace the incident spectral radiance L_λ

[1]Chapter 7 addresses the problem of determining the relative spectral responsivity $r(\lambda_o,\lambda)$ and the pass band $\Delta\lambda$ of a spectroradiometer containing a monochromator.

in the measurement equation by the desired radiometric quantity. An example of this is given in problem 2 where it was necessary to remove the flux responsivity R_Φ from within the integral over solid angle. This integral then became the spectral irradiance E_λ, the desired radiometric quantity. The substitution could be made only if R_Φ had the same value throughout ω at every point of A. A constancy such as this would have to be investigated and confirmed experimentally in an auxiliary measurement to an accuracy close to that finally desired. If this required condition were not found to exist, the instrument would have to be replaced by one in which it did exist.

6. Following the introduction of the desired quantity, the resulting equation is simplified so that a unique solution is possible. As illustrated in the examples, this usually requires

   that the unknown radiometric quantity be constant over one or more of the remaining integration intervals so that it can be removed from the integrals,

   resorting to averages, or

   utilizing a relative spectral responsivity function and a relative spectral distribution (or distribution with respect to other radiation parameters) of the unknown radiometric quantity.

7. Repeat steps 4, 5, and 6 with respect to the calibration standard to be used.

8. Combine the simplified equations for the unknown and the standard so as to eliminate the responsivity and associated geometrical factors common to both equations. This results in a solution for the unknown radiometric quantity in terms of the two measured output signals (unknown and standard) and the known value of the calibrated standard. Alternatively, some prefer the mathematically equivalent steps of first solving the equation for the calibration measurement of the standard, to evaluate the responsivity for the desired radiometric quantity (combining the flux responsivity and associated geometrical factors), and then using that responsivity to evaluate the unknown quantities represented by the output signals from measurements of those quantities. This alternative approach is used primarily when less frequent calibration measurements are made due to less stringent accuracy requirements.

An experienced radiometrist will carry out as much as possible of this entire procedure first as a "thought experiment" while planning the measurement. In this way he may be able to determine whether or not the measurement is feasible with the available instrumentation, or to determine what instruments will be needed. It enables him to plan ahead for the calibration measurements and for any auxiliary experiments that may be required. Thus, the measurement equation provides a systematic, comprehensive approach for planning how to make a particular measurement as well as for carrying it out and finally estimating its uncertainty.

Finally, when a unique solution is not possible because not even the relative incident distribution is known, we have seen that results of limited usefulness and significance can be obtained in the form of normalized quantities. Normalization is discussed immediately following the treatment of problem 3.

LIMITATIONS of the MEASUREMENT EQUATION DEVELOPED in THIS CHAPTER. Most of the limitations have already been pointed out. However, because of their importance, they are summarized here. The measurement equation developed in this chapter is incomplete. It does *not* include

the radiation parameters of time and polarization,

environmental and instrumental parameters, such as ambient temperature, humidity, or magnetic fields,

corrections for diffraction effects [as a perturbation or departure from pure geometrical (ray) optics], or

cases where the responsivity is non-linear.

Moreover, the sample problems are stated as simply as possible to illustrate the important concepts involved, and likely sources of error in carrying out actual measurements are not pointed out in most cases. We intend to incorporate these additional parameters and corrections and to go into greater detail in our treatment of the measurement-equation approach to radiometry in subsequent chapters.

SUMMARY of CHAPTER 5. An essential factor in the measurement equation is the flux responsivity, given here as a function of the spatial and spectral radiation parameters only, since we have not yet taken up the temporal and polarization parameters nor the environmental and instrumental parameters:

$$R_\Phi(x,y,\theta,\phi,\lambda) \equiv dS(x,y,\theta,\phi,\lambda)/d\Phi(x,y,\theta,\phi,\lambda) \; [S\cdot W^{-1}]. \qquad (5.1a)$$

In terms of this flux responsivity, the output-signal element produced by an incident-flux element $d\Phi(x,y,\theta,\phi,\lambda)$ [W], associated with an incident ray of spectral radiance $L_\lambda(x,y,\theta,\phi,\lambda)$ $[W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}]$, is

$$dS(x,y,\theta,\phi,\lambda) = R_\Phi(x,y,\theta,\phi,\lambda)\cdot d\Phi(x,y,\theta,\phi,\lambda)$$

$$= R_\Phi(x,y,\theta,\phi,\lambda)\cdot L_\lambda(x,y,\theta,\phi,\lambda)\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda \text{ [S]}. \quad (5.2a)$$

When "reducing" measurement data, some people prefer, instead, to use

$$d\Phi = dS/R_\Phi = K_\Phi\cdot dS \text{ [W]}, \quad (5.3)$$

where $K_\Phi \equiv 1/R_\Phi$ $[W\cdot S^{-1}]$ is sometimes called the calibration "constant" although, like the responsivity, it is obviously also a function of all of the radiation parameters and may be quite variable. More generally, the responsivity of a detector element, or of an entire radiometric instrumentation system, is defined as the output signal per unit input of any incident radiometric quantity [17].

The measurement equation, relating the output signal to the incident distribution of spectral radiance at the receiving aperture of a radiometric instrument, is introduced by deriving a simple equation, involving only spatial and spectral radiation parameters, for three illustrative problems: (1) transferring the spectral-radiance calibration of a tungsten-ribbon lamp, (2) determining the spectral irradiance near a large source, and (3) determining irradiance with a broad-band (broad-spectral-band) radiometer. It is evident from these examples that the measurement equation, in terms of just the three radiation parameters of position, direction, and spectrum (wavelength), has basically the following form for all radiometric measurements:

$$S(A,\omega,\Delta\lambda) = \int_{\Delta\lambda}\int_{A}\int_{\omega} R_\Phi(x,y,\theta,\phi,\lambda)\cdot L_\lambda(x,y,\theta,\phi,\lambda)\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda \text{ [S]}. \quad (5.30b)^1$$

$R_\Phi(x,y,\theta,\phi,\lambda)$ $[S\cdot W^{-1}]$ is the instrument spectral-ray flux responsivity for the flux element

$$d\Phi(x,y,\theta,\phi,\lambda) \equiv L_\lambda(x,y,\theta,\phi,\lambda)\cdot\cos\theta\cdot d\omega\cdot dA\cdot d\lambda \text{ [W]} \quad (3.10)$$

associated with the ray, of spectral radiance $L_\lambda(x,y,\theta,\phi,\lambda)$ $[W\cdot m^{-2}\cdot sr^{-1}\cdot nm^{-1}]$, incident on the receiving-aperture-area element dA $[m^2]$ at the point x,y within the solid-angle element $d\omega$ [sr] from the direction θ,ϕ and within the spectral-wavelength element $d\lambda$ [nm] at the wavelength λ. The integration limits or intervals $\Delta\lambda$, A, and ω [more explicitly, $\omega(x,y)$] are selected, based on the measurement configuration, to include all of the incident radiant flux that might possibly contribute to the instrument response (output signal). Caution is urged in excluding or ignoring parameter values for which either R_Φ or L_λ, alone, appears to be zero, particularly if it approaches zero gradually or asymptotically as a function of that parameter. Even though L_λ is very small, the integrated contribution over a wide wavelength band in which R_Φ is substantial may be

[1]This is identical to eq. (5.30) except that the functional dependence of R_Φ and L_λ on the radiation parameters is explicitly shown here.

surprisingly large, and the same may be true if R_Φ is very small and L_λ is substantial over such a parameter level.

Note that, in addition, both S and R_Φ may be functions of a wavelength setting λ_o, as in eqs. (5.4) and (5.5), when the instrument includes a monochromator. However, we have given eq. (5.30), in the more explicit form of eq. (5.30b) above, as the general relationship from the standpoint that each new setting of the monochromator, like the insertion of a new spectral filter in a filter radiometer, can be considered as defining a new instrument with a new spectral responsivity. It is the spectral responsivity of such an instrument, with a fixed wavelength setting or fixed filter, that we discuss in general terms in this summary.

Unfortunately, there is no unique general solution to the measurement equation. Even with $R_\Phi(x,y,\theta,\phi,\lambda)$ completely specified with respect to all parameters, there are unlimited possibilities for $L_\lambda(x,y,\theta,\phi,\lambda)$ that will satisfy eq. (5.30b). Furthermore, we cannot replace L_λ in eq. (5.30b) by the radiometric quantity we want to measure when it is an integral of just L_λ without R_Φ unless eq. (5.30b) can be modified to contain that integral, which requires that R_Φ be a constant with respect to one or more radiation parameters. In *principle*, if the parameter intervals are made small enough so that L_λ varies negligibly within the intervals, L_λ can be evaluated. If this is done for enough different parameter values to specify the function L_λ over some range, the integrals over that range can be evaluated. However, this is usually extremely difficult and tedious and is generally *impractical*.

The practical approach to solving the measurement equation is to select and adjust the instrumentation and the measurement configuration to make possible (a) substitution of the desired radiometric quantity and (b) simplification to obtain a unique solution. A complete and exhaustive procedure or algorithm covering every conceivable case is not possible. In fact, development of this subject will be our main concern throughout most of the rest of this Manual. For now, we suggest a set of steps for solving the measurement equation, eq. (5.30b), as a good way to start dealing with almost every case. These steps cover a systematic review of the pertinent features of the measurement configuration. They establish approximate values of the terms in eq. (5.30b) for estimating the expected output signal to assess the feasibility of a proposed measurement and the adequacy of proposed instrumentation. They review the constancy of R_Φ required for transformation to the desired radiometric quantity. They deal with the further simplification, which usually requires

that the unknown radiometric quantity be constant over one or more of the remaining integration intervals so that it can be removed from the integrals,

resorting to averages, or

utilizing a relative spectral responsivity function and a relative spectral distribution (or distribution with respect to other radiation parameters) of the unknown radiometric quantity.

When adequate simplification has been achieved, the simplified equations for the unknown, and for a calibration measurement of a standard, can be combined so as to eliminate the responsivity and associated geometrical factors, leaving the solution for the unknown radiometric quantity in terms of just the two measured output signals (unknown and standard) and the known value of the standard. Alternatively, the equation for the standard can be solved to evaluate the responsivity; then this calibrated responsivity can be used to evaluate unknown quantities from the signals produced when they are measured in the same way. Finally, if a unique solution is not possible because not even the *relative* incident distribution is known, results of limited usefulness and significance can be obtained in the form of normalized quantities. Normalization is briefly discussed following problem 3.

Readers are reminded that the measurement equation developed in this chapter is still not complete. It does not include the radiation parameters of time and polarization, environmental and instrumental parameters, corrections for diffraction effects, or cases of non-linear responsivity.

Regular use of the measurement-equation approach to radiometry is useful in planning for and estimating the feasibility of a proposed measurement, including calibration measurements and needed auxiliary experiments. It provides a systematic, comprehensive approach for carrying out a measurement and for estimating its uncertainty.

Appendix 3. PROJECTED SOLID ANGLES, THROUGHPUTS, and CONFIGURATION FACTORS

by Fred E. Nicodemus

When the spatial distribution of (radiant) flux is given, as we have presented it, in terms of ray-radiance, the beam-geometry configurations and measures are naturally given in terms of beam-intersecting projected areas and solid angles (or intersecting areas and projected solid angles), and throughputs (see Appendix 2 [5]). However, most of the heat-transfer and illumination engineering literature deals with source exitance, irradiance on a receiver, and what we'll call "configuration factors." Even though the configuration factors are usually defined in terms of radiant energy exchange (with simplifying assumptions that aren't always emphasized sufficiently), they are actually purely geometrical quantities that can be directly related to projected solid angles and throughputs. We develop those relations in this appendix because it is frequently useful to transform a projected solid angle or throughput to the equivalent configuration factor so that it can be evaluated by means of the fairly extensive published tables of configuration factors, thereby avoiding otherwise long and difficult computations.

The quantities involved in the engineering treatment are called by a wide variety of names: angle factor, angle ratio, configuration factor, geometrical factor, geometrical configuration factor, interchange factor, shape factor, or view factor, sometimes with "coefficient" in place of "factor". We prefer the term "geometrical configuration factor", or just "configuration factor", used by Siegel and Howell [22], but our notation and defining equations follow more nearly those for the corresponding "angle factor" of Sparrow and Cess [23]. As we use the term "configuration factor", then, it includes or is closely related to the CIE-IEC [6] terms and concepts of "configuration factor" c, "form factor" f, and "(mutual) exchange coefficient" g. Although we do not use or define here these CIE-IEC quantities, their relationships to each quantity that we do define is given for the benefit of those who may want to use them or material involving them.

It is important to understand the configuration factors not only because they are so widely used in the engineering literature but also, as already mentioned, because of the existence of extensive tables of formulas [22,23] that can be used in evaluating them for a very wide variety of beam configurations. By knowing the exact relationships between those factors and the corresponding projected solid angles and throughputs, the latter can also be evaluated from the tables with very substantial savings in time and effort. Furthermore, the formulas in the tables can be extended to still other configurations, in many instances, by the use of the so-called "configuration-factor algebra" [22] or "angle-factor algebra" [23].

We denote the configuration factor (angle factor) for the beam consisting of all rays between points on a reference surface of area A_i and a distant surface of area A_j as F_{i-j} or, more explicitly, $F_{A_i-A_j}$. Either, or both, of these surface areas A may be just an element of area dA. The reference surface is ordinarily the locus of the vertices of the solid angles or projected solid angles that combine with its area to make up the throughput between it and the distant surface area. We use these designations, "reference" and "distant" in place of the usual ones of "source" and "receiver" surfaces to emphasize the fact that we are concerned only with purely geometrical relationships between the two surfaces. Also, the reference surface is not necessarily, or always, the source; sometimes it is the receiver and the distant surface is the source. Furthermore, as indicated earlier, the configuration factor F_{i-j} is usually defined in terms of the interchange of radiant flux. That definition is as follows: when the reference surface is perfectly uniform and diffuse (lambertian), so that all rays through every point of the reference surface have the same radiance, F_{i-j} is the fraction of the total flux into (or from) the full hemisphere above the reference surface area that intercepts the distant surface area. (If the reference surface is an extended curved (non-planar) surface, this refers to the hemisphere above each surface element, taken element by element, that makes up the reference surface area, as will be clear in the defining relations given below.)

The purely mathematical (geometrical) defining relations for the four possible configurations shown in figures A3-1, -2, -3, and -4 are listed, respectively, in eqs. (A3-1), (A3-2), (A3-3), and (A3-4). They are given, each time, on the first line as they usually appear in the engineering literature. On the second and succeeding lines, in each case, are given the relationships to the corresponding projected solid angle $\Omega \equiv \int \cos\theta \cdot d\omega \equiv \iint \cos\theta \cdot \sin\theta \cdot d\theta \cdot d\phi$ and to the corresponding throughput $\Theta \equiv \iint dA \cdot d\Omega$, as well as to the CIE-IEC [6] "configuration factor" c, "form factor" f, and "(mutual) exchange coefficient" g. D_s is always the slant distance between the elements dA_i and dA_j.

(1) The configuration factor for the beam between two infinitesimal surface elements dA_i (reference) and dA_j (distant) (see figure A3-1, p. 96) is

$$dF_{dA_i-dA_j} \equiv \cos\theta_{ij} \cdot \cos\theta_{ji} \cdot dA_j/(\pi \cdot D_s^2)$$

$$= (1/\pi) \cdot \cos\theta_{ij} \cdot d\omega_{ij} = (1/\pi) \cdot d\Omega_{ij} = dc_{ij}$$

$$= [1/(\pi \cdot dA_i)] \cdot d\Theta = (1/dA_i) \cdot dg = df_{ij} \tag{A3-1}$$

(2) The configuration factor for the beam between an infinitesimal surface element dA_i (reference) and an extended surface area A_j (distant) (see figure A3-2, p. 97) is

$$dF_{dA_i-A_j} \equiv \int_{A_j} [\cos\theta_{ij} \cdot \cos\theta_{ji}/(\pi \cdot D_s^2)] \cdot dA_j$$

$$= (1/\pi) \cdot \int_{\omega_{ij}} \cos\theta_{ij}\, d\omega_{ij} = (1/\pi) \cdot \int_{\omega_{ij}} d\Omega_{ij} = (1/\pi) \cdot \Omega_{ij} = c_{ij}$$

$$= [1/(\pi \cdot dA_i)] \cdot d\Theta = (1/dA_i) \cdot dg = f_{ij} \tag{A3-2}$$

(3) The configuration factor for the beam between an extended surface area A_i (reference) and an infinitesimal surface element dA_j (distant) (see figure A3-3, p. 98) is

$$dF_{A_i-dA_j} \equiv (1/A_i)\cdot\int_{A_i} [\cos\theta_{ij}\cdot\cos\theta_{ji}\cdot dA_j/(\pi\cdot D_s^2)]\cdot dA_i$$

$$= [1/(\pi\cdot A_i)]\cdot\int_{A_i} (\cos\theta_{ij}\cdot d\omega_{ij})\cdot dA_i = [1/(\pi\cdot A_i)]\cdot\int_{A_i} d\Omega_{ij}\cdot dA_i$$

$$= (1/\pi)\cdot\overline{d\Omega_{ij}} = \overline{dc_{ij}} \quad \text{[both averaged over the area } A_i\text{]}$$

$$= [1/(\pi\cdot A_i)]\cdot d\Theta = (1/A_i)\cdot dg = df_{ij} \qquad \text{(A3-3)}$$

(4) The configuration factor for the beam between two extended surface areas A_i (reference) and A_j (distant) (see figure A3-4, p. 98) is

$$F_{A_i-A_j} \equiv (1/A_i)\cdot\int_{A_i}\int_{A_j} [\cos\theta_{ij}\cdot\cos\theta_{ji}/(\pi\cdot D_s^2)]\cdot dA_j\cdot dA_i$$

$$= [1/(\pi\cdot A_i)]\cdot\int_{A_i}\int_{\omega_{ij}} \cos\theta_{ij}\cdot d\omega_{ij}\cdot dA_i = [1/(\pi\cdot A_i)]\cdot\int_{A_i} \Omega_{ij}\cdot dA_i$$

$$= (1/\pi)\cdot\overline{\Omega_{ij}} = \overline{c_{ij}} \quad \text{[both averaged over the area } A_i\text{]}$$

$$= [1/(\pi\cdot A_i)]\cdot\Theta = (1/A_i)\cdot g = f_{ij} \qquad \text{(A3-4)}$$

It is also easily shown that the following reciprocity relations hold with respect to all of the configuration factors:

$$dA_i\cdot dF_{dA_i-dA_j} = dA_j\cdot dF_{dA_j-dA_i} \qquad \text{(A3-1a)}$$

$$dA_i\cdot dF_{dA_i-A_j} = A_j\cdot dF_{A_j-dA_i} \qquad \text{(A3-2a)}$$

$$A_i\cdot dF_{A_i-dA_j} = dA_j\cdot dF_{dA_j-A_i} \qquad \text{(A3-3a)}$$

$$A_i\cdot F_{A_i-A_j} = A_j\cdot F_{A_j-A_i} \qquad \text{(A3-4a)}$$

Note that eqs. (A3-2a) and (A3-3a) are redundant. Both are included here only to maintain the one-to-one relationship between this set of equations and the preceding set of defining equations [eqs. (A3-1), (A3-2), (A3-3), and (A3-4)].

In the engineering literature, as previously mentioned, there are extensive tables [22,23] of configuration factors, or equivalent quantities by other names (most of which are listed in the second paragraph of this appendix). These can also be readily combined to obtain the configuration factors for still other combinations of surfaces by application of the so-called "configuration-factor algebra" [22] or "angle-factor algebra" [23] that has been developed. Accordingly, by expressing projected solid angles and throughputs in terms of configuration factors, in other words, by reversing the relations of eq. (A3-1), (A3-2), (A3-3), and (A3-4), the tables of configuration factors can be used to evaluate many projected solid angles and throughputs without carrying out the integration for a projected solid angle (eq. (2.31) [5]

$$\Omega \equiv \int_{\omega} \cos\theta \cdot d\omega = \iint \cos\theta \cdot \sin\theta \cdot d\theta \cdot d\phi \ [\mathrm{sr}] \tag{A3-5}$$

or a throughput (eq. (2.27) [5]

$$\Theta \equiv \iint d\Omega \cdot dA = \iiiint \cos\theta \cdot \sin\theta \cdot d\theta \cdot d\phi \cdot dx \cdot dy \ [\mathrm{m}^2 \cdot \mathrm{sr}] \tag{A3-6}$$

which, while they are conceptually straightforward, often entail complex and difficult computations. The required <u>inverse relations, needed for using the configuration-factor tables to evaluate a projected solid angle or throughput</u>, are given in eqs. (A3-7) to (A3-12), inclusive:

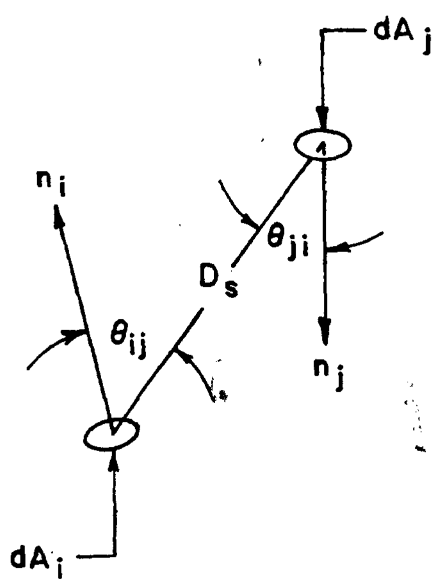

Figure A3-1. Configuration for interchange between two infinitesimal surface elements.

(a) The element of projected solid angle subtended at a surface element dA_i by a surface element dA_j (see figure A3-1) is

$$d\Omega_{ij} = \pi \cdot dF_{dA_i - dA_j} = \pi \cdot (dA_j / dA_i) \cdot dF_{dA_j - dA_i} \, [\mathrm{sr}]. \tag{A3-7}$$

The corresponding element of throughput between dA_i and dA_j is

$$d\Theta = dA_i \cdot d\Omega_{ij} = \pi \cdot dA_i \cdot dF_{dA_i - dA_j} = \pi \cdot dA_j \cdot dF_{dA_j - dA_i} \, [\mathrm{m}^2 \cdot \mathrm{sr}]. \tag{A3-8}$$

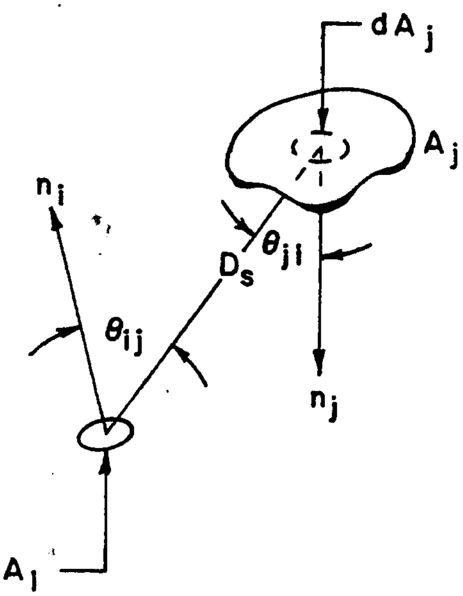

Figure A3-2. Configuration for interchange between an infinitesimal surface element and an extended surface.

(b) The projected solid angle subtended at a surface element dA_i by an extended surface area A_j (see figure A3-2) is

$$\Omega_{ij} = \pi \cdot F_{dA_i - A_j} = \pi \cdot (A_j / dA_i) \cdot dF_{A_j - dA_i} \, [\mathrm{sr}]. \tag{A3-9}$$

The corresponding element of throughput between dA_i and A_j is

$$d\Theta = dA_i \cdot \Omega_{ij} = \pi \cdot dA_i \cdot F_{dA_i - A_j} = \pi \cdot A_j \cdot dF_{A_j - dA_i} \, [\mathrm{m}^2 \cdot \mathrm{sr}]. \tag{A3-10}$$

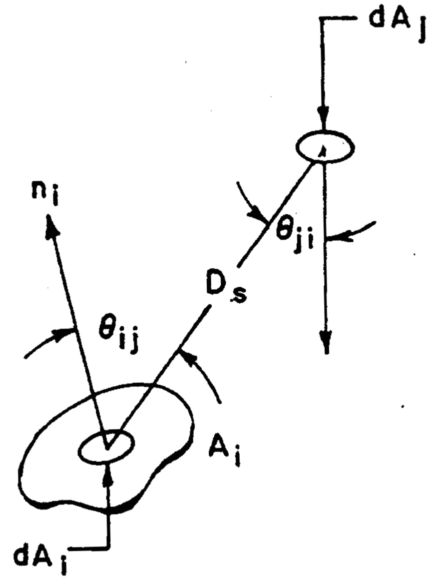

Figure A3-3. Configuration for interchange between an extended surface and an infinitesimal surface element.

(c) The element of throughput between an extended surface area A_i and a surface element dA_i (see figure A3-3) is

$$d\Theta = \pi \cdot A_i \cdot dF_{A_i - dA_j} = \pi \cdot dA_j \cdot F_{dA_j - A_i} [m^2 \cdot sr]. \quad \text{(A3-11)}$$

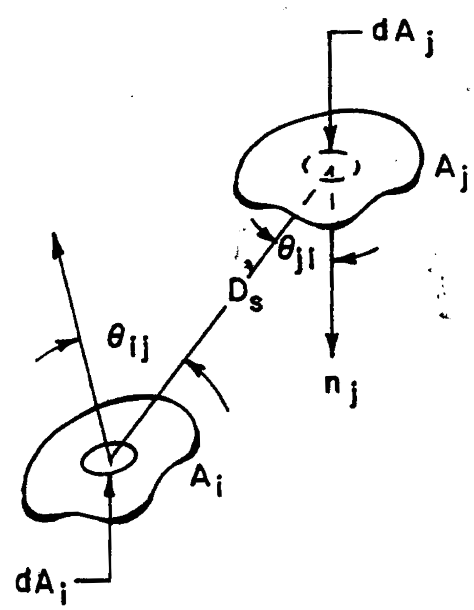

Figure A3-4. Configuration for interchange between two extended surfaces.

(d) The throughput between two extended surface areas A_i and A_j (see figure A3-4) is

$$\Theta = \pi \cdot A_i \cdot F_{A_i - A_j} = \pi \cdot A_j \cdot F_{A_j - A_i} \ [m^2 \cdot sr]. \tag{A3-12}$$

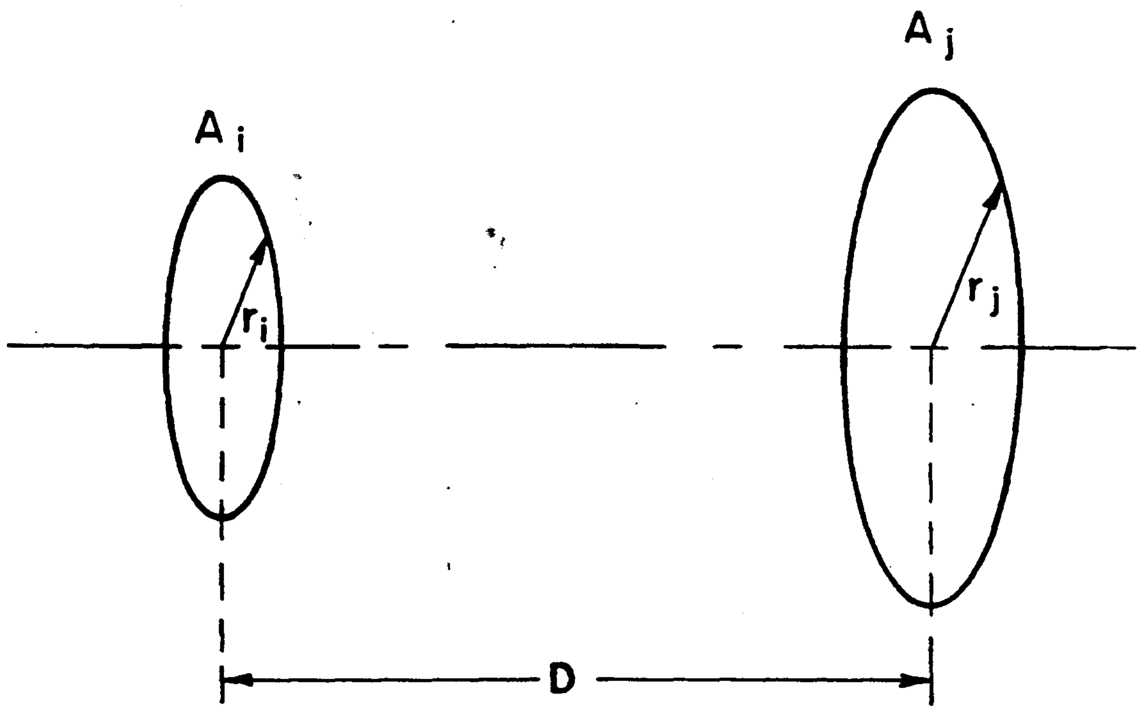

Figure A3-5. Two parallel, coaxial circular discs.

$A_i = \pi \cdot r_i^2; \quad A_j = \pi \cdot r_j^2;$

D is the perpendicular distance between centers along the common axis.

We will illustrate the use of configuration-factor tables by obtaining the formula for the throughput of a very common beam configuration, the beam between two parallel coaxial circles. Most optical systems have circular elements. In particular, they employ circular stops, so the images of those stops, the pupils and windows,[1] are also circular. Accordingly, it is quite common for a beam to be defined by two parallel, coaxial, circular areas A_i and A_j separated by a distance D along the axis between their centers, as shown in figure A3-5. For example, in figure 4.10, $A_i = \Delta A_s$ and $A_j = \Delta A_r$ or, again, in figure 4.13, we might designate the beam-defining apertures as $A_i = A_E$ and $A_j = A_r$ or else as $A_i = A_r$ and $A_j = A_F$. In any case, returning to figure A3-5, we designate the radius of

[1]See any standard textbook on geometrical optics [13,14].

A_i as r_i and that of A_j as r_j so that $A_i = \pi \cdot r_i^2$ and $A_j = \pi \cdot r_j^2$. We will use the first part of eq. (A3-12)

$$\Theta = \pi \cdot A_i \cdot F_{A_i - A_j} \ [m^2 \cdot sr], \tag{A3-12a}$$

so what we need from the tables is the formula or equation for evaluating the configuration factor $F_{A_i-A_j}$ for the configuration of figure A3-5.

In the "Angle Factor Catalogue" in Appendix A of [23], we find the desired geometrical configuration illustrated as "configuration 3" with the following entry (adjusting notation to correspond with figure A3-5):

$$X = r_j/D, \quad Y = D/r_i, \quad Z = 1 + (1 + X^2)Y^2,$$

$$F_{A_i-A_j} = (1/2)[Z - (Z^2 - 4X^2Y^2)^{1/2}]. \tag{A3-13}$$

Alternatively, in the "Catalog of Selected Configuration Factors" in Appendix C of [22], we also find the desired geometrical configuration illustrated in item 18, with the following entry (again we make straightforward substitutions of our notation):

$$R_i = r_i/D, \quad R_j = r_j/D, \quad X = 1 + (1 + R_j^2)/R_i^2,$$

$$F_{A_i-A_j} = (1/2)\{X - [X^2 - 4(R_j/R_i)^2]^{1/2}\}. \tag{A3-14}$$

Starting with either formula, eq. (A3-13) or (A3-14), for $F_{A_i-A_j}$, we can obtain, after some algebraic manipulation, the following expression for the desired throughput:

$$\Theta = (\pi^2/2) \cdot \{(r_i^2 + r_j^2 + D^2) - [(r_i^2 + r_j^2 + D^2)^2 - 4r_i^2 \cdot r_j^2]^{1/2}\} \ [m^2 \cdot sr]. \tag{A3-15}$$

Appendix 4. Some Nomenclature Considerations -- Signal S, Responsivity R [or $\mathcal{R}$], Reflectance Factor R. -- by Fred E. Nicodemus

Dr. G. Bauer, Chairman of the Vocabulary Subcommittee of the CIE Technical Committee TC-1.2, Photometry and Radiometry, has pointed out some instances where our nomenclature departs from, and may conflict with, the CIE-IEC International Lighting Vocabulary [6] and some revisions to that document that are being proposed by his Committee. First, although our use of the term "signal" and the symbol S (or s for the relative quantity) to designate, in general, the output of a radiometer or radiometric instrument is based on the IEC [20], the symbol S, at least, is also used by the CIE-IEC [6] to denote the "relative spectral energy [power] distribution (of radiation)". To date, we have not had occasion to make much use of that general designation, since we are usually concerned with the spectral distribution of a specific radiometric quantity, e.g., spectral radiance $L_{\lambda}(\lambda)$ or spectral radiant flux $\Phi_{\lambda}(\lambda)$. Furthermore, when we do want to refer to such quantities, in general, without designating any one of them specifically, we prefer to let the symbol X designate any such radiometric quantity, Φ, I, E, L, etc. as in [21]. Then the spectral distribution is given as $X_{\lambda}(\lambda)$, where the subscript λ is a clear reminder that this quantity is a derivative with respect to the spectral variable, something that is not brought out by the CIE-IEC symbol $S(\lambda)$ for a spectral distribution.

In choosing to use the IEC term "signal" and symbol S for the output of a radiometer, we are strongly motivated by the fact that this is the "language" of communications engineering. We feel that it is well to recognize that such an output is indeed a "signal" that is used to convey information about the input of incident radiation. In fact, that is the most significant thing about it, regardless of whether it is a voltage, a current, a photographic film density, or a displacement of a meter needle or of a line on a strip-chart record. Furthermore, the use of this nomenclature and point of view may facilitate access to powerful analysis tools in the literature of communications engineering that may usefully be brought to bear on some radiometric measurement problems.

Dr. Bauer also notes that they wish to recommend the use of the symbol s for responsivity (sensitivity) and that our use of R and r for responsivity may conflict with the use of R for "reflectance factor" by the CIE-IEC [6]. We strongly support the position of R. Clark Jones [24], that the use of the term sensitivity is to be deprecated because of its ambiguity and that the more explicit and unambiguous terms "responsivity" [output per unit input] and "detectivity" [reciprocal of noise-equivalent input] should always be used in its place. Accordingly, we prefer to avoid even the symbol s for responsivity, because of its association with the undesirable term "sensitivity." Furthermore, we do not anticipate serious difficulty with the possible confusion between R for responsivity (and r for relative responsivity) and for reflectance factor. The context will usually make it quite clear which use is intended. Also, while both may have angular or directional dependence, the directional dependence of the reflectance factor always involves two sets of directional designations, those for both the incident and reflected ray or beam geometry, e.g., $R(\theta_i,\phi_i;\theta_r,\phi_r)$ or, most generally, $R(\omega_i;\omega_r)$ [26]. The directional dependence of responsivity, on the other hand, involves only the direction of the incident rays $R(\theta,\phi)$. Neither, of course, is a derivative; both are weighting functions or weighting factors with respect to the radiometric quantities and related signals. In those instances where it may be necessary to use symbols for both responsivity and reflectance factor in the same discussion and the same equations, we would probably shift to a script $\mathcal{R}$ for responsivity, in order to keep them separate, but we do not believe that this will occur often enough to make it worthwhile to use the, otherwise less convenient, script $\mathcal{R}$ for responsivity all of the time.

We are grateful to Dr. Bauer for his comments and we understand that there may be quite different and, for them, valid reasons for the positions taken by him and his committee. However, for the reasons given above, we believe that we need to take the foregoing positions for this tutorial presentation. We urge that these reasons also be given consideration by those currently engaged in revising the CIE-IEC International Lighting Vocabulary [6]. It should be emphasized that these are the views of the author, only, and not necessarily those of the National Bureau of Standards or of any other group with which he may be associated.

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NBS TECHNICAL NOTE 910-1 -- ERRATA

Reverse of Title Page, last line of footnote:

price should be given as \$2.10.

p. 8, next to last line:

"directed" should read "detected".

p. 11, Figure 2.1:

ribbon filament should be extended downward so that point 1 is at the middle of the filament.

p. 21, last line of footnote:

first word should be "wish" (not "with").

p. 25, 2nd line of eq. (2.15):

insert θ_1 so that it reads: "$= L_1 \cdot \cos\theta_1 \cdot dA_1 \cdot \cos\theta_2 \cdot dA_2/D^2$ [W],".

p. 32, eq. (2.24):

insert multiplication dot between $d\omega$ and dA.

p. 32, 2nd line after eq. (2.24):

insert "e" in "includes".

p. 52, 1st line of eq. (3.10):

change "$\sin\theta$" to "$\cos\theta$".

p. 52, 1st line after eq. (3.10):

change "eqs. (2.23) and (2.28)" to "eqs. (2.24) and (2.29)."

p. 53, 1st and 3rd lines:

change "eq. (2.24)" to "eq. (2.25)".

p. 53, line 9:

change "eqs. (2.25) through (2.33)" to "eqs. (2.24) through (2.29)".

p. 69, next to last line of paragraph following eq. (A2-8):

change "f_t" to read "F_t".