SIM Metrology School: Pressure

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Why is pressure important?

Weather prediction

Aviation: Altimetry, air speed, jet engines

Transportation: engine performance

Health

Industrial processes: wafer production

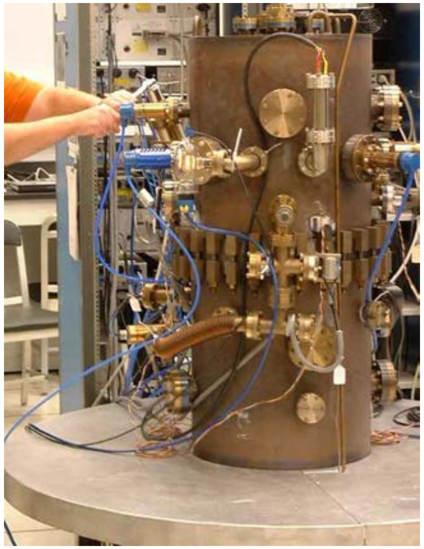
Energy: oil-well pressure, pipeline flow

Pressure is a derived unit

- Pressure = Force / Area = mass x acceleration / area = Mg/A
  - Force has units of kg × m/s^2, Area has units of m^2
- Traceability through mass (kilogram) and length (meter) (gravity requires time)
- Traceability depends on realization method
  - That is, what physical device (standard) establishes the pressure?
  - Does the standard require other measurements (example: temperature)?
- We will focus on Pressure Standards rather than Pressure Measurement

Common Pressure Standards

Piston gauges: 10 kPa to 500 MPa



Vacuum standards: 10^{-7} Pa to 10 Pa (10^{-9} torr to 0.1 torr)

Liquid column manometers: 1 mPa to 360 kPa

Note: barometric pressure is 100 kPa at sea level
Blood pressure is 10 kPa to 30 kPa

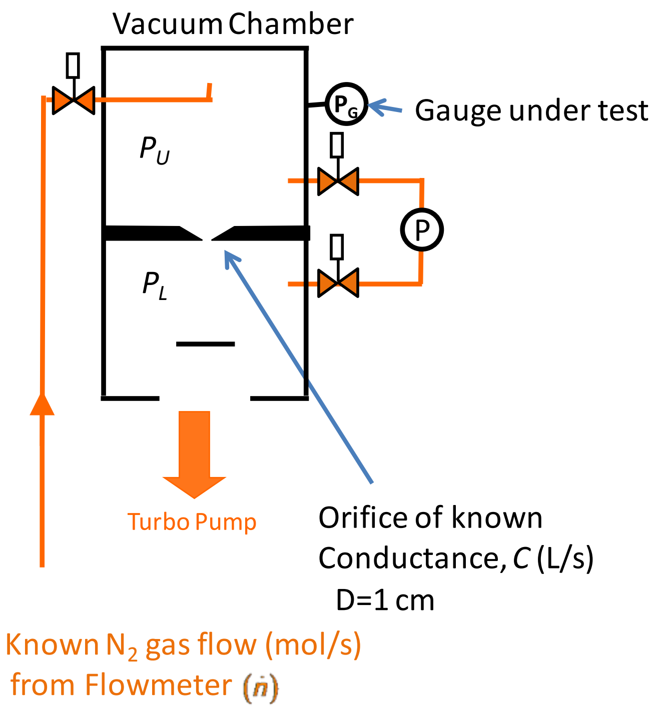
Mercury Manometers: NIST version

- Lowest uncertainties (5.2 ppm, *k*=2)
- Implementation is a length measurement
  - 1 mm Hg = 133 Pa
- Technically difficult to operate and maintain
- Commercial versions cost $500k
- Contain hazardous substance
- Not practical for most labs

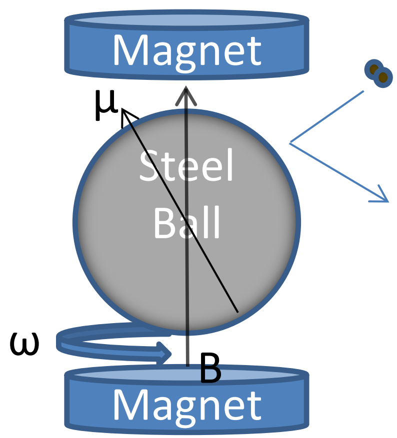
$$\Delta p = \rho g \Delta h$$

Vacuum Standards

NIST Dynamic Expansion Standard



- Uncertainties 0.3 % and up
- Requires expertise in vacuum technology
- Primary method requires gas flow measurement
- Comparative standards can be purchased
  - Vacuum chamber with calibrated gauge
- Not practical for most labs



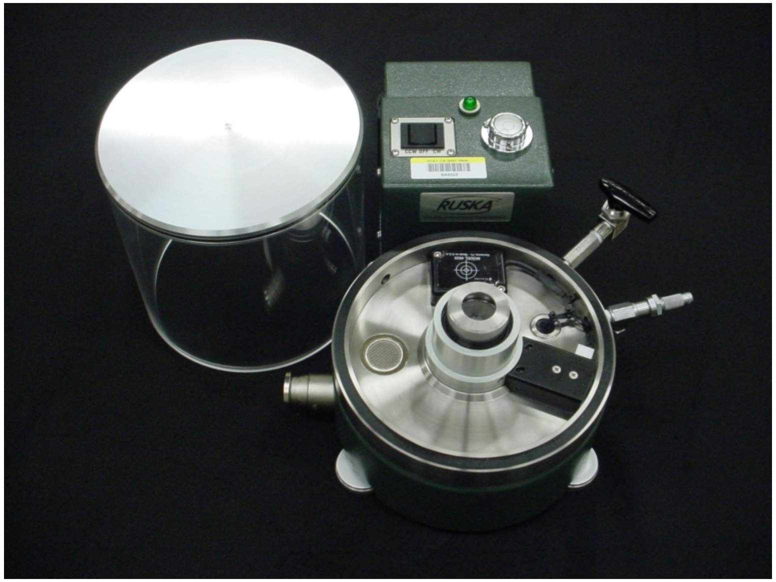
Spinning Rotor Gauge

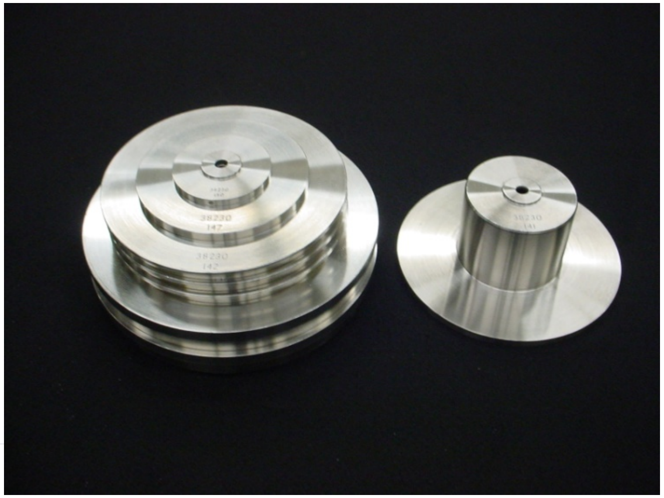
- Steel Ball is suspended & spun in electro-magnetic field
- Pressure determined from deceleration rate
- Range: 10^{-4} Pa to 1 Pa

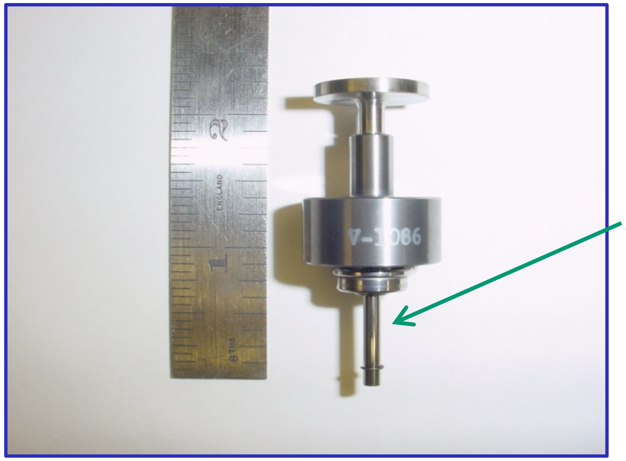
Piston Gauge Pressure Standards: This Course

- Can be used from 10 kPa to 500 MPa, liquid or gas
- Uncertainties (k=2) of 10 ppm to 40 ppm
- Commercially available, but highest quality from small number of manufacturers
- Portable, easy to operate
- Pressure set by increments of mass
- Systems have become more automated in last decade
- Wide level of expertise at National Metrology Institutes, calibration labs, industry in their use
- Piston gauge traceability by 2 methods
  - Dimensional measurement of metrological artifact (piston and cylinder) – or –
  - Comparison to another pressure standards (piston gauge or manometer)
- Piston gauges used to calibrate:
  - Other piston gauges
  - Electronic (or analogue) pressure instruments

Ruska 2465 Piston Gauge Base and Piston-Cylinder Assembly (V-series, 7 MPa)







3.3 mm OD

Various Piston Gauge Platforms (*other names* pressure balance, dead weight gauge, dead weight tester)

Temperature and piston position

Integrated Monitor:
T, h, rotation, fall rate

Automated Mass Handler

- Piston gauges are used to calibrate other piston gauges and electronic transducers.

Piston Gauge Operation and Performance

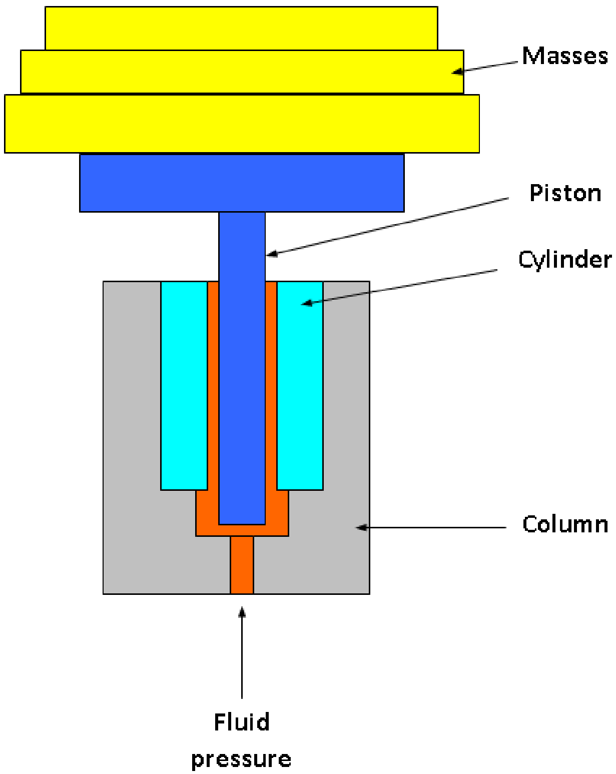
- Piston position: usually midstroke
  - Electronic sensing of position. Fluke 7000 series is built into base.
- Piston and mass rotation: 10 to 30 rpm
- Protection from air drafts
- Verticality of piston
  - Level piston or base
- Lab conditions: low vibration, steady temperature
- Piston temperature affects area and pressure
- Leak tight plumbing
- Good piston operation: observe rotation decay, fall rate, sensitivity
- Equilibration time at operating T and P
  - Pressure changes cause temperature changes;
  - Temperature changes cause pressure changes

Pressure from a Piston Gauge

Gravity forces from masses
+ surface tension

$$pA = Mg + \textbf{Surface Tension}$$

Vertical fluid pressure
forces action on piston



Pressure from a Piston Gauge

A_e = Effective area at T_R
m_i = mass of weights
g = gravity
ρ_a = ambient density surrounding wts
ρ_{mi} = density of weights
γ = surface tension
C = piston circumference
α_p, α_c = piston and cylinder thermal expansion coefficient
T = piston temperature
T_{REF} = reference temperature

$$\boldsymbol{p} = \frac{\sum_i \boldsymbol{m}_i \boldsymbol{g}\left(1-\frac{\rho_a}{\rho_{mi}}\right)+\gamma \boldsymbol{C}}{\boldsymbol{A}_e\left(1+\left(\alpha_p+\alpha_c\right)\left(\boldsymbol{T}-\boldsymbol{T}_{REF}\right)\right)}$$

Note: The calibration of one piston gauge against another piston gauge is called a crossfloat

Pressure from a Piston Gauge: Importance of Terms

- A_e Obtain from calibration certificate (or measure dimensions: much more difficult and expensive)
  - Largest uncertainty component (6 ppm to 40 ppm)
  - May be a function of pressure
- m Obtain form mass calibration
  - Second largest uncertainty, < 1 ppm to 10 ppm
- $(1-\rho_a/\rho_m)$ buoyance correction
  - 150 ppm for gauge mode, 0 ppm for absolute mode
  - Uncertainty < 2 ppm
  - Air density needs to be measured in gauge mode for a transducer calibration. Need P_{amb}, T_{amb}, rh
  - In a crossfloat calibration, each gauge has same buoyancy correction. Air density not so critical
- ρ_m Must be considered with mass calibration
  - Did mass lab include density uncertainty in mass uncertainty?

m_i was assigned using measured force and a value of ρ_{mi}

$$\boldsymbol{m_i g}\left(1-\frac{\rho_a}{\rho_{mi}}\right)=\boldsymbol{m_R g}\left(1-\frac{\rho_a}{\rho_{mR}}\right)$$

Pressure from a Piston Gauge: Importance of Terms

- $\alpha_p+\alpha_c$ ~ 9×10^{-6} m/m/K; thermal expansion 10 ppm if $T-T_R$ = 1 K
  - If your T is different from T_R where A_e was determined, uncertainty in thermal expansion coefficient may be important. Usually < 1 ppm
- T Thermometer must be traceable. 0.1 C uncertainty is 1 ppm in pressure
- g Needed at site of use. For a crossfloat, g cancels. For a transducer calibration, p sees full g uncertainty
- γC Surface tension = 0 for gas media. For a 10 mm D piston in oil, ST equivalent to 30 mg mass.

Pressure at the Transducer from a Piston Gauge

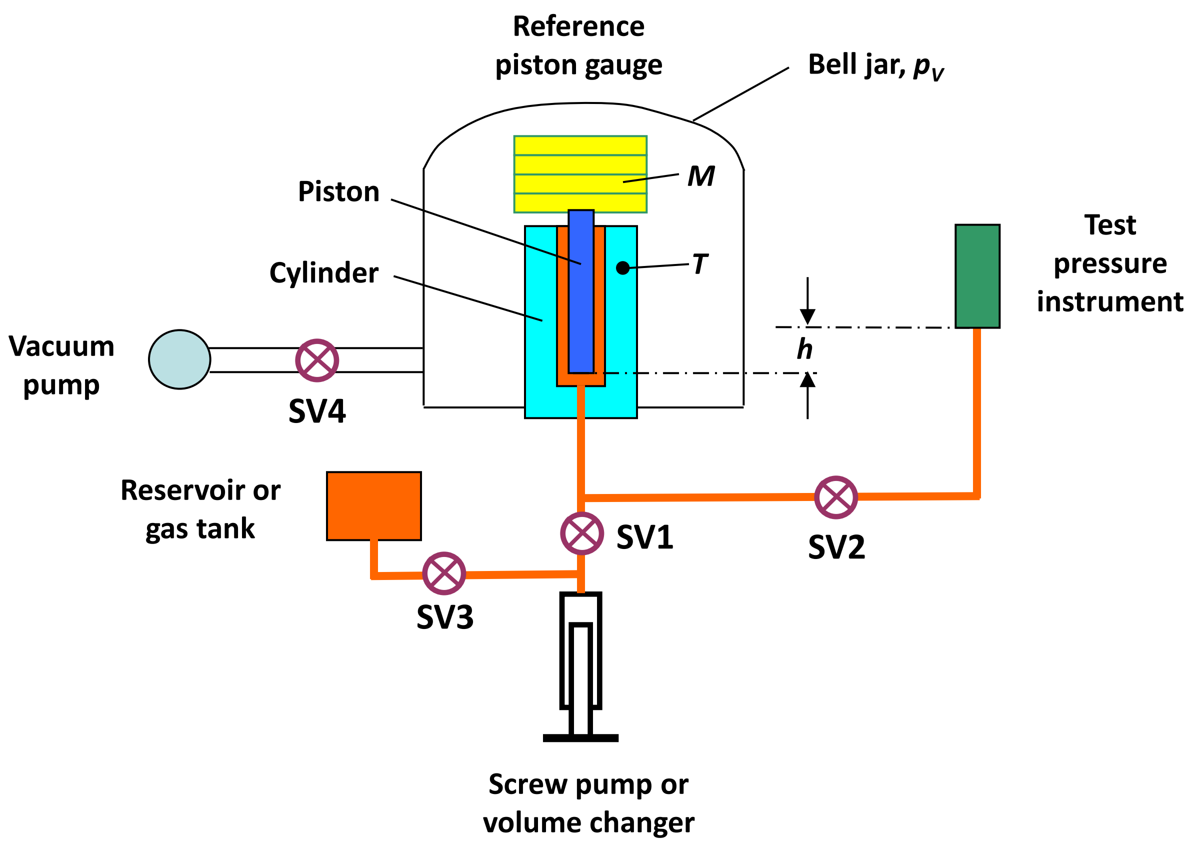
$$p_T = p_{PG} + (\rho_f - \rho_a) g h$$

Head correction

h = reference level difference between transducer and PG
ρ_f = density of pressurizing fluid

- For gas media, 1 cm in h corresponds to 1 ppm in P. Uncertainties in h rarely important
- For liquid media, 1 mm in h corresponds to 9 Pa. Head correction and uncertainty in h and ρ_f significant if P < 10 MPa, less important for high P.
- Piston height sensors provide h resolution to < 0.1 mm

Experimental Setup for a Pressure Calibration



Bell jar not evacuated for gauge mode

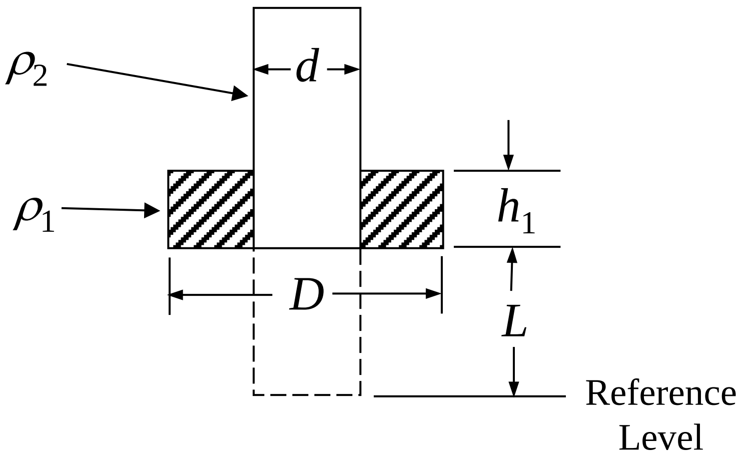
Calibration Procedure for a Crossfloat

1. Unpack gauge
2. Connect pressure port.
3. Check vertical alignment.
4 & 5.Determine reference level and differences
6. Check thermometer readings. Record ambient conditions (T, P_{amb}, rh)
7. Check or calibrate piston vertical position indicator
8. Add calibrated masses.
9. Apply pressure and elevate pistons.
10. Rotate masses. If rotation is difficult, investigate and resolve
11. Place covers over masses if required. Evacuate if absolute mode.
12. Check for leaks or excessive fall rates. Fix if needed.
13. Wait for pressure stability (5 to 30 minutes, depending on gauge, media or pressure change).
14. Determine balance condition. Adjust masses if needed. (More detail later)
    - This step not required for a transducer calibration
15. Record masses and temperatures. Record vacuum if absolute mode.
16. Lower pistons by reducing pressure.
17. Add new masses and repeat from step 9.

Reference Level of a Piston

- For solid, straight pistons, reference level is bottom of piston
- For irregular shape, need to account for additional fluid displacement at the density and area of the piston
- Example: flange on a piston

$$L = \frac{\left(D^2 - d^2\right)}{d^2}\frac{\rho_1}{\rho_2}h_1$$



Calibrating a Piston Gauge with another Piston Gauge

- The ***test*** and ***ref*** piston gauges each produce a pressure
- A fluid line connects ***test*** and ***ref*** gauges.
- When the gauges produce the same pressure at a common location in the fluid line (and no fluid movement), they are in equilibrium
- Pressure of the gauge is adjusted by changing mass

$$p_T = p_R - \left(\rho_f - \rho_a\right) gh$$

Recall that:

$$p = \frac{\sum_i m_i g\left(1 - \frac{\rho_a}{\rho_{mi}}\right) + \gamma C}{A_e\left(1 + \left(\alpha_p + \alpha_c\right)\left(T - T_{REF}\right)\right)}$$

Effective Area for a Piston Gauge

$$\frac{\sum m_{i,T}\, g\left(1-\frac{\rho_a}{\rho_{mi,T}}\right)+\gamma C_T}{A_{e,T}\left(1+\alpha_T\left(T_T-T_{REF}\right)\right)}=\frac{\sum m_{i,R}\, g\left(1-\frac{\rho_a}{\rho_{mi,R}}\right)+\gamma C_R}{A_{e,R}\left(1+\alpha_{STD}\left(T_R-T_{REF}\right)\right)}-\left(\rho_f-\rho_a\right)gh$$

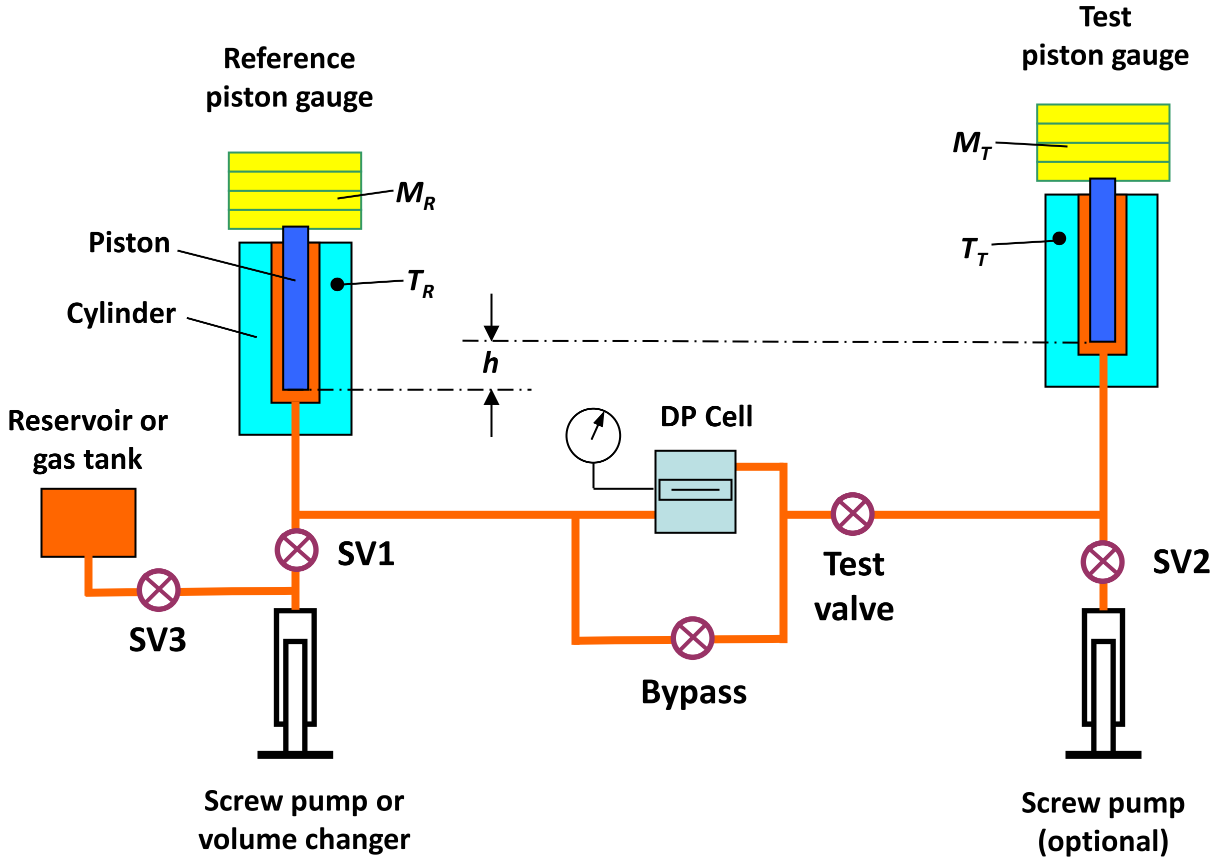
Pressure of Test PG Pressure of Ref Standard PG Head Correction

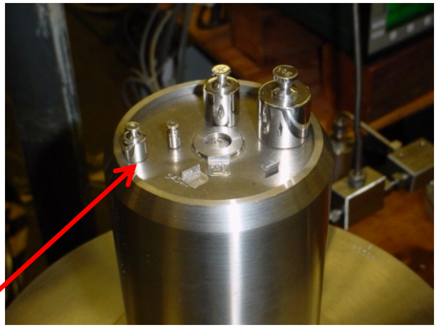
Rearrange to solve for $A_{e,T}$: ***The Measurement Equation for Piston Gauge Calibrations***

$$A_{e,T}=A_{e,R}\cdot\frac{\sum m_{i,T}\left(1-\frac{\rho_a}{\rho_{mi,T}}\right)+\frac{\gamma C_T}{g}}{\sum m_{i,R}\left(1-\frac{\rho_a}{\rho_{mi,R}}\right)+\frac{\gamma C_R}{g}}\cdot\frac{\left(1+\alpha_R\left(T_R-T_{REF}\right)\right)}{\left(1+\alpha_T\left(T_T-T_{REF}\right)\right)}\cdot\frac{1}{\left(1-\frac{\left(\rho_f-\rho_a\right)gh}{p}\right)}$$

Thermal expansion coefficients have been combined
Note g cancels out
Assumes pressure from two PGs are in equilibrium (balanced)

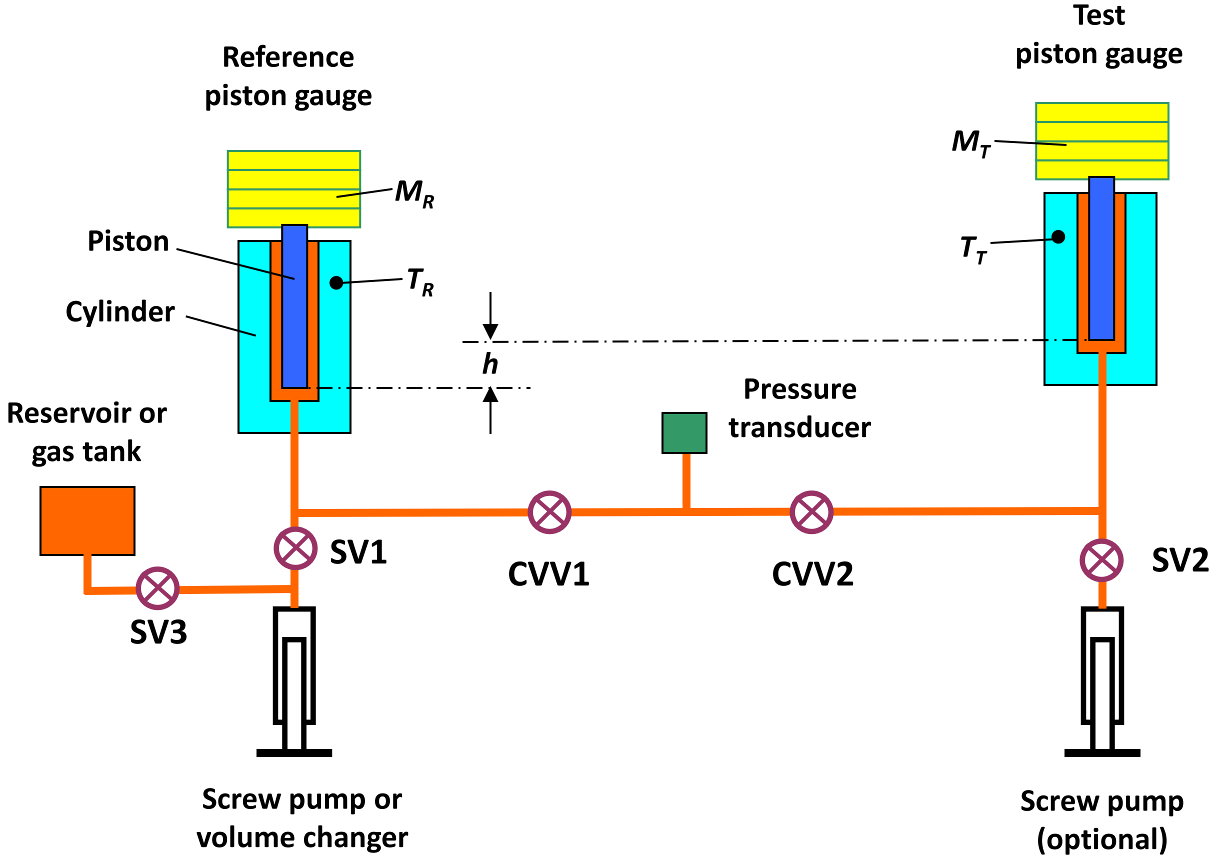
Experimental Setup for a Crossfloat (PG) Calibration DP Cell Method





Open bypass valve and close test valve to null DP Cell
Close bypass, open test to detect imbalance. Adjust mass

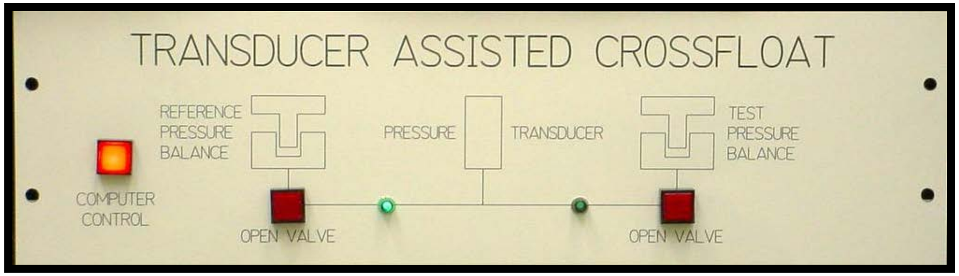
Experimental Setup for a Crossfloat (PG) Calibration
Fall Rate or Transducer Assisted Crossfloat Method



- Fall rate: measure piston fall rates when CVVs closed and open. Adjust mass
- Transducer Assisted Crossfloat (TAC): sequence PGs to transducer with alternate CCV1/CVV2 open and close. Equal pressures not necessary

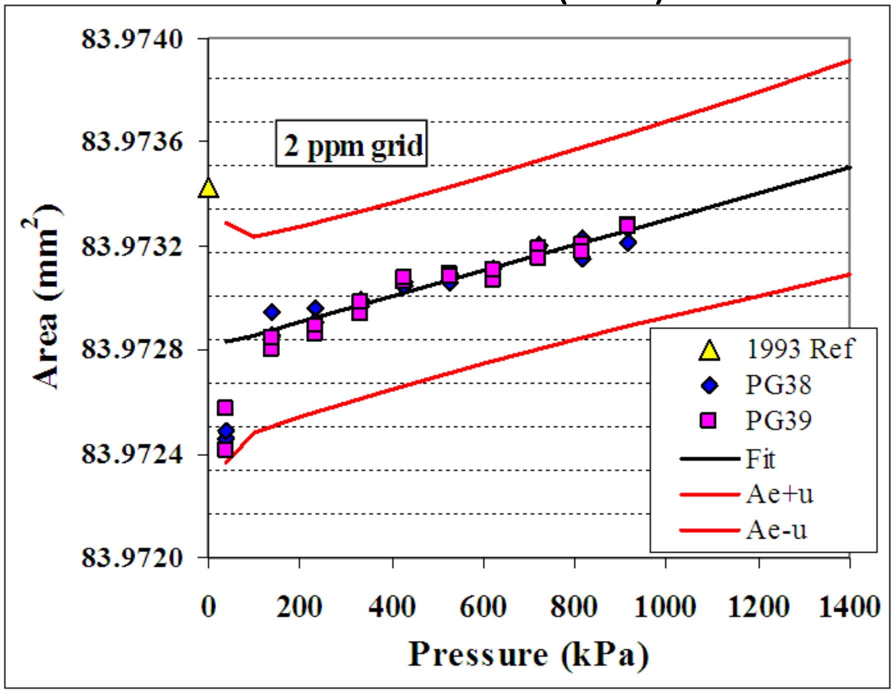
Relative Advantages of Balancing Methods

- DP Cell
  - Skilled technician required
  - Balance can be achieved quickly. Most widely used
  - Requires DP Cell to match pressure range
  - Computer not required
- Fall Rate
  - Can be used at any pressure
  - Need recording of piston height vs time (computer). Stopwatch/position indicator/computer
  - Slow if piston fall rate is slow.
  - Skilled technician; need to understand performance of PG vs h, etc.
- TAC
  - Need precise pressure transducer
  - Automated
  - Not dependent on technician skill
  - Equal pressures not required



Fitting Calibration Data of Piston Gauges

PG34 (C74)



- Measure A_e vs P at 10 pressures, space equally from low to high, plot results
- Fit to model of A_e vs P
- In the model below, A_0, b_1, b_2, and t are fitting coefficients
- Use linear least squares to determine coefficients

$$A_{e,f} = A_0(1 + b_1 p + b_2 p^2) - t / p$$

- Fit to several models (less terms to more). Look at standard deviation of residuals.
- If adding term does not decrease residual, don't include
- b_2 is nearly always insignificant
- b_1 (linear distortion) needed above 1 MPa.
- t accounts for hook at low pressure. Can indicate mass errors or residual forces. Try to fit without it.

Type A Uncertainty: Fitting Calibration Data

- Increasing ***n*** (observations) reduces Type A uncertainty, increases time and cost
- Regression in linear model below can be performed in Excel, where

$$A_{e,f} = A_0 + A_1 p \text{ , with } b_1 = A_1 / A_0$$

- The Type A uncertainty of the calibration is the uncertainty of the fitted line, not the uncertainty of the coefficients. For the linear model:

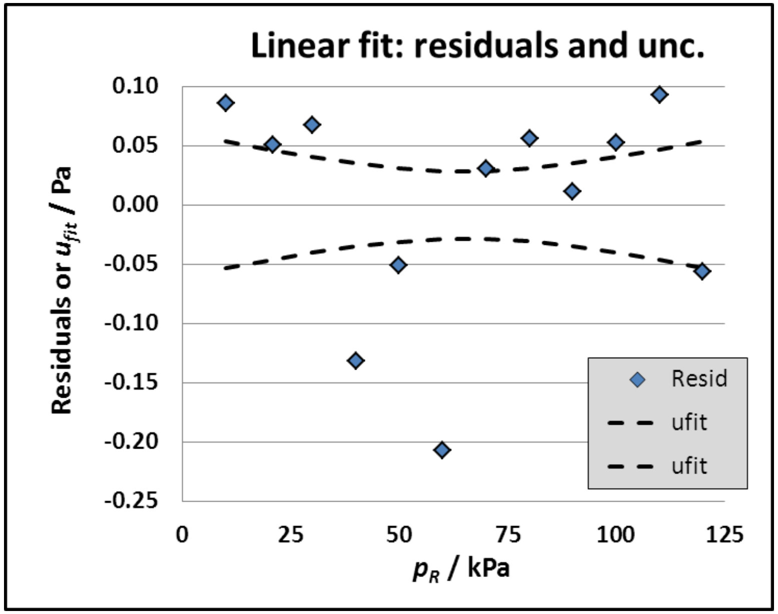
$$u_A(A_{e,f}) = \left(\frac{\sigma^2}{n} + \left(u(b_1)(p_{fit} - p_{ave}) \right)^2 \right)^{1/2}$$

Where:

$$\sigma = \left[\frac{1}{n-2} \sum_{i=1}^{n} \left(A_{e,f} - A_e \right)^2 \right]^{1/2}$$

Obtain from regression analysis

Average of ***n*** measured pressures



Type A Uncertainty: Fitting Calibration Data

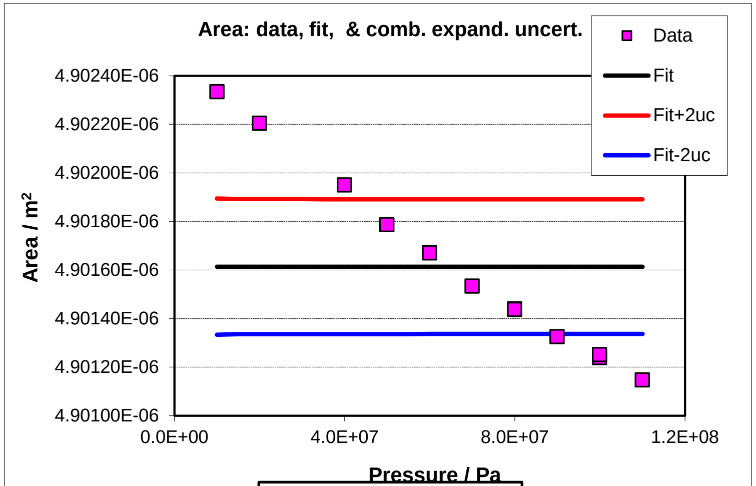
- Constant model

$$A_{e,f} = A_0 \ , A_0 = \frac{1}{n}\sum_{i=1}^{n} A_e$$

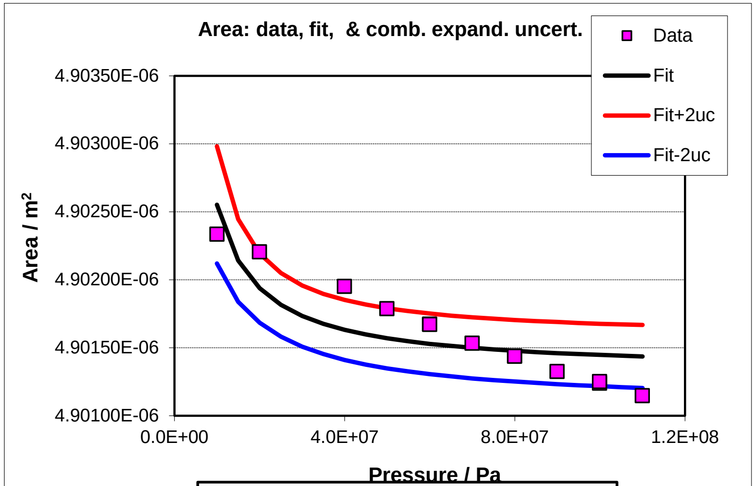
- The Type A uncertainty of the calibration is the uncertainty of A_0

$$u_A\left(A_0\right) = \left[\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(A_0 - A_e\right)^2\right]^{1/2}$$

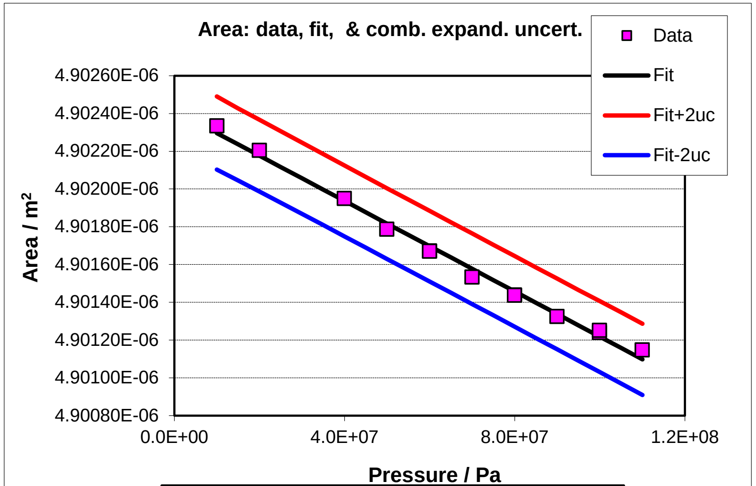
Fit Example: Oil gauge Calibration, 10 MPa to 110 MPa



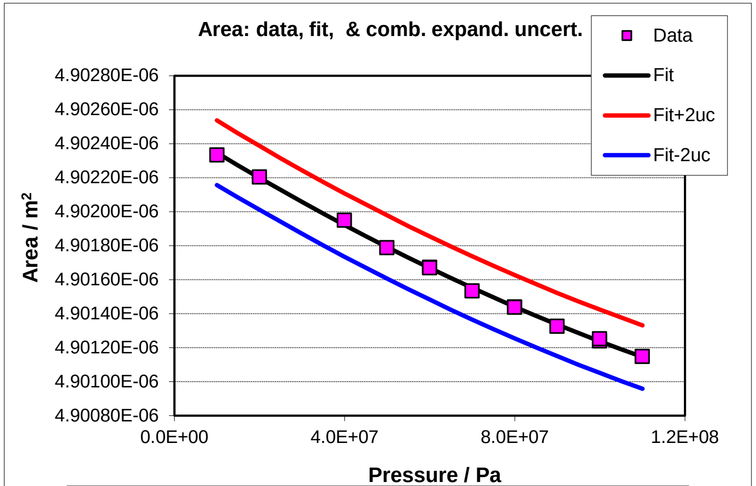
$$A_{e,\,fit1} = A_0$$



$$A_{e,\,fit2} = A_0 - t\,/\,p$$

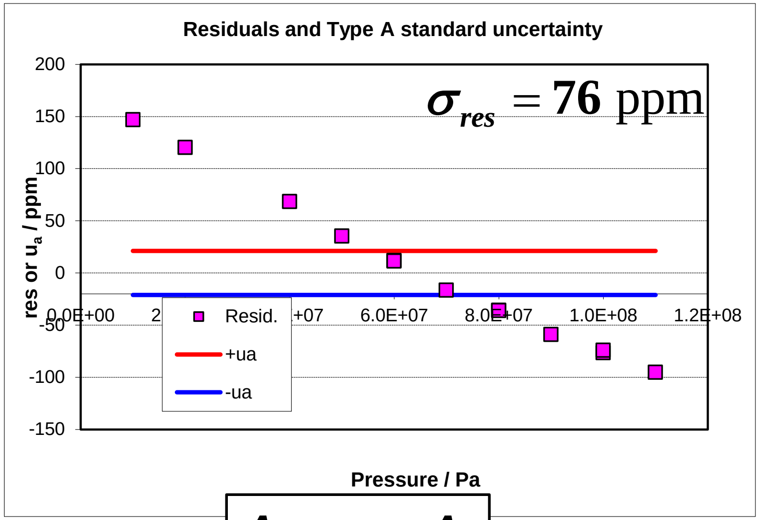


$$A_{e,\,fit3} = A_0(1 + b_1 p)$$

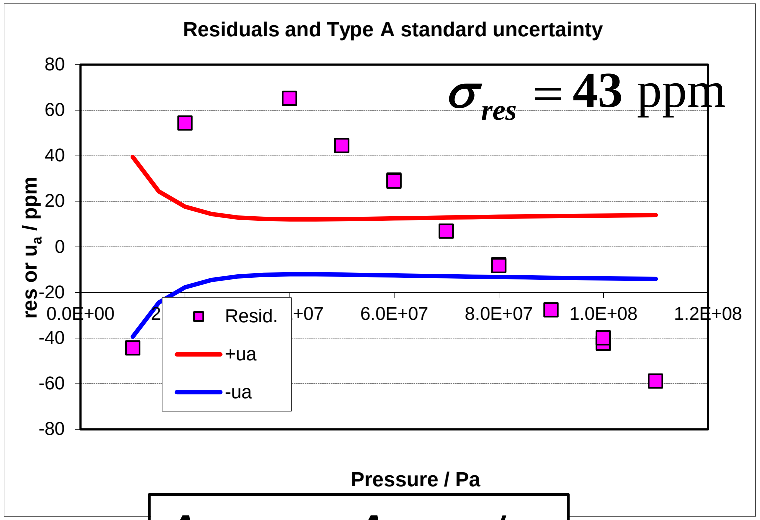


$$A_{e,\,fit5} = A_0(1 + b_1 p + b_2 p^2)$$

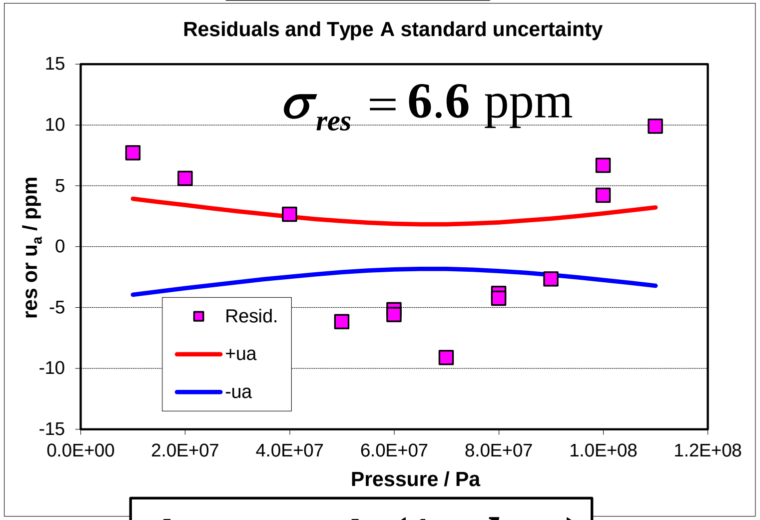
Fit Example: Oil Cal, 10 MPa to 110 MPa, Residual Plots



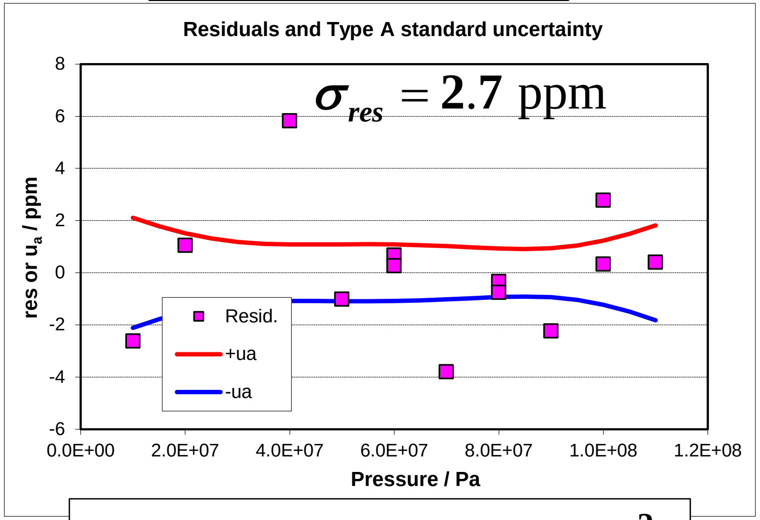
$$A_{e,fit1} = A_0$$



$$A_{e,fit2} = A_0 - t/p$$



$$A_{e,fit3} = A_0(1 + b_1 p)$$



$$A_{e,fit5} = A_0(1 + b_1 p + b_2 p^2)$$

Fit Example: Oil Cal, 10 MPa to 110 MPa

In this example, fit 5 is chosen because it produces the smallest data scatter with no structure in the residuals

$$A_{e,fit5} = A_0(1 + b_1 p + b_2 p^2)$$

A_0 (m^2) = 4.902501E-06
b_1 (Pa^{-1}) = -3.209E-12
b_2 (Pa^{-2}) = 6.310E-21

P (MPa)	A_{fit} (m^2)	u_A/A x10^6	u_B/A x10^6	$2u_C/A$ x10^6
10.00	4.902347E-06	2.1	19.3	38.9
15.00	4.902272E-06	1.8	19.1	38.4
20.00	4.902199E-06	1.5	19.1	38.2
25.00	4.902127E-06	1.3	19.0	38.1
30.00	4.902057E-06	1.2	19.0	38.1
35.00	4.901988E-06	1.1	19.0	38.1
40.00	4.901921E-06	1.1	19.0	38.0
45.00	4.901856E-06	1.1	19.0	38.0
50.00	4.901792E-06	1.1	19.0	38.0
55.00	4.901729E-06	1.1	19.0	38.0
60.00	4.901668E-06	1.1	19.0	38.0
65.00	4.901609E-06	1.1	19.0	38.0
70.00	4.901551E-06	1.0	19.0	38.0
75.00	4.901495E-06	1.0	19.0	38.0
80.00	4.901440E-06	0.9	19.0	38.0
85.00	4.901387E-06	0.9	19.0	38.0
90.00	4.901336E-06	0.9	19.0	38.0
95.00	4.901286E-06	1.0	19.0	38.0
100.00	4.901237E-06	1.2	19.0	38.0
105.00	4.901190E-06	1.5	19.0	38.1
110.00	4.901145E-06	1.8	19.0	38.1

Summary of Calibration Procedures

- Know your piston gauge platform and reference standard gauge
- Establish a calibration procedure and follow it
  - Leaks, unsteady lab temperatures, calibrating too quickly must be avoided
- Determine traceability of measurements affecting result
  - Area, mass, temperature for all cals
  - Vacuum for absolute P, air density for pressure cals
  - Ambient P if gauge mode piston gauge used to calibrate absolute mode transducer
- Measure A vs P (or p_T vs P) evenly over instrument span, 10 pts or more
- Fit data to appropriate models
  - Linear distortion for piston gauge
  - Polynomial fits for pressure transducers
- Use of data fitting and statistical software very useful
- Graphs of data aid choice of fits

Uncertainty Analysis Method

Type A

Determine from statistics of fit or repeated measurements

Type B

- Determine measurement equation
- Identify uncertainty components
- Define sensitivity coefficients
- Estimate standard uncertainties of components
- Sum the uncertainty components

1. Pressure standard: piston gauge. Device under test: Piston gauge

2. Pressure standard: piston gauge. Device under test: electronic pressure instrument (transducer). *Notes are provided, will not be discussed*

Basic Uncertainty Analysis

- Include Type A and Type B uncertainty components
- Type A estimated from repeated measurements or statistics of fit
- Type B estimated from law of propagation of uncertainty applied to measurement equation *and other relevant considerations*
- Add components at *k*=1 (standard) level
- Apply coverage factor (usually *k*=2) for expanded uncertainty

$$u_c = \left(u_A^2 + u_B^2\right)^{1/2}, \quad U_c = ku_c, \quad k = 2$$

$$A_e = A_e\left(x_1, x_2, \ldots., x_N\right), \quad u_B^2(A_e) = \sum_{i=1}^{N}\left(\frac{\partial A_e}{\partial x_i}\right)^2 u^2(x_i) + \text{correlated terms}$$

$$u_{xi} = \frac{\partial A_e}{\partial x_i}u(x_i), \quad u_B^2(A_e) = \left(u_{x1}^2 + u_{x2}^2 + u_{x3}^2 + \ldots. + u_{xN}^2\right)$$

x_i are the variables in the measurement equation (*A, m, T,* etc.)
dA_e / dx_i are the sensitivity coefficients
$u(x_i)$ are the uncertainties of the variables.

Case 1: DUT=piston gauge

Measurement Equation

$$A_{e,T} = A_{e,R} \cdot \frac{\sum m_{i,T}\left(1-\dfrac{\rho_a}{\rho_{mi,T}}\right)+\dfrac{\gamma C_T}{g}}{\sum m_{i,R}\left(1-\dfrac{\rho_a}{\rho_{mi,R}}\right)+\dfrac{\gamma C_R}{g}} \cdot \frac{\left(1+\bar{\alpha}_R\left(T_R-T_{REF}\right)\right)}{\left(1+\bar{\alpha}_T\left(T_T-T_{REF}\right)\right)} \cdot \frac{1}{\left(1-\dfrac{\left(\rho_f-\rho_a\right)gh}{p}\right)}$$

Uncertainty Terms for Type B

$$u_B^2\left(A_{e,T}\right) = u_{A_{e,R}}^2 + u_{M_R}^2 + u_{M_T}^2 + u_{\rho_a}^2 + u_{\rho_{MR}}^2 + u_{\rho_{MT}}^2 + u_{\gamma}^2 + u_{C_R}^2 + u_{C_T}^2 + u_g^2$$
$$+ u_{\bar{\alpha}_R}^2 + + u_{\bar{\alpha}_T}^2 + + u_{T_R}^2 + u_{T_T}^2 + u_{\rho_f}^2 + u_h^2$$

Area of standard and masses are usually most important

Case 1: Uncertainty due to Ref Piston Gauge Area, $u_{Ae,R}$

Sensitivity Coefficient

$$\frac{\partial \boldsymbol{A}_{e,T}}{\partial \boldsymbol{A}_{e,R}} = \frac{\boldsymbol{A}_{e,T}}{\boldsymbol{A}_{e,R}}$$

Standard uncertainty, $u(A_{e,R})$

- From calibration of piston gauge
- Usually the largest component

$u_{Ae,R}$ = 6 ppm to 20 ppm of $A_{e,R}$

Case 1: Uncertainty due to Ref Piston Gauge Masses, u_{MR}

Sensitivity Coefficient

$$\frac{\partial \boldsymbol{A}_{e,T}}{\partial \boldsymbol{M}_R} = -\frac{\boldsymbol{A}_{e,T}}{\boldsymbol{M}_R}$$

Standard uncertainty, $u(M_R)$

- From mass calibration
- Assume correlated, meaning uncertainty of the sum of masses are added geometrically rather than in quadrature $u(M_R)=u(m_{1R})+u(m_{2R})+\ldots+u(m_{nR})$
- Need to know if density uncertainty was included in mass uncertainty quoted by mass calibration

Case 1: Uncertainty due to Test Piston Gauge Masses, u_{MT}

Sensitivity Coefficient

$$\frac{\partial A_{e,T}}{\partial M_T} = \frac{A_{e,T}}{M_T}$$

Standard uncertainty, $u(M_T)$

- From mass calibration
- Assume correlated, meaning uncertainty of the sum of masses are added geometrically rather than in quadrature
  $u(M_T)=u(m_{1T})+u(m_{2T})+\ldots+u(m_{nT})$
- Need to know if density uncertainty was included in mass uncertainty quoted by mass calibration
- Assume not correlated with Ref masses

Case 1: Uncertainty due to Ref PG Mass Density, $u_{\rho MR}$

Sensitivity Coefficient

- assumes mass density uncertainty not included in mass calibration
- assumes mass calibration at ambient pressure

Gauge mode

$$\frac{\partial A_{e,T}}{\partial \rho_{M,R}} = A_{e,T} \frac{(\rho_{a,cal} - \rho_a)}{\rho_{M,R}^2}$$

Absolute mode

$$\frac{\partial A_{e,T}}{\partial \rho_{M,R}} = A_{e,T} \frac{\rho_{a,cal}}{\rho_{M,R}^2}$$

$\rho_{a,cal}$ is air density at time of mass cal. Air density diff ~ 0.03 kg/m^3
Sensitivity coefficient is larger in absolute mode

Standard uncertainty, $u(\rho_{M,R})$

- OIML R 111-1: guidelines to measure. If not measured, $u(\rho_{M,R})$ = 70 kg/m^3 for SS.
- If mass density uncertainty was included in mass calibration
  - Absolute mode: set = 0 to avoid double counting.
  - Gauge mode: ask for mass uncertainty without density uncertainty.

OIML R 111-1 at http://www.fundmetrology.ru/depository/04_IntDoc_all/R_E_111-1.pdf

Case 1: Uncertainty due to Test PG Mass Density, $u_{\rho MT}$

Sensitivity Coefficient

- assumes mass density uncertainty not included in mass calibration
- assumes mass calibration at ambient pressure

Gauge mode

$$\frac{\partial A_{e,T}}{\partial \rho_{M,T}} = A_{e,T} \frac{\left(\rho_a - \rho_{a,cal}\right)}{\rho_{M,T}^2}$$

Absolute mode

$$\frac{\partial A_{e,T}}{\partial \rho_{M,T}} = -A_{e,T} \frac{\rho_{a,cal}}{\rho_{M,T}^2}$$

$\rho_{a,cal}$ is air density at time of mass cal. Air density diff ~ 0.03 kg/m^3
Sensitivity coefficient is larger in absolute mode

Standard uncertainty, $u(\rho_{M,T})$

- OIML R 111-1: guidelines to measure. If not measured, $u(\rho_{M,T})$ = 70 kg/m^3 for SS.
- If mass density uncertainty was included in mass calibration
  - Absolute mode: set = 0 to avoid double counting.
  - Gauge mode: ask for mass uncertainty without density uncertainty.

OIML R 111-1 at http://www.fundmetrology.ru/depository/04_IntDoc_all/R_E_111-1.pdf

Case 1: Uncertainty due to Air Density, $u_{\rho A}$

Sensitivity Coefficient (0 for absolute mode)

- Assumes all ref mass densities the same, all test mass densities the same
- First term is due to buoyancy correction, second term is due to head correction

$$\frac{\partial A_{e,T}}{\partial \rho_a} = A_{e,T}\left[\frac{\rho_{M,T} - \rho_{M,R}}{\rho_{M,T} \cdot \rho_{M,R}} - \frac{gh}{p}\right]$$

Standard uncertainty, $u(\rho_a)$

- If ρ_a measured, $u(\rho_a)$ due to $u(P_{amb})$, $u(T_{amb})$. Likely < 0.01 kg/m^3
- If not measured, estimate ~0.03 kg/m^3

$u_{\rho A}$ nearly always < 0.1 ppm

Case 1: Uncertainty due to Surface Tension, u_γ

Sensitivity Coefficient (0 for gas media)

$$\frac{\partial A_{e,T}}{\partial \gamma} = A_{e,T} \cdot \frac{C_R}{M_R g}\left(\frac{D_R}{D_T} - 1\right)$$

Standard uncertainty, u(γ)

- Term is always insignificant even for oil

Case 1: Uncertainty due to Piston Circumference, as it contributes to Surface Tension Force, u_{CR} & u_{CT}

Sensitivity Coefficients for both C_R and C_T (0 for gas media)

$$\frac{\partial A_{e,T}}{\partial C_R} = -A_{e,T} \cdot \frac{\gamma}{M_R g} \qquad \frac{\partial A_{e,T}}{\partial C_T} = A_{e,T} \cdot \frac{\gamma}{M_T g}$$

Standard uncertainty, $u(C_R)$, $u(C_T)$

- Could be estimated from piston diameter uncertainty

Always insignificant

Case 1: Uncertainty due to Gravity, u_g

Sensitivity Coefficient

- Gravity cancels on Ref and Test piston gauge masses (not so for transducer cal)
- Only effect is head correction

$$\frac{\partial A_{e,T}}{\partial g} = A_{e,T} \frac{(\rho_f - \rho_a)h}{p}$$

Standard uncertainty, u(g)

- Could be estimated from gravity survey

Always insignificant for cals of piston gauges

Case 1: Uncertainty due to thermal expansion, $u_{\alpha R}$ & $u_{\alpha T}$

Sensitivity Coefficients for both Ref and Test piston gauges

$$\frac{\partial A_{e,T}}{\partial \bar{\alpha}_R} = A_{e,T}(T_R - T_{REF}) \qquad \frac{\partial A_{e,T}}{\partial \bar{\alpha}_T} = -A_{e,T}(T_T - T_{REF})$$

Standard uncertainty, $u(\alpha_R)$ and $u(\alpha_T)$

- Estimate as $u(\alpha) = 0.06\mathrm{x}\alpha$ (rectangular distr. of 10 %)
- Can be minimized by operating T close to reference temperature

If ΔT = 1 C, uncertainty is 0.5 ppm: small but not insignificant

Case 1: Uncertainty due to Temperature, u_{TR} & u_{TT}

Sensitivity Coefficients for both Ref and Test piston gauges

$$\frac{\partial A_{e,T}}{\partial T_R} = A_{e,T}\bar{\alpha}_R \qquad \frac{\partial A_{e,T}}{\partial T_T} = -A_{e,T}\bar{\alpha}_T$$

Standard uncertainty, $u(T_R)$ and $u(T_T)$

- Depends on thermometer calibration, placement of thermometer near piston

u_{TR}, u_{TT}: f $u(T)$ = 0.1 C, uncertainty is 1 ppm: small but not insignificant

Case 1: Uncertainty due to Fluid Density, $u_{\rho f}$

Sensitivity Coefficient

$$\frac{\partial A_{e,T}}{\partial \rho_f} = A_{e,T}\,\frac{gh}{p}$$

Standard uncertainty, $u(\rho_f)$

- Gas: insignificant
- Can be minimized by reducing h
- Oil: if not measured, estimate as 1 % of fluid density

$u_{\rho f}$ in oil: for h = 0.1 m, can be 10 Pa/***p***. Can become important at ***p*** < 10 MPa

Case 1: Uncertainty due to reference height difference, u_h

Sensitivity Coefficient

$$\frac{\partial A_{e,T}}{\partial h} = A_{e,T}\,\frac{(\rho_f - \rho_a)g}{p_T}$$

Standard uncertainty, u(h)

- Gas: insignificant
- Depends on position resolution of both pistons, measurement of reference level difference. At NIST, we take u(h) = 1 mm

u_h in oil: 10 Pa/***p***. Can become important at ***p*** < 10 MPa

Case 1: Summary of Type B uncertainties, 200 kPa Gas, gauge mode

Uncertainty term				Sensitivity coefficient divided by $A_{e,T}$			Standard uncertainty		Rel. unc.
Name	Symbol	Value	Units	Definition	Abs value	Units	Value	Units	on $A_{e,T}$
REF Area	$A_{e,R}$	8.397E-05	m^2	$1/A_{e,R}$	11909	m^{-2}	3.63E-10	m^2	**4.33E-06**
REF Mass	M_R	1.714	kg	$-1/M_R$	0.584	kg^{-1}	3.43E-06	kg	**2.00E-06**
TEST Mass	M_T	1.714	kg	$1/M_T$	0.583	kg^{-1}	3.43E-06	kg	**2.00E-06**
Ambient density	ρ_a	1.18	kg/m^3	$\Delta\rho_M/(\rho_{MT}\rho_{MR})$ - gh/p_T	9.80E-07	m^3/kg	0.03	kg/m^3	2.94E-08
REF mass dens.	ρ_{MR}	7800	kg/m^3	$(\rho_{a,cal}-\rho_a)/\rho_{MR}^2$	4.93E-10	m^3/kg	45.0	kg/m^3	2.22E-08
TEST mass dens.	ρ_{MT}	7800	kg/m^3	$(\rho_a-\rho_{a,cal})/\rho_{MT}^2$	4.93E-10	m^3/kg	45.0	kg/m^3	2.22E-08
Surface tension	γ	0.000	N/m	$C_R/(M_R g)(D_R/D_T-1)$	3.11E-07	m/N	0.000	N/m	0.000
REF piston circ.	C_R	3.25E-02	m	$-\gamma/(M_R g)$	0.000	1/m	3.25E-05	m	0.000
TEST piston circ.	C_T	3.25E-02	m	$\gamma/(M_T g)$	0.000	1/m	3.25E-05	m	0.000
Gravity	g	9.801011	m/s^2	$(\rho_f-\rho_a)h/p_T$	2.33E-07	s^2/m	2.00E-06	m/s^2	4.66E-13
REF therm. exp.	$\alpha_{p,R}+\alpha_{c,R}$	9.10E-06	C^{-1}	$T_R-23.00$	0.50	C	5.25E-07	C^{-1}	2.63E-07
TEST therm. exp.	$\alpha_{p,T}+\alpha_{c,T}$	9.10E-06	C^{-1}	$-(T_T-23.00)$	0.50	C	5.25E-07	C^{-1}	2.63E-07
REF temperature	T_R	22.50	C	$\alpha_{p,R}+\alpha_{c,R}$	9.10E-06	C^{-1}	0.058	C	5.25E-07
TEST temperature	T_T	22.50	C	$-(\alpha_{p,T}+\alpha_{c,T})$	9.10E-06	C^{-1}	0.058	C	5.25E-07
Fluid density	ρ_f	3.51	kg/m^3	gh/P_T	9.80E-07	m^3/kg	3.51E-03	kg/m^3	3.44E-09
Height difference	h	0.02	m	$(\rho_f-\rho_a)g/p_T$	1.14E-04	1/m	0.001	m	1.14E-07
Press. equilibrium	ΔP	0.00	Pa	$-1/p_T$	5.00E-06	1/Pa	0.12	Pa	5.84E-07
					Relative combinded standard unc.				**5.27E-06**

Remember, these are at k=1!

Case 1: Summary of Type B Uncertainties, 5 MPa oil, gauge mode

Uncertainty term				Sensitivity coefficient divided by $A_{e,T}$			Standard uncertainty		Rel. unc. on $A_{e,T}$
Name	Symbol	Value	Units	Definition	Abs Value	Units	Value	Units	
REF Area	$A_{e,R}$	8.402E-05	m^2	$1/A_{e,R}$	11901.89	m^{-2}	9.24E-10	m^2	**1.10E-05**
REF Mass	M_R	42.86	kg	$-1/M_R$	2.33E-02	kg^{-1}	1.24E-04	kg	**2.89E-06**
TEST Mass	M_T	42.85	kg	$1/M_T$	2.33E-02	kg^{-1}	1.24E-04	kg	**2.89E-06**
Ambient density	ρ_a	1.18	kg/m^3	$\Delta\rho_M/(\rho_{MT}\rho_{MR})$ - gh/p_T	9.80E-08	m^3/kg	0.030	kg/m^3	2.94E-09
REF mass dens.	ρ_{MR}	7800	kg/m^3	$(\rho_{a,cal}-\rho_a)/\rho_{MR}^2$	4.93E-10	m^3/kg	45.0	kg/m^3	2.22E-08
TEST mass dens.	ρ_{MT}	7800	kg/m^3	$(\rho_a-\rho_{a,cal})/\rho_{MT}^2$	4.93E-10	m^3/kg	45.0	kg/m^3	2.22E-08
Surface tension	γ	3.06E-02	N/m	$C_R/(M_R g)(D_R/D_T-1)$	9.33E-09	m/N	1.77E-03	N/m	1.65E-11
REF piston circ.	C_R	3.25E-02	m	$-\gamma/(M_R g)$	7.28E-05	1/m	3.25E-05	m	2.37E-09
TEST piston circ.	C_T	3.25E-02	m	$\gamma/(M_T g)$	7.29E-05	1/m	3.25E-05	m	2.37E-09
Gravity	g	9.801011	m/s^2	$(\rho_f-\rho_a)h/p_T$	8.57E-06	s^2/m	2.00E-06	m/s^2	1.71E-11
REF therm. exp.	$\alpha_{p,R}+\alpha_{c,R}$	9.10E-06	C^{-1}	$T_R-23.00$	0.50	C	5.25E-07	C^{-1}	2.63E-07
TEST therm. exp.	$\alpha_{p,T}+\alpha_{c,T}$	9.10E-06	C^{-1}	$-(T_T-23.00)$	0.50	C	5.25E-07	C^{-1}	2.63E-07
REF temperature	T_R	23.50	C	$\alpha_{p,R}+\alpha_{c,R}$	9.10E-06	C^{-1}	0.058	C	5.25E-07
TEST temperature	T_T	23.50	C	$-(\alpha_{p,T}+\alpha_{c,T})$	9.10E-06	C^{-1}	0.058	C	5.25E-07
Fluid density	ρ_f	857.8	kg/m^3	gh/p_T	9.80E-08	m^3/kg	9.91	kg/m^3	9.71E-07
Height difference	h	0.05	m	$(\rho_f-\rho_a)g/p_T$	1.68E-03	1/m	0.001	m	**1.68E-06**
Press. equilibrium	ΔP	0.00	Pa	$-1/p_T$	2.00E-07	1/Pa	5.83	Pa	**1.17E-06**
					Relative combinded standard unc.				**1.20E-05**

Remember, these are at k=1!

Case 1: Summary Comments for Type B

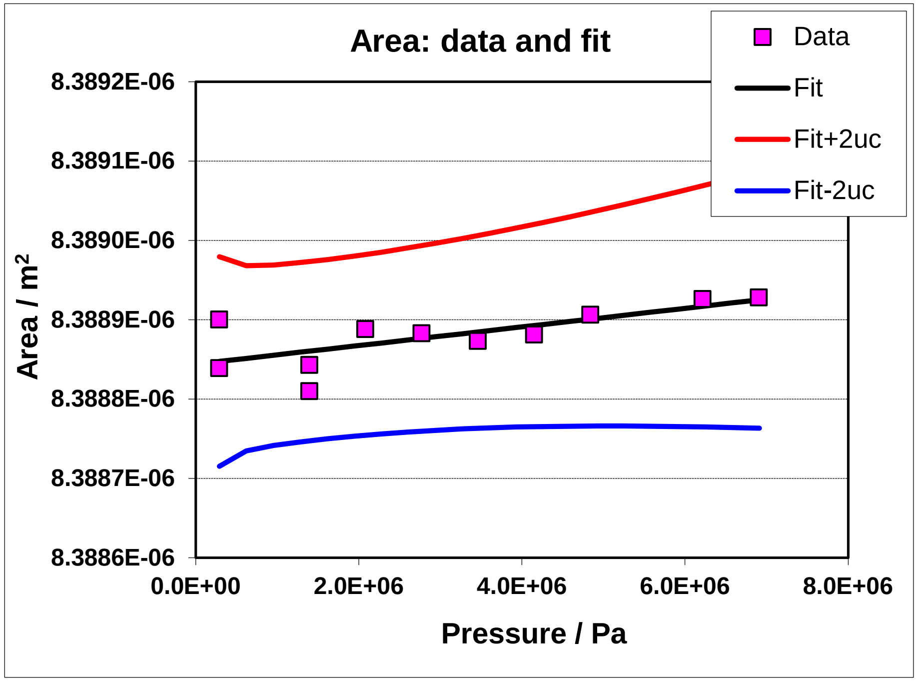
- For most cases, the largest components (Area of standard, masses, thermometer calibration) are not dependent on experimental conditions
- Can develop general Type B uncertainty expressions that cover most conditions
- E.g., NIST PG13 in gauge mode

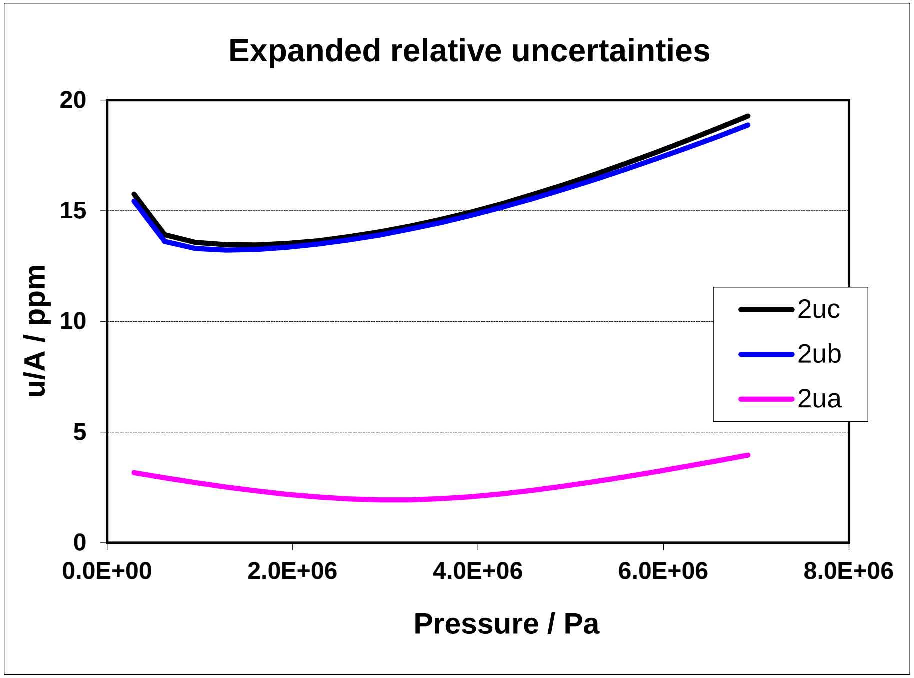
$$\frac{u_B}{A_e} = \left(\left(\frac{1.18}{p} \right)^2 + \left(6.53\text{E-}6 \right)^2 + \left(1.12\text{E-}12 \left(p - 828704 \right) \right)^2 + \left(\frac{0.29}{p} h \right)^2 \right)^{1/2}$$

- Need to consider long term stability of your piston gauge standard

Combined Uncertainty

$$u_c = \left(u_A^2 + u_B^2\right)^{1/2}, \quad U_c = ku_c, \quad k = 2$$





- Gas piston gauge calibration, gauge mode, 7 MPa
- Linear fit. Combined uncertainty dominated by Type B

Summary of Uncertainty Analysis

- Use appropriate measurement equation for the type of calibration
- Uncertainties in PG effective area and masses are always important
- Minimize differences in reference level, especially for oil calibrations below 10 MPa
- Air density and gravity are more important in a pressure calibration than an effective area calibration
- Need to know if mass calibration included uncertainty of the density of the masses
- Type B uncertainties for piston gauges are usually dominant
- If providing a calibration equation to characterize the DUT, Type A uncertainty should be that of the fitted equation

Case 2: DUT = pressure transducer

Measurement Equation

$$p_T = \frac{\sum_i m_i g\left(1-\frac{\rho_a}{\rho_{mi}}\right)+\gamma C}{A_e\left(1+\left(\alpha_p+\alpha_c\right)\left(T-T_{REF}\right)\right)}+p_V-\left(\rho_f-\rho_a\right)gh$$

Uncertainty Terms for Type B

$$u_B^2\left(p_T\right)=u_{A_e}^2+u_M^2+u_{\rho_a}^2+u_{\rho_M}^2+u_\gamma^2+u_C^2+u_g^2+u_{\bar{\alpha}}^2+u_T^2+u_{\rho_f}^2+u_h^2+u_{p_V}^2$$

Area of standard and masses are usually most important
Surface tension force insignificant
Gravity uncertainty contributes directly to pressure uncertainty
In gauge mode, 1 % unc. in air density contributes 1.3 ppm unc. in pressure

Case 2: Uncertainty due to Piston Gauge Area, u_{Ae}

Sensitivity Coefficient

$$\frac{\partial p_T}{\partial A_e} = -\frac{p_T}{A_e}$$

Standard uncertainty, $u(A_e)$

- From calibration of piston gauge
- Usually the largest component

u_{Ae} = 6 to 20 ppm of A_e

National Institute of Standards and Technology

Case 2: Uncertainty due to Piston Gauge Masses, u_{MR}

Sensitivity Coefficient

$$\frac{\partial \boldsymbol{p_T}}{\partial \boldsymbol{M}} = \frac{\boldsymbol{p_T}}{\boldsymbol{M}}$$

Standard uncertainty, u(M)

- From mass calibration
- Assume correlated, meaning uncertainty of the sum of masses are added geometrically rather than in quadrature $u(M)=u(m_1)+u(m_2)+\ldots+u(m_n)$
- Need to know if density uncertainty was included in mass uncertainty quoted by mass calibration

Case 2: Uncertainty due to Air Density, $u_{\rho A}$

Sensitivity Coefficient (0 for absolute mode)

- First term is due to buoyancy correction, second term is due to head correction

$$\frac{\partial p_T}{\partial \rho_a} = -\frac{p_T}{\rho_m} + gh$$

Standard uncertainty, $u(\rho_a)$

- Need to measure ρ_a ; $u(\rho_a)$ due to $u(P_{amb})$, $u(T_{amb})$. Likely < 0.01 kg/m^3

$u_{\rho A}$ 1.3 ppm if $u(\rho_a)$ = 0.01 kg/m^3

Case 2: Uncertainty due to PG Mass Density, $u_{\rho M}$

Sensitivity Coefficient

- assumes mass density uncertainty not included in mass calibration
- assumes mass calibration at ambient pressure

Gauge mode

$$\frac{\partial p_T}{\partial \rho_M} = p_T \frac{\left(\rho_a - \rho_{a,cal}\right)}{\rho_M^2}$$

Absolute mode

$$\frac{\partial p_T}{\partial \rho_M} = -p_T \frac{\rho_{a,cal}}{\rho_M^2}$$

$\rho_{a,cal}$ is air density at time of mass cal. Air density diff ~ 0.03 kg/m^3
Sensitivity coefficient is larger in absolute mode

Standard uncertainty, $u(\rho_{M,R})$

- OIML R 111-1: guidelines to measure. If not measured, $u(\rho_{M,R})$ = 70 kg/m^3 for SS.

$u_{\rho M}$ = 1.3 ppm (absolute mode, $u(\rho_{M,R})$ = 70 kg/m^3). Ignore in gauge mode

If mass density uncertainty was included in mass calibration

- Absolute mode: set = 0 to avoid double counting.
- Gauge mode: ask for mass uncertainty without density uncertainty.

OIML R 111-1 at http://www.fundmetrology.ru/depository/04_IntDoc_all/R_E_111-1.pdf

Case 2: Uncertainty due to Gravity, u_g

Sensitivity Coefficient

- Gravity more important than in piston gauge calibration
- Head correction ignored compared to mass force

$$\frac{\partial p_T}{\partial g} = \frac{p_T}{g}$$

Standard uncertainty, u(g)

- At NIST we use u(g) = 0.2 ppm. Will be larger if not measured.

Case 2: Uncertainty due to thermal expansion and temperature, u_α & u_T

Sensitivity Coefficients

$$\frac{\partial p_T}{\partial \bar{\alpha}} = -p_T(T - T_{REF})$$

$$\frac{\partial p_T}{\partial T} = -p_T \bar{\alpha}$$

Standard uncertainty, u(α) and u(T)

- Same considerations as for piston gauge calibration. Uncertainties can be 0.5 ppm to 1.0 ppm

Case 2: Uncertainty due to Fluid Density, $u_{\rho f}$

Sensitivity Coefficient

$$\frac{\partial p_T}{\partial \rho_f} = -gh$$

Standard uncertainty, $u(\rho_f)$

- Gas: insignificant
- Can be minimized by reducing h
- Oil: if not measured, estimate as 1 % of fluid density

$u_{\rho f}$ in oil: for h = 0.1 m, can be 10 Pa. Can become important at $\boldsymbol{p}$ < 10 MPa

Case 2: Uncertainty due to reference height difference, u_h

Sensitivity Coefficient

$$\frac{\partial p_T}{\partial h} = -\left(\rho_f - \rho_a\right) g$$

Standard uncertainty, u(h)

- Gas: insignificant
- Depends on position resolution of both pistons, measurement of reference level difference. At NIST, we take u(h) = 1 mm

u_h in oil: 10 Pa. Can become important at p < 10 MPa

Case 2: Uncertainty due to bell jar pressure, u_{PV}

Sensitivity Coefficient

$$\frac{\partial p_T}{\partial p_V} = 1$$

Standard uncertainty, $u(p_v)$

- Comes from vacuum gauge calibration
- If p_v= 2 Pa, $u(p_v)$ = 0.5 % of $p_{v,}$ this is insignificant except for p < 10 kPa

Case 2: Summary of uncertainties, 100 kPa, absolute mode

Uncertainty term				Sensitivity coefficient divided by p_T			Standard uncertainty		Rel. unc. on p_T
Name	Symbol	Value	Units	Definition	Abs value	Units	Value	Units	
PG Area	A_E	3.357E-04	m^2	$-1/A_E$	2979	m^{-2}	1.88E-09	m^2	**5.59E-06**
PG Mass	M	3.425	kg	$1/M$	0.292	kg^{-1}	6.85E-06	kg	**2.00E-06**
Ambient density	ρ_a	0.000	kg/m^3	$-1/\rho_M + gh/p_T$	1.09E-04	m^3/kg	1.14E-07	kg/m^3	1.24E-11
Mass density	ρ_M	7800	kg/m^3	$(\rho_a - \rho_{a,cal})/\rho_M^2$	1.94E-08	m^3/kg	45.03	kg/m^3	8.73E-07
Surface tension	γ	0.000	N/m	$C/(Mg)$	1.93E-03	m/N	0.000	N/m	0.000
PG circum.	C	6.50E-02	m	$\gamma/(Mg)$	0.000	1/m	6.50E-05	m	0.000
Gravity	g	9.801011	m/s^2	$1/g$	0.102	s^2/m	2.00E-06	m/s^2	2.04E-07
PG therm. exp.	$\alpha_p + \alpha_c$	1.46E-05	C^{-1}	$-(T - 23.00)$	0.50	C	8.40E-07	C^{-1}	4.20E-07
PG temperature	T	22.50	C	$-(\alpha_p + \alpha_c)$	1.46E-05	C^{-1}	0.0577	C	8.40E-07
Fluid density	ρ_f	1.14	kg/m^3	$-gh/p_T$	1.96E-05	m^3/kg	0.0011	kg/m^3	2.23E-08
Height difference	h	0.20	m	$-(\rho_f - \rho_a)g/p_T$	1.12E-04	1/m	8.20E-04	m	9.16E-08
Bell jar pressure	p_V	2.00	Pa	$1/p_T$	1.00E-05	1/Pa	0.010	Pa	1.00E-07
					Relative combinded standard unc.				**6.08E-06**