Self-Study Manual on Optical Radiation Measurements: Part 1 — Concepts, Chapter 11

Fred E. Nicodemus, Editor

Chapter 11. Linearity Considerations and Calibrations,
by John B. Shumaker

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PREFACE

Preparation of the NBS Self-Study Manual on Optical Radiation Measurements is an important part of our effort to meet the needs of the National Measurement System with respect to the measurement of optical electromagnetic radiation. Significant needs exist for measurements with uncertainties of about one percent, but the uncertainties actually achieved are often 5 to 10 percent, or even more. These relatively poor accuracies, as compared to those in many other types of physical measurements, result to a great extent from the multidimensional character of optical radiation; radiant power is distributed and may vary with position, direction, wavelength, polarization, and time. Measurement results are also affected by various instrumental and environmental parameters. In addition, many of those who make measurements of optical radiation have little or no training or experience in this field and are limited in the amount of time they can devote to acquiring the needed information, understanding, and proficiency with measurement techniques. Moreover, there are few schools that offer courses in radiometry, and there are almost no adequate texts or references dealing with this entire subject.

The idea of producing a Self-Study Manual at NBS to try to fill some of this void was developed by one of us (HJK) in the latter part of 1973. Detailed planning got under way in the summer of 1974 when a full-time Editor (FEN) was appointed. The first nine chapters were published in a series of four NBS Technical Notes (910- series) up to June 1979. However, subsequent progress was slowed by administrative uncertainties and problems that affected the availability of authors, causing several changes in scheduling, and that finally ended all direct NBS funding for the Project on September 30, 1979. Welcome transfers of funds from a number of other federal agencies were obtained to carry the Project to September 30, 1980, with the hope that it could then be restored to the NBS budget. When that failed to materialize, outside funding was again obtained to September 30, 1981 (through FY 81).

Meanwhile the Program Director (HJK) retired at the end of May 1981, and the Editor (FEN) on September 29, 1981. However, remaining FY 81 funds were used for a grant to Catholic University of America (CUA), with whom NBS negotiated a cooperative agreement under which CUA hired the full-time Editor (FEN) who continues, as before, at NBS (same address and telephone), working closely with the new Project Head (JBS) there. CUA is also engaging the services of three other former NBS employees: Dr. Kostkowski as part-time consultant throughout the Project and as author or co-author of one or two more chapters, Mr. Richmond as author or co-author of two more chapters, and Dr. Venable as author of one more chapter, as outlined in more detail below.

Our aim for the entire Manual has been to provide a comprehensive tutorial treatment that is complete enough for self instruction. That is what is meant in the title by "self-study"; the Manual does not contain explicitly programmed learning steps as that term often connotes. In addition, through detailed, yet concise, chapter summaries, the Manual is designed to serve also as a convenient reference source. Those already familiar with a topic should turn immediately to the summary at the end of the appropriate chapter. They can determine from that summary what, if any, of the body of the chapter they want to read for more details.

The material in the Manual is presented at the level of a college graduate in science or engineering, but even for those with facility in college mathematics and a first course in physics, it's not at all easy reading in spite of our best efforts at clarity and simplicity. This is an unavoidable result of the primary aim ("to make one-percent measurements commonplace") coupled with the fact that it must serve the needs of so many different fields, including astronomy and astrophysics, mechanical heat-transfer engineering, illumination engineering, photometry, meteorology, photo-biology and photo-chemistry, optical pyrometry, remote sensing, military infrared applications, etc.

Apparently it is very difficult for those who have not been directly involved to realize the full implications of the situation just discussed. Each of us tends to think of radiometry and radiometric measurements in terms of our own immediate experiences and requirements. What each of us would like to have is a set of simple, carefully designed procedures for making our own particular measurements, with appropriate cautions concerning likely sources of error. However, the next reader wants the same thing, but for *entirely different* measurements. The desired radiometric quantities to be measured are different, the instrumentation is different, the ambient conditions are different -- the possible ways in which significant differences may exist, in terms of the radiation parameters (position, direction, spectrum, time or frequency of modulation or fluctuation, and polarization) as well as instrumental and environmental parameters, are so numerous that any attempt to cover them all with a "cookbook" treatment of specific measurement procedures would be impossibly unwieldy and could never be completed within any conceivable budget limitations short of utopia. The only way in which we can hope to effectively assist *every* reader who needs to make one-percent measurements is to provide you, the reader, with material which, *with sufficient effort on your part*, will help you to develop sufficient understanding and grasp of basic principles to solve your own particular measurement problems. That's why we have concentrated on the basic material of Part I--Concepts and why it is not easy reading.

As an exception, we have published, in NBS TN 910-5, just one chapter of Part III--Applications (see list of chapters, past, present and future, below). It gives a detailed account of some very difficult high-accuracy field measurements of ultraviolet solar terrestrial spectral irradiance, with frequent references to applicable portions of the earlier chapters of Part I--Concepts to illustrate how they can be used to achieve improved measurement results. It also illustrates clearly the complexities of such high-accuracy measurements that make it impossible for anyone other than you yourself to determine the details of how best to make your particular measurements with your particular instruments and constraints and for your particular objectives.

In NBS TN 910-6, we returned to Part I-Concepts with a chapter on coherence, that, like Chapter 6 on polarization, attempts to present the subject as it pertains to radiometric measurements. There are many interesting and highly important features of coherence phenomena, as well as of polarization phenomena. However, we are not concerned with them here, as such, but only to the extent that they affect the measurement of the flow of power or flux in a beam of electromagnetic optical radiation. This Technical Note 910-6, with Chapter 10 of Part I--Concepts, is the first departure from classical (geometrical- or ray-optics) radiometry in this Manual. From the beginning, due primarily to the rapid growth in the use of lasers, there has been a clear and increasingly urgent need for a treatment of the radiometry of coherent radiation, where geometrical (ray) optics is often an inadequate approximation. However, when Dr. Shumaker started, almost three years earlier, to look into the possibility of writing a chapter on coherence in radiometry, it was still not clear whether the basic theory on which this chapter could be based had been developed sufficiently to make it feasible. Fortunately, the recent rapid progress in this area did take place and we were delighted to be able, at last, to respond to a number of repeated requests for the material in this chapter.

Now, in this Technical Note 910-7 with Chapter 11 of Part I--Concepts (on Linearity Considerations and Calibrations), we look more closely at another way in which the output from a real radiometric instrument may depart from the assumptions we have been making. So far in this Manual, we have assumed linearity (responsivity that does not vary with the level or amount of the incident radiometric quantity) but the output from real instruments will depart from linearity to varying degrees and in some cases they are deliberately made non-linear, e.g., with logarithimic response to cover a wider dynamic range without range switching. A thorough, comprehensive analysis is presented that shows how the results of a variety of linearity calibration measurements can be used to establish or determine an instrument response function $f(S')$ that transforms the actual (non-linear) output S' to a linearized output Y which, except for a possible scale factor, is equivalent to the linear output S that appears in all of our earlier equations, particularly in the measurement equation, first introduced in Chapter 5.

The basic approach and focal point of the treatment in this Manual is this measurement equation, first introduced in detail but limited to the radiation parameters of position, direction, and spectrum, in Chapter 5. We believe that every measurement problem should be addressed with such an equation, relating the quantity desired to the data obtained, through a detailed characterization of the instruments used and the radiation field observed, in terms of all of the relevant parameters. These parameters always include the radiation parameters (listed above), as well as environmental and instrumental parameters peculiar to each measurement configuration. The objective of the Manual is to develop the basic concepts required so that the reader will be able to use this measurement-equation approach. It is our belief that this is the only way that uncertainties in the measurement of optical radiation can generally be limited to one, or at most a few, percent.

The original, overall plan for the Manual organized it into three Parts: Part I--Concepts, Part II--Instrumentation, and Part III--Applications. That was our rather ambitious plan when we started out; limitations of support and available resources, particularly available authors, have determined how much, or how little, we could accomplish. So far, as indicated above, we have concentrated on Part I--Concepts. The Part I chapters already published are:

1. Introduction, by F. E. Nicodemus, H. J. Kostkowski, and A. T. Hattenburg

2. Distribution of Optical Radiation with Respect to Position and Direction--Radiance, by F. E. Nicodemus and H. J. Kostkowski

3. Spectral Distribution of Optical Radiation, by F. E. Nicodemus and H. J. Kostkowski

4. More on the Distribution of Optical Radiation with Respect to Position and Direction, by Fred E. Nicodemus

5. An Introduction to the Measurement Equation, by Henry J. Kostkowski and Fred E. Nicodemus

6. Distribution of Optical Radiation with Respect to Polarization, by John B. Shumaker

7. The Relative Spectral Responsivity and Slit-Scattering Function of a Spectroradiometer, by Henry J. Kostkowski

8. Deconvolution, by John B. Shumaker

9. Physically Defining Measurement-Beam Geometry by Using Opaque Barriers, by Fred E. Nicodemus.

Chapters 1, 2, and 3 were in TN 910-1; 4 and 5 in TN 910-2; 6 in TN 910-3; and 7, 8, and 9 in TN 910-4. Now published, as mentioned above, is TN 910-5 with Part III--Applications:

Chapter 1. Measurement of Solar Terrestrial Spectral Irradiance in the Ozone Cut-Off Region, by Henry J. Kostkowski, Robert D. Saunders, John F. Ward, Charles H. Popenoe, and A.E.S. Green.

In TN 910-6, we returned to Part I--Concepts:

10. Introduction to Coherence in Radiometry, by John B. Shumaker,

followed by TN 910-7 (the present volume):

11. Linearity Considerations and Calibrations, by John B. Shumaker.

We are now fully funded for FY 84 (to 9/30/84) to continue the current arrangements with Catholic University of America, as described above, in order to complete Part I--Concepts, for which the following chapters still remain to be completed:

Distribution of Optical Radiation with Respect to Time, by Fred E. Nicodemus

Spectrophotometry, by William H. Venable, Jr.

Blackbody Radiation and Temperature Scales, by Joseph C. Richmond

Physical Photometry by A. T. Hattenburg and/or Fred E. Nicodemus

Thermal Radiation Properties of Matter, by Joseph C. Richmond.

Finally, as stated above, the measurement-equation approach is central to the entire Manual. Accordingly, the material presented in each chapter needs to be related to the approach introduced in Chapter 5. Depending on how well this is accomplished in each chapter, there may be need at the end to have a final summary chapter for this purpose, tying up loose ends and putting the whole Part I into perspective. Also, examples of various categories of environmental and instrumental parameters and their significance could be usefully presented and discussed. But it's too soon to evaluate this need at present.

Incidentally, in preparing material for the Manual, we have had the pleasure of rediscovering the fact that the best way to learn anything is to try to teach it to someone else. The exercise of preparing tutorial material for such wide general application has required us to analyze our own measurement activities in a different way, broadening our understanding and resulting in improved methods and more accurate results. Note that all references to measurement accuracy or uncertainty in this preface are concerned not only with precision (relating to the repeatability of measurement results) but also with accuracy (relating to agreement with the "truth" which, while unknowable in the last analysis, is approximated by analyses and estimates based on the widest possible experience, including agreement with measurements of the same quantities by others, particularly when they have used different instrumentation and methods of measurement).

ACKNOWLEDGMENTS. As indicated above, we have been entirely supported by funds from other federal agencies since September 30, 1979. Furthermore, it has not been feasible to segregate the activities on this project in any way that makes it possible to consider that only a certain portion is being carried out under the support of any single agency. All of the following have contributed, directly or indirectly, to all phases of the preparation of the Manual, including this Technical Note 910-7, at some time since 9/30/79, and we want to acknowledge their most welcome support:

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There have been many others who, in these days of tight budgets, were not able to contribute financial support to this project when they wanted to but who helped very substantially in other ways. They recommended potential sources of financial support and urged those who were able to do so to participate. Just their words of encouragement and of constructive criticism were tremendously helpful. To all of these we are most sincerely grateful.

We are also deeply indebted to a great many individuals for invaluable "feedback", concerning the technical content of the Manual, that has encouraged us and has helped us to put these chapters together more effectively. We renew our strong invitation to all readers to submit comments, criticisms, and suggestions. In particular, we would welcome illustrative examples and problems from as widely different areas of application as possible.

We are especially grateful to Mrs. Betty Castle for the skillful and conscientious effort that produced the excellent typing and layout of still another difficult text.

Fred E. Nicodemus, Editor

John B. Shumaker, Project Head

Henry J. Kostkowski, Consultant

March 1984

Contents

SELF-STUDY MANUAL on OPTICAL RADIATION MEASUREMENTS

Part I. Concepts

This is the seventh in a series of Technical Notes (910-) entitled "Self-Study Manual on Optical Radiation Measurements". It contains Chapter 11 of Part I of this Manual. Additional chapters will continue to be published, similarly, as they are completed. The Manual is a comprehensive tutorial treatment of the measurement of optical radiation that is complete enough for self instruction. Detailed chapter summaries make it also a convenient authoritative reference source.

In this chapter we review the radiometric treatment of a non-linear radiometer. The emphasis is on the underlying radiometric principles and the experimental evaluation of a true response function so that such "real" radiometer-output signals can be used in the idealized equations appropriate for linear radiometers. Several common techniques are discussed: beam addition, beam attenuation, the inverse-square law, and a number of other techniques in which non-radiometric measurements provide some or all of the basis for the response-function calibration. Many references are given; they should permit the reader to pursue the experimental details of any of the techniques in greater depth.

Key Words: linearity; non-linearity; optical radiation; radiometric calibration; radiometry.

Chapter 11. Linearity Considerations and Calibrations

by John B. Shumaker

In this CHAPTER. We discuss the radiometric treatment of a radiometer whose output is not necessarily proportional to the incident flux. The emphasis is on the underlying radiometric principles and the experimental evaluation of a true response function so that such "real" radiometer-output signals can be used in the idealized equations of earlier chapters. We begin with an introduction which starts with the measurement equation, reviewing how it is related to desired radiometric quantities, such as spectral irradiance, and showing how it simplifies to the intuitive measurement relationships we usually use. We then examine how detector non-linearity complicates matters and introduce the idea of a linearized radiometer output signal Y (which is equivalent to the linear output signal S of earlier chapters) and the concept of the radiometric response function which relates Y to the actual (non-linear) instrument output signal S'. Finally, after these preliminaries, we come to the main theme of the chapter where we assume that the response function relationship can be expressed in the form of a polynomial or other appropriate function involving a small number of unknown coefficients. The remainder of the chapter is devoted to illustrating how these coefficients can be determined. Several common techniques are discussed: beam addition, beam attenuation, the inverse-square law and a number of other techniques in which non-radiometric measurements provide some or all of the basis for the response-function calibration. Many references are given; they should permit the reader to pursue the experimental details of any of the techniques in greater depth.

INTRODUCTION. In Chapter 5 [11.1][1] we introduced the measurement equation which relates the output signal of a radiometer to the distribution of radiant power entering the radiometer. This equation in its most general form is (see eq. (6.36) [11.2])

[1]Figures in brackets indicate literature references listed at the end of this Technical Note.

$$S = \int \int \int \int (R_{00} \cdot L_{\lambda 0} + R_{01} \cdot L_{\lambda 1} + R_{02} \cdot L_{\lambda 2} + R_{03} \cdot L_{\lambda 3}) \cdot \cos\theta \cdot d\omega \cdot dA \cdot dt \cdot d\lambda / \Delta t$$

where the $L_{\lambda i}$ are the Stokes polarization components of the spectral radiance at the entrance of the radiometer, the R_{0i} are the first row Mueller matrix elements of the radiometer flux responsivity, and S is the generated output signal (e.g. volts or centimeters of displacement on a strip chart recording). The integrations are over all the radiation parameters, x, y, θ, ϕ, t, and λ, in which the radiant beam is limited by the radiometer acceptance intervals: entrance pupil or receiving aperture ΔA, acceptance solid angle $\Delta\omega$, duration[1] of measurement Δt, and spectral pass band $\Delta\lambda$. Hereafter we will abbreviate the measurement equation as

$$S = \int \ldots \int R \cdot L_\lambda \cdot dx \cdot \ldots \cdot dt \cdot d\lambda / \Delta t \quad [S] \tag{11.1}$$

where it is to be understood that the multiple integral covers all appropriate variables mentioned above and moreover that one of the integrations is to be interpreted as a sum over the four Stokes components [11.2]. The flux responsivity R and spectral radiance L_λ are, in general, functions of all these variables of integration. If spectral radiance is not the radiometric quantity of interest, we can group the integrations over some of the variables to produce an approximation to the desired quantity. For example, if spectral irradiance is desired, we first define a mean spectral irradiance by

$$\bar{E}_\lambda = [\int_{\Delta\omega} R \cdot dE_\lambda] / \bar{R} = [\int_{\Delta\omega} R \cdot L_\lambda \cdot \cos\theta \cdot d\omega] / \bar{R} \quad [W \cdot m^{-2} \cdot nm^{-1}]$$

where

$$\bar{R} = [\int_{\Delta\omega} R \cdot \cos\theta \cdot d\omega] / \pi \quad [S \cdot W^{-1}].$$

Then, in terms of these quantities, the measurement equation becomes

$$S = \int \ldots \int \bar{R} \cdot \bar{E}_\lambda \cdot dx \cdot \ldots \cdot dt \cdot d\lambda / \Delta t \quad [S] \tag{11.2}$$

where the integrals no longer include the directional variables. Now, if R is truly independent of direction, we will have $\bar{R} = R$ and $\bar{E}_\lambda = E_\lambda$, and the instrument will correctly measure spectral irradiance. If R is somewhat directionally dependent, then only the weighted mean spectral irradiance $\bar{E}_\lambda$

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[1]No real measuring instrument or system responds with perfect time resolution to the instantaneous value of the incident radiometric quantity but rather to an average value of that quantity over some period Δt [s] that is a measure of its time-resolution capability. We indicate this rather crudely here by the notation

$$S(t) = \int_{\Delta t} R \cdot L_\lambda(t) \cdot dt / \Delta t \quad [S].$$

It will be seen in the chapter on the distribution of optical radiation with respect to time that the true expression is a convolution integral with a weighting-function responsivity that accounts for the dynamics of the contributions to $S(t)$ of the incident $L_\lambda(\tau)$, at all earlier times τ[s], as a function of the elapsed time $(t - \tau)$ [s].

can be measured and, unless some further independent information is available about the angular distribution of the field, this is the best that can be obtained from such an instrument. We notice that measurement equation (11.2) is still of the same form as eq. (11.1) -- only the names have been changed and some of the variables of integration have been suppressed. Thus, we can consider eq. (11.1) to be representative of the measurement equation for any radiometric quantity and, although we shall continue to write L_λ and speak of spectral radiance, our discussion will apply to any radiometric quantity and it is understood that the variables of integration are limited to those appropriate to the quantity in question.

The measurement equation [eq. (11.1)] is an integral equation which cannot be solved for the radiometric quantity of interest L_λ unless, as we saw in Chapter 5 [11.1], at least the relative dependence of both R and L_λ upon the variables $x, \ldots, \lambda$ is known. However, we can formally define an average spectral radiance and an average responsivity by:

$$\langle L_\lambda \rangle = \int \ldots \int R \cdot L_\lambda \cdot dx \cdot \ldots \cdot d\lambda / (\langle R \rangle \cdot \Theta) \quad [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}] \tag{11.3}$$

and

$$\langle R \rangle = \int \ldots \int R \cdot dx \cdot \ldots \cdot d\lambda / \Theta \quad [\mathrm{S \cdot W^{-1}}] \tag{11.4}$$

where $\Theta = \int \ldots \int dx \cdot \ldots \cdot d\lambda$ $[\mathrm{m^2 \cdot sr \cdot s \cdot nm}]$ is a generalized throughput for the radiometer. Then we can solve eq. (11.1) to give

$$\langle L_\lambda \rangle = S \cdot \Delta t / (\langle R \rangle \cdot \Theta) \quad [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}]. \tag{11.5}$$

Usually, for a well-designed radiometer, the responsivity can be approximated by a function which is nearly constant over a small range of most of the variables and drops abruptly to zero outside this range. When this is true, we can write

$$\langle R \rangle \approx R \quad [\mathrm{S \cdot W^{-1}}]$$

and

$$\langle L_\lambda \rangle \approx \int_{\Delta x} \ldots \int_{\Delta\lambda} L_\lambda dx \cdot \ldots \cdot d\lambda / \Theta \quad [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}]$$

where R is this constant responsivity within the small ranges $\Delta x, \ldots, \Delta\lambda$ which we have called the acceptance intervals of the radiometer. With this approximation, eq. (11.5) becomes

$$\langle L_\lambda \rangle \approx S \cdot \Delta t / (R \cdot \Theta) \quad [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}}]. \tag{11.6}$$

If the acceptance intervals are small enough that L_λ can also be assumed constant, then $\langle L_\lambda \rangle \approx L_\lambda$.

<u>NON-LINEARITY and the RADIOMETER RESPONSE FUNCTION</u>. We now turn to the complications which arise if the responsivity is non-linear, that is, if it depends upon the magnitude of incident spectral radiance L_λ (or other appropriate radiometric quantity). Let us consider in a general way the production of a

signal[1] S' by a radiometer exposed to a field of spectral radiance L_λ. We will not be overly careful about this; the object is merely to obtain an idea of the way in which the various undesirable contributions to the signal, such as dark current and unintentional non-linearities, might arise in the output. Somewhere within the instrument the photons are absorbed and converted (by a transducer or detector) to some other kind of signal, usually electronic, such as electric current or a countable pulse train. To be definite, let us assume it's an electric current. This current is then subsequently amplified, perhaps, and displayed or recorded. At the place in the instrument where the conversion from radiant flux to current has taken place, the current will contain a contribution due to the spectral radiance of interest given by

$$\int\ldots\int R^{o}\cdot L_\lambda\cdot dx\cdot\ldots\cdot d\lambda/\Delta t$$

where the responsivity $R^{o}(x,\ldots,\lambda)$ includes the transmittance of instrumental optical components and the efficiency of the transducer conversion but is independent of L_λ. There may be other contributions -- non-linear contributions -- dependent upon the magnitude of L_λ and, additionally, there may be contributions to the current here from thermal emission, gamma rays, light beams in the environment other than the one of interest (background radiation), and probably other things that we haven't thought of. We will lump all these together into a contribution

$$\eta(\Delta A,\ldots,\Delta\lambda,L_\lambda)$$

which, as indicated, may depend in an arbitrary way upon all of the radiometer acceptance intervals and upon the measured field itself. The sum of these two contributions is the converted intermediate signal which is then processed and displayed by, for example, an amplifier and recorder, to yield the final output. Let us write this final transformation as

$$S' = G[\int\ldots\int R^{o}\cdot L_\lambda\cdot dx\cdot\ldots\cdot d\lambda/\Delta t + \eta(\Delta A,\ldots,\Delta\lambda,L_\lambda)] \tag{11.7}$$

where $G(u)$ is some function which describes what the amplifier and display or recorder do to that intermediate current. For an ideal linear amplifier $G(u)$ will simply be $g\cdot u$ but, more generally, we can include logarithmic amplification $G(u) = g\cdot\log(u)$ or other non-linear transformation of the intermediate photocurrent. The quantity closest to our interest is the multiple integral buried in eq. (11.7). We can formally solve for it as

$$\int\ldots\int R^{o}\cdot L_\lambda\cdot dx\cdot\ldots\cdot d\lambda/\Delta t = G^{-1}(S') - \eta(\Delta A,\ldots,\Delta\lambda,L_\lambda) \tag{11.8}$$

where $G^{-1}(S')$ is just the mathematical inverse of function $G(u)$, whatever it may be. That is, if $S' = G(u)$ then $u = G^{-1}(S')$.

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[1]We will hereafter use the symbol S' for an output signal which may not be linearly related to the radiometric field. In this way the earlier equations in S, in which linearity was assumed, are distinguished from the present more general equations.

If everything on the right of eq. (11.8) were known, this would pretty much be the equivalent of eq. (11.1). In principle, $G(u)$ could be measured quite accurately, since this is just an electronic problem, but the function η is almost unknowable. However, if we can ignore the dependence of η on other quantities and consider it a function of S' alone, then, as we shall see, the treatment of non-linearity becomes quite tractable. The consequence of this approach is, of course, that, since η is probably also dependent upon the acceptance intervals of the radiometer, a recalibration to re-establish or reconfirm η is required each time that an acceptance interval is changed. Normally, one expects to have to perform a limited recalibration anyway whenever an acceptance interval, such as spectral pass band or duration of measurement, is changed, so this is not of much significance. If η is assumed to depend upon S' instead of upon the field distribution L_λ, this means that it depends upon some kind of average of L_λ over the acceptance intervals of the radiometer, since this is, after all, what uniquely determines S', we hope. This will be correct if L_λ is uniform over the acceptance intervals. Otherwise errors may be introduced by this assumption. Thus, if the detector saturates in one small portion of its area or during a portion of the measurement time, our treatment will fail. Whether non-uniformity of the spectral radiance over some acceptance interval will cause such difficulties or not will depend upon details of the radiometer construction -- where in the signal processing the non-linearity originates (in the function η or the function G), what detector is used, where diffusers or lenses are located, etc. For example, a diffuser just in front of the detector may provide sufficient flux averaging to prevent local detector saturation. As always in radiometry, the consequences of shortcomings in instrumentation and data reduction are minimized if all instrument characterization and calibration is carried out with incident beams which mimic, as far as possible, the flux levels and the spatial, angular, temporal, spectral, and polarization distributions of the unknown beams which will be measured.

With η being treated as a function of S' alone, then, the right hand side of eq. (11.8) becomes a function of S' only, which we will denote by $f(S')$. The integral on the left hand side is directly proportional to the radiometric quantity of interest; that is, doubling the flux will double the value of this integral. Let us call this integral Y and, by defining measurement averages as in eqs. (11.3) and (11.4), we can always write $Y = \langle R^o \rangle \cdot \langle L_\lambda \rangle \cdot \Theta / \Delta t$. We can think of Y as the output of an imaginary radiometer which differs from the real one only in not suffering from non-linearity. Its output will never saturate, for example, or, if the real radiometer is non-linear because of a logarithmic amplifier then Y is proportional to the intermediate linear signal ahead of the amplifier. Thus, we have

$$Y = \int \ldots \int R^o \cdot L_\lambda \cdot dx \cdot \ldots \cdot d\lambda / \Delta t = f(S') \tag{11.9}$$

where $f(S')$ $[= G^{-1}(S') - \eta(S')]$ is to be regarded as simply some unknown function of S' whose nature will be elicited from suitable measurements carried out with the radiometer itself. If we can evaluate $f(S')$, we will have reduced the problem of the non-linear radiometer to the simpler problem of the linear radiometer measurement equation, eq. (11.1).

EXPERIMENTAL CHARACTERIZATION of LINEARITY. For want of a better name we shall refer to the function $Y = f(S')$ as the response function of the radiometer. There are a few situations when the response function can be predicted from theoretical considerations: absolute detectors in which the signal output is the electrical power required to duplicate the temperature change in the detector when subjected to radiant heating from the test beam, silicon detectors for which a well-developed quantum theory exists, and possibly some photon-counting detector systems. Even in these instances, however, it has been customary to perform some independent confirmation of the response function before relying upon the theoretical analysis exclusively. Of course the determination of the response function is relatively trivial if one has a source already calibrated at the desired flux levels or if one has a suitable set of calibrated attenuators or another detector of known response function. Therefore we shall be concerned here with measuring the response function independently, so far as possible, of externally supplied radiometric standards and calibrations: the kind of bootstrapping detector characterizations which standards laboratories routinely carry out. We will first consider the determination of the response function by radiometric measurements alone. Later we will consider techniques in which some other kind of measurement, such as length or temperature, provides the basis for evaluating the response function.

Although the response function may not be known exactly, we can certainly assume that we can select an adequately approximate functional form for $f(S')$ which will depend upon a number of adjustable coefficients. The main task of this chapter then will be to consider the experimental evaluation of these coefficients. We shall dwell extensively on the particular functional form

$$Y = f(S') = F\cdot(f_0+f_1\cdot S'+f_2\cdot S'^2+ \ . \ . \ . \ +f_n\cdot S'^n) \tag{11.10}$$

because it is easy to work with and is especially appropriate for radiometers which depart but little from linearity. Most useful detectors exhibit a range of essentially linear behavior and, within this range, one would expect very small values for the coefficients $f_2, f_3, \ . \ . \ . \ , f_n$. Notice that there is a redundancy in eq. (11.10): the factor F could be absorbed into the coefficients f_i, hence its value, or alternatively a common scale factor in the f_i's, can be chosen arbitrarily. We shall make use of this arbitrariness to normalize some of the f_i's when it is convenient to do so. Another functional form for the response function which may be suitable for radiometers which are designed to have a logarithmic response is

$$Y = \exp[F\cdot(f_0+f_1\cdot S'+f_2\cdot S'^2+ \ . \ . \ . \ +f_n\cdot S'^n)]. \tag{11.11}$$

Again, if the system is well-designed, we would expect the higher order coefficients $f_2, f_3, \ . \ . \ . \ , f_n$ to be small, and again we've included a redundant coefficient F, for later convenience. Equations (11.10) and (11.11) do not, of course, exhaust the kinds of response functions which one might choose. Depending upon the detector, its application, and the technique used for evaluating the coefficients, other forms may be found useful. For the NBS photoelectric pyrometer, for example, the form

$$Y = (f_0+f_1\cdot S'+ \ . \ . \ . \ +f_5\cdot S'^5)\cdot\exp(30.15S'-13.02S'^2) \tag{11.12}$$

is employed, where the exponential factor provides the general shape of the response function (within about a factor of 2) as S' changes from 0.5 to 1 and Y by a factor of 500 in this range. In this case, S' is the current, in the filament of an internally mounted strip lamp, required to produce a radiance match between that strip filament and an image of the unknown external source.

Even the best "linear" detector system becomes non-linear at sufficiently high radiant power densities (when it is vaporized by the incident radiant field, for example). Frequently this non-linearity begins with the output signal starting to saturate reversibly (usually caused by the amplifier rather than the transducer itself) at high inputs. The response functions which we illustrate in this chapter [e.g. eq. (11.10)] are not capable of describing saturation behavior but such an instrument will in any case not be useful as a radiometer in the neighborhood of saturation since its output there is, at best, weakly dependent upon the incident flux level. This is an example of a situation where even exact knowledge of the detailed non-linear behavior may not permit useful radiometric measurements to be made. Clearly it is not profitable to include measurements under saturation conditions in the experimental evaluation of the response function except under unusual conditions, such as in a study of detector mechanisms.

Generally speaking, there are two kinds of purely radiometric measurements that furnish useful information for response-function determinations: beam-addition measurements and attenuation measurements. Beam addition involves the measurement of the response of the radiometer to each of two or more beams separately and then its response to the sum of the beams. By beam attenuation we mean the measurement of the effect on the response of a radiometer of a repeatable beam attenuator, such as a filter. In either case, many measurements are carried out for varying beam flux levels and from these measurements one attempts to obtain the response function $f(S')$.

BEAM ADDITION. Let us turn first to a detailed treatment of beam addition measurements, since this technique is generally felt to be somewhat more fundamental and flexible than the attenuation measurement technique. We require a beam-adding device in which two or more light beams (hereafter we will assume just two beams), whose flux levels can be independently varied, are combined to yield a single beam which is measured by the radiometer under test. The flux level variations can be carried out in a variety of ways, such as by adjusting lamp currents, interposing filters, or changing aperture sizes. The combining of the beams can be accomplished by a beam splitter [11.3-11.6] or an integrating sphere [11.7,11.8], or the beams can simply be allowed to enter the radiometer to be tested at different locations or from different angles [11.9,11.10] if the instrument spatial or angular responsivity is not in question. We assume that the two beams originate in independent sources so that their fluxes can be added without interference effects. Even if the two beams have a common origin, the interference effects will usually be negligible [11.8] but, for highest accuracy, they must be estimated, since incoherent flux addition is an essential assumption of this technique. We consider a two-beam adder, with beams labeled A and B, and a near-linear radiometer for which a response function of the form of eq. (11.10) is adequate. Consider a pair of measurements in which the flux level of beam B is held constant and beam A is set at two different flux levels. Since the quantity Y [eq. (11.9)] is, aside from the proportionality constant $\langle R^0 \rangle \cdot \Theta / \Delta t$, equal to the average spectral radiance (or other appropriate radiometric quantity) entering the instrument and, since the total incident spectral radiance is the sum of the spectral radiances from the two beams, we can write the equations

$$Y_1^{(A)} + Y_1^{(B)} = f(S'_{11}) \tag{11.13}$$

and

$$Y_2^{(A)} + Y_1^{(B)} = f(S'_{21}) \tag{11.14}$$

to describe the addition of the spectral radiances in beams A and B to produce the output signals S'_{11} and S'_{21}. Now we repeat this pair of measurements using the same two levels of beam A but a different level of beam B:

$$Y_1^{(A)} + Y_2^{(B)} = f(S'_{12}) \tag{11.15}$$

$$Y_2^{(A)} + Y_2^{(B)} = f(S'_{22}). \tag{11.16}$$

From equations (11.13) and (11.14), we obtain

$$Y_1^{(A)} - Y_2^{(A)} = f(S'_{11}) - f(S'_{21}) \tag{11.17}$$

and, from eqs. (11.15) and (11.16), we obtain

$$Y_1^{(A)} - Y_2^{(A)} = f(S'_{12}) - f(S'_{22}). \tag{11.18}$$

Consequently, we can write $f(S'_{11}) - f(S'_{21}) = f(S'_{12}) - f(S'_{22})$ or, if we substitute from eq. (11.10) and regroup the terms:

$$\begin{aligned} & f_1\cdot(S'_{11}+S'_{22}-S'_{12}-S'_{21}) + f_2\cdot(S'^2_{11}+S'^2_{22}-S'^2_{12}-S'^2_{21}) \\ & + f_3\cdot(S'^3_{11}+S'^3_{22}-S'^3_{12}-S'^3_{21}) + \; . \; . \; . + f_n\cdot(S'^n_{11}+S'^n_{22}-S'^n_{12}-S'^n_{21}) = 0. \end{aligned} \tag{11.19}$$

The values of S'_{11}, S'_{12}, S'_{21}, and S'_{22} are the observed radiometric readings so everything in this equation is known except the coefficients $f_1, \ldots f_n$. The coefficients f_0 and F have dropped out of these equations. A repetition of such a set of measurements for another n-1 different combinations of flux levels will provide n equations in these n unknown coefficient values. Unfortunately, since the right-hand sides of all of these equations are zero, we can solve only for ratios of these coefficient values. This is obvious if one observes that, if a particular set of values of the f_i's satisfies the set of equations of the form of eq. (11.19), so will any multiple of these f_i's. At this point, then, we can arbitrarily choose the value of one of these coefficients or impose some other arbitrary condition upon them. Probably a good condition to impose is:

$$f_1 + f_2 + \; . \; . \; . + f_n = 1. \tag{11.20}$$

An alternative is to set the most important f_i, usually f_1, equal to 1. However, there is always the risk in setting some $f_i = 1$ that one will accidentally have chosen a coefficient whose value is actually zero and then the rest of the analysis becomes nonsense. We can now solve for the values of the coefficients f_1 through f_n. Actually we have one more equation than we need, but, in practice one usually takes many times as many measurements as needed and uses least-squares minimization to solve the set of equations for these coefficients. There still remain two undetermined coefficients, f_0 and F. These cannot be evaluated by additional measurements with the beam adder but require supplemental measurements of some other kind, such as measurements of known fluxes. The first one f_0 is easily evaluated in the visible by simply setting the flux to zero with a black shutter, i.e., setting $Y = 0$ and reading the dark signal S'_0:

$$0 = f_0 + f_1 \cdot S'_0 + f_2 \cdot S'^2_0 + \ldots + f_n \cdot S'^n_0. \tag{11.21}$$

Since f_0 is now the only unknown in this equation, it is easily solved for. Usually such a dark signal measurement will already have been included among those used in determining the non-linear coefficients f_2 through f_n. At this point we have reduced the problem to that of calibrating a linear radiometer, for, if we define

$$\begin{aligned} S &= Y/F \\ &= f_0 + f_1 \cdot S' + \ldots + f_n \cdot S'^n \end{aligned} \tag{11.22}$$

and $R = R^o/F$,

we can write

$$S = \int \ldots \int R \cdot L_\lambda \cdot dx \cdot \ldots \cdot d\lambda / \Delta t, \tag{11.23}$$

which is just eq. (11.1). For any instrument reading S', then, S can be calculated from eq. (11.22) and the measurement of the responsivity R can be carried out normally using eq. (11.23) and known radiant inputs. If $Y=0$ cannot be achieved, as, for example, in the infrared, then a supplemental measurement with some other known flux is one way to determine the value of the unknown constant f_0 remaining in eq. (11.23). We will mention another way when we discuss beam-attenuation measurements.

Perhaps a simple numerical example will help to clarify the application of eqs. (11.19) and (11.21). Suppose we have a nominally linear radiometer and a means of irradiating it with either or both of two sources. Assume that we obtain the following measurements:

Dark current:	S'_{11} =	-0.0100
Lamp #1:	S'_{21} =	0.7779
Lamp #2:	S'_{12} =	0.9711
Both Lamps:	S'_{22} =	1.7301

We assume that the radiometer response function can be approximated by

$$f(S') = f_0 + S' + f_2 \cdot S'^2$$

where we have included the arbitrary normalization condition $f_1 = 1$. Equations (11.19) and (11.21) yield directly

$$f_2 = -(S'_{11}+S'_{22}-S'_{21}-S'_{12})/(S'^2_{11}+S'^2_{22}-S'^2_{21}-S'^2_{12})$$

and

$$f_0 = -S'_{11}(1 + f_2 \cdot S'_{11}).$$

When we insert the values for S' we find $f_0 = 0.0100$ and $f_2 = 0.0200$. Thus, the radiometer is slightly non linear $(f_2 \neq 0)$. Luckily equations (11.19) and (11.21) are linear in the unknown coefficients and their solution is straightforward and unique. For other arbitrary response functions, such as eq. (11.11), the equivalent of eq. (11.19) will not be linear in the coefficients. Then the set of equations may require iterative numerical techniques for its solution and will generally yield multiple solutions. We will see an example of this when we discuss the beam attenuation technique.

An interesting variation of this basic technique involves chopping one of the beams and using a phase-sensitive or lock-in amplifier to extract the signal contribution from that beam [11.11]. There are several ways such measurements could be carried out. One way is to use a constant primary source in beam A which is chopped at some convenient fixed frequency while the flux level in beam B is slowly increased from zero. Two signals are simultaneously recorded (on an X-Y recorder, for example): the mean DC signal output from the radiometer and the AC component of the signal at the chopping frequency. To understand this technique let us return briefly to eqs. (11.17) and (11.18). We see that both of these equations are of the form

$$Y_1^{(A)} - Y_2^{(A)} = f(S'_{1i}) - f(S'_{2i}) \tag{11.24}$$

where the subscript i stands for a measurement made at the ith level of beam B. Let subscript 1 stand for the state when the chopper admits beam A to the radiometer and subscript 2 for the blocked state. Then $\Delta Y = Y_1^{(A)} - Y_2^{(A)}$ is proportional to the constant flux in beam A ahead of the chopper. Now we suppose that ΔY is small, so that (since we are dealing with a nearly linear radiometer) $\Delta S' = S'_{1i} - S'_{2i}$ will also be small, and we can write eq. (11.24) as

$$\Delta Y \approx \Delta S' \cdot \frac{df(S')}{dS'} . \tag{11.25}$$

In the measurements, then, the AC signal is proportional to $\Delta S'$ and the DC signal is just $S' + \Delta S'/2 \approx S'$, the signal due to the slowly increasing flux in beam B. Since the quantity ΔY is a constant, we can obtain the response function from the integral of eq. (11.25),

$$f(S') \approx \Delta Y \cdot \int \frac{dS'}{\Delta S'} . \tag{11.26}$$

Aside from a proportionality constant and a constant of integration, this is just the integral of the reciprocal of the AC component regarded as a function of the DC component S'. The proportionality constant and the constant of integration must be determined, as before, by supplemental measurements, for example, of sources of known output, one of which may be zero. The main requirements for the validity of this technique are that ΔY be small enough that $\Delta Y/\Delta S' \approx dY/dS'$ and that the response function $f(S')$ be qualitatively the same for chopped and DC fluxes. We emphasize this latter condition because, since two quite different measurement durations are involved, it may not be possible to minimize errors by matching these calibration measurement times to the test measurement times as easily as in the more tedious DC beam addition procedure described above. However, if the form of $f(S')$ can be assumed to be independent of the duration of a measurement, then this AC technique is particularly attractive as a quick test for linearity: when the AC component $\Delta S'$ of the signal is constant over the useful range of the DC component S', then

$$f(S') \approx S' \cdot \Delta Y/\Delta S' + \text{const.} \tag{11.27}$$

and the linearity of the radiometer is thus established.

BEAM ATTENUATION. We now turn to a consideration of the use of attenuation measurements in determining the response function $f(S')$. The equipment requirements for this technique are so modest -- just a few filters or one filter and an adjustable lamp -- that it deserves much more serious consideration than it usually receives. Let us imagine the measurement of the transmittance τ of a filter by recording the radiometer output S' when the filter is in the beam and, again, when it has been removed. As before, since Y [eq. (11.9)] is the average spectral radiance entering the radiometer apart from the proportionality constant $\langle R^0 \rangle \cdot \Theta/\Delta t$, we can write the ratio

$$\tau = \frac{Y_1}{Y_2} = \frac{f(S_1')}{f(S_2')} \tag{11.28}$$

to describe the relationship between the instrument readings S_1' and S_2' and the filter transmittance τ. We assume that we don't know the value of this transmittance but that it is constant, independent of flux level. We therefore change the flux level of the beam by any convenient means, such as by inserting another filter[1] into the beam and remeasure τ:

$$\tau = \frac{Y_3}{Y_4} = \frac{f(S_3')}{f(S_4')} \, . \tag{11.29}$$

This gives us the one equation

$$\frac{f(S_1')}{f(S_2')} = \frac{f(S_3')}{f(S_4')} \tag{11.30}$$

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[1]When an additional filter is added to an optical system it is customary to tilt it a few degrees away from the normal to the optic axis in order to deflect any filter-filter interreflections out of the optical path. If this precaution is not taken, there may be an apparent increase of filter transmittance due to contributions from these interreflections. Of course, the insertion of a tilted filter will cause a lateral beam displacement which may require a compensating lateral adjustment of the source or radiometer.

independent of the unknown τ. If we now substitute into eq. (11.30) a model functional form for the response function $f(S')$, such as eq. (11.10), we obtain one equation in the unknown coefficients f_0, $f_1, \ldots,$ and f_n and involving the recorded instrument readings S_1', S_2', S_3', and S_4'

$$\frac{f_0 + f_1 \cdot S_1' + \ldots + f_n \cdot S_1'^n}{f_0 + f_1 \cdot S_2' + \ldots + f_n \cdot S_2'^n} = \frac{f_0 + f_1 \cdot S_3' + \ldots + f_n \cdot S_3'^n}{f_0 + f_1 \cdot S_4' + \ldots + f_n \cdot S_4'^n}$$

or

$$f_0 \cdot f_1 \cdot (S_1' - S_2' - S_3' + S_4') + \ldots + f_0 \cdot f_n \cdot (S_1'^n - S_2'^n - S_3'^n + S_4'^n)$$

$$+ f_1^2 \cdot (S_1' \cdot S_4' - S_2' \cdot S_3') + \ldots + f_1 \cdot f_n \cdot (S_1' \cdot S_4'^n - S_2' \cdot S_3'^n - S_2'^n \cdot S_3' + S_1'^n \cdot S_4')$$

$$+ \ldots + f_n^2 \cdot (S_1'^n \cdot S_4'^n - S_2'^n \cdot S_3'^n) = 0 . \qquad (11.31)$$

A few repetitions of this basic measurement scheme, with the same filter at additional beam flux levels or with different filters, will give enough such equations to solve for the f_i's. As in the beam-adding situation [eq. (11.19)] the redundant scaling factor F drops out. Also, because the right hand sides are all zero, we can only obtain relative f_i values from these equations. It is therefore convenient again to include in the set of simultaneous equations an additional normalizing equation such as eq. (11.20). With our choice of eq. (11.10) for the functional form of $f(S')$ we obtain equations which are quadratic in the f_i's by these attenuation measurements. This means that there will be multiple solutions to the set of equations of the form of eq. (11.31)[1]. Thus, additional measurements of a different kind may be required to select the correct solution from among all these candidates.

A simple numerical example may again be helpful in illustrating these equations. Suppose we use the same nominally linear radiometer as in the previous section to measure the transmittance of a filter with two different sources as follows:

Dark Current:	S_0' =	-0.0100
Lamp #1 + Filter:	S_1' =	0.3870
Lamp #1:	S_2' =	0.7779
Lamp #2 + Filter:	S_3' =	0.4853
Lamp #2:	S_4' =	0.9711

[1]The fact that $\tau = [f(S_1')]^k/[f(S_2')]^k = [f(S_3')]^k/[f(S_4')]^k$ leads to the same set of equations [eq. (11.31)] and, consequently, the same solution regardless of the value of k accounts for some of this multiplicity of solutions: some of the solutions will simply represent approximations to integral powers of one another.

Again let us assume the response function $f(S) = f_0 + S + f_2 \cdot S^2$ with the normalizing condition $f_1 = 1$. The dark current measurement gives us [eq. (11.21)]

$$0 = f_0 + S_0' + f_2 \cdot S_0'^2 .$$

If we solve for f_0 and substitute into eq. (11.31) we will obtain, after some rearranging,

$$a \cdot f_2^2 + b \cdot f_2 + c = 0$$

where $$a = S_2^2 \cdot S_3^2 - S_1^2 \cdot S_4^2 + S_0^2 \cdot D_2$$

$$b = S_2 \cdot S_3^2 + S_3 \cdot S_2^2 - S_1 \cdot S_4^2 - S_4 \cdot S_1^2 + S_0 \cdot D_2$$

$$c = S_2 \cdot S_3 - S_1 \cdot S_4 + S_0 \cdot D_1$$

and $$D_n = S_1^n + S_4^n - S_2^n - S_3^n .$$

This leads to the two solutions

$$f_0 = 0.0100, \quad f_2 = 0.0218$$

and $$f_0 = 0.0074, \quad f_2 = 26.38.$$

The first solution gives us a filter transmittance of 0.4996 while the second solution gives 0.2594. If this is all the information we have, then we can only conclude that the radiometer is either nearly linear (i.e., the first solution is correct) or that it is extremely non-linear (the second solution) and, if non-linear, we must take more data in order to investigate the contributions of the higher order coefficients, f_3, f_4, etc., -- a very difficult undertaking by this technique because of the necessity of solving the simultaneous quadratic equations and sorting out the resulting multiplicity of solutions. However, if we have reason to believe that the radiometer is nearly linear (perhaps the data were taken at very low flux levels) we can discard the second solution with confidence and we have obtained an expression for the response function and a value for the filter transmittance. This transmittance can then be used to identify the correct solution at other flux levels where linearity is more in doubt. In this way the evolution of the response function with flux level may be studied at least as long as the higher order terms, $f_3 \cdot S^3$, etc., remain negligible. In summary, we see that beam attenuation measurements can readily determine the range over which a radiometer is nearly linear and produce an accurate measurement of the response function over that range -- provided we possess a *soupçon* of faith.

The complication of multiple solutions can be avoided by a fortuitous choice of a model for $f(S')$. For example let us use eq. (11.11) for $f(S')$ in eq. (11.30). We then have

$$\frac{\exp[F\cdot(f_0+f_1\cdot S_1'+\ldots+f_n\cdot S_1'^n)]}{\exp[F\cdot(f_0+f_1\cdot S_2'+\ldots+f_n\cdot S_2'^n)]} = \frac{\exp[F\cdot(f_0+f_1\cdot S_3'+\ldots+f_n\cdot S_3'^u)]}{\exp[F\cdot(f_0+f_1\cdot S_4'+\ldots+f_n\cdot S_4'^n)]} \tag{11.32}$$

instead of eq. (11.31). Then, when we take logarithms of both sides, we obtain an equation exactly like eq. (11.19) of the beam-adding technique, which is linear in the coefficients f_1, $f_2,\ldots,$ and f_n. This shows the essential similarity of the two techniques: a multitude of filter measurements with different beam fluxes or different filters leads to the same kinds of equations in the same unknowns as with the beam adder. There is one practical minor difference: the exponential expression for $f(S')$ of eq. (11.11) doesn't accommodate a zero flux level measurement so that such measurements cannot be included in determining the values of the f_i's and will not be within the range of validity of the resulting response function. Aside from this difference, we can conclude that there is no reason in principle to prefer the beam addition technique to the beam attenuation technique. However, for a nearly linear radiometer for which a polynomial response function seems appropriate, the beam addition technique leads to far simpler equations with a unique solution.

Beam-attenuation measurements of the response function are often viewed with suspicion due, probably, to confusion over the fact that, in order to establish the linearity of a radiometer by such measurements, it is necessary but not sufficient that observed "indicated" filter transmittances be independent of beam flux level. Suppose that $f(S') = f_n\cdot S'^n$. Then, the filter transmittance measured at two different flux levels gives

$$\frac{S_1'^n}{S_2'^n} = \frac{S_3'^n}{S_4'^n}$$

or, upon taking the n^{th} root of each side of the equation,

$$\frac{S_1'}{S_2'} = \frac{S_3'}{S_4'} \quad . \tag{11.33}$$

In this case the observed "indicated" filter transmittance $(S_1'/S_2'$, for example) is independent of flux level whether the response function is linear $(n = 1)$ or is described by some other power of S'. The "indicated" transmittance, of course, will agree with the actual filter transmittance only if $n = 1$. Notice that, if eq. (11.33) is <u>not</u> satisfied by transmittance measurements of a filter at two levels, we can positively conclude that the radiometer is <u>not</u> linear.

Although eq. (11.31) is normally relatively intractable due to its quadratic dependence upon the f_i's, there is one circumstance where this is not a problem. Notice that eq. (11.31) includes terms in f_0 and is linear in this unknown (the f_0^2 terms cancel in deriving this equation), whereas in the treatment of beam addition using the same polynomial expression for $f(S')$ all the terms in f_0 cancel. Thus attenuation measurements yielding an equation such as eq. (11.31) provide a convenient technique for obtaining f_0 after f_1, f_2, etc., have been determined by beam addition in those cases where a dark-current measurement $(Y = 0)$ is impractical.

While we are on the subject of dark current, we should probably digress to say a few words about dark-current offsets and their role in our treatment. It is generally considered good practice to correct every output signal S' by subtracting the value of the output obtained in the absence of any intentional input radiant flux and to measure this dark signal as close, in a temporal sense, to the measurement S' as practical. The reason for this is that frequently this dark signal is a bias which varies little during the time between successive measurements but may vary significantly and erratically (perhaps depending upon ambient temperature) over the course of a day. Thus, measurements corrected for a continuously varying dark current will generally exhibit better precision than measurements not so corrected or measurements corrected for a long-term average dark current. If this dark-current model is correct and the radiometer is nearly linear, it is clear that dark-current subtraction is a reasonable procedure. However, if the model is incorrect or if the radiometer is highly non-linear, it is not obvious that simple subtraction of the dark current is meaningful[1]. Perhaps the best that can be said is that it is probably rarely harmful. In any case, it is immaterial in our equations whether a dark-current correction has been made or not. If the values of S' do not include dark-current correction, then the coefficients of a response function, such as eq. (11.10), computed from them will effectively include allowance for a constant dark current, and if the dark-current model above is correct, the coefficients will carry a somewhat greater statistical uncertainty than the corresponding, but different, coefficients computed from data corrected for a continuously varying dark current. Of course, whether S' is interpreted as including dark-current corrections or not, it must always be consistently interpreted throughout the computation and later application of the response function $f(S')$. Finally, if S' is not corrected for dark current, eq. (11.33), equating the transmittance measurements of a filter at two flux levels, will not be satisfied because this requires proportionality -- not simply linearity. Usually, in the treatment of detector linearity, dark-current correction is assumed and then linearity and proportionality are synonymous.

In summary, we see that both beam-addition and beam-attenuation measurements produce useful data for analysis of the dependence of a radiometer response function upon flux level. We choose a suitable functional form for the relationship between the flux entering the instrument and the signal output, one which includes a small number of unknown coefficients to be determined by experiment, and then express the physics of the addition or attenuation in terms of this function and any necessary additional parameters, such as transmittances. Next, we experimentally carry out a particular beam addition or attenuation at two different flux levels, so as to permit the mathematical elimination of all unknown parameters except the coefficients of the response function. Finally, such sets of measurements are carried out under enough different combinations of flux levels to permit solution of the resulting simultaneous equations for these coefficients. An excess of measurements in this step is desirable and allows a least-squares solution to be obtained[2]. In any case, one or two coefficients will remain undetermined

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[1]If one had a good model for the signal processing within the radiometer, say, like eq. (11.8), and, if one knew whether the dark current originated in the detector (η) or electronics (G), one could work out the proper dark current correction procedure.

[2]One of the advantages of a least-squares solution is that it will provide information about the optimum number n of coefficients f_i which should be retained. All that is needed is to perform the

(Footnote continued on page 16)

by this process. One of them will always be a scaling factor, which can only be determined by absolute instrument calibration with a source of known flux, just as for a simple radiometer with linear response. The other coefficient can also be evaluated by an absolute calibration at a second level, such as a zero level (dark current), but can also be evaluated by including both beam additions and beam attenuations among the experimental measurements [11.12]. Once the coefficients of the response function are known, the values of the additional experimental parameters, such as transmittances, relative aperture areas, etc., can be computed from the measurements, if they are wanted.

INVERSE-SQUARE LAW. We have discussed methods of determining the response function of a radiometer by making only radiometric measurements. There are also important techniques which utilize some non-radiometric measurements. We will discuss some of these methods now.

One of the most common techniques is to make use of the inverse-square law. In this approach the irradiance at a radiometer is varied predictably by varying the measured distance between a source and the radiometer entrance pupil or receiving aperture [11.13]. If radiance is required instead of irradiance a diffuser can be interposed in front of the radiometer as a new source, and the distance in question is then the distance from the original source to the diffuser. We have already introduced the inverse-square law and its applications in some detail in Chapter 4 (pp. 31-40) [11.14]. In its ideal form the inverse square law is simply

$$Y = f(S') = C/\ell^2$$

where C is a constant and ℓ is the source-to-radiometer distance. However, for good accuracy one would usually want to include corrections for the geometry of the source and radiometer. Let us return to eq. (11.9), the linear-response measurement equation, and rewrite it in terms of the spectral radiance $L_\lambda^{(s)}$ in the source plane by substituting $dx^{(s)}\cdot dy^{(s)}\cdot\cos\theta^{(s)}/\ell_s{}^2$ for the element of solid angle subtended at the radiometer by the element of source area $dx^{(s)}\cdot dy^{(s)}$. We have

$$Y = \int\ldots\iint_{A^{(s)}} L_\lambda^{(s)}\cdot R^o\cdot\ell_s{}^{-2}\cdot\cos\theta^{(s)}\cdot\cos\theta\cdot dx\cdot dy\cdot dx^{(s)}\cdot dy^{(s)}\cdot dt\cdot\ldots\cdot d\lambda/\Delta t \qquad (11.34)$$

where $(x^{(s)},y^{(s)})$ is a point in the source, $\theta^{(s)}$ is the angle between the normal to the source plane and the direction to the point (x,y) in the radiometer entrance aperture, and ℓ_s is the distance between these two source and aperture points. If we assume that the source plane is parallel

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[2](cont.)calculations for a range of values of n and to observe the accuracy of the fitting and the statistical uncertainties in the fitted coefficients. Usually one can locate a value of n (generally $n < 7$, in our experience) beyond which little, if any, significant improvement in the accuracy of the fitting occurs and beyond which the computed uncertainties of additional coefficients are comparable to the absolute magnitudes of the coefficients themselves. This implies that these additional coefficients could be set equal to zero with no appreciable loss of accuracy; in other words, more coefficients have been computed than are justified by the precision of the data.

to the radiometer aperture plane then $\theta^{(s)} = \theta$ and the slant distance ℓ_s between the source point $(x^{(s)},y^{(s)})$ and the aperture point (x,y) is given by

$$\ell_s{}^2 = \ell^2 + (x^{(s)} - x)^2 + (y^{(s)} - y)^2 \tag{11.35}$$

where ℓ is the perpendicular source-to-radiometer-aperture distance which is measured. Under the usual assumptions of spatially and angularly uniform spectral radiance and responsivity eq. (11.34) becomes

$$Y = [\int\ldots\int L_\lambda^{(s)}\cdot R^o\cdot dt\cdot\ldots\cdot d\lambda/(\Delta t)]\cdot\iiiint\limits_{A\ A^{(s)}} \ell_s{}^{-2}\cdot\cos^2\theta\cdot dx\cdot dy\cdot dx^{(s)}\cdot dy^{(s)}. \tag{11.36}$$

The final integration over the radiometer aperture (receiving aperture or entrance pupil) A, and the source area $A^{(s)}$ that is entirely within the instrument throughput, can be carried out exactly for simple source and aperture shapes (see appendix 3 in [11.15]) and the result gives the dependence of Y upon the source-to-radiometer distance ℓ apart from the constant factor

$$\int\ldots\int L_\lambda^{(s)}\cdot R^o\cdot dt\cdot\ldots\cdot d\lambda/\Delta t.$$

Having expressed $Y(\ell)$ as a function of ℓ, apart from a constant scale factor, we can evaluate the unknown coefficients in whatever form of response-function approximation we choose [e.g. eq. (11.10)] from

$$f(S') = Y(\ell) \tag{11.37}$$

by recording radiometer signals S' for a sufficient number of distances ℓ. Normally one would employ an excess of measurements and use least-squares fitting to determine the coefficients. The final multiplicative scaling factor must, as always, be obtained by an absolute calibration with a source of known radiometric output.

Instead of attempting to evaluate the integrals in eq. (11.34) it is tempting to assume a functional form for $Y(\ell)$, say

$$Y(\ell) = Y_0\cdot\ell^{-2} + Y_1\cdot\ell^{-3} + \ldots + Y_m\cdot\ell^{-m-2} \tag{11.38}$$

for use in eq. (11.37) and to use the measurements of radiometric response at various source distances to determine the coefficients $Y_0, Y_1, \ldots Y_m$ as well as the coefficients $f_0, f_1, \ldots f_n$ in $F(S')$. In the general case of an arbitrary function with an arbitrary number of coefficients of no particular physical significance like eq. (11.38) this is not possible. The difficulty is that for any given reasonable degree of accuracy many different sets of f_i and Y_i coefficients can be calculated depending upon the choice of m, say. In other words, for a particular set of data we can add more coefficients Y_i and thereby reduce the number of coefficients f_i required to achieve a given accuracy of fit, and *vice versa*. This lack of uniqueness of the response function comes about because by introducing an arbitrary function on the right hand side of eq. (11.37) we no longer can be assured that the two sides are proportional to radiant flux. There are, however, special circumstances where this technique can succeed. When the functional form on the right is known and contains a fixed number of Y_i coefficients of physical significance, then these can be determined along with the coefficients f_i whose number is not known *a priori*. Such a situation could arise, for example, if the location of the entrance pupil or receiving aperture within the radiometer were unknown so that the distance ℓ is the sum of a measured distance ℓ' and a constant, but unknown, offset ℓ_0. Then eq. (11.34) can be evaluated exactly (or approximately) in terms of $\ell = \ell' + \ell_0$, and ℓ_0 can be determined along with the coefficients of $f(S')$ by least squares fitting of eq. (11.37) to the (S', ℓ') measurement pairs.

One of the drawbacks of the inverse-square technique is the long light path required if a large range of flux levels is desired. One way to shorten this distance is to cascade two inverse-square-law devices by mounting a small diffuser to serve simultaneously as the receiver of the first device and the source of the second. Reference [11.16] describes such an installation in which the light path is folded back, parallel to itself, at the diffuser and the source-diffuser distance equals the diffuser-radiometer distance. The resulting inverse-fourth-law apparatus covers six orders of magnitude of flux levels with only 3 [m] of diffuser travel.

<u>BLACKBODY-SOURCE TEMPERATURE</u>. Another technique for measuring a radiometer response function, which is particularly useful in the infrared, is based upon measurement of the temperature of a blackbody source. One simply measures the temperature and then calculates the blackbody spectral radiance from the Planck equation. At temperatures below the gold point (1338 [K]) the International Practical Temperature Scale (IPTS) is defined non-radiometrically -- by means of the voltage or resistance of physical probes -- so, in this range, the temperature measurement of a variable-temperature blackbody provides a means of realizing sources of non-radiometrically determined, arbitrary spectral radiance. The subject of blackbodies and temperature is treated in another chapter of this Manual. For now, we merely note that the spectral radiance of a blackbody is given by

$$L_\lambda^{(b)}(\lambda) = \frac{c_1}{\pi \cdot n^2 \cdot \lambda^5 \cdot \{\exp[c_2/(n \cdot \lambda \cdot T)]-1\}} \quad [\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot m^{-1}}] \tag{11.39}$$

where $c_1 = 3.742 \cdot 10^{-16}$ $[\mathrm{W \cdot m^2}]$ is known as the first radiation constant, $c_2 = .01439$ $[\mathrm{m \cdot K}]$ is the second radiation constant, n is the index of refraction in the medium -- usually air -- in which the radiant field is being measured, λ [m] is the wavelength in this medium, and T [K] is the absolute temperature of the source. Assuming blackbody radiance and radiometer responsivity both uniformly distributed in position, direction, time, and polarization over the acceptance intervals of the radiometer, we obtain from eq. (11.9)

$$Y = \int \ldots \int dx \cdot \ldots \cdot dt \cdot \int_{\Delta\lambda} R^0(\lambda) \cdot L_\lambda^{(b)}(\lambda) \cdot d\lambda / \Delta t \tag{11.40}$$

$$\approx R^0(\lambda_0) \cdot L_\lambda^{(b)}(\lambda_0) \cdot \Theta / \Delta t$$

where $L_\lambda^{(b)}(\lambda)$ is given by eq. (11.39), λ_0 is some wavelength within the band $\Delta\lambda$, and Θ is the generalized throughput defined in connection with equations (11.3) and (11.4). We assume either that the spectral pass band $\Delta\lambda$ of the radiometer is sufficiently narrow that $L_\lambda^{(b)}(\lambda)$ can be taken as constant [as in the final expression of eq. (11.40)], or in any case that different points of the detector are not exposed to different spectral components of the incoming flux, leading, for example, to the possibility of local detector saturation.

Apart from a constant factor, then, eq. (11.40) gives the dependence of Y upon T. However, the evaluation of the λ-integral of that equation requires knowledge of the dependence of the responsivity R^o upon λ. This dependence will usually be available only if the spectral response of the instrument is dominated by a spectral filter function or mechanical slit function whose shape can be measured or estimated independently of the detector and electronics of the radiometer. In this case, or if the pass band $\Delta\lambda$ is known to be very narrow, we can calculate the relative dependence of Y upon T. We then can proceed to evaluate the unknown coefficients in a suitably chosen response function from measurements of the radiometer output at various blackbody temperatures using

$$f(S') = Y(T). \tag{11.41}$$

Although, in principle, as in the case of the inverse-square law, it is not essential that all of the physical parameters defining the integral in eq. (11.40) be known, it is difficult to think of a realistic situation in which the evaluation of such parameters from the radiometric measurements simultaneously with the evaluation of the coefficients of $f(S')$ would be useful.

The difficulty with using blackbody sources below the gold point in the visible is that, for tenth-percent radiance accuracy, wavelengths must be accurate to 10^{-2} [nm], or better, and the measured thermodynamic temperature [in terms of which the Planck function, eq. (11.39), is expressed] must be more accurate than the IPTS value (which is the best contemporary approximation to the thermodynamic temperature).

PREDICTABLE ATTENUATORS.

Absorbing Filters

There are a number of ways of predictably attenuating a beam and we will discuss them together here because they have many common features. We will first consider absorbing filters, consisting of different thicknesses of colored glass, or filters composed of transparent cells containing pure fluids or liquid solutions. The internal spectral transmittance of such a material is given by

$$\tau^{(i)}(\lambda) = \exp[-a(\lambda)\cdot\ell] \tag{11.42}$$

where $a(\lambda)$ is the spectral linear absorption coefficient and ℓ is the thickness of the absorbing layer. The overall or external filter spectral transmittance is then obtained from the internal spectral transmittance by allowing, in addition, for reflections (and interreflections) at the front and back surfaces of the filter:

$$\tau(\lambda) = \frac{[1-\rho(\lambda)]^2 \cdot \tau^{(i)}(\lambda)}{1-\rho^2(\lambda)\cdot[\tau^{(i)}(\lambda)]^2} \ . \qquad (11.43)$$

In this equation, $\rho(\lambda)$ is this interface spectral reflectance per surface, and we have assumed plane-parallel surfaces and equal reflectances at front and back. For glass-air interfaces in the visible, $\rho(\lambda)$ can usually be taken as 0.04. (We have ignored the usually small contributions from the glass-liquid interfaces in liquid-cell filters.) For a derivation of eq. (11.43) and more details, see [11.17].[1] In solutions, the spectral absorption coefficient can often be approximated by

$$a(\lambda) = a_c(\lambda)\cdot c \qquad (11.44)$$

where c is the concentration of solute and $a_c(\lambda)$ is the spectral absorption coefficient per unit depth and per unit concentration of solute, which generally depends upon the solvent as well as upon the solute. Equation (11.44) is known as Beer's law. Like all the laws which predict a linear dependence of a solution property upon concentration, Beer's law will frequently fail, especially at high concentrations.

The spectral linear absorption coefficients $a(\lambda)$ and $a_c(\lambda)$ can be found in the literature [11.17] for many systems. Thus, by varying the thickness or concentration of these systems we can construct filters with a certain degree of predictability. A measurement of the output of the test radiometer, when one of these filters is in the optical path, yields an equation

$$f(S') = C\cdot\tau(\lambda) \qquad (11.45)$$

where $f(S')$ is a suitable response function, such as eq. (10), and C is a proportionality factor which, assuming a constant primary flux source, will be the same for all measurements. If filters with enough different values of $\tau(\lambda)$ are available, a sufficient number of measurements can be taken to solve these equations for the relative values of the unknown coefficients in the selected response-function form. Obviously, varying the solute concentration is the easiest way of providing the wide range of transmittances desirable for response-function measurements. The parameters $a(\lambda)$, $a_c(\lambda)$, and $\rho(\lambda)$ can be evaluated simultaneously with the coefficients of $f(S')$, if necessary. As usual, the value of C and any multiplicative scaling factor in $f(S')$ can only be obtained from supplemental measurements, including at least one measurement of a source of known flux.

Three-Polarizer Attenuators

The three-polarizer attenuator has already been discussed in Chapter 6 (p. 35) [11.2]. This attenuator consists of a rotatable polarizer sandwiched between two polarizers oriented with their polarization directions parallel [11.19-11.21]. The transmittance of the device is a function of the

[1] Note that the spectral (linear) absorption coefficient given in ref. [11.17] is a factor of $1/\ln(10)$ times the coefficient $a(\lambda)$ in eq. (11.42). We follow the ANSI/IES RP-16-1980 [11.18] notation for the spectral absorptance $\alpha(\lambda) = 1-\tau(\lambda) = 1-\exp[-a(\lambda)\cdot\ell]$ in non-scattering, linearly absorbing media. See chapter on Spectrophotometry and its appendix on nomenclature (in preparation as of March 1984).

angle of rotation of the middle polarizer (or half-wave plate). For many sheet polarizers the principal transmittances, k_1, and k_2, are known and, assuming that the three polarizers are identical and that their retardance is negligible, then the expression in eq. (6.53) [11.2] for the transmittance of the three-polarizer train reduces to

$$\tau = (k_1+k_2)\cdot[(k_1-k_2)^2\cdot\cos^4\phi + k_1\cdot k_2]/2 \tag{11.46}$$

where ϕ is the angular orientation of the middle polarizer. Measurements of radiometer output for various angular settings of such an attenuator will again lead to equations of the form of eq. (11.45), with $\tau(\phi)$ given by eq. (11.46). These equations can be solved for the unknown coefficients in $f(S')$, and for k_1 and k_2, if they are also unknown.

The parameters k_1 and k_2, like the parameters $a(\lambda)$ and $a_c(\lambda)$ of the previous attenuation method are likely to exhibit a relatively strong wavelength dependence. This means that linearity assessment, using either of these methods, is best carried out with narrow spectral pass bands. Otherwise a problem, similar to that discussed in connection with blackbody sources, is encountered in which an integral over λ, like that of eq. (11.40), must be evaluated. Unlike the blackbody situation, however, in these cases, if the radiometer is broad band, a narrow-spectral-band primary source can be used to avoid the integration problem.

Calibrated Apertures

Apertures of known area can be used in a spatially uniform beam to produce predictable beam attenuation. In irradiance measurements the apertures are used to limit the size of a diffuse source; in radiance measurements they are placed in a collimated beam, for example, between two lenses which, together, image the source plane onto the radiometer field stop. Frequently, many apertures of different sizes, each with its own shutter, are mounted on the same aperture plate. Then measurements are taken with the radiometer while different apertures are opened singly, in pairs, three at a time, etc. If the measurement S'_k is acquired when apertures of total area A_k are open then the flux is proportional to

$$f(S'_k) = C\cdot A_k \tag{11.47}$$

for each measurement. The constant C is a measure of the flux per unit area at the aperture and is assumed to be the same for all measurements. These equations are the same as eq. (11.45) and can be solved for the relative values of the coefficients f_i. As in all of these techniques, it is preferable to take a large excess of data and solve the system of equations by least squares minimization.

If the apertures are arranged as described above, so that more than one can be opened simultaneously, then the device functions as a beam adder and the data can be analyzed without regard to the known aperture areas. Actually this is the usual mode of operation [11.7-11.10, 11.22, 11.23]. If we define an occupation variable ξ_i, where $\xi_i = 1$ if the ith aperture is open during a measurement and $\xi_i = 0$ if it is closed, then a measurement S' with more than one aperture open satisfies

$$f(S') = \xi_1\cdot f(S'_1) + \xi_2\cdot f(S'_2) + \xi_3\cdot f(S'_3) + \ldots \tag{11.48}$$

where S_i' is the measured signal when aperture i alone is open. If enough apertures are available a large enough set of equations can be constructed to solve for the coefficients of the response function $f(S')$. Otherwise the measurements must be repeated at additional flux levels of the primary source, achieved by inserting filters, for example, to provide enough independent equations. This, then, is a generalization of the analysis of beam-addition measurements for more than two beams. It provides a useful means of testing some of the uncertainties in the use of calibrated apertures: beam uniformity, interference effects (diffraction), and accuracy of aperture-area measurements.

Spinning Sector Wheels

Sector wheels are simply chopper wheels in which the angular openings are carefully measured and possess sharp edges exactly aligned in the radial direction [11.5]. When rotated rapidly such wheels attenuate a beam in the time dimension much as multiple calibrated apertures do in the solid angle or area dimension. In both cases a flux distribution is constructed which is non-uniform in some dimension and, as indicated earlier in this chapter, if the non-linear contributions are dependent upon this distribution and not simply upon S' then the function $f(S')$ is not sufficient to characterize the radiometer response completely. In the case of variable apertures, the uniformity of the beam is easily restored with a diffuser but we know of no practical means of restoring temporal uniformity to a chopped beam[1]. Specifically, the problem with sector wheels is that a fast detector, such as a photomultiplier, will usually be able to follow the on-off beam interruptions, and the steady observed radiometer output signal is produced by some later stage in the electronics which smooths the detector pulses. Only the electronics following this smoothing stage experiences the intermediate level of signal which the sector wheel is designed to produce; all the preceding electronics, including the detector, experiences only the two extreme on or off signal levels. On the other hand, for an intrinsically slow detector, such as a thermopile [11.25], the smoothing is performed within the detector itself and the entire radiometer responds to the intended time-averaged sector-wheel transmittance. Thus the use of sector wheels does not provide a test of overall radiometer linearity unless one is satisfied that the initial step in the radiant power conversion process is slow compared to the chopper period. Notice that the requirement that the detector temporal resolution be inadequate to resolve the chopping frequency for sector wheels is just the opposite of the requirement for application of the AC technique described in connection with beam addition. There it was necessary that the detector resolve the chopped beam enough to produce an accurately measurable AC output.

An approach similar to the sector-wheel technique is to generate a pulsed source electronically by supplying pulsed power to a light emitting diode (LED) or, for example, by interrupting a steady source with a Kerr cell. The pulse width is generally held fixed and the pulsing frequency is varied to achieve a varying average flux level proportional to this frequency. This is in contrast to the sector-wheel technique in which, in effect, the flux-pulse width is varied while the pulse frequency is nominally fixed. Again one should confirm that the radiometer response to this kind of pulsing flux is

[1]R. D. Lee developed sector wheels in which the angular openings penetrate to the center of the wheels, similar to the sector wheel described in ref. [11.24]. These wheels are supported and rotated by their rims and are centered on the axis of an axisymmetric optical system. Such sector wheels used with a radiometer of uniform spatial responsivity should not encounter the objection of temporal non-uniformity, provided the sector wheels are not imaged on the detector.

equivalent to its response to the kinds of measurements for which the response function is being determined. In addition, one should check that the flux-pulse shape doesn't change with frequency, for example, due to heating of the LED at higher duty cycles.

Of course there are situations, such as studies of the mechanism of detector behavior, where there is no question of trying to duplicate as closely as possible some practical routine testing condition. Many linearity characterization techniques, especially those using choppers or pulsed sources, provide valuable clues about detector mechanisms and are employed for that purpose [11.26-11.28]. We have assumed in this chapter, however, that the purpose of the linearity characterization is to provide a unique response function for an existing radiometer as used for a specific testing or measurement program. In that application one must, as usual, be cautious in applying radiometer calibrations or characterizations to measurements taken under significantly different experimental conditions.

PHOTON COUNTING. We have discussed the assessment of radiometer linearity by purely radiometric measurements and by methods in which radiometric measurements are supplemented by some other measurements such as distance or temperature. Now we wish to consider, in somewhat more detail, those detector systems mentioned earlier in this chapter where the response function can be predicted without any radiometric measurements. Let us first examine photon counting.

When a photon is absorbed by a photomultiplier, it has a certain probability of generating a free electron at the photocathode. This electron is accelerated by the electric fields within the photomultiplier to produce, by a series of collisions, a shower of electrons which forms a photomultiplier output pulse. In DC operation the radiometer output signal is a function of the time-averaged current in these photomultiplier output pulses. In photon counting, the individual current pulses are simply counted as separate events, and the radiometer output is the total count of such events during the radiometer measurement time Δt. Unfortunately, if two photoelectrons are generated nearly simultaneously, their individual current pulses may not be resolvable by the amplifying and counting circuitry and the second pulse may be ignored. The minimum time between photoelectrons which is required for proper counting of all pulses is called the dead time of the instrument. Thus, in photon counting, the primary source of (unintentional) non-linearity is the loss of photoelectron counts during dead times.

We will associate the quantity Y with the mean photoelectron generation rate. That is, Y would be the counting rate of the radiometer if the dead time were zero. Each photoelectron pulse will be counted provided it does not occur during the dead time following the previous pulse. So the actual counting rate which will be observed is given by

$$S' = Y \cdot p(t_d, Y) \tag{11.49}$$

where $p(t_d,Y)$ is the probability of no photoelectrons being generated within the dead time t_d. We have assumed that no further mathematical operations are performed on the signal by the radiometer [$G(u) = u$, in the notation of eq. (11.7)]. The temporal distribution of photoelectrons depends upon the temporal distribution of photons arriving at the photomultiplier and this, in turn, depends upon the coherence state of the radiation there. For highly coherent radiation the photons obey a Poisson distribution and the probability that no second photoelectron will be produced during the dead time is given by:

$$p(t_d, Y) = \exp(-t_d \cdot Y) \ . \tag{11.50c}$$

For a thermal light source (if the photomultiplier were in the interior of a blackbody) the photons obey the Bose-Einstein distribution and

$$p(t_d, Y) = (1 - t_d \cdot Y)^{-1} \ . \tag{11.50t}$$

Thus we have the two extreme cases:

$$S' = Y \cdot \exp(-t_d \cdot Y) \quad \text{(coherent)} \tag{11.51c}$$

and

$$S' = Y/(1 - t_d \cdot Y) \quad \text{(thermal)}. \tag{11.51t}$$

These equations can be expanded and solved for Y:

$$Y = S' + t_d \cdot S'^2 + (3/2) \cdot t_d^2 \cdot S'^3 + \ . \ . \ . = f(S') \tag{11.52c}$$

and

$$Y = S' + t_d \cdot S'^2 + t_d^2 \cdot S'^3 + \ . \ . \ . = f(S') \ . \tag{11.52t}$$

Thus, the response function $f(S')$ is non-linear and, up to terms quadratic in S', is identical for both models. For low signal levels where the first two terms will be sufficiently accurate, then, an electronic measurement of the dead time t_d will suffice to determine the response function, including a good estimate of the contribution of the third term. We have assumed no dark current in our derivation. Although photon counting significantly reduces the dark count, it is unlikely to eliminate it completely. A dark contribution will simply be a constant offset to Y, which will result in the addition of a constant term in $f(S')$ in eqs. (11.52).

SILICON PHOTODIODES. Detectors for radiometers are generally classified either as photon detectors or thermal detectors. By this we mean that the detector output results either from changes in the number (or mobility) of charge carriers produced by the incident photons or from temperature changes caused by the incident radiation. With proper design and under suitable conditions either kind of detector can serve as an absolute detector whose responsivity can be determined independently of any source of known radiometric output. Thus such absolute detectors furnish another approach to linearity characterizations. We will first consider the recently developed, absolute, silicon photodiode detector.

When photons fall on a photon detector, some will be reflected, some absorbed, and some possibly transmitted through the detector without any interaction. Of those that are absorbed, some will give rise to the charge carriers which can ultimately be measured by the external electronics. Probably a number of materials possess a potential range of operating conditions within which each absorbed photon will almost always produce exactly one free electron or electron-hole pair which, in turn, can be collected by the external electrical circuitry before being lost through recapture or recombination.

Under these conditions such a detector possesses a linear responsivity which is essentially the absorptance of the detector. The difficulty is to find the range of wavelengths, flux levels, and electronic conditions under which such ideal behavior is exhibited, then to measure the absorptance so that the total incident flux can be related to the generation of charge carriers, and finally to measure electronically the fraction of generated charge carriers which is actually collected in normal operation.

At the present time the only photon detectors which are sufficiently well understood to permit this kind of characterization are silicon photodiodes operated as current sources with the external load being a virtual short circuit. The evaluation of the responsivity by means of reflectance measurements and simple electrical measurements has been described in a series of papers by Geist, Zalewski, and Schaefer [11.29-11.33]. For our purposes, however, thinking of a silicon diode as a component of a radiometer whose overall responsivity is needed, the responsivity of the detector alone is largely irrelevant. It is sufficient to know that, in the wavelength range from 400 [nm] to 800 [nm] and for flux densities of 10 [$mW \cdot cm^{-2}$] or less, the responsivity of such absolute silicon diodes appears to be linear to within present-day measurement accuracy (0.01%) and to vary from about 0.2 [$A \cdot W^{-1}$] at 400 [nm] to 0.6 [$A \cdot W^{-1}$] at 800 [nm]. If the radiometer is used within these bounds and if the amplifier and readout electronics are linear (which can be checked electronically), then the radiometer is linear. As indicated earlier in this chapter, the radiometrist of little faith may still wish to perform some limited linearity tests. For this purpose, probably either the pulsed LED technique mentioned above or the verification of Bouguer's law (that the transmittance of a series of filters is the product of their individual transmittances) using neutral density filters would suffice. Although neither test is wholly satisfactory by itself, either one taken in conjunction with the reputation for linearity of these detectors [11.7] should convince any but the most dedicated skeptic.

ELECTRICALLY CALIBRATED DETECTORS. The other class of absolute detector is the black absorber whose temperature is monitored while it is subjected alternately to heating by the radiant beam to be measured and to electrical heating. A direct measure of the radiant power is the electrical power required to produce the same thermal behavior as the incident radiant beam produces. Such detectors range from small, portable, pyroelectric radiometers[1] [11.34,11.35], which are commercially available, to research installations containing black cavity detectors operated in high vacuum at liquid helium temperatures [11.36,11.37]. Among the major uncertainties in the use of electrically calibrated detectors are nonblackness and the fact that the radiant heating occurs in a slightly different physical location in the device than the electrical heating. However, careful design and thorough instrument characterization permit the corrections for such imperfections to be determined with sufficient accuracy to yield absolute radiometric measurements over broad spectral and flux level ranges with overall uncertainties of 0.1%. Theoretical and practical considerations in the design of electrically calibrated detectors and techniques for measuring these corrections are given in references [11.38] and [11.39]. These detector corrections, based upon an analysis of the optical, electrical, and thermal properties of the detector, constitute a complete determination of the responsivity of the detector. To the extent that some of the characterization procedures require the measurement of radiant fluxes, one could object that electrically calibrated detectors do not represent an *ab initio* means of establishing a detector response

[1] In the case of pyroelectric detectors it is the rate of temperature change which is sensed instead of temperature itself.

function because a quantitative detector must already be available. While, strictly speaking, this is true, the corrections involved are so small (of the order of a percent) that only a relatively low order of accuracy is required to produce a high accuracy response function. Thus, an uncharacterized electrically calibrated detector can be used for its own characterization [11.38,11.39].

GENERAL CONSIDERATIONS on LINEARITY CHARACTERIZATION TECHNIQUES. Many devices or installations designed to evaluate the response function of a radiometer (or simply test for linearity) span about a factor of 10^3 in flux levels. One of the reasons for this is that this is roughly the maximum signal-level range which can be covered with radiometric precision (0.1%) when one is limited to contemporary 6-digit readout devices. To cover a wider flux level range, it is usually necessary to change an amplifier feed-back resistor, the DVM range setting, or make some other detector system configuration change (assuming that such changes are under the control of the radiometrist) that may modify the system response function. Thus one will generally be satisfied with different overlapping response functions for different detector-system configurations, each of which covers a flux level range of about 10^3. These individual response functions may appear to show little resemblance to one another unless they are based upon a valid physical model in which the coefficients have physical significance. However, each of these overlapping response functions should be correct for radiometric measurements within the useful range of the detector-system configuration to which it applies and, if the same measurement is carried out under two different detector-system configurations, the fluxes computed using the corresponding two response functions should agree satisfactorily.

In eqs. (11.22) and (11.23) we showed one way to define a linear signal S in terms of the response function $f(S')$ and to write a measurement equation for S. The value of S, computed according to eq. (11.22) from the radiometer output signal S', is, strictly speaking, only correct for the range of values of L_λ and for the distribution of L_λ with respect to position, direction, polarization, wavelength, and time which was used in evaluating the coefficients f_0, f_1, etc. Conceivably, a totally different field might lead to a different set of coefficients f_i. If there is reason to suspect that this may happen, then a linearity characterization technique should be chosen which provides a calibration-field distribution in the pertinent dimension (or dimensions) which matches the anticipated distribution of the unknown fields to be measured. This may rule out, for example, pulsed techniques if steady sources are to be measured, or spectrally continuous sources if lasers are to be measured. In addition to this consideration, there are some obvious limitations of some of the techniques, some of which have already been touched upon, such as the unreliability of Beer's law and the difficulties of using variable temperature blackbodies in the visible. Beyond these considerations, there is little reason for preferring one technique over another; all are capable of covering a useful range of flux levels with good accuracy.

Probably the most popular technique is beam addition in which apertures are used to control the beam flux levels. A range of flux levels of three decades or so can readily be spanned and the accuracy attainable with a well-designed system is unsurpassed. As suggested earlier, a major requirement for the applicability of this technique is that the radiometer response be insensitive to beam direction. Second place in popularity probably belongs to the inverse-square technique. This technique is frequently mentioned in passing in the literature but never described in any detail, perhaps because the applications, precautions, and equipment requirements seem so self-evident. Reference [11.16] provides a discussion of some of these aspects in the context of an inverse-fourth apparatus. For a different review of linearity calibration techniques including a review of techniques for improving linearity see reference [11.40].

SUMMARY of CHAPTER 11. In this chapter we have considered only the simplest possible kind of non-linearity in which the radiometer signal output behaves as though it has suffered a simple non-linear amplification. Thus our treatment cannot handle fatigue effects or situations where the radiometric measurements require comparing different relative spatial or temporal distributions of detector irradiation which may, for example, lead to accidental local detector saturation at one point or at one moment of some measurements. However, this treatment will generally be valid in the absence of fatigue if the non-linearity is produced in the electronics during amplification of the collected electrical signal or if the relative spatial and temporal distributions of irradiation on the detector surface are the same for all measurements.

For radiometer non-linearity of this type it is convenient to write a measurement equation

$$Y = \int\ldots\int R^{o}\cdot L_{\lambda}\cdot dx\cdot\ldots\cdot dt\cdot d\lambda/\Delta t \tag{11.53}$$

in terms of a linearized radiometer flux responsivity R^{o} which is essentially the true non-linear responsivity except for its dependence upon the level of incident flux. It could, for example, be taken as

$$R^{o}(x,\ldots\lambda) = R(x,\ldots,\lambda,L_{\lambda}^{o}) \tag{11.54}$$

where L_{λ}^{o} is any arbitrary level such as $L_{\lambda}^{o} = 0$. The integral defining Y is proportional to the flux incident on the radiometer. Y itself can be interpreted as the signal output of an imaginary radiometer which differs from the true one primarily by being linear. Although it is customary to think of the true output signal S' as being produced by the incident flux and therefore to write S' as some function of Y, it is more useful for our purposes to invert this functional relationship and set

$$Y = \int\ldots\int R^{o}\cdot L_{\lambda}\cdot dx\cdot\ldots\cdot d\lambda/\Delta t = f(S') \tag{11.9}$$

We call $f(S')$ the response function of the radiometer.

The main concern of this chapter is the examination of several techniques for determining the response function $f(S')$. In general a technique involves assuming some functional form for $f(S')$ which depends upon a small number of unknown parameters. For example, for a radiometer believed to be nearly linear, an appropriate form would be

$$f(S') = f_{o} + f_{1}\cdot S' + f_{2}\cdot S'^{2} + \ldots + f_{n}\cdot S'^{n}. \tag{11.10}$$

Then suitable measurements are made of the radiometer output signal S' under conditions in which the various fluxes incident on the radiometer are related to each other or to other non-radiometric measurements. Since Y, or $f(S')$, is proportional to the incident flux, this permits writing a set of simultaneous equations in the unknown response-function parameters f_i. Usually an excess of measurements is desirable in order to allow least-squares fitting and an estimation of the adequacy of the selected functional form for $f(S')$. Once the parameters f_i are known, the linearized radiometer output Y can be calculated from eqs. (11.9) and (11.10) for any measurement S', and this reduces the non-linear radiometer problem essentially to that of a linear radiometer obeying the measurement equation, eq. (11.53).

One of the most common techniques uses the superposition or addition of two or more beams incident on the radiometer. Each beam is measured alone by the radiometer, with the other beam extinguished, and then the combined superposition is measured. If the response function is assumed to be of the form of eq. (11.10), then such measurements lead to simple linear simultaneous equations in the unknown coefficients f_i. The beam superposition is usually achieved using beam splitters, or two or more beam apertures which can be opened or closed independently. For routine use, beam adders based upon either of these approaches can be readily automated.

In principle one can obtain similar information with fixed, but unknown, attenuators, such as neutral density filters, by using the radiometer to measure the apparent transmittances of the attenuators singly, in pairs, etc. Unfortunately, as a stand-alone technique, when applied to nearly linear radiometers, the method yields non-linear simultaneous equations and ambiguous results. It is best employed in combination with some other technique such as beam addition, mentioned above, or the inverse-square law technique, as an easy experimental means of extending the range of a response-function calibration or of introducing additional intermediate points.

Among techniques for determining the response function which depend upon a non-radiometric scale measurement, probably the most commonly used is the inverse-square technique. Here the radiometer is used to measure the apparent irradiance from a steady source as a function of the distance from the source to the radiometer. Again one assumes a functional form for the response function such as eq. (11.10) and attempts to fit the measurements to

$$f(S') = C/\ell^2 \tag{11.55}$$

where ℓ is the source-to-radiometer distance. For good accuracy, corrections must be included [eq. (11.36)] for the sizes and shapes of the source and radiometer entrance aperture. It is also possible to include additional parameters such as an unknown zero offset for the source-to-radiometer distance under certain circumstances and to evaluate these parameters from the radiometric measurements along with the unknown parameters of the selected response function.

Another common technique used extensively for calibrations in the infrared is to measure the radiometer output signal as the temperature of a blackbody source is varied. Using the Planck equation [eq. (11.39)] one can calculate the spectral radiance or, if the geometry is known, the spectral irradiance at the radiometer from measurements of the thermodynamic temperature of the blackbody. At temperatures below about the gold point (1337 [K]) the relationship between the thermodynamic temperature and the response of thermocouples or platinum resistance thermometers is well enough known to permit accurate radiometric calibration. Unfortunately, in the visible at these low temperatures, blackbody radiation is too feeble and too strongly wavelength dependent for this technique to be practical.

Other techniques which involve non-radiometric measurements are to vary the incident flux predictably by the use of calculable filters (calculating surface reflectance from the index of refraction, and absorption using Beer's law, etc.), polarizers at various orientations, limiting apertures of known size, or rapidly spinning sector disks with known angular openings. The application of most of these ideas and the associated precautions are obvious. Spinning sector disks, however, present a not-so-obvious difficulty: since a sector disk at any instant is usually either admitting the full flux or none at all, it really is not providing the kind of intermediate level, steady flux at the detector which is implied by the signal output. Consequently caution must be exercised in applying calibrations based upon pulsed techniques to steady-state-measurement situations and vice versa.

Finally, we come to linearity calibrations which are really not calibrations at all in any radiometric sense since this aspect of the calibration is more-or-less avoided by a detailed analysis of the physics of the detection process. First there is photon counting in which, ideally, absorbed photons are individually counted one-by-one. All of the non-ideal effects except the quantum efficiency of the detector can be obtained by electronic measurements. Second, there are electrically calibrated absolute detectors in which the thermal effect of the absorption of radiation is imitated by electrical heating of the same detector. The 'output' of the radiometer is the electrical power required to match the radiant heating effects. Again most of the corrections for the small difference in heat-loss mechanisms and the like between radiant and electrical heating can be predicted from the results of separate measurements of the mechanical, electrical, and thermal characteristics of the device. And last, there are silicon photodiodes which exhibit 100% internal quantum efficiencies over a wide range of operating conditions. The determination of the admissible range of operating conditions -- wavelengths, electrical bias, etc., -- requires separate electronic and optical measurement procedures.

References

[11.1] Henry J. Kostkowski and Fred E. Nicodemus, "An Introduction to the Measurement Equation", Chapter 5 of "Self-Study Manual on Optical Radiation Measurements: Part I--Concepts", Nat. Bur. Stand. (U.S.) Tech. Note 910-2, 118 pages (February 1978) pp. 58-92.

[11.2] John B. Shumaker, "Distribution of Optical Radiation with Respect to Polarization", Chapter 6 of "Self-Study Manual on Optical Radiation Measurements: Part I--Concepts", Nat. Bur. Stand. (U.S.) Tech. Note 910-3, 62 pages (June 1977).

[11.3] Diamond E. Erminy, "Scheme for Obtaining Integral and Fractional Multiples of a Given Radiance", J. Opt. Soc. Am., Vol. 53, No. 12 (Dec. 1963) pp. 1448-1449.

[11.4] Alfred Reule, "Testing Spectrophotometer Linearity", Appl. Optics, Vol. 7, No. 6 (June 1968) pp. 1023-1028.

[11.5] H. Kunz, "Representation of the Temperature Scale above 1337.58 K with Photoelectric Direct Current Pyrometers", Metrologia, Vol. 5, No. 3 (July 1969) pp. 88-102.

[11.6] L. Coslovi and F. Righini, "Fast Determination of the Nonlinearity of Photodetectors", Appl. Optics, Vol. 19, No. 18 (15 Sept. 1980) pp. 3200-3203.

[11.7] W. Budde, "Multidecade Linearity Measurements on SI Photodiodes", Appl. Optics, Vol. 18, No. 10 (15 May 1979) pp. 1555-1558.

[11.8] Klaus D. Mielenz and Ken L. Eckerle, "Spectrophotometer Linearity Testing Using the Double-Aperture Method", Appl. Optics, Vol. 11, No. 10 (Oct. 1972) pp. 2294-2303.

[11.9] Kurt Bischoff, "Die Messung des Proportionalitätsverhaltens von Strahlungsempfängern über grosse Bestrahlungsstärkebereiche", Z. Instr., Vol. 69 (1961) No. 5, pp. 143-147.

[11.10] C. L. Sanders, "A Photocell Linearity Tester", Appl. Optics, Vol. 1, No. 3 (May 1962) pp. 207-211.

[11.11] Michael A. Lind, "Measurement of the Absolute Spectral Response of Detectors", Proc. of the 1976 EOSD Conf. (Sept. 1976) pp. 55 ff.

[11.12] R. C. Hawes, "Technique for Measuring Photometric Accuracy", Appl. Optics, Vol. 10, No. 6 (June 1971) pp. 1246-1253.

[11.13] Fred E. Nicodemus, "Physically Defining Measurement-Beam Geometry by Using Opaque Barriers", Chapter 9 of "Self-Study Manual on Optical Radiation Measurements: Part I--Concepts", Nat. Bur. Stand. (U.S.) Tech. Note 910-4, 134 pages (June 1979) pp. 91-119.

[11.14] Fred E. Nicodemus, "More on the Distribution of Optical Radiation with respect to Position and Direction", Chapter 4 of "Self-Study Manual on Optical Radiation Measurements: Part I--Concepts", Nat. Bur. Stand. (U.S.) Tech. Note 910-2 (Feb. 1978) pp. 1-57.

[11.15] Fred E. Nicodemus, "Projected Solid Angles, Throughputs, and Configuration Factors", Appendix 3 (to Chapters 4 & 5) of "Self-Study Manual on Optical Radiation Measurements: Part I--Concepts", Nat. Bur. Stand. (U.S.) Tech. Note 910-2 (Feb. 1978) pp. 93-100.

[11.16] Dennis A. Swyt and James G. LaRock, "Inverse-Fourth Apparatus for Photometric Calibrations", Rev. Sci. Instrum., Vol. 49, No. 8 (Aug. 1978) pp. 1083-1089.

[11.17] J. A. Dobrowolski, "Coatings and Filters", Section 8 in "Handbook of Optics", sponsored by the Optical Society of America (McGraw-Hill Book Company, New York, 1978) pp. 8-1 to 8-124.

[11.18] American National Standard, "Nomenclature and Definitions for Illuminating Engineering", RP-16, ANSI/IES RP-16-1980 [Revision of ANSI Z7.1-1967 (R 1973)], Illuminating Engineering Society of North America, 345 East 47th Street, New York, NY 10017.

[11.19] H. E. Bennett, "Accurate Method for Determining Photometric Linearity", Appl. Optics, Vol. 5, No. 8 (Aug. 1966) pp. 1265-1270.

[11.20] G. Ruffino, "Precise Continuous Optical Attenuator", J. Res. Nat. Bur. Stand. (U.S.), Vol. 74C, Nos. 1 & 2 (Jan.-June 1970) pp. 9-13.

[11.21] K. D. Mielenz and K. L. Eckerle, "Accuracy of Polarization Attenuators", Appl. Optics, Vol. 11, No. 3 (Mar. 1972) pp. 594-603.

[11.22] J. S. Preston and F. W. Cuckow, "A Photoelectric Spectrophotometer of High Accuracy", Proc. Phys. Soc. (London), Vol. 48 (1936) pp. 869-880.

[11.23] Mamoru Nonaka and Takeo Kashima, "Linearity Characterisics of Multiplier Phototubes", Jap. J. Appl. Phys., Vol. 2, No. 12 (Dec. 1963) pp. 785-791.

[11.24] G. M. Pool, "Die Messung der Absorption im ultravioletten Spektrum", Z. Physik, Vol. 29 (1924) No. 6, pp. 311-314.

[11.25] G. Cappuccio, A. D'Amico, S. D'Angelo, and C. Ranghiasci, "Photometric Linearity Test for IR Spectrophotometers by means of a Rotating Sector Disk Attenuator", Appl. Optics, Vol. 21, No. 20 (15 Oct. 1982) pp. 3619-3622.

[11.26] H. J. Jung, "Eine dynamische Methode zur Messung der Nichtlinearitäten fotoelektrischer Strahlungsempfänger", Z. angew. Physik, Vol. 30 (1971) No. 5, pp. 338-341.

[11.27] H. J. Jung, "Kompensation von Nichtlinearitäten bei photoelektrischen Strahlungsmessungen", Z. angew. Physik, Vol. 31 (1971) No. 3, pp. 170-176.

[11.28] G. Sauerbrey, "Linearitätsabweichungen bei Strahlungsmessungen mit Photovervielfachern", Appl. Optics, Vol. 11, No. 11 (Nov. 1972) pp. 2576-2583.

[11.29] E. F. Zalewski and J. Geist, "Silicon Photodiode Absolute Spectral Response Self-Calibration", Appl. Optics, Vol. 19, No. 8 (15 April 1980) pp. 1214-1216.

[11.30] Jon Geist, "Silicon Photodiode Front Region Collection Efficiency Models", J. Appl. Phys., Vol. 51, No. 7 (July 1980) pp. 3993-3995.

[11.31] J. Geist, E. F. Zalewski, and A. R. Schaefer, "Spectral Response Self-Calibration and Interpolation of Silicon Photodiodes", Appl. Optics, Vol. 19, No. 22 (15 Nov. 1980) pp. 3795-3799.

[11.32] A. R. Schaefer and J. Geist, "Spatial Uniformity of Quantum Efficiency of a Silicon Photovoltaic Detector", Appl. Optics, Vol. 18, No. 12 (15 June 1979) pp. 1933-1936.

[11.33] A. R. Schaefer, E. F. Zalewski, and Jon Geist, "Silicon Detector Nonlinearity and Related Effects", Appl. Optics, Vol. 22, No. 8 (15 April 1983) pp. 1232-1236.

[11.34] Jon Geist and W. R. Blevin, "Chopper-Stabilized Null Radiometer Based Upon an Electrically Calibrated Pyroelectric Detector", Appl. Optics, Vol. 12, No. 11 (Nov. 1973) pp. 2532-2535.

[11.35] W. R. Blevin and Jon Geist, "Influence of Black Coatings on Pyroelectric Detectors", Appl. Optics, Vol. 13, No. 5 (May 1974) pp. 1171-1178.

[11.36] D. C. Ginnings and M. L. Reilly, "Calorimetric Measurement of Thermodynamic Temperatures Above 0°C Using Total Blackbody Radiation", in "Temperature--Its Measurement and Control in Science and Industry", Vol. 4, Part 1 (1973) pp. 339-348.

[11.37] T. J. Quinn and J. E. Martin, "Radiometric Measurement of Thermodynamic Temperature between 327 and 365 K", in "Temperature--Its Measurement and Control in Science and Industry", James F. Schooley, Editor-in-Chief (American Institute of Physics, New York, 1982) Vol. 5, Part 1, pp. 103-107.

[11.38] Jon Geist, "Fundamental Principles of Absolute Radiometry and the Philosophy of This NBS Program (1968-1971)", Nat. Bur. Stand. (U.S.) Tech. Note 594-1, 59 pages (June 1972).

[11.39] F. Hengstberger, "An Improved Theory of the Instrumental Corrections for Absolute Radiometers", Metrologia, Vol. 13 (1977) pp. 69-78.

[11.40] C. L. Sanders, "Accurate Measurements of and Corrections for Non-linearities in Radiometers", J. Res. Nat. Bur. Stand. (U.S.), Vol. 76A, No. 5 (Sept.-Oct. 1972) pp. 437-453.