The Observed Properties of Liquid Helium at the Saturated Vapor Pressure

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The equilibrium and transport properties of liquid ^{4}He are deduced from experimental observations at the saturated vapor pressure. In each case, the bibliography lists all known measurements. Quantities reported here include density, thermal expansion coefficient, dielectric constant, superfluid and normal fluid densities, first, second, third, and fourth sound velocities, specific heat, enthalpy, entropy, surface tension, ion mobilities, mutual friction, viscosity and kinematic viscosity, dispersion curve, structure factor, thermal conductivity, latent heat, saturated vapor pressure, thermal diffusivity and Prandtl number of helium I, and displacement length and vortex core parameter in helium II.
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Key words: helium I; helium II; observed equilibrium and transport properties of liquid ^{4}He.

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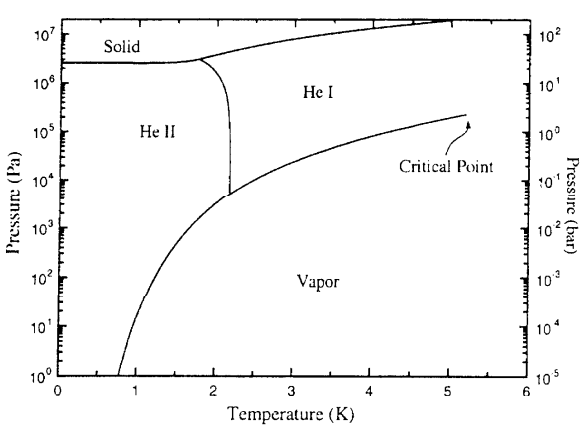
Introduction

Liquid helium has been studied intensively for more than three quarters of a century: indeed one may advance the claim that it is the most studied pure substance in the history of science. Our group, pursuing certain theoretical topics in the 1960's, became aware of the unwelcome impact of faulty data in testing ideas on the theory of superfluidity. We therefore began a collection of well-documented data which first appeared in the book by Donnelly[1] and a further set of useful tables appeared in the book by Wilks.[2] As research progressed, it became evident that the pressure of liquid helium is just as significant a parameter as temperature, and it was recognized that it would be decades before enough data would exist to make a critical compilation of the properties of liquid helium as a function of pressure and temperature.

A ray of hope emerged when inelastic neutron scattering studies began to appear, and one gradually learned the true experimental nature of the dispersion curve for elementary excitations in He II. Early work by Bendt, Cowan and Yarnell[3] showed that the thermodynamic properties of He II along the vapor pressure line could be extracted from neutron scattering measurements with reasonable success. Unfortunately, it soon became apparent that the parameters of the spectrum are both temperature dependent and pressure dependent. The temperature dependence was especially troublesome because the standard formulas of statistical mechanics, and indeed quantum mechanics, have no direct provision for handling temperature-dependent energy levels. This problem was studied by Donnelly and Roberts[4] who produced an approximate method for the computation of thermodynamic quantities which can be used when the energy levels are known from the experiment. The methods of Donnelly and Roberts then were used by Brooks and Donnelly[5] to produce their report "The Calculated Thermodynamic Properties of Superfluid Helium-4" which evolved through a thesis, a preliminary edition, and finally a publication over the period 1972–1977. Brooks and Donnelly's tables of equilibrium properties have the advantage that they are thermodynamically self-consistent over the entire temperature–pressure (T,P) plane; that is: they obey Maxwell's relations; they have the disadvantage that they are not useful near the λ transition. Further, they are based on some *ad hoc* assumptions such as the concept of an "effective sharp spectrum" whose validity is not known except by the test of utility. Nevertheless, those calculated tables still stand as the most reliable general guide to the equilibrium properties of He II over the whole (T,P) plane as shown in the diagram below.



With the issue of this study, we are presenting a comprehensive collection of the experimental data for the equilibrium and transport properties of pure liquid ^{4}He.

The format used will generally be the following:

(1) Chronological bibliography of all known measurements (placed at the end of each section).
(2) Selection of an "adopted database" obtained by subjective judgment. Note that the temperature ranges given in the adopted table base are in the temperature scale used by the authors, not necessarily T_{90}.
(3) To facilitate the description of the adopted database we have chosen a table format. The key # is used to give the origin of the data in the adopted database and leads to the literature citation. To save space we have used $E+2$, $E+1$, $E+0$, $E-1$ instead of 10^2, 10, 1, 10^{-1} etc.
(4) Table of the adopted database. Temperature values in the adopted database are in T_{90} Note that the ITS-90 is not defined below 0.65 K. Values below 0.65 K are on the thermodynamic scale.
(5) Table of recommended values obtained by the cubic spline fits of the data.
(6) Curves showing the recommended values and the fractional deviation of values of the adopted database from the recommended values.
If $f(T)$ is the quantity being discussed we define the fractional deviation Δ in percent as

$$\Delta=\frac{f_{\text{measured}}-f_{\text{calculated}}}{f_{\text{measured}}}\times 100.$$

(7) Tables of recommended values.

In most cases we have represented the data by cubic splines. A routine to evaluate these splines is included as an appendix.

A word is required about significant figures in this study. Most (but certainly not all) of the data here is on the order of 1% in accuracy so that three figures should be adequate to represent it. Furthermore, the spline fit returns all the figures yielded by the calculation. Nevertheless we have usually given recommended data to four figures simply as a compromise among data of varying precision and accuracy.

A second problem is that some data such as specific heat spans an enormous range: over 5 decades in $T-T_\lambda$ (see the recommended values in Sec. 7).

Temperature resolution is another problem. Modern thermometry resolves temperatures throughout the region to 4 K with millikelvin to microkelvin precision. However the absolute temperature is not known to anything like this resolution. Most superfluidity researchers have used the T_{58} vapor pressure scale and their data is keyed to it for better or for worse. However the T_{90} scale is now used to calibrate thermometers, and we have undertaken the somewhat risky task of converting our T_{58} tables, which were circulated as a Report of the Department of Physics, to T_{90}. The method of doing this was outlined in an MS thesis by Ling Lui in 1992.[6]

In some cases we have recalculated temperatures ourselves after discussion with the authors. Thus a few figures appear which are not exactly those originally published. Naturally, the biggest changes in data, and greatest sensitivity to the T_{90}, occur near the lambda transition.

An even more bizarre situation appears in representing data near the lambda transition. Here the difference in temperature from the lambda transition is what is really measured, and these days, that can be done in some cases to

nanokelvin resolution, especially above 2.17 K. Thus in the last 100 mk below and above the lambda point we have had to carry many significant figures because our spline fits the temperature. It is the joy of least squares cubic splines that they can represent data near a singular point like the lambda point. The down side is that we are printing data on temperature with many figures which were really measured as deviations. The reader will appreciate that any strict treatment of printing only significant figures will lead to a typographic nightmare.

We can summarize the situation by stating that we are usually erring on the side of carrying too many figures, and that the antidote is a quick glance at the normalized deviation curve.

In making the spline fits, we have tried to get as good a fit to the data as possible, especially near the lambda point. But these fits should under no circumstances be relied upon for scaling investigations.

Note that we have listed a number of quantities in the units normally used by the low temperature physics community. Conversion to SI units has been made in several cases in this article and conversion factors to SI are given wherever needed.

Our initial work was done using HP BASIC. We list spline evaluation programs in the Appendix in HP BASIC. In addition, FORTRAN, C, visual Basic, and HP48G subroutines are given.

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[6] L. Lui, MS thesis, University of Oregon (unpublished).

1. Density, Thermal Expansion, and Dielectric Constant

The density of liquid ^{4}He at saturated vapor pressure (SVP) has been measured by a number of authors (see the chronological bibliography at the end of this chapter). We have chosen as our source the recent work of Niemela and Donnelly,[40] who measured the dielectric constant ε of liquid helium and resolved many of the discrepancies which existed in the older literature.

Dielectric constants were converted to density using the Clausius–Mossotti relation:

$$\frac{(\varepsilon-1)}{(\varepsilon+2)}=\frac{4\pi\alpha_M\rho}{3M}, \tag{1.1}$$

where $M=4.0026$ g/mol is the molecular weight of helium and α_M is the molar polarizability. They adopted the value $\alpha_M=0.123\,296$ cm^3/mol deduced by Harris-Lowe and Smee[30] from their measurements of dielectric constant and Kerr and Taylor's[18] measurement of the density.

Although Niemela and Donnelly did not make any measurements below the density minimum, they provided a continuous representation of the density and expansion coefficient to zero temperature using theoretical methods described in detail in their paper obtaining values of the density which are described by the following equation up to 1.344 K:

$$\rho-\rho_0=\sum_{i=1}^{4} m_i T^{(i+1)}, \tag{1.2}$$

where $\rho_0=0.145\,139\,7$ g/cm^3. The coefficients of Eq. (2) are given in Table 1.1. The authors fit the reduced density

$$\Delta\rho=\frac{\rho-\rho_\lambda}{\rho_\lambda} \tag{1.3}$$

to a function of the form:

$$\Delta\rho=\sum_{i=1}^{2} a_i t^i \ln|t|+\sum_{i=1}^{m} b_i t^i, \tag{1.4}$$

where $t=T-T_\lambda$. Here, $T_\lambda=2.1768$ K and $\rho_\lambda=0.146\,108\,7$ g/cm^3. Because they wished to tabulate the expansion coefficient over the entire range of temperatures, it was desirable that not only the density, but also its first derivative should smoothly join the calculated values given by Eq. (1.2). This was best achieved by using Eq. (1.2) up to 1.344 K and using Eq. (1.4) for data in the ranges $1.344\ \text{K}\leq T\leq T_\lambda$ and $T_\lambda<T<4.9$ K. The resulting set of equations and their derivatives provide a continuous representation of the density and expansion coefficient from near 0 to 4.9 K. The coefficients for Eq. (1.4) are given in Table 1.1. The mean fractional deviation of the density data from the fit is 0.5×10^{-6}.

The density is tabulated in g/cm^3. The entries must be multiplied by 1000 to convert to kg/m^3.

TABLE 1.1. Coefficients for Eqs. (2), (4), (5) and (6)

	$a_i\times10^3$		$b_i\times10^3$		$m_i\times10^5$	$s_i\times10^3$
i	1.344 K–T_λ	T_λ–4.9 K	1.344 K–T_λ	T_λ–4.9 K	0–1.344 K	0–1.344 K
1	−7.575 37	−7.946 05	3.799 37	−30.3511	−1.269 35	−0.117 818
2	6.874 83	5.07 051	1.865 57	−10.2326	7.124 13	1.640 45
3			4.883 45	−3.006 36	−16.746 1	−6.187 50
4			0	0.240 720	8.753 42	13.4293
5			0	−2.457 49		−11.3971
6			0	1.534 54		2.941 76
7			0	−0.308 182		

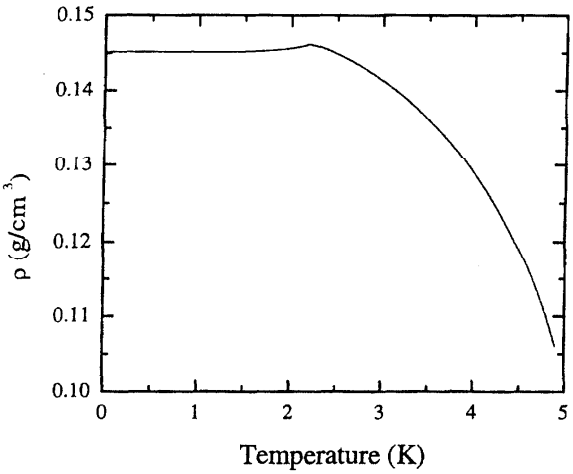

FIG. 1.1. The recommended values of the density of liquid ^{4}He as a function of temperature at the saturated vapor pressure.

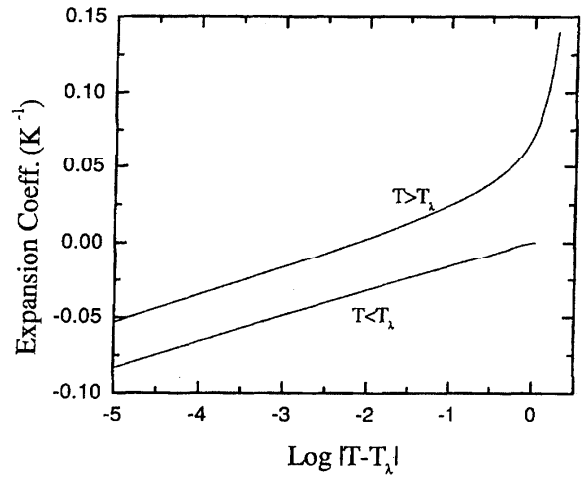

FIG. 1.3. Detail of the recommended values for the thermal expansion coefficient of liquid ^{4}He near the lambda transition.

In Fig. 1.1 we plot Eqs. (1.2) and (1.4) for the density. The following expressions give the deduced thermal expansion coefficient: For $0.15 \leq T < 1.344$:

$$\alpha = \sum_{i=1}^{6} s_i T^i. \tag{1.5}$$

For $1.344 \leq T < T_\lambda$ and $T_\lambda < T < 4.9$ K,

$$\alpha = \frac{-\rho_\lambda}{\rho}\left((a_1+b_1)+a_1 \ln|t|+(a_2+2b_2)t+2a_2 t \ln|t| + \sum_{n=3}^{7} n b_n t^{(n-1)}\right). \tag{1.6}$$

The coefficients are listed in Table 1.1. The method of obtaining Eq. (1.5) is explained by Niemela and Donnelly. Note that the average fractional deviation of Eq. (1.5) from the theoretical values on which it is based is 1×10^{-6}, so that its valid temperature range begins 150 mK above absolute zero. Equation (1.6) was derived from Eq. (1.4). Also be aware that these data were not taken in great temperature detail near the lambda point.

Figure 1.2 shows the expansion coefficient over the entire range of temperatures. The dielectric constant $\varepsilon(T)$ can be obtained as follows:

$$\varepsilon(T) = \frac{1+2\eta(T)}{1-\eta(T)}. \tag{1.7}$$

Here,

$$\eta(T) = \frac{4\pi\alpha_M}{3M}\rho(T). \tag{1.8}$$

This roundabout method of getting the dielectric constant arises because Niemela and Donnelly fitted the density (their main goal) after converting individual dielectric constant measurements to density by means of Eq. (1.1).

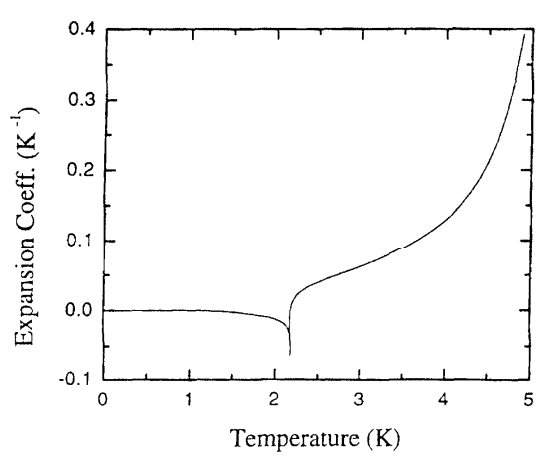

FIG. 1.2. The recommended values of the thermal expansion coefficient of liquid ^{4}He at the saturated vapor pressure as a function of temperature.

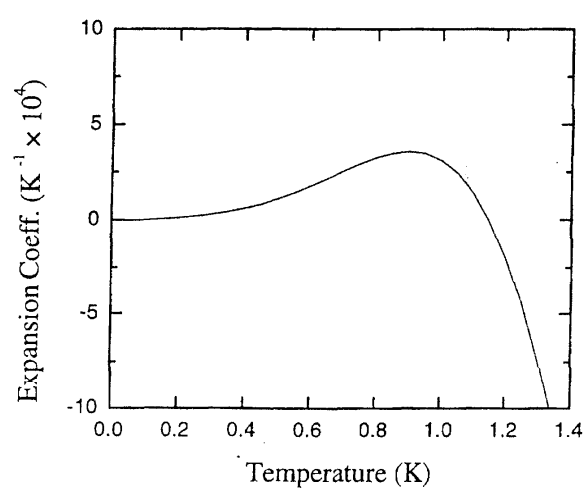

FIG. 1.4. The recommended values of the thermal expansion coefficient of liquid ^{4}He at low temperatures.

TABLE 1.2. Recommended values of the dielectric constant, density and thermal expansion coefficient of liquid ^{4}He at saturated vapor pressure

T_{90} (K)	ε	ρ (g/cm^3)	$10^3\alpha$ (K^{-1})	T_{90} (K)	ε	ρ (g/cm^3)	$10^3\alpha$ (K^{-1})
0.00	1.057 255	1.451 397$E-1$	0.000	2.50	1.057 135	1.448 402$E-1$	39.4
0.10	1.057 255	1.451 397$E-1$	0.001	2.55	1.057 017	1.445 467$E-1$	41.8
0.15	1.057 255	1.451 396$E-1$	0.004	2.60	1.056 892	1.442 368$E-1$	44.1
0.20	1.057 255	1.451 395$E-1$	0.011	2.65	1.056 761	1.439 114$E-1$	46.3
0.25	1.057 255	1.451 395$E-1$	0.018	2.70	1.056 625	1.435 712$E-1$	48.4
0.30	1.057 255	1.451 393$E-1$	0.028	2.75	1.056 482	1.432 164$E-1$	50.6
0.35	1.057 255	1.451 391$E-1$	0.042	2.80	1.056 334	1.428 472$E-1$	52.7
0.40	1.057 255	1.451 388$E-1$	0.058	2.85	1.056 180	1.424 638$E-1$	54.8
0.45	1.057 254	1.451 384$E-1$	0.080	2.90	1.056 020	1.420 661$E-1$	57.0
0.50	1.057 254	1.451 377$E-1$	0.107	2.95	1.055 854	1.416 538$E-1$	59.2
0.55	1.057 254	1.451 368$E-1$	0.139	3.00	1.055 683	1.412 269$E-1$	61.5
0.60	1.057 253	1.451 356$E-1$	0.175	3.05	1.055 505	1.407 850$E-1$	63.9
0.65	1.057 253	1.451 342$E-1$	0.214	3.10	1.055 322	1.403 279$E-1$	66.3
0.70	1.057 252	1.451 324$E-1$	0.254	3.15	1.055 132	1.398 551$E-1$	68.7
0.75	1.057 251	1.451 304$E-1$	0.292	3.20	1.054 936	1.393 663$E-1$	71.3
0.80	1.057 250	1.451 281$E-1$	0.325	3.25	1.054 733	1.388 611$E-1$	74.0
0.85	1.057 249	1.451 257$E-1$	0.348	3.30	1.054 523	1.383 390$E-1$	76.7
0.90	1.057 248	1.451 232$E-1$	0.357	3.35	1.054 307	1.377 997$E-1$	79.5
0.95	1.057 247	1.451 207$E-1$	0.345	3.40	1.054 084	1.372 427$E-1$	82.5
1.00	1.057 246	1.451 183$E-1$	0.309	3.45	1.053 853	1.366 675$E-1$	85.5
1.05	1.057 246	1.451 163$E-1$	0.242	3.50	1.053 615	1.360 736$E-1$	88.7
1.10	1.057 245	1.451 150$E-1$	0.138	3.55	1.053 369	1.354 605$E-1$	92.0
1.15	1.057 245	1.451 144$E-1$	−0.008	3.60	1.053 115	1.348 278$E-1$	95.3
1.20	1.057 245	1.451 151$E-1$	−0.200	3.65	1.052 853	1.341 748$E-1$	98.9
1.25	1.057 246	1.451 173$E-1$	−0.442	3.70	1.052 583	1.335 009$E-1$	103
1.30	1.057 248	1.451 215$E-1$	−0.737	3.75	1.052 305	1.328 054$E-1$	106
1.35	1.057 250	1.451 281$E-1$	−1.08	3.80	1.052 017	1.320 877$E-1$	110
1.40	1.057 254	1.451 373$E-1$	−1.45	3.85	1.051 720	1.313 467$E-1$	115
1.45	1.057 259	1.451 493$E-1$	−1.87	3.90	1.051 414	1.305 817$E-1$	119
1.50	1.057 265	1.451 646$E-1$	−2.36	3.95	1.051 097	1.297 914$E-1$	124
1.55	1.057 273	1.451 837$E-1$	−2.91	4.00	1.050 770	1.289 745$E-1$	129
1.60	1.057 282	1.452 071$E-1$	−3.53	4.05	1.050 432	1.281 296$E-1$	134
1.65	1.057 293	1.452 352$E-1$	−4.23	4.10	1.050 082	1.272 549$E-1$	140
1.70	1.057 307	1.452 686$E-1$	−4.99	4.15	1.049 719	1.263 483$E-1$	146
1.75	1.057 323	1.453 079$E-1$	−5.84	4.20	1.049 343	1.254 075$E-1$	153
1.80	1.057 341	1.453 538$E-1$	−6.79	4.25	1.048 952	1.244 297$E-1$	160
1.85	1.057 362	1.454 070$E-1$	−7.86	4.30	1.048 545	1.234 117$E-1$	168
1.90	1.057 387	1.454 684$E-1$	−9.07	4.35	1.048 121	1.223 498$E-1$	177
1.95	1.057 416	1.455 394$E-1$	−10.5	4.40	1.047 677	1.212 398$E-1$	187
2.00	1.057 449	1.456 217$E-1$	−12.2	4.45	1.047 213	1.200 768$E-1$	198
2.05	1.057 488	1.457 181$E-1$	−14.4	4.50	1.046 725	1.188 552$E-1$	211
2.10	1.057 534	1.458 340$E-1$	−17.7	4.55	1.046 211	1.175 686$E-1$	225
2.15	1.057 594	1.459 840$E-1$	−24.7	4.60	1.045 669	1.162 098$E-1$	241
2.20	1.057 643	1.461 049$E-1$	9.64	4.65	1.045 095	1.147 706$E-1$	258
2.25	1.057 596	1.459 877$E-1$	20.7	4.70	1.044 485	1.132 419$E-1$	279
2.30	1.057 526	1.458 148$E-1$	26.4	4.75	1.043 836	1.116 131$E-1$	301
2.35	1.057 443	1.456 071$E-1$	30.5	4.80	1.043 143	1.098 727$E-1$	328
2.40	1.057 349	1.453 727$E-1$	33.8	4.85	1.042 400	1.080 076$E-1$	358
2.45	1.057 246	1.451 162$E-1$	36.7	4.90	1.041 602	1.060 033$E-1$	392

1.1. Chronological Bibliography for Density and Expansion Coefficient

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1.2. Chronological Bibliography for Dielectric Constant

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2. Superfluid and Normal Fluid Densities

Adopted Database

Author(s)	Key #	Method	Range (K)	Uncertainty (%)
Landau Theory	1	Integration	$0.1 \leq T \leq 1.25$	
Maynard	2	u_2 & u_4	$1.2 \leq T \leq 2.15$	<0.5
Tam & Ahlers	3	u_2 & u_4	$1.553 \leq T \leq 2.15917$	<0.5
Singsass & Ahlers	4	Asymptotic formula	$T \geq 2.072$	see paper

Comments and Key to Authors

(1) We have generated 24 points of ρ_n, the normal fluid density from 0.1 to 1.25 K by integrating over the dispersion curve of Sec. 13. This method is very good at low temperatures, but by 1 K starts to degrade because the spectrum becomes temperature dependent. We used the total density from Sec. 1 to generate the superfluid density ρ_s.

(2) Data from Ref. 22.

(3) Data from Ref. 25.

(4) Singsass and Ahlers, (Ref. 24) give the asymptotic formula $\rho_s/\rho = k_0(1+k_1|t|)|t|^{\zeta}(1+D_\rho|t|^{\Delta})$, with $t = 1-(T/T_\lambda)$, $k_0 = 2.403$, $k_1 = -1.46$, $D_\rho = 0.33$, $\zeta = 0.6717$, and $\Delta = 0.5$. The formula is based on new precision entropy data. We generated 18 points from 2.1 to 2.1768 K.

(5) The total density from Sec. 1 is used to compute quantities such as $\rho_n = \rho - \rho_s$, ρ_s/ρ, and ρ_n/ρ.

(6) The densities are tabulated in g/cm^3. Multiply entries by 1000 to convert to kg/m^3.

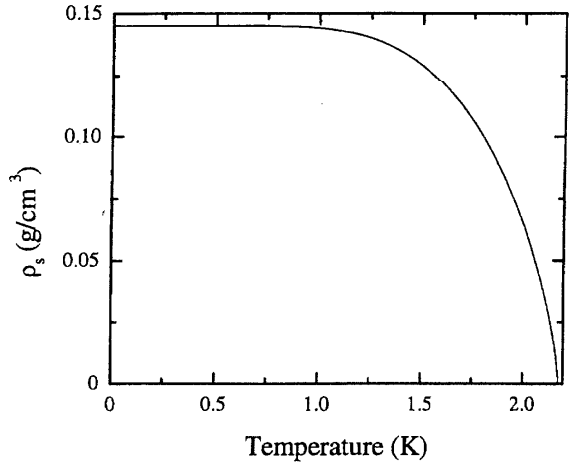

FIG. 2.1. Recommended values of the superfluid density of helium II as a function of temperature.

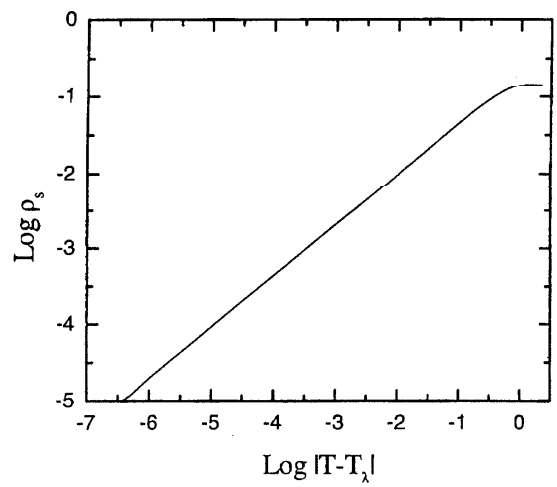

FIG. 2.2. Recommended values of the superfluid density of helium II near the lambda transition.

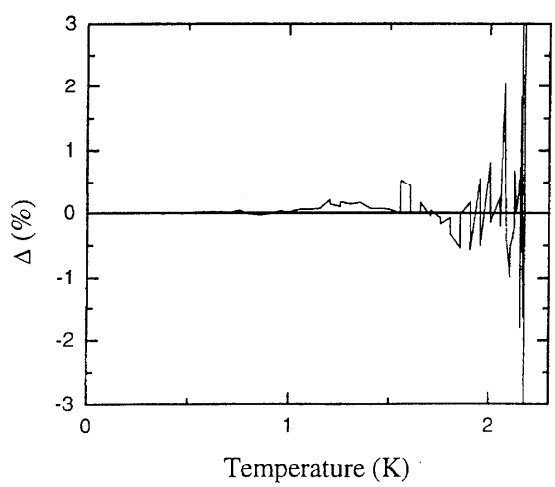

FIG. 2.3. The fractional deviation of the adopted database from the recommended values for the superfluid density of helium II expressed in percent.

TABLE 2.1. The adopted database for the superfluid density of helium II as a function of temperature

T_{90} (K)	ρ_s (g/cm^3)	Key	T_{90} (K)	ρ_s (g/cm^3)	Key
0	0.145 14	1	1.854 119 00	0.091 93	2
0.100 36490	0.1451	1	1.904 151 00	0.083 59	2
0.150 53830	0.1451	1	1.904 151 00	0.084 19	3
0.200 70610	0.1451	1	1.904 151 00	0.084 21	3
0.250 86810	0.1451	1	1.954 189 00	0.074 2	2
0.301 02560	0.1451	1	1.954 189 00	0.074 93	3
0.351 17790	0.1451	1	1.954 189 00	0.074 99	3
0.401 32560	0.1451	1	2.004 234 00	0.063 6	2
0.451 46850	0.1451	1	2.004 234 00	0.064 21	3
0.501 60770	0.1451	1	2.054 293 00	0.051 45	2
0.551 74260	0.1451	1	2.054 293 00	0.051 64	3
0.601 87360	0.1451	1	2.054 293 00	0.051 68	3
0.651 99130	0.1451	1	2.076 800 00	0.044 19	4
0.702 12460	0.1451	1	2.084 348 00	0.042 97	3
0.752 24480	0.145	1	2.104 399 00	0.036 69	2
0.802 36180	0.145	1	2.104 399 00	0.036 51	3
0.852 47570	0.1449	1	2.104 399 00	0.036 52	3
0.902 58630	0.1447	1	2.114 429 00	0.033 02	3
0.952 69420	0.1444	1	2.124 462 00	0.029 35	3
1.002 77700	0.1441	1	2.126 800 00	0.028 21	4
1.052 86100	0.1436	1	2.134 497 00	0.025 39	3
1.102 95500	0.143	1	2.144 531 00	0.021 08	3
1.153 02900	0.1422	1	2.149 547 00	0.018 73	3
1.203 26700	0.141	2	2.154 563 00	0.016 33	3
1.203 26700	0.1411	1	2.154 563 00	0.016 28	3
1.253 20300	0.1399	1	2.154 563 00	0.016 7	2
1.253 20300	0.1398	2	2.158 575 00	0.014 09	3
1.303 32400	0.1383	2	2.160 580 00	0.013 08	3
1.353 44300	0.1364	2	2.162 586 00	0.012 18	3
1.403 51500	0.1343	2	2.163 589 00	0.011 2	3
1.453 55700	0.1317	2	2.166 800 00	0.009 59	4
1.503 59800	0.1287	2	2.170 800 00	0.006 79	4
1.553 63700	0.1253	2	2.173 800 00	0.004 25	4
1.556 64000	0.1244	3	2.175 800 00	0.002 02	4
1.603 69000	0.1208	3	2.176 200 00	0.001 43	4
1.603 69000	0.1213	2	2.176 500 00	8.990 48$E-4$	4
1.653 77900	0.1168	2	2.176 700 00	4.291 95$E-4$	4
1.653 77900	0.1166	3	2.176 740 00	3.043 92$E-4$	4
1.703 88200	0.1117	2	2.176 770 00	1.909 95$E-4$	4
1.703 88200	0.1116	3	2.176 790 00	9.126 85$E-5$	4
1.753 98100	0.1059	2	2.176 795 00	5.728 37$E-5$	4
1.753 98100	0.106	3	2.176 799 00	1.942 74$E-5$	4
1.804 06500	0.099 33	2	2.176 799 50	1.219 51$E-5$	4
1.804 06500	0.099 58	3	2.176 799 90	4.136 64$E-6$	4
1.854 11900	0.092 41	3	2.176 800 00	0	

TABLE 2.2. Knots and coefficients of the spline fit of the superfluid density of helium II

Knots	Coefficients
$K(1)=0.0$	$C(1)=1.451\,275\,432\,822\,459E-1$
$K(2)=0.0$	$C(2)=1.451\,334\,563\,362\,309E-1$
$K(3)=0.0$	$C(3)=1.449\,759\,191\,497\,576E-1$
$K(4)=0.0$	$C(4)=1.455\,008\,000\,684\,433E-1$
$K(5)=0.443$	$C(5)=1.407\,5E-1$
$K(6)=0.9012$	$C(6)=1.095\,E-1$
$K(7)=1.5419$	$C(7)=8.15E-2$
$K(8)=1.7540$	$C(8)=5.30E-2$
$K(9)=1.918$	$C(9)=2.1E-2$
$K(10)=2.111$	$C(10)=8.904\,576E-3$
$K(11)=2.156\,991$	$C(11)=3.053\,214E-3$
$K(12)=2.173\,218$	$C(12)=1.494\,043E-3$
$K(13)=2.175\,647$	$C(13)=8.342\,826E-4$
$K(14)=2.176\,358$	$C(14)=5.106\,86E-4$
$K(15)=2.176\,568$	$C(15)=2.837\,9E-4$
$K(16)=2.176\,692$	$C(16)=1.287\,426E-4$
$K(17)=2.176\,766$	$C(17)=5.202\,569E-5$
$K(18)=2.176\,791$	$C(18)=2.153\,580E-5$
$K(19)=2.176\,798$	$C(19)=8.564\,206E-6$
$K(20)=2.176\,799$	$C(20)=3.567\,958E-6$
$K(21)=2.176\,799\,99$	$C(21)=0$
$K(22)=2.1768$	
$K(23)=2.1768$	
$K(24)=2.1768$	
$K(25)=2.1768$	

TABLE 2.3. Recommended values of the normal and superfluid densities of helium II

T_{90} (K)	ρ_s (g/cm^3)	ρ_n (g/cm^3)	T_{90} (K)	ρ_s (g/cm^3)	ρ_n (g/cm^3)
0	0.145 13	0	1.45	0.131 99	0.013 16
0.05	0.145 13	$2.95E-9$	1.5	0.129 00	0.016 17
0.1	0.145 12	$5.9E-9$	1.55	0.125 56	0.019 63
0.15	0.145 11	$8.85E-9$	1.6	0.121 63	0.023 58
0.2	0.145 10	$2.771E-8$	1.65	0.117 15	0.028 09
0.25	0.145 09	$6.687E-8$	1.7	0.112 06	0.033 21
0.3	0.145 09	$1.368E-7$	1.75	0.106 30	0.039 00
0.35	0.145 08	$2.502E-7$	1.8	0.099 82	0.045 54
0.4	0.145 08	$4.242E-7$	1.85	0.092 54	0.052 86
0.45	0.145 09	$7.03E-7$	1.9	0.084 44	0.061 03
0.5	0.145 11	$1.249E-6$	1.95	0.075 42	0.070 12
0.55	0.145 12	$2.588E-6$	2	0.065 07	0.080 55
0.6	0.145 13	$6.069E-6$	2.05	0.052 75	0.092 97
0.65	0.145 13	$1.452E-5$	2.1	0.037 79	0.108 04
0.7	0.145 11	$3.295E-5$	2.11	0.034 43	0.111 43
0.75	0.145 06	$6.923E-5$	2.12	0.030 91	0.114 97
0.8	0.144 98	$1.345E-4$	2.13	0.027 18	0.118 74
0.85	0.144 86	$2.431E-4$	2.14	0.023 11	0.122 84
0.9	0.144 69	$4.127E-4$	2.15	0.018 62	0.127 37
0.95	0.144 46	$6.636E-4$	2.16	0.013 59	0.132 43
1	0.144 14	0.001 02	2.17	0.007 29	0.138 78
1.05	0.143 71	0.001 41	2.171	0.006 52	0.139 55
1.1	0.143 13	0.001 99	2.172	0.005 72	0.140 36
1.15	0.142 35	0.002 76	2.173	0.004 89	0.141 20
1.2	0.141 37	0.003 75	2.174	0.004 01	0.142 08
1.25	0.140 12	0.004 99	2.175	0.003 00	0.143 10
1.3	0.138 60	0.006 52	2.176	0.001 75	0.144 36
1.35	0.136 76	0.008 37	2.1768	0	0.146 11
1.4	0.134 57	0.010 57			

TABLE 2.4. Recommended values of superfluid and normal fluid density ratios for helium II as a function of temperature

T_{90} (K)	ρ_s/ρ	ρ_n/ρ	T_{90} (K)	ρ_s/ρ	ρ_n/ρ
0	1.000	0	1.45	0.909	0.091
0.05	1.000	$2.03E-8$	1.50	0.889	0.111
0.1	1.000	$4.06E-8$	1.55	0.865	0.135
0.15	1.000	$6.10E-8$	1.60	0.838	0.162
0.2	1.000	$1.91E-7$	1.65	0.807	0.193
0.25	1.000	$4.60E-7$	1.70	0.771	0.229
0.3	1.000	$9.42E-7$	1.75	0.732	0.268
0.35	1.000	$1.72E-6$	1.80	0.687	0.313
0.4	1.000	$2.92E-6$	1.85	0.636	0.364
0.45	1.000	$4.84E-6$	1.90	0.580	0.420
0.5	1.000	$8.61E-6$	1.95	0.518	0.482
0.55	1.000	$1.78E-5$	2.00	0.447	0.553
0.6	1.000	$4.18E-5$	2.05	0.362	0.638
0.65	1.000	$1.00E-4$	2.10	0.259	0.741
0.7	1.000	$2.27E-4$	2.11	0.236	0.764
0.75	1.000	$4.77E-4$	2.12	0.212	0.788
0.8	0.999	$9.27E-4$	2.13	0.186	0.814
0.85	0.998	$2.00E-3$	2.14	0.158	0.842
0.9	0.997	$3.00E-3$	2.15	0.128	0.872
0.95	0.995	$5.00E-3$	2.16	0.093	0.907
1	0.993	$7.00E-3$	2.17	0.050	0.950
1.05	0.990	$1.00E-2$	2.171	0.045	0.955
1.1	0.986	1.40-2	2.172	0.039	0.961
1.15	0.981	$1.9E-2$	2.173	0.033	0.967
1.2	0.974	$2.6E-2$	2.174	0.027	0.973
1.25	0.966	$3.4E-2$	2.175	0.021	0.979
1.3	0.955	$4.5E-2$	2.176	0.012	0.988
1.35	0.942	$5.8E-2$	2.1768	0	1.000

2.1. Chronological Bibliography for Superfluid Density

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3. First Sound Velocity

Adopted Database

Author(s)	Key #	Method	Range (K)	Uncertainty (%)
Van Itterbeek & Forrez	1	Resonance	$2.5 \leq T \leq 4.24$	<0.26
Vignos and Fairbank	2	Pulse	$2.5 \leq T \leq 4$	<0.3
Whitney and Chase	3	Pulse	$0.15 \leq T \leq 1.8$	0.1
Barmatz and Rudnick	4	Resonance	$1.8 \leq T \leq 2.5$	0.2
Heiserman *et al.*	5	Resonance	$1.187 \leq T \leq 1.804$	0.2
Tam and Ahlers	6	Resonance	$1.5 \leq T \leq 2.16$	0.1

Comments and Key to Authors

(1) Reference 10. Points above 2.5 K with errors less than 0.3%. Corrected to T_{58}.
(2) Reference 20. Smoothed data.
(3) Reference 17. Values of the "reduced time delay" defined in terms of Whitney and Chase's value for u_1 at 0 K, $u_1(0) = 238\ 30(13)$ cm/s, have been converted to velocities using our value of $u_1(0) = 238\ 21$ cm/s obtained from an equation of state.[24] This point was included in the data set. In the range $0.2\ \mathrm{K} < T < 1.1\ \mathrm{K}$, the results are frequency dependent with variations as great as 20 cm/s between velocities measured at 1 and 11.9 MHz.
(4) Reference 22. A constant 54 cm/s has been subtracted from the data to produce absolute agreement with the data of Tam and Ahlers. About 6 mK above T_λ, the data show an anomaly in the temperature measurement due to the density maximum (change of sign of the expansion coefficient which affects the convective mechanism for thermal equilibrium).
(5) Reference 25. Simultaneously measured u_1, u_2, u_4.
(6) Reference 26. Simultaneously measured u_1, u_2, u_4.
(7) References 27, 28 and 29 are more modern papers covering not only the velocity of sound, but also attenuation and dispersion, topics not contained in this compilation.

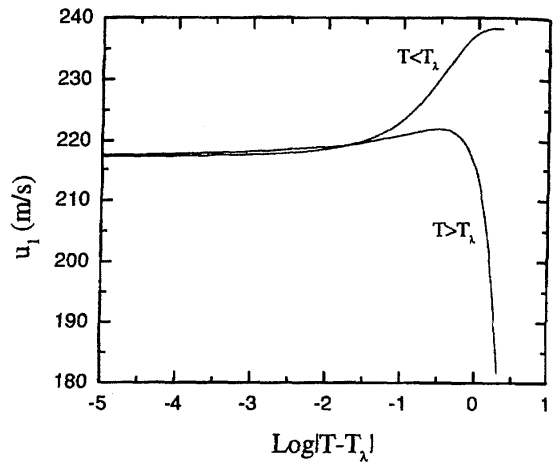

FIG. 3.2. Detail of the recommended values for the first sound velocity of liquid ^{4}He about the lambda transition.

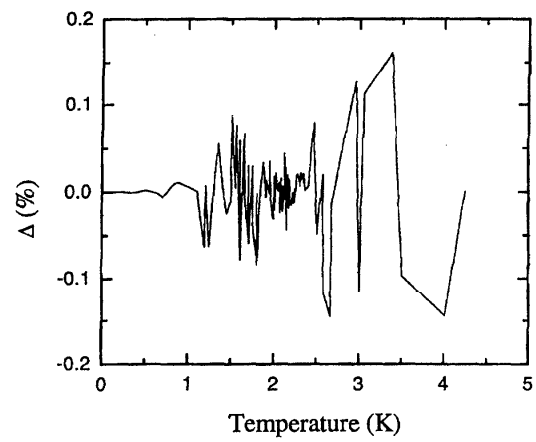

FIG. 3.3. The fractional deviation of the adopted database from the recommended values of the velocity of first sound in liquid ^{4}He expressed in percent.

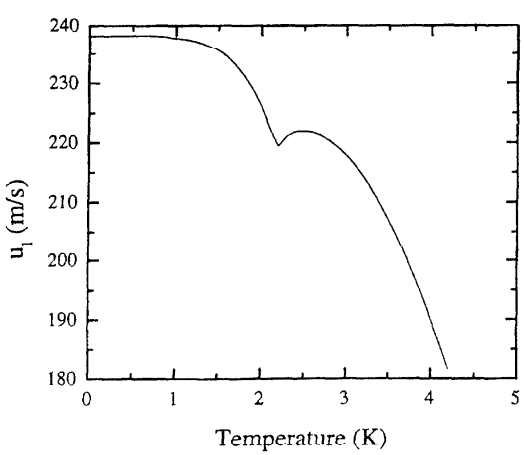

FIG. 3.1. The recommended values for the first sound velocity of liquid ^{4}He as a function of temperature at saturated vapor pressure.

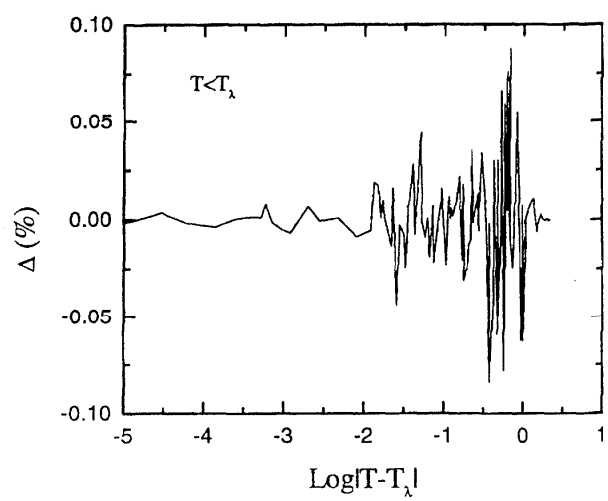

FIG. 3.4. The fractional deviation of the adopted database from the recommended values of the velocity of first sound in liquid ^{4}He expressed in percent for $T < T_\lambda$.

TABLE 3.1. Adopted database for first sound velocity of liquid ^{4}He

T_{90} (K)	u_1 (m/s)	Key	T_{90} (K)	u_1 (m/s)	Key	T_{90} (K)	u_1 (m/s)	Key	T_{90} (K)	u_1 (m/s)	Key
0.000 000	2.382 1E+2	3	1.877 030	2.295 3E+2	4	2.154 563	2.194 3E+2	6	2.177 377	2.181 0E+2	4
0.100 365	2.382 1E+2	3	1.895 914	2.291 1E+2	4	2.155 158	2.194 3E+2	4	2.177 470	2.181 5E+2	4
0.200 706	2.382 1E+2	3	1.904 151	2.289 3E+2	6	2.158 715	2.191 9E+2	6	2.177 561	2.182 1E+2	4
0.301 026	2.382 2E+2	3	1.904 151	2.289 5E+2	6	2.159 827	2.191 0E+2	4	2.177 564	2.181 7E+2	4
0.401 326	2.382 3E+2	3	1.912 537	2.286 9E+2	4	2.160 600	2.190 7E+2	6	2.177 670	2.182 0E+2	4
0.501 608	2.382 4E+2	3	1.930 458	2.282 3E+2	4	2.162 195	2.189 3E+2	6	2.177 779	2.182 3E+2	4
0.601 874	2.382 6E+2	3	1.946 329	2.278 0E+2	4	2.163 759	2.188 2E+2	6	2.177 874	2.182 5E+2	4
0.651 991	2.382 6E+2	3	1.954 189	2.275 0E+2	6	2.164 925	2.187 9E+2	4	2.177 970	2.182 7E+2	4
0.702 125	2.382 5E+2	3	1.954 189	2.274 9E+2	6	2.168 944	2.184 8E+2	4	2.178 070	2.182 8E+2	4
0.752 245	2.382 0E+2	3	1.961 447	2.273 8E+2	4	2.172 074	2.181 7E+2	4	2.178 225	2.183 1E+2	4
0.802 362	2.381 3E+2	3	1.974 516	2.269 9E+2	4	2.174 028	2.179 6E+2	4	2.178 331	2.183 3E+2	4
0.852 476	2.380 5E+2	3	1.986 397	2.266 5E+2	4	2.174 796	2.178 5E+2	4	2.179 156	2.184 5E+2	4
0.902 586	2.379 7E+2	3	1.998 495	2.262 7E+2	4	2.175 628	2.177 7E+2	4	2.182 933	2.188 2E+2	4
1.102 955	2.375 9E+2	3	2.004 234	2.259 7E+2	6	2.175 886	2.177 3E+2	4	2.186 902	2.190 0E+2	4
1.190 093	2.375 0E+2	5	2.009 518	2.258 8E+2	4	2.176 097	2.176 9E+2	4	2.206 879	2.195 5E+2	4
1.203 267	2.372 9E+2	3	2.022 169	2.253 5E+2	4	2.176 207	2.176 5E+2	4	2.211 285	2.196 6E+2	4
1.246 176	2.373 0E+2	5	2.035 571	2.249 0E+2	4	2.176 278	2.176 5E+2	4	2.218 489	2.197 8E+2	4
1.301 319	2.369 0E+2	5	2.047 786	2.244 6E+2	4	2.176 439	2.176 1E+2	4	2.226 982	2.199 6E+2	4
1.303 324	2.368 7E+2	3	2.054 293	2.242 0E+2	6	2.176 557	2.175 7E+2	4	2.233 971	2.201 0E+2	4
1.354 445	2.365 0E+2	5	2.054 293	2.242 0E+2	6	2.176 655	2.175 3E+2	4	2.244 515	2.202 5E+2	4
1.403 515	2.363 1E+2	3	2.056 655	2.241 2E+2	4	2.176 742	2.174 7E+2	4	2.254 519	2.204 2E+2	4
1.405 517	2.363 0E+2	5	2.063 579	2.238 2E+2	4	2.176 760	2.174 5E+2	4	2.264 751	2.205 4E+2	4
1.454 557	2.360 0E+2	5	2.072 554	2.235 3E+2	4	2.176 772	2.174 3E+2	4	2.277 122	2.207 1E+2	4
1.503 598	2.355 4E+2	3	2.079 160	2.232 0E+2	4	2.176 786	2.174 1E+2	4	2.292 627	2.209 2E+2	4
1.504 599	2.353 0E+2	5	2.084 348	2.229 4E+2	6	2.176 791	2.173 9E+2	4	2.302 212	2.210 5E+2	4
1.553 637	2.350 0E+2	5	2.086 849	2.228 5E+2	4	2.176 796	2.173 7E+2	4	2.314 606	2.211 5E+2	4
1.556 640	2.348 0E+2	6	2.095 511	2.224 9E+2	4	2.176 802	2.175 2E+2	4	2.327 541	2.212 9E+2	4
1.603 690	2.345 3E+2	3	2.103 851	2.221 4E+2	4	2.176 804	2.175 9E+2	4	2.344 900	2.214 2E+2	4
1.603 690	2.342 9E+2	6	2.104 399	2.220 9E+2	6	2.176 808	2.176 3E+2	4	2.363 446	2.215 9E+2	4
1.604 692	2.346 0E+2	5	2.104 399	2.220 5E+2	6	2.176 817	2.176 8E+2	4	2.405 301	2.217 8E+2	4
1.653 779	2.336 1E+2	6	2.111 657	2.217 6E+2	4	2.176 825	2.177 1E+2	4	2.462 922	2.217 3E+2	4
1.658 789	2.337 0E+2	5	2.114 429	2.215 8E+2	6	2.176 834	2.177 3E+2	4	2.505 156	2.220 0E+2	2
1.703 882	2.331 3E+2	3	2.119 899	2.213 3E+2	4	2.176 842	2.177 5E+2	4	2.576 309	2.217 0E+2	1
1.703 882	2.329 4E+2	6	2.124 462	2.210 8E+2	6	2.176 855	2.177 6E+2	4	2.577 311	2.220 0E+2	1
1.706 888	2.331 0E+2	5	2.126 417	2.208 8E+2	4	2.176 867	2.177 9E+2	4	2.660 552	2.217 0E+2	1
1.753 981	2.320 9E+2	6	2.133 460	2.206 1E+2	4	2.176 876	2.178 0E+2	4	2.678 609	2.213 0E+2	1
1.756 986	2.322 0E+2	5	2.134 497	2.205 7E+2	6	2.176 883	2.178 3E+2	4	2.956 223	2.182 0E+2	1
1.804 065	2.314 0E+2	3	2.136 395	2.203 9E+2	4	2.176 916	2.178 7E+2	4	3.006 300	2.180 0E+2	2
1.804 065	2.312 1E+2	6	2.141 056	2.201 8E+2	4	2.176 936	2.179 0E+2	4	3.050 364	2.168 0E+2	1
1.808 071	2.313 0E+2	5	2.144 531	2.200 6E+2	6	2.177 020	2.179 6E+2	4	3.377 817	2.099 0E+2	1
1.844 214	2.303 5E+2	4	2.145 827	2.199 5E+2	4	2.177 114	2.179 9E+2	4	3.506 953	2.070 0E+2	2
1.854 119	2.301 1E+2	6	2.149 547	2.197 3E+2	6	2.177 171	2.180 2E+2	4	4.007 034	1.900 0E+2	2
1.861 054	2.299 4E+2	4	2.151 463	2.197 1E+2	4	2.177 277	2.180 7E+2	4	4.248 035	1.795 0E+2	1
1.869 474	2.297 3E+2	4	2.154 563	2.194 0E+2	6						

TABLE 3.2. Knots and coefficients for the spline fit of first sound velocity of liquid ^{4}He

Knots	Coefficients
$K(1)=0.000\,000$	$C(1)=2.382\,100E+2$
$K(2)=0.000\,000$	$C(2)=2.382\,041E+2$
$K(3)=0.000\,000$	$C(3)=2.382\,203E+2$
$K(4)=0.000\,000$	$C(4)=2.382\,958E+2$
$K(5)=0.5.01\,6077$	$C(5)=2.375\,414E+2$
$K(6)=0.702\,1246$	$C(6)=2.364\,020E+2$
$K(7)=1.002\,777$	$C(7)=2.270\,033E+2$
$K(8)=1.804\,065$	$C(8)=2.221\,461E+2$
$K(9)=2.004\,234$	$C(9)=2.187\,201E+2$
$K(10)=2.154\,563$	$C(10)=2.180\,010E+2$
$K(11)=2.169\,604$	$C(11)=2.176\,634E+2$
$K(12)=2.175\,800$	$C(12)=2.175\,663E+2$
$K(13)=2.176\,300$	$C(13)=2.174\,367E+2$
$K(14)=2.176\,750$	$C(14)=2.173\,641E+2$
$K(15)=2.176\,797$	$C(15)=2.175\,712E+2$
$K(16)=2.176\,797$	$C(16)=2.176\,713E+2$
$K(17)=2.176\,797$	$C(17)=2.178\,168E+2$
$K(18)=2.176\,810$	$C(18)=2.179\,702E+2$
$K(19)=2.176\,830$	$C(19)=2.181\,464E+2$
$K(20)=2.176\,950$	$C(20)=2.187\,212E+2$
$K(21)=2.177\,300$	$C(21)=2.193\,106E+2$
$K(22)=2.178\,000$	$C(22)=2.216\,496E+2$
$K(23)=2.184\,625$	$C(23)=2.238\,243E+2$
$K(24)=2.224\,629$	$C(24)=2.056\,185E+2$
$K(25)=2.505\,156$	$C(25)=1.795\,017E+2$
$K(26)-K(29)=4.248035$	

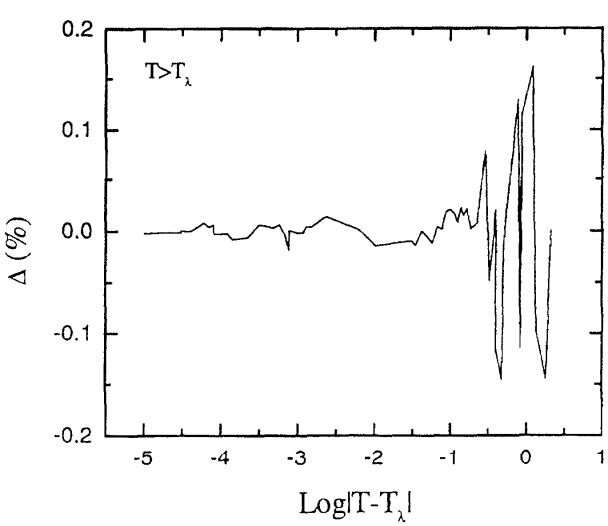

FIG. 3.5. The fractional deviation of the adopted database from the recommended values of the velocity of first sound in liquid ^{4}He expressed in percent for $T>T_\lambda$.

TABLE 3.3. Recommended values of first sound velocity in liquid ^{4}He

T_{90} (K)	u_1 (m/s)	T_{90} (K)	u_1 (m/s)
0.0000	$2.382E+2$	2.1765	$2.176E+2$
0.0500	$2.382E+2$	2.1766	$2.176E+2$
0.1000	$2.382E+2$	2.1767	$2.175E+2$
0.1500	$2.382E+2$	2.1768	$2.175E+2$
0.2000	$2.382E+2$	2.1769	$2.178E+2$
0.2500	$2.382E+2$	2.1770	$2.179E+2$
0.3000	$2.382E+2$	2.1800	$2.186E+2$
0.3500	$2.382E+2$	2.2000	$2.194E+2$
0.4000	$2.382E+2$	2.2500	$2.203E+2$
0.4500	$2.382E+2$	2.3000	$2.210E+2$
0.5000	$2.382E+2$	2.3500	$2.215E+2$
0.5500	$2.383E+2$	2.4000	$2.218E+2$
0.6000	$2.383E+2$	2.4500	$2.219E+2$
0.6500	$2.383E+2$	2.5000	$2.219E+2$
0.7000	$2.382E+2$	2.5500	$2.218E+2$
0.7500	$2.382E+2$	2.6000	$2.217E+2$
0.8000	$2.381E+2$	2.6500	$2.214E+2$
0.8500	$2.381E+2$	2.7000	$2.211E+2$
0.9000	$2.380E+2$	2.7500	$2.208E+2$
0.9500	$2.379E+2$	2.8000	$2.203E+2$
1.0000	$2.378E+2$	2.8500	$2.198E+2$
1.0500	$2.377E+2$	2.9000	$2.192E+2$
1.1000	$2.376E+2$	2.9500	$2.186E+2$
1.1500	$2.375E+2$	3.0000	$2.178E+2$
1.2000	$2.373E+2$	3.0500	$2.171E+2$
1.2500	$2.371E+2$	3.1000	$2.162E+2$
1.3000	$2.369E+2$	3.1500	$2.153E+2$
1.3500	$2.367E+2$	3.2000	$2.143E+2$
1.4000	$2.363E+2$	3.2500	$2.132E+2$
1.4500	$2.360E+2$	3.3000	$2.121E+2$
1.5000	$2.355E+2$	3.3500	$2.109E+2$
1.5500	$2.350E+2$	3.4000	$2.097E+2$
1.6000	$2.345E+2$	3.4500	$2.084E+2$
1.6500	$2.338E+2$	3.5000	$2.070E+2$
1.7000	$2.331E+2$	3.5500	$2.056E+2$
1.7500	$2.322E+2$	3.6000	$2.041E+2$
1.8000	$2.313E+2$	3.6500	$2.025E+2$
1.8500	$2.302E+2$	3.7000	$2.009E+2$
1.9000	$2.290E+2$	3.7500	$1.992E+2$
1.9500	$2.277E+2$	3.8000	$1.975E+2$
2.0000	$2.262E+2$	3.8500	$1.957E+2$
2.0500	$2.244E+2$	3.9000	$1.939E+2$
2.1000	$2.223E+2$	3.9500	$1.920E+2$
2.1500	$2.197E+2$	4.0000	$1.900E+2$
2.1760	$2.177E+2$	4.0500	$1.880E+2$
2.1761	$2.177E+2$	4.1000	$1.859E+2$
2.1762	$2.177E+2$	4.1500	$1.838E+2$
2.1763	$2.176E+2$	4.2000	$1.816E+2$
2.1764	$2.176E+2$		

3.1. Chronological Bibliography for First Sound Velocity

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4. Second Sound Velocity

Adopted Database

Author(s)	Key #	Method	Range (K)	Uncertainty (%)
Heiserman *et al.*	1	Resonance	$1.187 \leq T \leq 2.150$	<0.2
Greywall & Ahlers	2	Resonance	$2.16 \leq T \leq 2.172$	<0.2
Tam & Ahlers	3	Resonance	$1.5 \leq T \leq 2.16$	<0.2
Peshkov	4	Resonance	$0.5 \leq T \leq 1.15$	0.75
Wang *et al.*	5	Resonance	$1.13 \leq T \leq 2.09$	0.07
Lipa & Marek	6	Resonance		see comment 6

Comments and Key to Authors

(1) Reference 23. Points at 2.05 and 2.15 K omitted as inconsistent with later measurements.
(2) Reference 19. Data above 2.145 K omitted in favor of data of Greywall and Ahlers.
(3) Reference 26.
(4) Reference 8. Data used only below 1.15 K, higher temperature data is inconsistent with later measurements.
(5) Reference 27.
(6) Reference 28. Consists of points normalized to measurements of Greywall and Ahlers at $t = 1 - T/T_\lambda = 10^{-3}$. Authors note that for $t < 3 \times 10^{-6}$ tests of the calculation of superfluid density from second sound begin to deviate from the best fit function.
(7) The velocities are listed in m/s. Multiply by 100 to convert to cm/s.

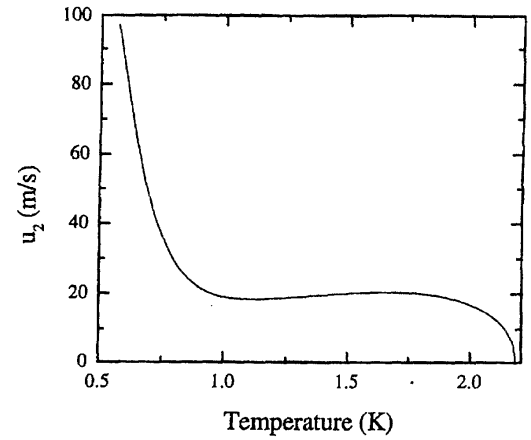

FIG. 4.1. The recommended values for the velocity of second sound in helium II as a function of temperature at saturated vapor pressure.

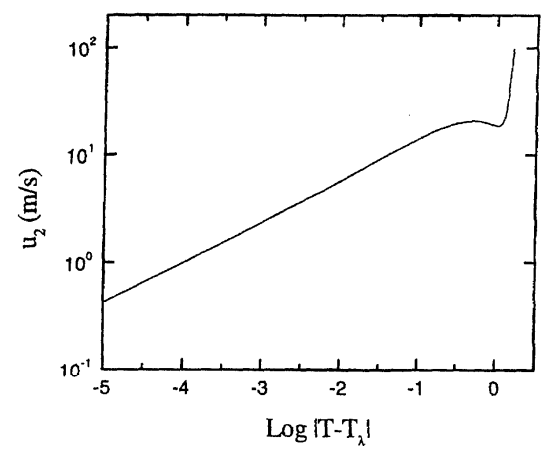

FIG. 4.2. Detail of the recommended values for the second sound velocity in helium II about the lambda transition.

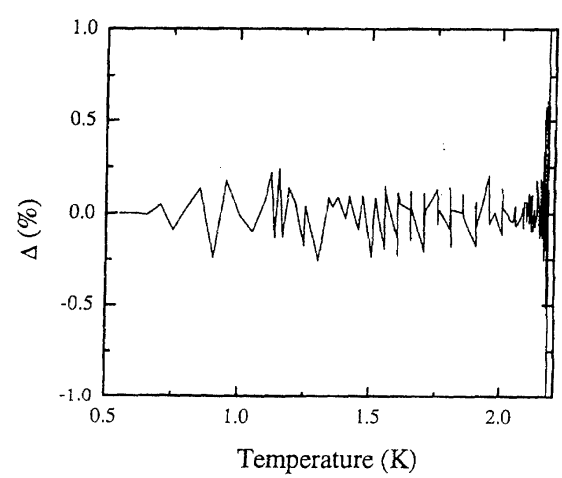

FIG. 4.3. The fractional deviation of values of the adopted data base from the recommended values for the velocity of second sound in helium II expressed in percent.

TABLE 4.1. Adopted database for second sound velocity in helium II

T_{90} (K)	u_2 (m/s)	Key	T_{90} (K)	u_2 (m/s)	Key	T_{90} (K)	u_2 (m/s)	Key	T_{90} (K)	u_2 (m/s)	Key
0.551 743	1.050 00E+2	4	1.804 065	1.989 00E+1	2	2.163 463	6.447 30E+0	6	2.176 370	1.709 20E+0	6
0.601 874	8.300 00E+1	4	1.804 065	1.984 50E+1	3	2.163 548	6.428 30E+0	2	2.176 405	1.649 80E+0	2
0.651 991	6.200 00E+1	4	1.808 071	1.988 00E+1	1	2.163 759	6.344 00E+0	3	2.176 416	1.634 89E+0	6
0.702 125	4.650 00E+1	4	1.808 271	1.983 95E+1	5	2.164 086	6.324 29E+0	6	2.176 478	1.527 60E+0	2
0.752 245	3.600 00E+1	4	1.808 371	1.983 81E+1	5	2.164 086	6.329 41E+0	6	2.176 492	1.504 71E+0	6
0.802 362	2.900 00E+1	4	1.854 119	1.942 00E+1	1	2.164 859	6.165 42E+0	6	2.176 495	1.499 07E+0	6
0.852 476	2.440 00E+1	4	1.854 119	1.940 00E+1	3	2.165 154	6.100 70E+0	2	2.176 521	1.448 33E+0	6
0.902 586	2.160 00E+1	4	1.857 822	1.938 32E+1	5	2.165 684	5.993 73E+0	6	2.176 533	1.422 30E+0	2
0.952 694	1.980 00E+1	4	1.904 151	1.880 00E+1	1	2.166 426	5.827 16E+0	6	2.176 576	1.333 53E+0	6
1.002 777	1.890 00E+1	4	1.904 151	1.878 00E+1	2	2.166 576	5.791 00E+0	2	2.176 576	1.330 60E+0	2
1.052 861	1.850 00E+1	4	1.904 151	1.875 50E+1	3	2.167 091	5.673 41E+0	6	2.176 587	1.308 93E+0	6
1.102 955	1.833 00E+1	4	1.904 151	1.876 20E+1	3	2.167 334	5.611 91E+0	6	2.176 590	1.299 71E+0	6
1.122 986	1.830 00E+1	4	1.907 353	1.873 14E+1	5	2.167 822	5.499 40E+0	2	2.176 610	1.249 90E+0	2
1.137 607	1.837 54E+1	5	1.954 189	1.784 00E+1	1	2.168 334	5.371 05E+0	6	2.176 614	1.242 31E+0	6
1.137 607	1.837 42E+1	5	1.954 189	1.786 40E+1	3	2.168 924	5.215 70E+0	2	2.176 625	1.212 07E+0	6
1.137 607	1.837 39E+1	5	1.954 189	1.787 00E+1	3	2.169 001	5.191 67E+0	6	2.176 637	1.178 40E+0	2
1.153 029	1.833 00E+1	4	1.957 192	1.782 45E+1	5	2.169 524	5.053 28E+0	6	2.176 641	1.170 56E+0	6
1.167 152	1.842 92E+1	5	1.976 908	1.737 49E+1	5	2.169 730	4.993 84E+0	6	2.176 649	1.145 96E+0	6
1.190 093	1.845 00E+1	1	2.004 234	1.668 00E+1	1	2.169 878	4.950 10E+0	2	2.176 650	1.143 91E+0	6
1.215 134	1.856 36E+1	5	2.004 234	1.663 80E+1	3	2.170 387	4.797 02E+0	6	2.176 660	1.115 21E+0	6
1.246 176	1.875 00E+1	1	2.006 736	1.658 32E+1	5	2.170 722	4.696 40E+0	2	2.176 660	1.118 28E+0	6
1.251 399	1.873 66E+1	5	2.026 757	1.595 88E+1	5	2.171 109	4.571 53E+0	6	2.176 660	1.114 90E+0	2
1.301 319	1.906 00E+1	1	2.036 769	1.561 37E+1	5	2.171 274	4.518 60E+0	2	2.176 672	1.076 26E+0	6
1.337 208	1.920 12E+1	5	2.046 682	1.524 24E+1	5	2.171 601	4.407 53E+0	6	2.176 678	1.057 50E+0	2
1.337 409	1.919 54E+1	5	2.054 293	1.492 60E+1	3	2.172 127	4.288 90E+0	2	2.176 693	1.006 00E+0	2
1.354 445	1.930 00E+1	1	2.054 293	1.493 50E+1	3	2.172 516	4.149 23E+0	6	2.176 697	9.942 56E−1	6
1.374 079	1.939 80E+1	5	2.056 697	1.484 29E+1	5	2.172 678	4.083 62E+0	6	2.176 706	9.592 00E−1	2
1.405 517	1.959 00E+1	1	2.066 614	1.441 25E+1	5	2.172 714	4.069 10E+0	2	2.176 710	9.430 07E−1	6
1.418 729	1.963 54E+1	5	2.076 632	1.394 51E+1	5	2.173 031	3.946 28E+0	6	2.176 717	9.165 00E−1	2
1.418 829	1.963 93E+1	5	2.084 349	1.355 50E+1	3	2.173 227	3.861 40E+0	2	2.176 726	8.775 00E−1	2
1.454 558	1.985 00E+1	1	2.086 133	1.347 78E+1	2	2.173 323	3.819 18E+0	6	2.176 729	8.620 34E−1	6
1.468 770	1.987 97E+1	5	2.086 553	1.343 70E+1	5	2.173 667	3.667 60E+0	2	2.176 733	8.418 00E−1	2
1.504 599	2.010 00E+1	1	2.096 678	1.287 91E+1	5	2.173 702	3.654 15E+0	6	2.176 737	8.250 00E−1	2
1.517 108	2.008 28E+1	5	2.096 777	1.288 65E+1	2	2.174 015	3.505 52E+0	6	2.176 743	7.974 61E−1	6
1.553 637	2.026 00E+1	1	2.104 399	1.241 20E+1	3	2.174 071	3.475 80E+0	2	2.176 744	7.932 00E−1	2
1.556 640	2.020 00E+1	3	2.104 399	1.240 70E+1	3	2.174 404	3.302 00E+0	2	2.176 754	7.390 40E−1	6
1.563 245	2.023 38E+1	5	2.106 209	1.231 12E+1	2	2.174 434	3.290 27E+0	6	2.176 764	6.693 48E−1	6
1.603 690	2.036 00E+1	1	2.114 429	1.173 90E+1	3	2.174 695	3.140 20E+0	2	2.176 764	6.713 98E−1	6
1.603 690	2.038 00E+1	2	2.114 572	1.175 19E+1	2	2.174 712	3.132 42E+0	6	2.176 775	5.894 18E−1	6
1.603 690	2.031 00E+1	3	2.121 978	1.120 45E+1	2	2.174 894	3.028 89E+0	6	2.176 777	5.658 51E−1	6
1.611 903	2.033 36E+1	5	2.124 462	1.099 80E+1	3	2.174 954	2.984 40E+0	2	2.176 785	4.839 08E−1	6
1.612 103	2.033 39E+1	5	2.128 512	1.067 92E+1	2	2.175 059	2.920 25E+0	6	2.176 788	4.511 48E−1	6
1.646 264	2.036 43E+1	5	2.134 262	1.017 38E+1	2	2.175 174	2.840 60E+0	2	2.176 792	3.826 45E−1	6
1.647 266	2.036 45E+1	5	2.134 497	1.013 30E+1	3	2.175 303	2.752 14E+0	6	2.176 795	3.245 63E−1	6
1.648 168	2.036 46E+1	5	2.143 812	9.218 90E+0	2	2.175 395	2.682 80E+0	2	2.176 797	2.720 95E−1	6
1.649 170	2.036 47E+1	5	2.144 531	9.152 00E+0	3	2.175 417	2.671 68E+0	6	2.176 797	2.660 44E−1	6
1.650 172	2.036 47E+1	5	2.147 768	8.771 50E+0	2	2.175 579	2.542 90E+0	2	2.176 797	2.610 90E−1	6
1.651 174	2.036 47E+1	5	2.149 547	8.533 00E+0	3	2.175 637	2.501 02E+0	6	2.176 797	2.481 80E−1	6
1.652 076	2.036 47E+1	5	2.151 231	8.344 00E+0	2	2.175 726	2.421 10E+0	2	2.176 798	2.422 06E−1	6
1.653 078	2.036 48E+1	5	2.154 307	7.932 50E+0	2	2.175 757	2.396 47E+0	6	2.176 798	2.362 61E−1	6
1.653 779	2.040 00E+1	1	2.154 563	7.873 00E+0	3	2.175 873	2.285 30E+0	2	2.176 798	2.355 18E−1	6
1.653 779	2.034 50E+1	3	2.154 563	7.885 00E+0	3	2.175 873	2.290 89E+0	6	2.176 798	2.365 55E−1	6
1.654 080	2.036 47E+1	5	2.157 027	7.537 00E+0	2	2.175 965	2.199 15E+0	6	2.176 798	2.313 85E−1	6
1.655 082	2.036 45E+1	5	2.158 715	7.242 00E+0	3	2.175 982	2.179 40E+0	2	2.176 798	2.317 31E−1	6
1.656 084	2.036 44E+1	5	2.159 393	7.165 40E+0	2	2.176 020	2.144 31E+0	6	2.176 798	2.255 18E−1	6
1.657 086	2.036 42E+1	5	2.160 303	7.021 30E+0	6	2.176 052	2.111 51E+0	6	2.176 798	2.288 20E−1	6
1.703 882	2.036 00E+1	2	2.160 601	6.920 00E+0	3	2.176 092	2.061 50E+0	2	2.176 798	2.227 28E−1	6
1.703 882	2.029 50E+1	3	2.160 728	6.949 55E+0	6	2.176 134	2.013 60E+0	2	2.176 798	2.269 67E−1	6
1.706 888	2.033 00E+1	1	2.161 555	6.806 03E+0	6	2.176 173	1.971 09E+0	6	2.176 798	2.238 68E−1	6
1.708 993	2.030 30E+1	5	2.161 688	6.773 90E+0	2	2.176 176	1.964 00E+0	2	2.176 798	2.230 47E−1	6
1.753 981	2.013 50E+1	3	2.162 195	6.637 00E+0	3	2.176 282	1.834 76E+0	6	2.176 798	2.190 49E−1	6
1.756 986	2.016 00E+1	1	2.162 543	6.621 54E+0	6	2.176 309	1.793 20E+0	2	2.176 798	2.200 92E−1	6
1.758 690	2.013 64E+1	5	2.162 797	6.575 42E+0	6	2.176 312	1.790 69E+0	6	2.176 798	2.212 36E−1	6

TABLE 4.1. Adopted database for second sound velocity in helium II—Continued

T_{90} (K)	u_2 (m/s)	Key	T_{90} (K)	u_2 (m/s)	Key	T_{90} (K)	u_2 (m/s)	Key	T_{90} (K)	u_2 (m/s)	Key
2.176 798	2.181 35$E-1$	6	2.176 798	2.078 52$E-1$	6	2.176 799	1.942 12$E-1$	6	2.176 799	2.014 35$E-1$	6
2.176 798	2.172 16$E-1$	6	2.176 798	2.078 52$E-1$	6	2.176 799	1.942 12$E-1$	6	2.176 799	2.014 35$E-1$	6
2.176 798	2.172 16$E-1$	6	2.176 798	2.059 51$E-1$	6	2.176 799	1.952 74$E-1$	6	2.176 799	2.024 90$E-1$	6
2.176 798	2.172 16$E-1$	6	2.176 798	2.050 93$E-1$	6	2.176 799	1.924 27$E-1$	6	2.176 799	1.760 16$E-1$	6
2.176 798	2.172 16$E-1$	6	2.176 799	1.995 87$E-1$	6	2.176 799	1.917 00$E-1$	6	2.176 799	1.726 43$E-1$	6
2.176 798	2.153 04$E-1$	6	2.176 799	1.987 75$E-1$	6	2.176 799	1.917 00$E-1$	6	2.176 799	1.738 34$E-1$	6
2.176 798	2.143 97$E-1$	6	2.176 799	1.998 32$E-1$	6	2.176 799	1.883 24$E-1$	6	2.176 799	1.695 86$E-1$	6
2.176 798	2.133 86$E-1$	6	2.176 799	1.949 11$E-1$	6	2.176 799	1.873 55$E-1$	6	2.176 799	1.707 02$E-1$	6
2.176 798	2.133 86$E-1$	6	2.176 799	1.951 92$E-1$	6	2.176 799	1.873 55$E-1$	6	2.176 799	1.677 00$E-1$	6
2.176 798	2.125 02$E-1$	6	2.176 799	1.785 38$E-1$	6	2.176 799	1.884 26$E-1$	6	2.176 799	1.678 19$E-1$	6
2.176 798	2.125 02$E-1$	6	2.176 799	1.785 38$E-1$	6	2.176 799	1.840 11$E-1$	6	2.176 800	0.000 00	0
2.176 798	2.105 58$E-1$	6	2.176 799	1.771 68$E-1$	6	2.176 799	1.809 88$E-1$	6			
2.176 798	2.086 86$E-1$	6	2.176 799	1.772 11$E-1$	6	2.176 799	2.022 14$E-1$	6			
2.176 798	2.078 04$E-1$	6	2.176 799	1.973 14$E-1$	6	2.176 799	2.024 34$E-1$	6			

TABLE 4.2. Knots and coefficients for the spline fit for the velocity of second sound. The spline returns the velocity in m/s

Knots	Coefficients
$K(1)=0.551\,7426$	$C(1)=1.050\,000E+02$
$K(2)=0.551\,7426$	$C(2)=9.287\,425E+01$
$K(3)=0.551\,7426$	$C(3)=4.973\,642E+01$
$K(4)=0.551\,7426$	$C(4)=2.712\,778E+01$
$K(5)=0.641\,9700$	$C(5)=1.931\,258E+01$
$K(6)=0.806\,2300$	$C(6)=1.795\,336E+01$
$K(7)=0.929\,7000$	$C(7)=1.923\,863E+01$
$K(8)=1.069\,640$	$C(8)=2.115\,625E+01$
$K(9)=1.285\,500$	$C(9)=1.921\,708E+01$
$K(10)=1.720\,200$	$C(10)=1.575\,245E+01$
$K(11)=1.940\,430$	$C(11)=1.192\,157E+01$
$K(12)=2.068\,850$	$C(12)=8.953\,388E+00$
$K(13)=2.126\,220$	$C(13)=6.771\,458E+00$
$K(14)=2.153\,079$	$C(14)=5.506\,380E+00$
$K(15)=2.164\,207$	$C(15)=4.456\,879E+00$
$K(16)=2.169\,182$	$C(16)=3.840\,343E+00$
$K(17)=2.170\,551$	$C(17)=2.542\,421E+00$
$K(18)=2.174\,832$	$C(18)=1.957\,438E+00$
$K(19)=2.175\,738$	$C(19)=1.427\,086E+00$
$K(20)=2.176\,319$	$C(20)=1.001\,273E+00$
$K(21)=2.176\,600$	$C(21)=0.674\,3104E-01$
$K(22)=2.176\,735$	$C(22)=0.456\,5972E-01$
$K(23)=2.176\,780$	$C(23)=0.351\,9492E-01$
$K(24)=2.176\,787$	$C(24)=0.208\,8277E-01$
$K(25)=2.176\,797$	$C(25)=0.105\,9874E-01$
$K(26)-K(30)$	$C(26)=0.000\,000E+00$
$=2.176\,800$	

TABLE 4.3. Recommended values of second sound velocity in helium II

T_{90} (K)	u_2 (m/s)	T_{90} (K)	u_2 (m/s)
0.6000	8.385$E+1$	1.6500	2.037$E+1$
0.6500	6.273$E+1$	1.7000	2.033$E+1$
0.7000	4.707$E+1$	1.7500	2.018$E+1$
0.7500	3.635$E+1$	1.8000	1.990$E+1$
0.8000	2.928$E+1$	1.8500	1.946$E+1$
0.8500	2.461$E+1$	1.9000	1.883$E+1$
0.9000	2.166$E+1$	1.9500	1.796$E+1$
0.9500	1.990$E+1$	2.0000	1.678$E+1$
1.0000	1.893$E+1$	2.0500	1.511$E+1$
1.0500	1.849$E+1$	2.1000	1.269$E+1$
1.1000	1.835$E+1$	2.1500	8.492$E+0$
1.1500	1.837$E+1$	2.1760	2.164$E+0$
1.2000	1.851$E+1$	2.1761	2.055$E+0$
1.2500	1.874$E+1$	2.1762	1.937$E+0$
1.3000	1.900$E+1$	2.1763	1.806$E+0$
1.3500	1.928$E+1$	2.1764	1.660$E+0$
1.4000	1.956$E+1$	2.1765	1.489$E+0$
1.4500	1.981$E+1$	2.1766	1.275$E+0$
1.5000	2.003$E+1$	2.1767	9.809$E-1$
1.5500	2.021$E+1$	2.1768	0.000$E+0$
1.6000	2.033$E+1$		

4.1. Chronological Bibliography for Second Sound Velocity

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5. Third Sound Velocity

The velocity of third sound in helium II for wavelength long compared to the thickness of the film d is given to good approximation by

$$u_3^2=\left(\frac{\langle\rho_s\rangle}{\rho}\right)fd\left(1+\frac{TS}{L}\right)^2, \tag{5.1}$$

where $\langle\rho_s\rangle/\rho$ is the effective superfluid density in the film, f is the restoring force per unit mass, S is the entropy, T is the thermodynamic temperature and L is the latent heat. The restoring force is

$$f=\alpha\beta(3\beta+4d)/[d^4(d+\beta)^2], \tag{5.2}$$

where α is the van der Waals's attraction between a ^{4}He atom and the substrate and β is a retardation parameter.

The effective superfluid density is less than the bulk density because of healing effects near the walls, such that

$$\frac{\langle\rho_s\rangle}{\rho}=\frac{\rho_s}{\rho}\left(1-\frac{D}{d}\right), \tag{5.3}$$

where ρ_s/ρ is the bulk value and D is a temperature-dependent parameter which has been determined to have the form

$$D=a+bT\rho/\rho_s, \tag{5.4}$$

with $a=0.5$ layers/K and $b=1.13$ for glass. Here D and d are in units of atomic layers. Both constants are determined from the experiment. The thickness scale is determined by the partial pressure P in the sample chamber from the relationship

$$d^3=\alpha/[T\ \ln(P/P_0)], \tag{5.5}$$

where P_0 is the saturated vapor pressure at temperature T.

For very thin films ($d<10$ atomic layers, where 1 atomic layer$=3.6$ Å) retardation effects are negligible and the restoring force can be approximated by $f=3\alpha/d^4$, so that to first order the velocity becomes

$$u_3^2\sim\frac{3\alpha}{d^3}\frac{(\langle\rho_s\rangle)}{\rho}(1+TS/L)^2. \tag{5.6}$$

The parameter D can then be determined by making a plot of

$$\Gamma=d-D=\frac{u_3^2d^4\rho}{3\alpha\rho_s(1+TS/L)^2} \tag{5.7}$$

as a function of d with slope unity and intercept D.

For a substrate which is reasonably smooth on the microscopic scale, the third sound velocity is relatively independent of the substrate on which it is measured because

$$u_3^2\sim3\alpha/d^3, \tag{5.8}$$

and

$$d^3\sim\alpha[T\ \ln(P_0/P)], \tag{5.9}$$

so that the first order u_3^2 is independent of α.

5.1. Chronological Bibliography for Third Sound Velocity

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6. Fourth Sound Velocity

Adopted Database

Author(s)	Key #	Range (K)
Heiserman *et al.*	1	$1.187\leq T\leq2.15$
Tam & Ahlers	2	$1.553\leq T\leq2.15917$

Comments and Key to Authors

(1) Reference 8. Simultaneously measured u_1, u_2, and u_4.
(2) Reference 9. Simultaneously measured u_1, u_2, and u_4. Data of both sets of authors agree to within a few tenths percent for u_4.

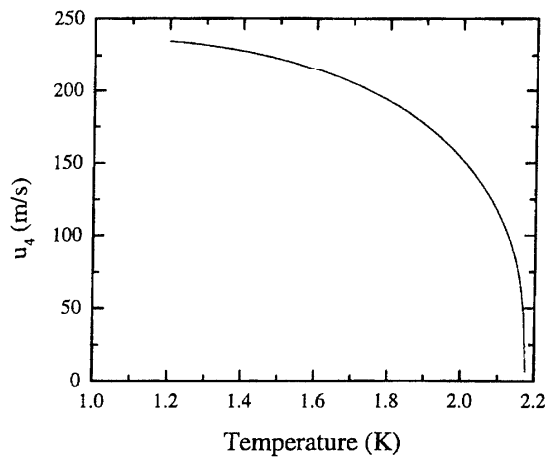

FIG. 6.1. The recommended values for the velocity of fourth sound as a function of temperature at the saturated vapor pressure.

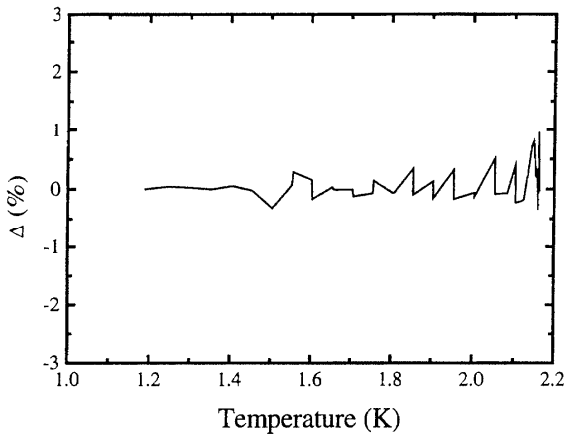

FIG. 6.2. The fractional deviation of values of the adopted database from the recommended values for the velocity of fourth sound in helium II expressed in percent.

TABLE 6.1. Adopted database for fourth sound velocity in helium II

T_{90} (K)	u_4 (m/s)	Key	T_{90}	u_4 (m/s)	Key
1.1901	2.347E+2	1	1.9042	1.752E+2	2
1.2462	2.333E+2	1	1.9542	1.636E+2	1
1.3013	2.317E+2	1	1.9542	1.643E+2	2
1.3544	2.298E+2	1	1.9542	1.644E+2	2
1.4055	2.274E+2	1	2.0042	1.512E+2	1
1.4546	2.250E+2	1	2.0042	1.513E+2	2
1.5046	2.226E+2	1	2.0543	1.340E+2	1
1.5536	2.183E+2	1	2.0543	1.348E+2	2
1.5566	2.176E+2	2	2.0843	1.224E+2	2
1.6037	2.141E+2	2	2.1044	1.118E+2	1
1.6047	2.147E+2	1	2.1044	1.125E+2	2
1.6558	2.098E+2	2	2.1144	1.068E+2	2
1.6588	2.094E+2	1	2.1245	1.005E+2	2
1.7039	2.047E+2	2	2.1345	9.330E+1	2
1.7069	2.046E+2	1	2.1445	8.490E+1	2
1.7540	1.989E+2	2	2.1495	7.990E+1	2
1.7570	1.981E+2	1	2.1546	7.440E+1	2
1.8041	1.921E+2	2	2.1546	7.450E+1	2
1.8081	1.915E+2	1	2.1586	6.920E+1	2
1.8541	1.835E+2	1	2.1606	6.660E+1	2
1.8541	1.843E+2	2	2.1616	6.420E+1	2
1.9042	1.747E+2	1	2.1636	6.160E+1	2
1.9042	1.751E+2	2	2.1768	0.000	

TABLE 6.2. Knots and coefficients for the spline fit of the velocity of fourth sound

Knots	Coefficients
$K(1)=1.190\,093$	$C(1)=2.346\,992E+2$
$K(2)=1.190\,093$	$C(2)=2.317\,649E+2$
$K(3)=1.190\,093$	$C(3)=2.235\,997E+2$
$K(4)=1.190\,093$	$C(4)=1.972\,721E+2$
$K(5)=1.622\,630$	$C(5)=1.746\,328E+2$
$K(6)=1.770\,580$	$C(6)=1.437\,044E+2$
$K(7)=1.936\,770$	$C(7)=1.120\,775E+2$
$K(8)=2.042\,310$	$C(8)=7.758\,563E+1$
$K(9)=2.129\,610$	$C(9)=4.689\,201E+1$
$K(10)=2.161\,789$	$C(10)=-1.786\,538E-08$
$K(11)=2.176\,800$	
$K(12)=2.176\,800$	
$K(13)=2.176\,800$	
$K(14)=2.176\,800$	

TABLE 6.3. Recommended values of fourth sound velocity in helium II

T_{90} (K)	u_4 (m/s)
1.2000	2.345E+2
1.2500	2.333E+2
1.3000	2.318E+2
1.3500	2.300E+2
1.4000	2.278E+2
1.4500	2.252E+2
1.5000	2.222E+2
1.5500	2.187E+2
1.6000	2.147E+2
1.6500	2.102E+2
1.7000	2.051E+2
1.7500	1.993E+2
1.8000	1.926E+2
1.8500	1.848E+2
1.9000	1.757E+2
1.9500	1.651E+2
2.0000	1.523E+2
2.0500	1.363E+2
2.1000	1.146E+2
2.1500	8.000E+1
2.1761	6.321E+0
2.1762	5.447E+0
2.1763	4.563E+0
2.1764	3.670E+0
2.1765	2.767E+0
2.1766	1.855E+0
2.1767	9.322E−1
2.1768	0.000E+0

6.1. Chronological Bibliography for Fourth Sound Velocity

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haus, W. J. O'Sullivan, and E. F. Hammel (Plenum, New York, 1974), Vol. I, pp. 279–282.
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7. Heat Capacity and Enthalpy

Adopted Database

Author(s)	Key #	Method	Range (K)	Uncertainty (%)
Donnelly	1	Theory	$0.01 \leq T \leq 0.06$	NA
Greywall	2	Calorimeter	$0.0606 \leq T \leq 1.024$	1
Wiebes	3	Calorimeter	$0.3 \leq T \leq 1.6$	2
Phillips	4	Calorimeter	$0.321\,61 \leq T \leq 2.178\,05$	1
Buckingham	5	Calorimeter	$2.072 \leq T \leq 2.272$	3
Ahlers	6	Calorimeter	$2.172\,82 \leq T \leq 2.178\,05$	1
Lipa	7	Calorimeter	$2.169\,910\,3 \leq T \leq 2.174\,027\,7$	1.5
Hill	8	Calorimeter	$2.1992 \leq T \leq 5.505\,43$	1

Comments and Key to Authors

(1) Reference 14. Calculated from the Landau theory.
(2) References 15 and 16. Temperatures taken on 1962 ^{3}He scale.
(3) Reference 9.
(4) Reference 10.
(5) Reference 6, page 85.
(6) Reference 11.
(7) Reference 17. Temperatures taken on the EPT-76 scale.
(8) Reference 3. Corrected to T_{58}.
(9) The spline is actually fit to $\log_{10} C_s$, where C_s is the heat capacity at saturation pressure.
(10) Lipa *et al.* (Ref. 16) have measured C_s very near the lambda point in a microgravity experiment. They fit their data using functions of the type $C_s = At^{-\alpha}(1 + Dt^{\Delta} + Et) + \beta$, where $t = |1 - T/T_\lambda|$. Δ was set at 0.5 above and below T_λ. The results were as follows:

	α	A	B	D	E
$T<T_\lambda$	−0.012 850	5.7015	456.28	−0.0228	0.323
$T>T_\lambda$	−0.012 850	6.0092	456.28	−0.0228	0.323

The range of data for $T<T_\lambda$ was $t = 10^{-2} - 10^{-9}$, and for $T>T_\lambda$ was $t = 10^{-6} - 10^{-9}$. The data above T_λ are not considered as accurate as the data below T_λ.
(11) The enthalpy H of liquid helium can be deduced from the heat capacity using the relation:

$$H = \int_0^T C_s\, dT + \int_0^{\mathrm{SVP}} V\, dp,$$

where the second term arises because we are following along the saturated vapor pressure line. H is the difference between the total enthalpy and the ground state enthalpy [see J. S. Brooks and R. J. Donnelly, J. Phys. Chem. Ref. Data, p. 51 (1977), Section 4.3.d].

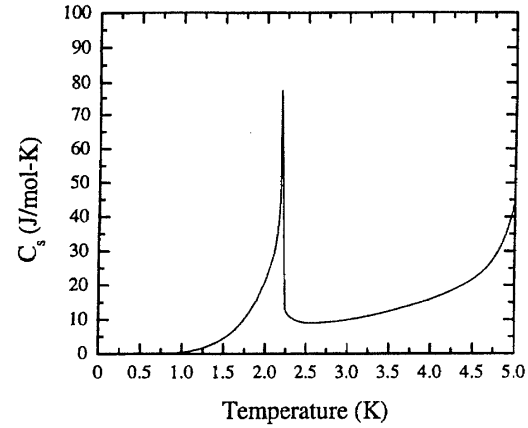

FIG. 7.1. The recommended values for the heat capacity of liquid ^{4}He as a function of temperature at saturated vapor pressure.

(12) We list recommended values of enthalpy directly from the above integration, so the uncertainties reflect uncertainties in heat capacity. We have also given a spline of the enthalpy for convenience of the reader. The spline does not return the exact recommended values (see Fig. 7.7).
(13) Most heat capacity measurements are reported in Joules/mol·K. To convert data from J/g·K to J/mol·K multiply by 4.0026. To convert from Joules/mol·K to J/kg·K multiply by 249.837. The same factor is used to convert the units for enthalpy.

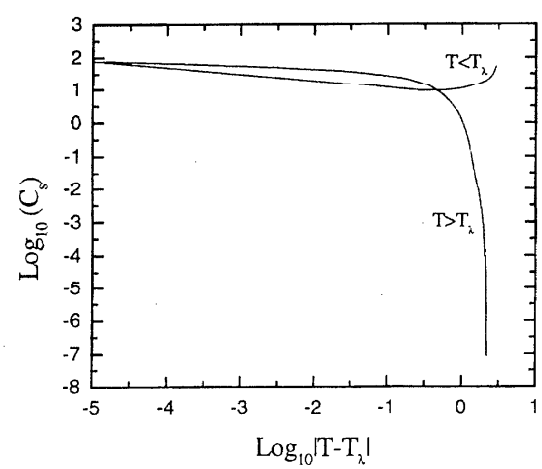

FIG. 7.2. The recommended values for the heat capacity of liquid ^{4}He at saturated vapor pressure near T_λ expressed as $\log_{10} C_s$ vs $\log_{10}|T - T_\lambda|$.

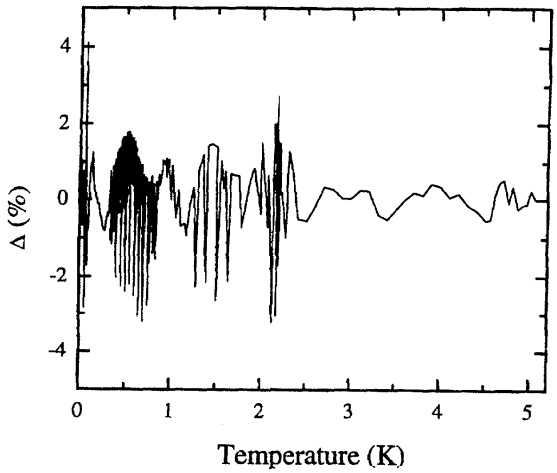

FIG. 7.3. The fractional deviation of values of the adopted database from the recommended values for the heat capacity of liquid ^{4}He expressed in percent.

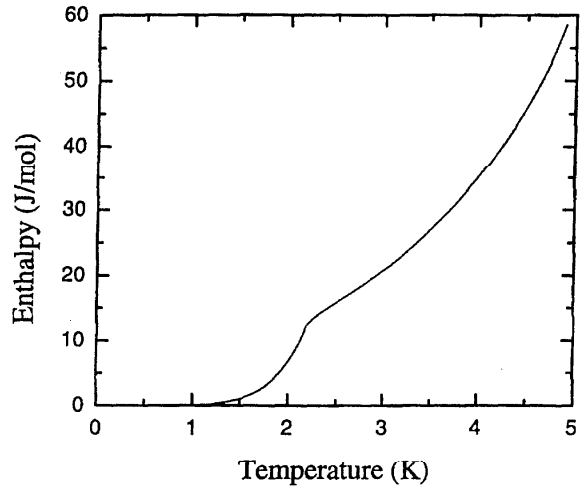

FIG. 7.6. Values for the enthalpy of liquid ^{4}He at saturated vapor pressure, obtained by integration of the recommended values of the heat capacity.

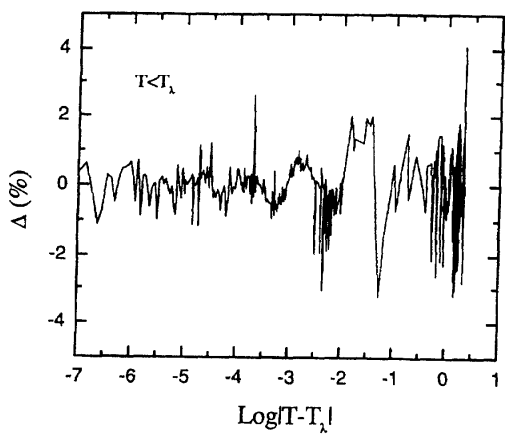

FIG. 7.4. The fractional deviation of values of the adopted database from the recommended values for the heat capacity of liquid ^{4}He near the lambda transition for $T<T_\lambda$.

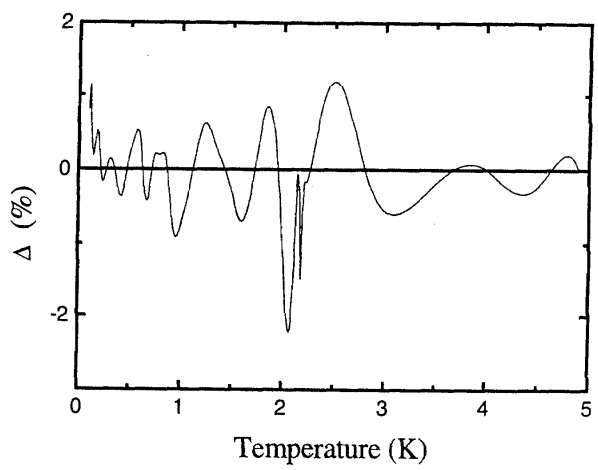

FIG. 7.7. Fractional deviation of the spline fit of the recommended values of the enthalpy from the enthalpy integrated from recommended values of the heat capacity.

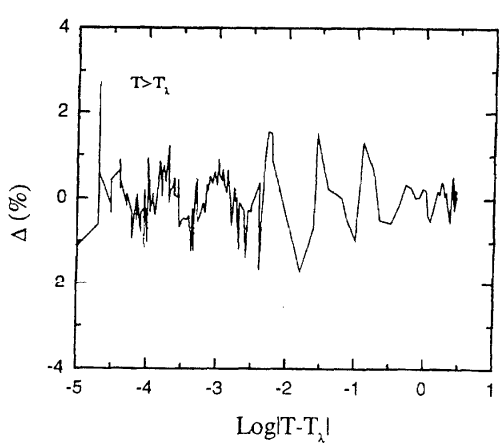

FIG. 7.5. The fractional deviation of values of the adopted database from the recommended values for the heat capacity of liquid ^{4}He near the lambda transition for $T>T_\lambda$.

TABLE 7.1. Adopted database for heat capacity of liquid ^{4}He at saturated vapor pressure

T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key
0.010 000 000	8.317 00E−08	1	0.332 900 000	2.952 00E−03	2	0.626 807 284	2.263 00E−02	4	1.388 218 288	3.019 00E+00	4
0.015 000 000	2.806 70E−07	1	0.337 236 048	3.052 00E−03	4	0.628 517 095	2.256 28E−02	2	1.403 515 049	3.090 01E+00	3
0.020 075 015	6.652 00E−07	1	0.339 201 980	3.161 00E−03	4	0.645 456 333	2.551 63E−02	2	1.453 556 614	3.726 42E+00	3
0.025 093 607	1.299 00E−06	1	0.349 400 000	3.397 00E−03	2	0.648 852 852	2.657 00E−02	4	1.503 597 910	4.462 90E+00	3
0.030 112 134	2.244 56E−06	1	0.351 177 931	3.482 26E−03	3	0.651 991 318	2.789 81E−02	3	1.505 409 334	4.580 00E+00	4
0.035 130 597	3.563 60E−06	1	0.353 755 696	3.583 00E−03	4	0.662 934 853	2.909 34E−02	2	1.505 899 719	4.685 00E+00	4
0.040 148 997	5.318 00E−06	1	0.358 810 557	3.730 00E−03	4	0.681 197 115	3.347 62E−02	2	1.553 637 123	5.323 46E+00	3
0.045 167 333	7.570 50E−06	1	0.366 700 000	3.902 00E−03	2	0.691 277 949	3.704 00E−02	4	1.582 153 782	5.906 00E+00	4
0.050 185 606	1.038 00E−05	1	0.371 879 445	4.139 00E−03	4	0.700 368 882	3.906 65E−02	2	1.603 690 373	6.316 10E+00	3
0.055 203 817	1.381 40E−05	1	0.380 223 962	4.403 00E−03	4	0.702 124 626	4.118 68E−02	3	1.635 233 195	7.208 00E+00	4
0.060 221 963	1.792 90E−05	1	0.384 700 000	4.487 00E−03	2	0.708 499 999	4.257 00E−02	4	1.658 829 565	7.564 00E+00	4
0.060 610 000	1.705 00E−05	2	0.391 476 993	4.795 00E−03	4	0.720 452 604	4.597 91E−02	2	1.754 802 154	1.023 00E+01	4
0.063 540 000	1.976 00E−05	2	0.401 325 554	5.243 41E−03	3	0.741 502 751	5.473 28E−02	2	1.776 611 822	1.108 00E+01	4
0.066 640 000	2.317 00E−05	2	0.401 800 000	5.090 00E−03	2	0.750 478 191	6.050 00E−02	4	1.865 627 080	1.432 00E+01	4
0.069 870 000	2.713 00E−05	2	0.403 451 755	5.256 00E−03	4	0.752 244 790	6.184 02E−02	3	1.916 469 079	1.655 00E+01	4
0.073 270 000	3.168 00E−05	2	0.412 929 109	5.591 00E−03	4	0.763 992 981	6.603 81E−02	2	1.992 102 080	2.088 00E+01	4
0.076 860 000	3.674 00E−05	2	0.417 900 000	5.710 00E−03	2	0.766 438 104	6.883 00E−02	4	1.996 426 104	2.075 00E+01	4
0.080 670 000	4.282 00E−05	2	0.428 112 594	6.237 00E−03	4	0.787 875 344	8.057 73E−02	2	2.052 220 307	2.474 00E+01	4
0.084 630 000	4.961 00E−05	2	0.433 600 000	6.364 00E−03	2	0.802 361 834	9.286 03E−02	3	2.062 756 913	2.564 00E+01	4
0.088 820 000	5.736 00E−05	2	0.436 847 534	6.596 00E−03	4	0.814 481 920	1.009 59E−01	2	2.063 718 535	2.568 00E+01	4
0.093 220 000	6.667 00E−05	2	0.448 800 000	7.039 00E−03	2	0.815 742 366	1.037 00E−01	4	2.065 000 728	2.581 00E+01	4
0.097 810 000	7.707 00E−05	2	0.451 468 481	7.444 84E−03	3	0.836 419 447	1.236 00E−01	4	2.066 713 713	2.594 00E+01	4
0.102 800 000	8.990 00E−05	2	0.454 136 258	7.442 00E−03	4	0.841 734 070	1.265 61E−01	2	2.068 767 375	2.614 00E+01	4
0.108 000 000	1.046 00E−04	2	0.463 700 000	7.759 00E−03	2	0.852 475 684	1.388 90E−01	3	2.070 811 114	2.630 00E+01	4
0.113 500 000	1.212 00E−04	2	0.464 605 602	7.946 00E−03	4	0.871 304 640	1.608 08E−01	2	2.076 331 708	2.637 71E+01	5
0.119 400 000	1.415 00E−04	2	0.478 200 000	8.516 00E−03	2	0.877 089 828	1.689 00E−01	4	2.113 325 831	3.065 99E+01	5
0.125 500 000	1.648 00E−04	2	0.483 407 690	8.972 00E−03	4	0.902 586 267	2.057 34E−01	3	2.123 709 861	3.267 00E+01	4
0.132 000 000	1.910 00E−04	2	0.492 800 000	9.342 00E−03	2	0.903 629 130	2.071 31E−01	2	2.136 704 232	3.366 19E+01	5
0.138 800 000	2.225 00E−04	2	0.497 085 329	9.782 00E−03	4	0.906 374 498	2.110 00E−01	4	2.140 798 148	3.404 00E+01	6
0.146 200 000	2.605 00E−04	2	0.501 607 656	1.028 67E−02	3	0.939 237 630	2.700 84E−01	2	2.144 510 586	3.494 00E+01	6
0.153 900 000	3.029 00E−04	2	0.507 100 000	1.021 00E−02	2	0.943 765 008	2.786 00E−01	4	2.148 814 681	3.589 00E+01	6
0.161 900 000	3.527 00E−04	2	0.514 462 524	1.089 00E−02	4	0.952 694 176	2.985 94E−01	3	2.151 453 098	3.682 39E+01	5
0.170 200 000	4.092 00E−04	2	0.521 500 000	1.117 00E−02	2	0.975 031 504	3.473 00E−01	4	2.159 768 125	3.917 00E+01	6
0.179 200 000	4.772 00E−04	2	0.523 617 428	1.151 00E−02	4	0.978 888 050	3.577 73E−01	2	2.160 680 705	3.958 57E+01	5
0.188 800 000	5.567 00E−04	2	0.535 900 000	1.222 00E−02	2	1.002 776 612	4.230 75E−01	3	2.160 690 733	3.948 00E+01	6
0.198 800 000	6.490 00E−04	2	0.546 077 483	1.320 00E−02	4	1.012 518 339	4.467 00E−01	4	2.162 395 451	3.976 00E+01	6
0.209 500 000	7.568 00E−04	2	0.550 700 000	1.342 05E−02	2	1.023 530 400	4.811 50E−01	2	2.165 754 329	4.176 00E+01	6
0.220 600 000	8.826 00E−04	2	0.551 742 562	1.404 91E−02	3	1.052 861 275	5.835 79E−01	3	2.166 596 462	4.230 00E+01	6
0.231 700 000	1.019 00E−03	2	0.555 683 078	1.403 00E−02	4	1.088 881 255	7.170 00E−01	4	2.166 596 462	4.222 74E+01	5
0.244 100 000	1.189 00E−03	2	0.565 687 962	1.476 75E−02	2	1.102 955 222	7.841 09E−01	3	2.167 318 259	4.306 00E+01	4
0.257 300 000	1.389 00E−03	2	0.576 517 791	1.603 00E−02	4	1.153 029 266	1.028 67E+00	3	2.167 439 558	4.276 00E+01	6
0.272 200 000	1.641 00E−03	2	0.580 874 318	1.628 19E−02	2	1.172 802 215	1.143 00E+00	4	2.168 030 001	4.330 00E+01	4
0.286 700 000	1.909 00E−03	2	0.588 478 962	1.733 00E−02	4	1.203 266 846	1.320 86E+00	3	2.168 932 168	4.409 00E+01	4
0.301 025 619	2.201 43E−03	3	0.596 225 336	1.802 15E−02	2	1.253 202 675	1.665 08E+00	3	2.169 107 584	4.389 00E+01	6
0.301 900 000	2.223 00E−03	2	0.601 873 568	1.949 27E−02	3	1.277 869 986	1.910 00E+00	4	2.169 543 609	4.456 00E+01	4
0.317 100 000	2.570 00E−03	2	0.612 103 323	2.008 69E−02	2	1.303 323 993	2.069 34E+00	3	2.169 944 542	4.460 00E+01	6
0.322 702 023	2.698 00E−03	4	0.612 310 364	2.046 00E−02	4	1.353 442 881	2.541 65E+00	3	2.170 072 838	4.467 00E+01	6

TABLE 7.1. Adopted database for heat capacity of liquid ^{4}He at saturated vapor pressure—Continued

T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key
2.170 305 372	4.486 91E+01	5	2.175 579 900	5.350 00E+01	7	2.176 583 200	6.240 00E+01	6	2.176 783 965	7.570 00E+01	7
2.170 405 602	4.552 00E+01	4	2.175 647 000	5.402 00E+01	6	2.176 589 940	6.270 00E+01	7	2.176 784 972	7.600 00E+01	7
2.170 406 604	4.492 00E+01	6	2.175 670 300	5.400 00E+01	7	2.176 600 000	6.123 98E+01	5	2.176 785 500	7.600 00E+01	6
2.170 744 372	4.525 00E+01	6	2.175 717 000	5.428 00E+01	6	2.176 606 060	6.320 00E+01	7	2.176 785 700	7.710 00E+01	6
2.170 780 454	4.530 00E+01	6	2.175 729 000	5.428 00E+01	6	2.176 615 000	6.329 00E+01	6	2.176 786 010	7.610 00E+01	7
2.170 936 807	4.618 00E+01	4	2.175 745 000	5.444 00E+01	6	2.176 619 900	6.370 00E+01	6	2.176 787 021	7.650 00E+01	7
2.171 087 144	4.554 00E+01	6	2.175 760 500	5.430 00E+01	7	2.176 620 470	6.320 00E+01	7	2.176 787 994	7.690 00E+01	7
2.171 287 593	4.581 00E+01	4	2.175 792 000	5.460 00E+01	6	2.176 631 500	6.388 00E+01	6	2.176 788 956	7.730 00E+01	7
2.171 438 929	4.593 00E+01	6	2.175 800 000	5.455 54E+01	5	2.176 636 230	6.370 00E+01	7	2.176 789 967	7.760 00E+01	7
2.171 528 127	4.677 00E+01	4	2.175 831 310	5.480 00E+01	7	2.176 645 370	6.400 00E+01	7	2.176 790 000	7.770 00E+01	6
2.171 973 000	4.627 00E+01	6	2.175 899 950	5.520 00E+01	7	2.176 655 790	6.460 00E+01	7	2.176 790 000	7.765 04E+01	5
2.172 030 000	4.705 00E+01	4	2.175 911 900	5.522 00E+01	6	2.176 665 910	6.460 00E+01	7	2.176 790 200	7.800 00E+01	6
2.172 151 000	4.652 00E+01	6	2.175 960 800	5.556 00E+01	6	2.176 671 800	6.528 00E+01	6	2.176 790 792	7.770 00E+01	7
2.172 320 000	4.820 00E+01	4	2.175 970 830	5.560 00E+01	7	2.176 675 720	6.530 00E+01	7	2.176 791 479	7.870 00E+01	7
2.172 428 000	4.686 00E+01	6	2.176 002 800	5.592 00E+01	6	2.176 678 000	6.551 00E+01	6	2.176 792 234	7.880 00E+01	7
2.172 645 000	4.707 00E+01	6	2.176 029 750	5.590 00E+01	7	2.176 678 500	6.537 00E+01	6	2.176 792 744	7.870 00E+01	7
2.172 820 000	4.711 06E+01	5	2.176 050 500	5.620 00E+01	6	2.176 685 000	6.570 00E+01	7	2.176 793 259	8.020 00E+01	7
2.172 830 000	4.829 00E+01	4	2.176 080 300	5.636 00E+01	6	2.176 695 450	6.590 00E+01	7	2.176 793 869	8.050 00E+01	7
2.172 983 000	4.760 00E+01	6	2.176 090 610	5.620 00E+01	7	2.176 700 000	6.608 29E+01	5	2.176 794 309	8.040 00E+01	7
2.173 111 000	4.779 00E+01	6	2.176 129 200	5.672 00E+01	6	2.176 704 242	6.650 00E+01	7	2.176 794 761	8.100 00E+01	7
2.173 322 000	4.806 00E+01	6	2.176 141 490	5.670 00E+01	7	2.176 710 628	6.700 00E+01	7	2.176 795 229	8.130 00E+01	7
2.173 664 000	4.861 00E+01	6	2.176 175 600	5.703 00E+01	6	2.176 716 977	6.730 00E+01	7	2.176 795 659	8.160 00E+01	7
2.173 710 000	4.973 00E+01	4	2.176 180 490	5.690 00E+01	7	2.176 723 343	6.780 00E+01	7	2.176 795 979	8.210 00E+01	7
2.173 767 000	4.886 00E+01	6	2.176 230 510	5.760 00E+01	7	2.176 728 740	6.760 00E+01	7	2.176 796 282	8.230 00E+01	7
2.174 008 000	4.923 00E+01	6	2.176 281 960	5.790 00E+01	7	2.176 734 305	6.850 00E+01	7	2.176 796 663	8.290 00E+01	7
2.174 173 000	4.961 00E+01	6	2.176 282 500	5.797 00E+01	6	2.176 738 974	6.920 00E+01	7	2.176 796 931	8.420 00E+01	7
2.174 290 000	4.971 23E+01	5	2.176 297 000	5.830 00E+01	6	2.176 744 134	6.900 00E+01	7	2.176 797 220	8.380 00E+01	7
2.174 340 000	4.988 00E+01	6	2.176 300 000	5.759 74E+01	5	2.176 748 459	6.950 00E+01	7	2.176 797 379	8.410 00E+01	7
2.174 673 000	5.065 00E+01	6	2.176 302 700	5.832 00E+01	6	2.176 750 000	6.964 52E+01	5	2.176 797 628	8.470 00E+01	7
2.174 710 300	5.060 00E+01	7	2.176 313 500	5.834 00E+01	6	2.176 751 982	7.020 00E+01	7	2.176 797 839	8.560 00E+01	7
2.174 843 000	5.112 00E+01	6	2.176 319 470	5.830 00E+01	7	2.176 755 459	7.030 00E+01	7	2.176 798 079	8.560 00E+01	7
2.174 860 600	5.120 00E+01	7	2.176 326 100	5.846 00E+01	6	2.176 758 801	7.060 00E+01	7	2.176 798 200	8.580 00E+01	7
2.175 010 100	5.170 00E+01	7	2.176 350 160	5.850 00E+01	7	2.176 762 052	7.120 00E+01	7	2.176 798 379	8.640 00E+01	7
2.175 044 000	5.171 00E+01	6	2.176 351 800	5.898 00E+01	6	2.176 764 908	7.150 00E+01	7	2.176 798 527	8.790 00E+01	7
2.175 075 000	5.179 00E+01	6	2.176 380 650	5.910 00E+01	7	2.176 765 790	7.180 00E+01	6	2.176 798 680	8.710 00E+01	7
2.175 079 000	5.184 00E+01	6	2.176 410 970	5.950 00E+01	7	2.176 767 701	7.180 00E+01	7	2.176 798 835	8.880 00E+01	7
2.175 140 500	5.210 00E+01	7	2.176 441 060	5.990 00E+01	7	2.176 769 257	7.130 00E+01	7	2.176 798 931	8.860 00E+01	7
2.175 210 000	5.235 40E+01	5	2.176 446 700	6.004 00E+01	6	2.176 771 289	7.260 00E+01	7	2.176 799 028	8.870 00E+01	7
2.175 244 000	5.228 00E+01	6	2.176 465 200	6.031 00E+01	6	2.176 773 790	7.270 00E+01	7	2.176 799 128	8.930 00E+01	7
2.175 261 600	5.240 00E+01	7	2.176 471 930	6.020 00E+01	7	2.176 775 441	7.340 00E+01	7	2.176 799 229	8.990 00E+01	7
2.175 318 000	5.260 00E+01	6	2.176 482 200	6.050 00E+01	6	2.176 777 369	7.360 00E+01	7	2.176 799 328	9.050 00E+01	7
2.175 370 300	5.260 00E+01	7	2.176 489 950	6.040 00E+01	7	2.176 779 885	7.440 00E+01	7	2.176 799 429	9.140 00E+01	7
2.175 383 000	5.285 00E+01	6	2.176 511 500	6.105 00E+01	6	2.176 780 000	7.384 80E+01	5	2.176 799 523	9.260 00E+01	7
2.175 411 000	5.293 00E+01	6	2.176 517 430	6.110 00E+01	7	2.176 780 922	7.470 00E+01	7	2.176 799 584	9.240 00E+01	7
2.175 480 300	5.310 00E+01	7	2.176 537 170	6.140 00E+01	7	2.176 781 000	7.400 00E+01	6	2.176 799 650	9.280 00E+01	7
2.175 514 000	5.332 00E+01	6	2.176 556 820	6.180 00E+01	7	2.176 781 600	7.590 00E+01	6	2.176 799 725	9.470 00E+01	7
2.175 578 000	5.358 00E+01	6	2.176 572 560	6.200 00E+01	7	2.176 782 511	7.510 00E+01	7	2.176 799 768	9.590 00E+01	7

TABLE 7.1. Adopted database for heat capacity of liquid ^{4}He at saturated vapor pressure—Continued

T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key
2.176 799 822	9.650 00E+01	7	2.176 807 642	5.950 00E+01	7	2.176 900 000	4.526 94E+01	5	2.178 240 900	3.150 00E+01	7
2.176 799 860	9.720 00E+01	7	2.176 808 443	5.920 00E+01	7	2.176 905 090	4.560 00E+01	7	2.178 341 000	3.120 00E+01	7
2.176 799 909	1.000 00E+02	7	2.176 809 033	5.850 00E+01	7	2.176 915 210	4.500 00E+01	7	2.178 390 000	3.118 03E+01	5
2.176 799 965	1.041 00E+02	7	2.176 809 600	5.680 00E+01	6	2.176 924 970	4.470 00E+01	7	2.178 450 300	3.090 00E+01	7
2.176 800 132	7.950 00E+01	7	2.176 809 698	5.810 00E+01	7	2.176 934 900	4.450 00E+01	7	2.178 524 000	3.043 00E+01	6
2.176 800 144	7.970 00E+01	7	2.176 810 000	5.760 00E+01	6	2.176 945 050	4.360 00E+01	7	2.178 590 400	3.030 00E+01	7
2.176 800 164	7.990 00E+01	7	2.176 810 000	5.763 74E+01	5	2.176 955 460	4.360 00E+01	7	2.178 678 000	2.999 00E+01	6
2.176 800 185	7.950 00E+01	7	2.176 810 569	5.780 00E+01	7	2.176 964 320	4.310 00E+01	7	2.178 740 200	3.000 00E+01	7
2.176 800 220	7.890 00E+01	7	2.176 811 400	5.710 00E+01	6	2.176 972 800	4.275 00E+01	6	2.178 827 700	2.990 00E+01	7
2.176 800 269	7.670 00E+01	7	2.176 811 505	5.710 00E+01	7	2.176 979 650	4.270 00E+01	7	2.178 869 000	2.946 00E+01	6
2.176 800 293	7.600 00E+01	7	2.176 812 520	5.690 00E+01	7	2.176 995 040	4.190 00E+01	7	2.179 214 000	2.864 00E+01	6
2.176 800 320	7.680 00E+01	7	2.176 813 463	5.630 00E+01	7	2.177 000 000	4.230 75E+01	5	2.179 310 000	2.873 87E+01	5
2.176 800 371	7.560 00E+01	7	2.176 814 493	5.640 00E+01	7	2.177 010 690	4.190 00E+01	7	2.179 553 000	2.793 00E+01	6
2.176 800 400	7.360 00E+01	7	2.176 815 612	5.580 00E+01	7	2.177 028 050	4.140 00E+01	7	2.179 814 000	2.748 00E+01	6
2.176 800 420	7.330 00E+01	7	2.176 816 537	5.520 00E+01	7	2.177 037 900	4.108 00E+01	6	2.179 880 000	2.733 00E+01	6
2.176 800 494	7.350 00E+01	7	2.176 816 800	5.370 00E+01	6	2.177 043 600	4.110 00E+01	7	2.180 204 000	2.680 00E+01	6
2.176 800 593	7.240 00E+01	7	2.176 817 900	5.460 00E+01	6	2.177 065 760	4.060 00E+01	7	2.180 749 000	2.604 00E+01	6
2.176 800 692	7.150 00E+01	7	2.176 818 091	5.480 00E+01	7	2.177 066 000	4.043 00E+01	6	2.180 780 000	2.653 72E+01	5
2.176 800 797	7.070 00E+01	7	2.176 819 577	5.450 00E+01	7	2.177 087 900	4.040 00E+01	7	2.181 431 000	2.520 00E+01	6
2.176 800 896	7.040 00E+01	7	2.176 820 000	5.463 55E+01	5	2.177 109 540	3.990 00E+01	7	2.181 963 955	2.444 00E+01	6
2.176 800 998	6.990 00E+01	7	2.176 821 081	5.420 00E+01	7	2.177 129 670	3.950 00E+01	7	2.182 673 184	2.377 00E+01	6
2.176 801 097	6.950 00E+01	7	2.176 823 067	5.350 00E+01	7	2.177 159 490	3.900 00E+01	7	2.182 925 239	2.369 54E+01	5
2.176 801 171	6.880 00E+01	7	2.176 825 087	5.350 00E+01	7	2.177 189 620	3.850 00E+01	7	2.186 624 998	2.121 38E+01	5
2.176 801 296	6.830 00E+01	7	2.176 827 095	5.300 00E+01	7	2.177 219 030	3.840 00E+01	7	2.192 523 901	1.877 22E+01	5
2.176 801 363	6.840 00E+01	7	2.176 829 050	5.260 00E+01	7	2.177 238 300	3.778 00E+01	6	2.201 723 640	1.657 08E+01	5
2.176 801 494	6.760 00E+01	7	2.176 831 035	5.250 00E+01	7	2.177 250 520	3.800 00E+01	7	2.203 823 802	1.593 04E+01	8
2.176 801 568	6.790 00E+01	7	2.176 832 778	5.180 00E+01	7	2.177 282 540	3.710 00E+01	7	2.216 426 239	1.465 35E+01	5
2.176 801 724	6.810 00E+01	7	2.176 835 750	5.120 00E+01	7	2.177 300 000	3.682 39E+01	5	2.239 736 243	1.283 63E+01	5
2.176 801 849	6.620 00E+01	7	2.176 837 100	5.100 00E+01	6	2.177 309 700	3.698 00E+01	6	2.253 845 448	1.216 79E+01	8
2.176 801 969	6.640 00E+01	7	2.176 838 888	5.060 00E+01	7	2.177 319 440	3.700 00E+01	7	2.276 666 284	1.141 94E+01	5
2.176 802 149	6.660 00E+01	7	2.176 841 885	5.050 00E+01	7	2.177 369 760	3.650 00E+01	7	2.304 205 527	1.056 69E+01	8
2.176 802 342	6.580 00E+01	7	2.176 844 784	5.000 00E+01	7	2.177 421 430	3.600 00E+01	7	2.354 513 418	9.886 47E+00	8
2.176 802 516	6.520 00E+01	7	2.176 848 510	4.980 00E+01	7	2.177 461 300	3.580 00E+01	7	2.404 936 229	9.526 20E+00	8
2.176 802 675	6.440 00E+01	7	2.176 850 000	4.963 22E+01	5	2.177 485 500	3.541 00E+01	6	2.505 557 267	9.125 90E+00	8
2.176 802 906	6.400 00E+01	7	2.176 852 300	4.930 00E+01	6	2.177 510 100	3.530 00E+01	7	2.606 086 904	9.085 90E+00	8
2.176 803 147	6.370 00E+01	7	2.176 852 561	4.920 00E+01	7	2.177 570 830	3.480 00E+01	7	2.706 596 360	9.165 90E+00	8
2.176 803 331	6.350 00E+01	7	2.176 856 247	4.900 00E+01	7	2.177 628 930	3.450 00E+01	7	2.807 058 930	9.366 10E+00	8
2.176 803 593	6.310 00E+01	7	2.176 861 043	4.880 00E+01	7	2.177 700 280	3.400 00E+01	7	2.907 241 720	9.646 30E+00	8
2.176 803 900	6.280 00E+01	7	2.176 866 602	4.780 00E+01	7	2.177 771 260	3.370 00E+01	7	3.007 401 739	9.966 50E+00	8
2.176 804 196	6.250 00E+01	7	2.176 871 084	4.770 00E+01	7	2.177 800 000	3.346 17E+01	5	3.107 543 262	1.032 67E+01	8
2.176 804 544	6.180 00E+01	7	2.176 872 600	4.730 00E+01	6	2.177 825 000	3.322 00E+01	6	3.207 478 906	1.076 70E+01	8
2.176 804 944	6.140 00E+01	7	2.176 876 677	4.740 00E+01	7	2.177 850 200	3.320 00E+01	7	3.307 424 279	1.132 74E+01	8
2.176 805 319	6.130 00E+01	7	2.176 882 637	4.680 00E+01	7	2.177 938 900	3.280 00E+01	7	3.407 351 902	1.188 77E+01	8
2.176 805 743	6.080 00E+01	7	2.176 888 300	4.640 00E+01	6	2.177 984 000	3.250 00E+01	6	3.507 252 795	1.244 81E+01	8
2.176 806 207	6.020 00E+01	7	2.176 888 888	4.680 00E+01	7	2.178 029 200	3.240 00E+01	7	3.607 129 693	1.304 85E+01	8
2.176 806 697	6.000 00E+01	7	2.176 889 500	4.614 00E+01	6	2.178 129 400	3.200 00E+01	7	3.707 066 600	1.368 89E+01	8
2.176 807 203	5.950 00E+01	7	2.176 897 038	4.630 00E+01	7	2.178 178 000	3.163 00E+01	6	3.807 062 091	1.440 94E+01	8

TABLE 7.1. Adopted database for heat capacity of liquid ^{4}He at saturated vapor pressure—Continued

T_{90} (K)	C_s (J/mol·K)	Key	T_{90} (K)	C_s (J/mol·K)	Key
3.907 137 696	1.512 99E+01	8	4.659 780 505	2.489 60E+01	8
3.907 233 994	1.597 04E+01	8	4.710 078 203	2.629 70E+01	8
4.107 360 133	1.693 11E+01	8	4.760 314 859	2.817 80E+01	8
4.207 682 570	1.793 17E+01	8	4.810 536 961	3.014 00E+01	8
4.307 978 467	1.913 20E+01	8	4.860 791 389	3.290 10E+01	8
4.408 382 309	2.045 30E+01	8	4.911 025 444	3.614 30E+01	8
4.508 963 741	2.201 40E+01	8	4.961 186 960	4.042 60E+01	8
4.559 218 620	2.289 50E+01	8	5.011 324 184	4.603 00E+01	8
4.609 475 809	2.377 50E+01	8	5.061 435 817	5.403 50E+01	8

TABLE 7.2. Knots and coefficients for the spline fit of $\log_{10} C_s$ for $T<T_\lambda$

Knots	Coefficients
$K(1)=1.000\,000\,000\,00E-2$	$C(1)=-7.080\,029E+0$
$K(2)=1.000\,000\,000\,00E-2$	$C(2)=-6.461\,204E+0$
$K(3)=1.000\,000\,000\,00E-2$	$C(3)=-5.407\,887E+0$
$K(4)=1.000\,000\,000\,00E-2$	$C(4)=-4.971\,098E+0$
$K(5)=2.497\,389\,907\,40E-2$	$C(5)=-4.092\,648E+0$
$K(6)=6.076\,411\,059\,90E-2$	$C(6)=-3.430\,290E+0$
$K(7)=6.365\,286\,537\,70E-2$	$C(7)=-2.300\,493E+0$
$K(8)=1.541\,571\,790\,76E-1$	$C(8)=-1.865\,551E+0$
$K(9)=2.492\,991\,354\,47E-1$	$C(9)=-6.120\,157E-1$
$K(10)=6.344\,970\,776\,44E-1$	$C(10)=1.800\,753E-1$
$K(11)=9.044\,653\,806\,69E-1$	$C(11)=8.200\,057E-1$
$K(12)=1.174\,745\,643\,85E+0$	$C(12)=1.231\,093E+0$
$K(13)=1.542\,559\,108\,67E+0$	$C(13)=1.494\,099E+0$
$K(14)=2.064\,990\,887\,20E+0$	$C(14)=1.633\,977E+0$
$K(15)=2.160\,709\,307\,46E+0$	$C(15)=1.694\,854E+0$
$K(16)=2.171\,575\,251\,25E+0$	$C(16)=1.760\,367E+0$
$K(17)=2.175\,906\,292\,00E+0$	$C(17)=1.813\,528E+0$
$K(18)=2.176\,553\,159\,00E+0$	$C(18)=1.849\,639E+0$
$K(19)=2.176\,728\,499\,00E+0$	$C(19)=1.900\,049E+0$
$K(20)=2.176\,787\,881\,00E+0$	$C(20)=1.935\,770E+0$
$K(21)=2.176\,796\,632\,00E+0$	$C(21)=1.974\,983E+0$
$K(22)=2.176\,799\,582\,00E+0$	$C(22)=2.017\,442E+0$
$K(23)=2.176\,799\,964\,68E+0$	
$K(24)=2.176\,799\,964\,68E+0$	
$K(25)=2.176\,799\,964\,68E+0$	
$K(26)=2.176\,799\,964\,68E+0$	

TABLE 7.3. Knots and coefficients for the spline fit of $\log_{10} C_s$ for $T>T_\lambda$

Knots	Coefficients
$K(1)=2.176\,800\,132\,00E+0$	$C(1)=1.900\,539E+0$
$K(2)=2.176\,800\,132\,00E+0$	$C(2)=1.862\,865E+0$
$K(3)=2.176\,800\,132\,00E+0$	$C(3)=1.822\,431E+0$
$K(4)=2.176\,800\,132\,00E+0$	$C(4)=1.759\,056E+0$
$K(5)=2.176\,801\,012\,00E+0$	$C(5)=1.679\,157E+0$
$K(6)=2.176\,803\,495\,00E+0$	$C(6)=1.584\,631E+0$
$K(7)=2.176\,818\,018\,00E+0$	$C(7)=1.464\,344E+0$
$K(8)=2.176\,924\,206\,00E+0$	$C(8)=1.274\,043E+0$
$K(9)=2.177\,507\,556\,00E+0$	$C(9)=1.095\,050E+0$
$K(10)=2.180\,863\,971\,00E+0$	$C(10)=9.546\,039E-1$
$K(11)=2.203\,740\,207\,94E+0$	$C(11)=9.503\,531E-1$
$K(12)=2.303\,680\,588\,65E+0$	$C(12)=1.100\,724E+0$
$K(13)=2.608\,490\,381\,28E+0$	$C(13)=1.260\,996E+0$
$K(14)=3.603\,357\,638\,94E+0$	$C(14)=1.421\,360E+0$
$K(15)=4.413\,584\,694\,22E+0$	$C(15)=1.609\,738E+0$
$K(16)=4.814\,045\,770\,42E+0$	$C(16)=1.732\,672E+0$
$K(17)=5.061\,435\,816\,98E+0$	
$K(18)=5.061\,435\,816\,98E+0$	
$K(19)=5.061\,435\,816\,98E+0$	
$K(20)=5.061\,435\,816\,98E+0$	

TABLE 7.4. Recommended values of the heat capacity of liquid ^{4}He

T_{90} (K)	C_s (J/mol·K)	T_{90} (K)	C_s (J/mol·K)
0.0000	0	2.2000	1.673E+1
0.0500	1.023E−05	2.2500	1.228E+1
0.1000	8.250E−05	2.3000	1.078E+1
0.1500	2.824E−4	2.3500	1.001E+1
0.2000	6.597E−4	2.4000	9.515E+0
0.2500	1.267E−3	2.4500	9.225E+0
0.3000	2.172E−3	2.5000	9.083E+0
0.3500	3.431E−3	2.5500	9.043E+0
0.4000	5.086E−3	2.6000	9.066E+0
0.4500	7.206E−3	2.6500	9.117E+0
0.5000	9.939E−3	2.7000	9.186E+0
0.5500	1.359E−2	2.7500	9.271E+0
0.6000	1.877E−2	2.8000	9.374E+0
0.6500	2.665E−2	2.8500	9.493E+0
0.7000	3.919E−2	2.9000	9.628E+0
0.7500	5.901E−2	2.9500	9.778E+0
0.8000	8.976E−2	3.0000	9.944E+0
0.8500	1.361E−1	3.0500	1.012E+1
0.9000	2.030E−1	3.1000	1.032E+1
0.9500	2.946E−1	3.1500	1.053E+1
1.0000	4.154E−1	3.2000	1.075E+1
1.0500	5.707E−1	3.2500	1.099E+1
1.1000	7.658E−1	3.3000	1.124E+1
1.1500	1.006E+0	3.3500	1.151E+1
1.2000	1.297E+0	3.4000	1.178E+1
1.2500	1.646E+0	3.4500	1.207E+1
1.3000	2.057E+0	3.5000	1.237E+1
1.3500	2.537E+0	3.5500	1.268E+1
1.4000	3.092E+0	3.6000	1.300E+1
1.4500	3.732E+0	3.6500	1.333E+1
1.5000	4.468E+0	3.7000	1.366E+1
1.5500	5.313E+0	3.7500	1.401E+1
1.6000	6.285E+0	3.8000	1.437E+1
1.6500	7.401E+0	3.8500	1.475E+1
1.7000	8.678E+0	3.9000	1.513E+1
1.7500	1.014E+1	3.9500	1.554E+1
1.8000	1.181E+1	4.0000	1.596E+1
1.8500	1.372E+1	4.0500	1.640E+1
1.9000	1.590E+1	4.1000	1.687E+1
1.9500	1.841E+1	4.1500	1.736E+1
2.0000	2.128E+1	4.2000	1.788E+1
2.0500	2.459E+1	4.2500	1.84E+01
2.1000	2.864E+1	4.3000	1.90E+01
2.1500	3.689E+1	4.3500	1.96E+01
2.1760	5.569E+1	4.4000	2.03E+01
2.1761	5.623E+1	4.4500	2.10E+01
2.1762	5.692E+1	4.5000	2.18E+01
2.1763	5.784E+1	4.5500	2.26E+01
2.1764	5.906E+1	4.6000	2.36E+01
2.1765	6.068E+1	4.6500	2.48E+01
2.1766	6.283E+1	4.7000	2.61E+01
2.1767	6.640E+1	4.7500	2.78E+01
2.1768	—	4.8000	2.98E+01
2.1769	4.569E+1	4.8500	3.22E+01
2.1770	4.227E+1	4.9000	3.53E+01
2.1775	3.543E+1	4.9500	3.93E+01
2.1780	3.264E+1	5.0000	4.47E+01

TABLE 7.5. Knots and coefficients for the spline fit to the enthalpy of liquid ^{4}He

Knots	Coefficients
$K(1)=0.000\,000$	$C(1)=0.000\,000E+00$
$K(2)=0.000\,000$	$C(2)=0.000\,000E+00$
$K(3)=0.000\,000$	$C(3)=-1.650\,000E-06$
$K(4)=0.000\,000$	$C(4)=1.801\,000E-05$
$K(5)=0.100\,000$	$C(5)=1.185\,000E-04$
$K(6)=0.200\,000$	$C(6)=4.108\,550E-04$
$K(7)=0.300\,000$	$C(7)=1.033\,200E-03$
$K(8)=0.370\,000$	$C(8)=2.710\,000E-03$
$K(9)=0.500\,000$	$C(9)=6.090\,000E-03$
$K(10)=0.610\,000$	$C(10)=1.784\,000E-02$
$K(11)=0.743\,950$	$C(11)=6.032\,740E-02$
$K(12)=0.870\,000$	$C(12)=2.400\,500E-01$
$K(13)=1.027\,550$	$C(13)=8.643\,720E-01$
$K(14)=1.250\,000$	$C(14)=2.445\,400E+00$
$K(15)=1.500\,000$	$C(15)=5.554\,360E+00$
$K(16)=1.750\,000$	$C(16)=8.800\,000E+00$
$K(17)=2.000\,000$	$C(17)=1.220\,130E+01$
$K(18)=2.130\,000$	$C(18)=1.558\,100E+01$
$K(19)=2.190\,120$	$C(19)=2.004\,000E+01$
$K(20)=2.232\,940$	$C(20)=3.223\,100E+01$
$K(21)=2.854\,200$	$C(21)=4.704\,640E+01$
$K(22)=4.000\,000$	$C(22)=5.848\,930E+01$
$K(23)=4.900\,000$	
$K(24)=4.900\,000$	
$K(25)=4.900\,000$	
$K(26)=4.900\,000$	

TABLE 7.6. Recommended values of the enthalpy of liquid ^{4}He

T_{90} (K)	H (J mol^{-1})	T_{90} (K)	H (J mol^{-1})
0.10	$2.039E-06$	2.60	16.876
0.15	$1.056E-05$	2.65	17.332
0.20	$3.334E-05$	2.70	17.784
0.25	$8.018E-05$	2.75	18.237
0.30	$1.651E-04$	2.80	18.696
0.35	$3.035E-04$	2.85	19.168
0.40	$5.130E-04$	2.90	19.656
0.45	$8.178E-04$	2.95	20.162
0.50	$1.250E-03$	3.00	20.686
0.55	$1.840E-03$	3.05	21.228
0.60	$2.640E-03$	3.10	21.789
0.65	$3.740E-03$	3.15	22.367
0.70	$5.370E-03$	3.20	22.964
0.75	$7.830E-03$	3.25	23.580
0.80	$1.152E-02$	3.30	24.213
0.85	$1.713E-02$	3.35	24.866
0.90	$2.543E-02$	3.40	25.537
0.95	$3.765E-02$	3.45	26.227
1.00	$5.535E-02$	3.50	26.935
1.05	$8.014E-02$	3.55	27.662
1.10	$1.140E-01$	3.60	28.409
1.15	$1.591E-01$	3.65	29.174
1.20	$2.180E-01$	3.70	29.958
1.25	$2.928E-01$	3.75	30.762
1.30	$3.863E-01$	3.80	31.584
1.35	$5.020E-01$	3.85	32.426
1.40	$6.436E-01$	3.90	33.288
1.45	$8.150E-01$	3.95	34.168
1.50	1.020	4.00	35.069
1.55	1.263	4.05	35.989
1.60	1.553	4.10	36.933
1.65	1.899	4.15	37.905
1.70	2.311	4.20	38.908
1.75	2.799	4.25	39.946
1.80	3.370	4.30	41.024
1.85	4.028	4.35	42.146
1.90	4.775	4.40	43.315
1.95	5.615	4.45	44.535
2.00	6.548	4.50	45.811
2.05	7.607	4.55	47.146
2.10	8.945	4.60	48.545
2.15	10.724	4.65	50.011
2.20	12.340	4.70	51.548
2.25	13.104	4.75	53.160
2.30	13.743	4.80	54.852
2.35	14.340	4.85	56.627
2.40	14.899	4.90	58.489
2.45	15.427	4.95	60.443
2.50	15.929	5.00	62.491
2.55	16.410		

7.1. Chronological Bibliography for Heat Capacity and Enthalpy

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8. Entropy

Adopted Database

Author(s)	Key #	Method	Range (K)
Singsass Table 1	1	Fountain pressure	$1.59 \leq T \leq 2.1705$
Van den Meijdenberg *et al.*	2	Fountain pressure	$1.15 \leq T \leq 2.05$
Singsass Table 4	3	Fountain pressure	$2.15 \leq T \leq 2.168$
Singsass Table 2	4	Fountain pressure	$1.6 \leq T \leq 2.159$
Singsass Table 5	5	Fountain pressure	$(T_\lambda - T)/T_\lambda = 0.1007$ -0.00970

Comments and Key to Authors

(1),(3),(4),(5) Singsass and Ahlers; Tables from Singsass Thesis, Ref. 7.

(2) Reference 4.

(6) The spline has been adjusted to agree approximately with Singsass and Ahlers at the lambda point. The spline returns $S_\lambda = 1.579$ J/g·K and $(dS/dt)_\lambda = 3.025$ J/g·K^2.

(7) Uncertainties: Singsass and Ahlers quote 0.1% precision, 0.5% accuracy. Van den Meijdenberg *et al.* quote 1% precision, 3% accuracy.

(8) Following the fountain pressure entropy spline are the knots and coefficients of the fit to entropy data integrated from the recommended values of heat capacity by

$$S = \int_0^T \frac{C_s}{T} dT.$$

This spline fit provides entropy data over both helium I and helium II regions.

(9) Most heat capacity measurements are reported in J/mol·K. Entropy, however is usually quoted in gram units. To convert data from J/g·K to J/mol·K multiply by 4.00 26. To convert from J/g·K to J/kg·K multiply by 1000.

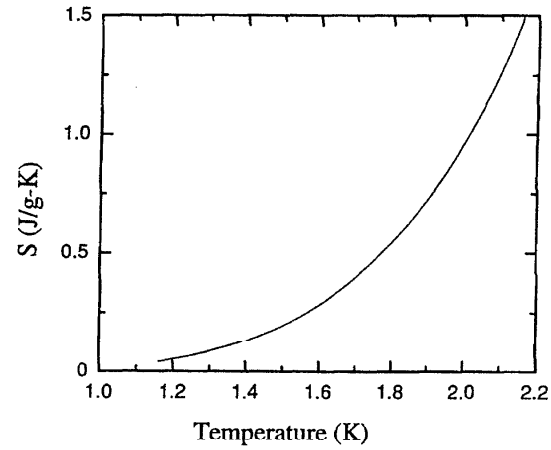

FIG. 8.1. The recommended values of the fountain pressure entropy of liquid ^{4}He as a function of temperature at saturated vapor pressure.

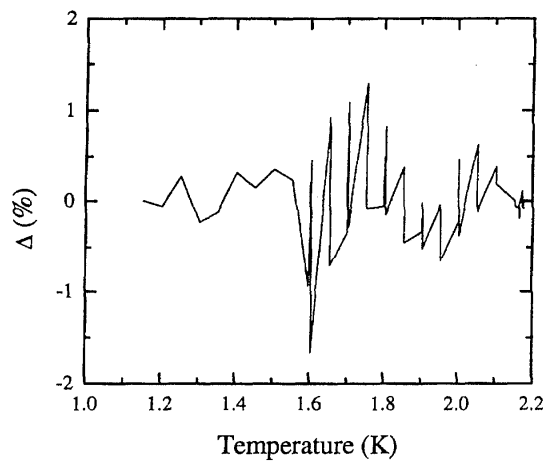

FIG. 8.2. The fractional deviation of values of the adopted database from the recommended values for the fountain pressure entropy of liquid ^{4}He expressed in percent.

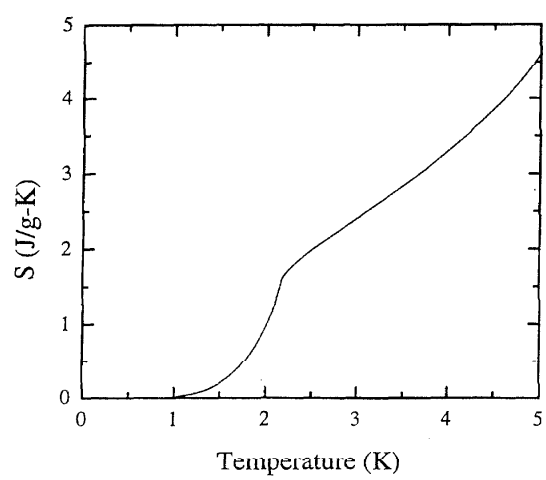

FIG. 8.3. The recommended values of the entropy of liquid ^{4}He obtained by integration of the heat capacity spline.

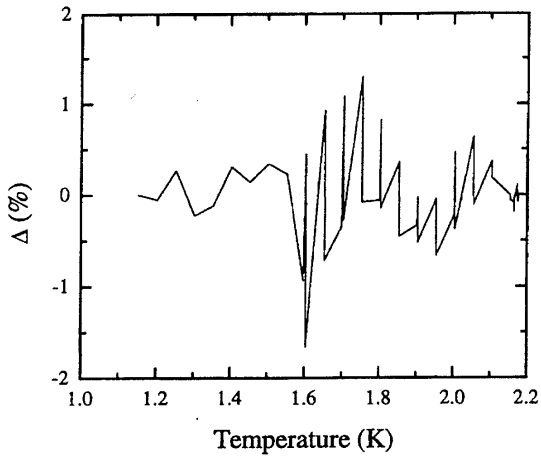

FIG. 8.4. The fractional deviation of values of the adopted database from the recommended values for the entropy integrated from the heat capacity spline.

TABLE 8.1. Adopted database for the fountain pressure entropy of liquid ^{4}He

T_{90} (K)	S (J/g·K)	Key	T_{90} (K)	S (J/g-K)	Key
1.153 029 27	3.950 000$E-2$	2	1.904 150 59	7.320 000$E-1$	2
1.203 266 85	5.150 000$E-2$	2	1.904 150 59	7.356 230$E-1$	4
1.253 202 67	6.650 000$E-2$	2	1.954 188 95	8.420 000$E-1$	2
1.303 323 99	8.550 000$E-2$	2	1.954 188 95	8.472 770$E-1$	4
1.353 442 88	1.0750 00$E-1$	2	2.003 032 41	9.663 170$E-1$	1
1.403 515 05	1.325 000$E-1$	2	2.004 233 58	9.630 000$E-1$	2
1.453 556 61	1.620 000$E-1$	2	2.004 233 58	9.711 100$E-1$	4
1.503 597 91	1.960 000$E-1$	2	2.054 293 41	1.105 000$E+0$	2
1.553 637 12	2.370 000$E-1$	2	2.054 293 41	1.113 220$E+0$	4
1.597 081 38	2.812 150$E-1$	1	2.103 897 49	1.271 440$E+0$	1
1.603 690 37	2.840 000$E-1$	2	2.104 398 92	1.275 540$E+0$	4
1.603 690 37	2.900 930$E-1$	4	2.154 562 75	1.468 060$E+0$	4
1.653 779 36	3.370 000$E-1$	2	2.154 663 05	1.469 430$E+0$	1
1.653 779 36	3.425 750$E-1$	4	2.154 691 10	1.469 300$E+0$	3
1.699 373 08	3.977 850$E-1$	1	2.164 290 53	1.512 190$E+0$	1
1.703 882 28	3.980 000$E-1$	2	2.164 290 53	1.513 560$E+0$	4
1.703 882 28	4.034 210$E-1$	4	2.164 292 67	1.511 520$E+0$	3
1.753 980 60	4.660 000$E-1$	2	2.169 703 54	1.540 000$E+0$	3
1.754 080 79	4.725 490$E-1$	4	2.169 703 98	1.540 690$E+0$	1
1.801 060 80	5.449 310$E-1$	1	2.172 900 00	1.557 640$E+0$	1
1.804 065 09	5.450 000$E-1$	2	2.172 903 43	1.557 990$E+0$	3
1.804 065 09	5.502 870$E-1$	4	2.174 600 00	1.570 540$E+0$	1
1.854 118 84	6.330 000$E-1$	2	2.175 100 00	1.571 870$E+0$	1
1.854 118 84	6.381 940$E-1$	4	2.175 300 00	1.573 910$E+0$	1
1.902 149 31	7.301 360$E-1$	1	2.176 800 00	1.578 970$E+0$	5

TABLE 8.2. Knots and coefficients for the spline fit of the fountain pressure entropy

Knots	Coefficients
$K(1)=1.153\,029$	$C(1)=3.949\,990E-2$
$K(2)=1.153\,029$	$C(2)=5.660\,103E-2$
$K(3)=1.153\,029$	$C(3)=9.933\,250E-2$
$K(4)=1.153\,029$	$C(4)=2.321\,544E-1$
$K(5)=1.402\,666$	$C(5)=5.183\,673E-1$
$K(6)=1.501\,863$	$C(6)=9.994\,261E-1$
$K(7)=1.801\,948$	$C(7)=1.506\,163E+0$
$K(8)=2.154\,592$	$C(8)=1.549\,628E+0$
$K(9)=2.164\,268$	$C(9)=1.575\,018E+0$
$K(10)=2.172\,874$	$C(10)=1.578\,977E+0$
$K(11)=2.176\,800$	
$K(12)=2.176\,800$	
$K(13)=2.176\,800$	
$K(14)=2.176\,800$	

TABLE 8.3. Recommended values of the fountain pressure entropy of liquid ^{4}He

T_{90} (K)	S (J/g·K)
1.20	5.059$E-2$
1.25	6.560$E-2$
1.30	8.397$E-2$
1.35	1.057$E-1$
1.40	1.310$E-1$
1.45	1.600$E-1$
1.50	1.940$E-1$
1.55	2.343$E-1$
1.60	2.815$E-1$
1.65	3.357$E-1$
1.70	3.972$E-1$
1.75	4.662$E-1$
1.80	5.429$E-1$
1.85	6.279$E-1$
1.90	7.233$E-1$
1.95	8.319$E-1$
2.00	9.561$E-1$
2.05	1.099$E+0$
2.10	1.262$E+0$
2.15	1.450$E+0$
2.1761	1.577$E+0$
2.1762	1.577$E+0$
2.1763	1.577$E+0$
2.1764	1.578$E+0$
2.1765	1.578$E+0$
2.1766	1.578$E+0$
2.1767	1.579$E+0$
2.1768	1.579$E+0$

TABLE 8.4. Knots and coefficients for the spline fit of entropy integrated from the recommended values of heat capacity

Knots	Coefficients
$K(1)=0.100\,00$	$C(1)=6.60E-6$
$K(2)=0.100\,00$	$C(2)=7.60E-6$
$K(3)=0.100\,00$	$C(3)=1.60E-5$
$K(4)=0.100\,00$	$C(4)=4.614\,13E-5$
$K(5)=0.111\,23$	$C(5)=1.840\,0E-4$
$K(6)=0.216\,25$	$C(6)=4.686\,75E-4$
$K(7)=0.276\,72$	$C(7)=0.001\,21$
$K(8)=0.448\,17$	$C(8)=0.002\,36$
$K(9)=0.563\,69$	$C(9)=0.005\,61$
$K(10)=0.719\,69$	$C(10)-0.019\,78$
$K(11)=0.853\,60$	$C(11)=0.083\,37$
$K(12)=1.001\,86$	$C(12)=0.358\,24$
$K(13)=1.401\,84$	$C(13)=0.768\,82$
$K(14)=1.729\,12$	$C(14)=1.178\,03$
$K(15)=1.998\,87$	$C(15)=1.386\,29$
$K(16)=2.077\,75$	$C(16)=1.536\,13$
$K(17)=2.153\,84$	$C(17)=1.582\,71$
$K(18)=2.176\,80$	$C(18)=1.624\,19$
$K(19)=2.176\,80$	$C(19)=1.722\,61$
$K(20)=2.176\,80$	$C(20)=1.918\,62$
$K(21)=2.211\,60$	$C(21)=2.535\,29$
$K(22)=2.348\,20$	$C(22)=3.166\,01$
$K(23)=2.690\,75$	$C(23)=4.103\,79$
$K(24)=4.382\,77$	$C(24)=4.440\,77$
$K(25)=4.763\,61$	$C(25)=4.615\,70$
$K(26)-K(29)$ $=5.000\,000$	

TABLE 8.5. Recommended values of entropy of liquid ^{4}He obtained by integration of the heat capacity spline

T_{90} (K)	S (J/g·K)	T_{90} (K)	S (J/g·K)
0.10	6.600$E-6$	2.25	1.720
0.15	2.298$E-5$	2.30	1.783
0.20	5.506$E-5$	2.35	1.838
0.25	1.074$E-4$	2.40	1.890
0.30	1.841$E-4$	2.45	1.939
0.35	2.898$E-4$	2.50	1.985
0.40	4.298$E-4$	2.55	2.030
0.45	6.093$E-4$	2.60	2.074
0.50	8.343$E-4$	2.65	2.116
0.55	1.113$E-3$	2.70	2.158
0.60	1.456$E-3$	2.75	2.200
0.65	1.904$E-3$	2.80	2.242
0.70	2.512$E-3$	2.85	2.284
0.75	3.339$E-3$	2.90	2.326
0.80	4.512$E-3$	2.95	2.367
0.85	6.206$E-3$	3.00	2.409
0.90	8.605$E-3$	3.05	2.451
0.95	1.194$E-2$	3.10	2.492
1.00	1.644$E-2$	3.15	2.534
1.05	2.236$E-2$	3.20	2.576
1.10	3.004$E-2$	3.25	2.619
1.15	3.982$E-2$	3.30	2.661
1.20	5.204$E-2$	3.35	2.704
1.25	6.706$E-2$	3.40	2.747
1.30	8.521$E-2$	3.45	2.790
1.35	1.068$E-1$	3.50	2.834
1.40	1.323$E-1$	3.55	2.878
1.45	1.620$E-1$	3.60	2.923
1.50	1.965$E-1$	3.70	3.014
1.55	2.364$E-1$	3.75	3.060
1.60	2.824$E-1$	3.80	3.107
1.65	3.350$E-1$	3.85	3.154
1.70	3.950$E-1$	3.90	3.203
1.75	4.630$E-1$	3.95	3.251
1.80	5.399$E-1$	4.00	3.301
1.85	6.270$E-1$	4.05	3.351
1.90	7.255$E-1$	4.10	3.402
1.95	8.367$E-1$	4.15	3.454
2.00	9.621$E-1$	4.20	3.507
2.05	1.103	4.25	3.561
2.10	1.263	4.30	3.616
2.1760	1.578	4.35	3.672
2.1762	1.579	4.40	3.728
2.1763	1.580	4.45	3.786
2.1764	1.580	4.50	3.846
2.1765	1.581	4.75	4.175
2.1766	1.581	4.80	4.250
2.1767	1.582	4.85	4.330
2.1768	1.583	4.90	4.416
2.1769	1.583	4.95	4.511
2.1770	1.583	5.00	4.616
2.20	1.643		

8.1. Chronological Bibliography for Entropy

[1] H. C. Kramers, J. D. Wasscher, and C. J. Gorter, Physica **18**, 329 (1952).
[2] G. R. Hercus and J. Wilks, Philos. Mag. **45**, 1163 (1954).
[3] O. V. Lounasmaa and E. Kojo, Physica **36**, 1 (1959).
[4] C. J. N. Van den Meijdenberg, K. W. Taconis, and R. De Bruyn Ouboter, Physica **27**, 197 (1961).
[5] J. Wiebes, Ph.D. thesis, Kammerlingh Onnes Laboratory, 1969 (unpublished).
[6] A. L. Singsaas and G. Ahlers, Phys. Rev. B **29**, 4951 (1983).
[7] A. L. Singsaas, Ph.D. thesis, University of California, 1984 (unpublished).

9. Surface Tension

Adopted Database

Author(s)	Key #	Method	Range (K)
Iino *et al.*	1	Surface waves	$0.35 \leq T \leq 5.199$
Magerlein & Sanders	2	Capillary rise	$-0.05 < t < 0.05$
			$t = 1 - T/T_\lambda$

Comments and Key to Authors

(1) Reference 17. Data are absolute and revise the value of surface tension at $T=0$ K to 354.4±0.5 mdyne/cm. Note that the temperature dependence of the surface tension agrees with earlier data of Ekhart *et al.*[16] almost exactly. Uncertainty is ±0.3 mdyne/cm.

(2) References 13 and 15. We have incorporated this data set by adjusting the absolute surface tension to fit the only two points overlapping Iino *et al.* Uncertainty is ±0.03%.

(3) On the original database, T_c was taken to be 5.199 K on the T_{58} scale. It has been corrected to 5.189 K. Upon conversion to ITS-90, $T_c = 5.1958$ K.

(4) According to Moldover[18] around $T_c = 5.189$ K (on the T_{58} scale), the surface tension can be expressed as

$$\sigma = 633(t)^{1.26}\ \mu\text{N/m},$$

where $t = (T_c - T)/T_c$.

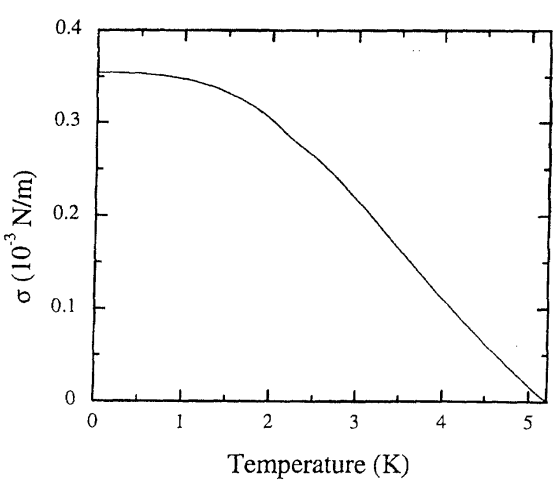

FIG. 9.1. The recommended values of the surface tension of liquid ^{4}He as a function of temperature at saturated vapor pressure.

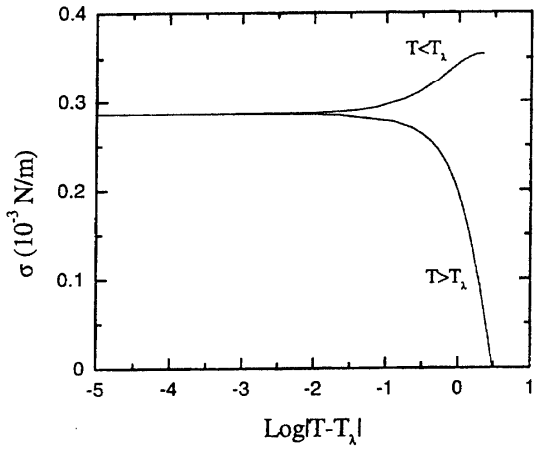

FIG. 9.2. Detail of the surface tension about the lambda transition.

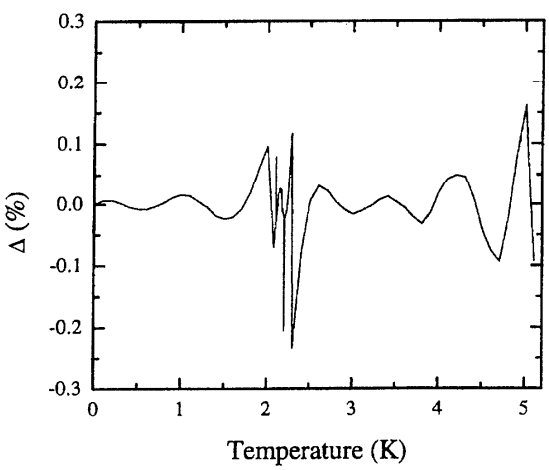

FIG. 9.3. The fractional deviation of the values of the adopted database from the recommended values of the surface tension of liquid ^{4}He expressed in percent.

TABLE 9.1. Adopted database for the surface tension of liquid ^{4}He

T_{90} (K)	σ (N/m)	Key	T_{90} (K)	σ (N/m)	Key
0.000 000	3.544 00$E-4$	1	2.177 827	2.860 72$E-4$	2
0.100 365	3.543 70$E-4$	1	2.178 246	2.860 40$E-4$	2
0.200 706	3.542 40$E-4$	1	2.178 666	2.860 05$E-4$	2
0.301 026	3.539 90$E-4$	1	2.179 057	2.859 73$E-4$	2
0.401 326	3.536 00$E-4$	1	2.179 459	2.859 45$E-4$	2
0.501 608	3.530 40$E-4$	1	2.180 023	2.858 96$E-4$	2
0.601 874	3.523 00$E-4$	1	2.180 618	2.858 49$E-4$	2
0.702 125	3.513 60$E-4$	1	2.180 915	2.858 32$E-4$	2
0.802 362	3.502 00$E-4$	1	2.181 194	2.858 04$E-4$	2
0.902 586	3.487 70$E-4$	1	2.181 721	2.857 47$E-4$	2
1.002 777	3.470 40$E-4$	1	2.182 695	2.856 74$E-4$	2
1.102 955	3.449 70$E-4$	1	2.183 738	2.855 94$E-4$	2
1.203 267	3.425 10$E-4$	1	2.185 086	2.854 87$E-4$	2
1.303 324	3.395 90$E-4$	1	2.186 419	2.853 83$E-4$	2
1.403 515	3.361 70$E-4$	1	2.187 845	2.852 75$E-4$	2
1.503 598	3.321 60$E-4$	1	2.189 845	2.851 19$E-4$	2
1.603 690	3.275 10$E-4$	1	2.191 956	2.849 54$E-4$	2
1.703 882	3.221 60$E-4$	1	2.194 914	2.847 29$E-4$	2
1.804 065	3.160 60$E-4$	1	2.197 996	2.844 85$E-4$	2
1.904 151	3.091 60$E-4$	1	2.201 022	2.842 51$E-4$	2
2.004 234	3.014 70$E-4$	1	2.204 070	2.840 13$E-4$	2
2.079 869	2.956 24$E-4$	2	2.204 624	2.845 00$E-4$	1
2.082 083	2.954 31$E-4$	2	2.207 112	2.837 84$E-4$	2
2.087 213	2.949 47$E-4$	2	2.212 391	2.833 75$E-4$	2
2.092 275	2.944 69$E-4$	2	2.217 529	2.829 80$E-4$	2
2.097 406	2.939 74$E-4$	2	2.222 839	2.825 71$E-4$	2
2.102 724	2.934 66$E-4$	2	2.228 186	2.821 54$E-4$	2
2.104 399	2.929 80$E-4$	1	2.233 636	2.817 37$E-4$	2
2.107 993	2.929 60$E-4$	2	2.239 167	2.813 09$E-4$	2
2.113 455	2.924 30$E-4$	2	2.246 959	2.807 07$E-4$	2
2.118 900	2.918 94$E-4$	2	2.247 050	2.807 04$E-4$	2
2.124 429	2.913 42$E-4$	2	2.254 904	2.800 95$E-4$	2
2.128 622	2.909 27$E-4$	2	2.263 157	2.794 40$E-4$	2
2.132 810	2.905 04$E-4$	2	2.271 323	2.788 02$E-4$	2
2.136 982	2.900 92$E-4$	2	2.279 781	2.781 43$E-4$	2
2.141 155	2.896 76$E-4$	2	2.287 614	2.775 14$E-4$	2
2.143 769	2.894 18$E-4$	2	2.287 627	2.775 16$E-4$	2
2.146 368	2.891 54$E-4$	2	2.304 706	2.772 90$E-4$	1
2.149 038	2.888 83$E-4$	2	2.404 936	2.697 80$E-4$	1
2.151 739	2.886 15$E-4$	2	2.505 156	2.618 70$E-4$	1
2.154 326	2.883 58$E-4$	2	2.605 385	2.535 00$E-4$	1
2.156 094	2.881 80$E-4$	2	2.705 694	2.446 60$E-4$	1
2.157 890	2.880 01$E-4$	2	2.805 957	2.353 60$E-4$	1
2.159 730	2.878 17$E-4$	2	2.906 140	2.256 30$E-4$	1
2.161 561	2.876 33$E-4$	2	3.006 300	2.155 30$E-4$	1
2.163 420	2.874 44$E-4$	2	3.106 442	2.051 10$E-4$	1
2.165 243	2.872 63$E-4$	2	3.206 578	1.944 60$E-4$	1
2.166 176	2.871 63$E-4$	2	3.306 723	1.836 40$E-4$	1
2.167 094	2.870 73$E-4$	2	3.406 851	1.727 30$E-4$	1
2.167 131	2.870 69$E-4$	2	3.506 953	1.618 10$E-4$	1
2.168 069	2.869 77$E-4$	2	3.607 030	1.509 20$E-4$	1
2.168 971	2.868 87$E-4$	2	3.707 067	1.401 30$E-4$	1
2.169 895	2.867 92$E-4$	2	3.807 062	1.294 90$E-4$	1
2.170 832	2.866 96$E-4$	2	3.907 038	1.190 20$E-4$	1
2.171 979	2.865 96$E-4$	2	4.007 034	1.087 40$E-4$	1
2.172 908	2.865 04$E-4$	2	4.107 060	9.867 00$E-5$	1
2.173 388	2.864 56$E-4$	2	4.207 083	8.880 00$E-5$	1
2.173 840	2.864 15$E-4$	2	4.306 979	7.912 00$E-5$	1
2.174 321	2.863 67$E-4$	2	4.406 982	6.961 00$E-5$	1
2.174 679	2.863 34$E-4$	2	4.507 062	6.024 00$E-5$	1
2.175 047	2.863 00$E-4$	2	4.607 173	5.098 00$E-5$	1
2.175 435	2.862 76$E-4$	2	4.707 276	4.183 00$E-5$	1
2.175 795	2.862 40$E-4$	2	4.807 336	3.276 00$E-5$	1
2.176 193	2.862 10$E-4$	2	4.907 327	2.380 00$E-5$	1
2.176 591	2.861 79$E-4$	2	5.007 230	1.505 00$E-5$	1
2.177 009	2.861 41$E-4$	2	5.107 032	6.750 00$E-6$	1
2.177 009	2.861 37$E-4$	2	5.195 767	0	
2.177 399	2.861 07$E-4$	2			

TABLE 9.2. Knots and coefficients for the spline fit of the surface tension of liquid ^{4}He

Knots	Coefficients
$K(1)=0.00\,0000$	$C(1)=3.543\,998E-4$
$K(2)=0.00\,0000$	$C(2)=3.545\,090E-4$
$K(3)=0.00\,0000$	$C(3)=3.524\,383E-4$
$K(4)=0.00\,0000$	$C(4)=3.366\,694E-4$
$K(5)=6.482\,413E-4$	$C(5)=2.921\,292E-4$
$K(6)=2.004\,234$	$C(6)=2.860\,852E-4$
$K(7)=2.176\,800$	$C(7)=2.796\,835E-4$
$K(8)=2.176\,800$	$C(8)=2.747\,804E-4$
$K(9)=2.178\,377$	$C(9)=2.346\,234E-4$
$K(10)=2.41\,2388$	$C(10)=1.357\,632E-4$
$K(11)=2.41\,2896$	$C(11)=5.361\,123E-5$
$K(12)=3.80\,7062$	$C(12)=1.263\,761E-5$
$K(13)=4.86\,8895$	$C(13)=3.795\,854E-6$
$K(14)=5.03\,5567$	$C(14)=5.599\,775E-12$
$K(15)=5.19\,5767$	
$K(16)=5.19\,5767$	
$K(17)=5.19\,5767$	
$K(18)=5.19\,5767$	

TABLE 9.3. Recommended values of the surface tension of liquid ^{4}He

T_{90} (K)	σ (N/m)	T_{90} (K)	σ (N/m)
0.1000	$3.544E-4$	2.3000	$2.770E-4$
0.1500	$3.543E-4$	2.3500	$2.735E-4$
0.2000	$3.543E-4$	2.4000	$2.699E-4$
0.2500	$3.541E-4$	2.4500	$2.662E-4$
0.3000	$3.540E-4$	2.5000	$2.623E-4$
0.3500	$3.538E-4$	2.5500	$2.582E-4$
0.4000	$3.536E-4$	2.6000	$2.540E-4$
0.4500	$3.533E-4$	2.6500	$2.497E-4$
0.5000	$3.530E-4$	2.7000	$2.452E-4$
0.5500	$3.527E-4$	2.7500	$2.406E-4$
0.6000	$3.523E-4$	2.8000	$2.359E-4$
0.6500	$3.519E-4$	2.8500	$2.311E-4$
0.7000	$3.514E-4$	2.9000	$2.262E-4$
0.7500	$3.508E-4$	2.9500	$2.212E-4$
0.8000	$3.502E-4$	3.0000	$2.161E-4$
0.8500	$3.496E-4$	3.0500	$2.110E-4$
0.9000	$3.488E-4$	3.1000	$2.058E-4$
0.9500	$3.480E-4$	3.2000	$1.952E-4$
1.0000	$3.471E-4$	3.3000	$1.844E-4$
1.1000	$3.451E-4$	3.4000	$1.735E-4$
1.2000	$3.426E-4$	3.5000	$1.626E-4$
1.3000	$3.397E-4$	3.6000	$1.517E-4$
1.4000	$3.362E-4$	3.6500	$1.463E-4$
1.5000	$3.322E-4$	3.7000	$1.409E-4$
1.5500	$3.300E-4$	3.7500	$1.355E-4$
1.6000	$3.276E-4$	3.8000	$1.302E-4$
1.6500	$3.251E-4$	3.8500	$1.249E-4$
1.7000	$3.224E-4$	3.9000	$1.197E-4$
1.7500	$3.195E-4$	3.9500	$1.146E-4$
1.8000	$3.164E-4$	4.0000	$1.095E-4$
1.8500	$3.131E-4$	4.0500	$1.044E-4$
1.9000	$3.096E-4$	4.1000	$9.942E-5$
1.9500	$3.060E-4$	4.1500	$9.445E-5$
2.0000	$3.021E-4$	4.2000	$8.954E-5$
2.0500	$2.980E-4$	4.2500	$8.466E-5$
2.1000	$2.936E-4$	4.3000	$7.983E-5$
2.1500	$2.889E-4$	4.3500	$7.503E-5$
2.1760	$2.862E-4$	4.4000	$7.028E-5$
2.1761	$2.862E-4$	4.4500	$6.556E-5$
2.1762	$2.862E-4$	4.5000	$6.087E-5$
2.1763	$2.862E-4$	4.5500	$5.622E-5$
2.1764	$2.862E-4$	4.6000	$5.160E-5$
2.1765	$2.862E-4$	4.6500	$4.701E-5$
2.1766	$2.862E-4$	4.7000	$4.245E-5$
2.1767	$2.862E-4$	4.7500	$3.792E-5$
2.1768	$2.861E-4$	4.8000	$3.341E-5$
2.1769	$2.861E-4$	4.8500	$2.893E-5$
2.1770	$2.861E-4$	4.9000	$2.447E-5$
2.2000	$2.843E-4$	4.9500	$2.005E-5$
2.2500	$2.805E-4$	5.0000	$1.570E-5$

9.1. Chronological Bibliography for Surface Tension

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10. Ion Mobilities

Adopted Database for μ_+

Author(s)	Key #	Method	Range (K)
Barenghi *et al.*	1	Ion pool	$0.2388 \leq T \leq 0.8462$
Schwarz	2	Drift velocity	$0.371 \leq T \leq 5.18$

Adopted Database for μ_-

Author(s)	Key #	Method	Range (K)
Barenghi *et al.*	1	Ion pool	$0.0528 \leq T \leq 0.9632$
Schwarz	2	Drift velocity	$0.247 \leq T \leq 5.18$

Comments and Key to Authors

(1) Reference 18. Uncertainties: for negative ions, >270 mK, and for positive ions, >370 mK, results are in good agreement with Ref. 14. Below these temperatures the mobilities depend on the experimental situation and the paper should be consulted for details.
(2) Reference 14. Uncertainty better than 2%.
(3) The splines are fit to the $\log_{10}$ of the mobility.
(4) To convert mobilities from $m^2/V \cdot s$ to $cm^2/V \cdot s$, multiply by 10^4.

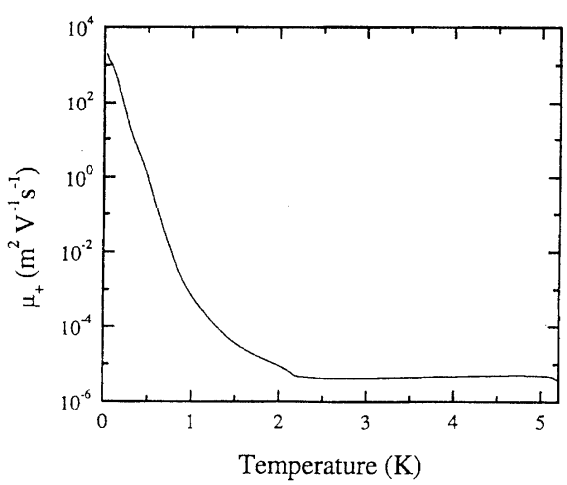

FIG. 10.1. Positive ion mobility as a function of temperature along the saturated vapor pressure line.

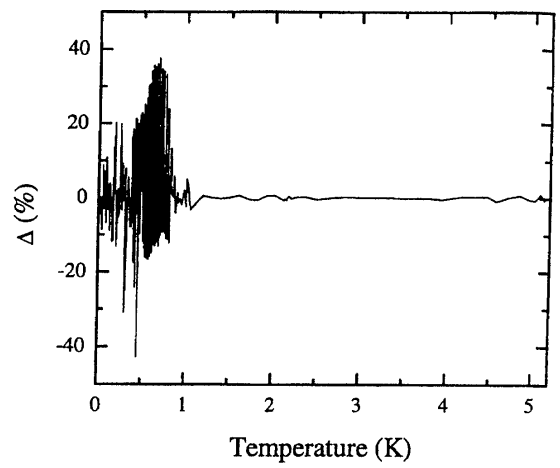

FIG. 10.2. Deviation of the database from the spline fit to $\log_{10} \mu_-$. (Note: The large deviations at low temperatures reflect a divergence in magnitudes reported in Refs. 14 and 18.)

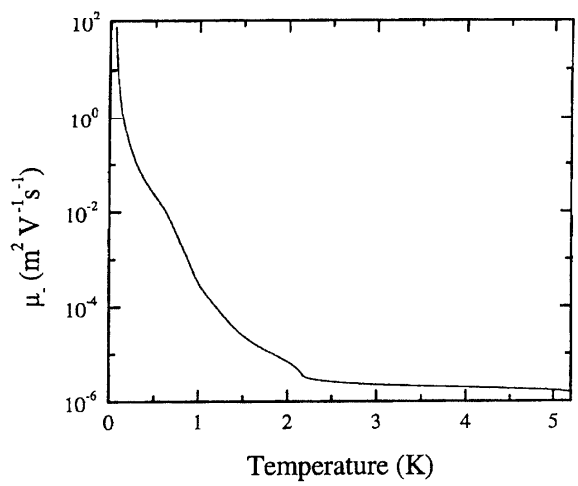

FIG. 10.3. The recommended values of the negative ion mobility as a function of temperature at saturated vapor pressure.

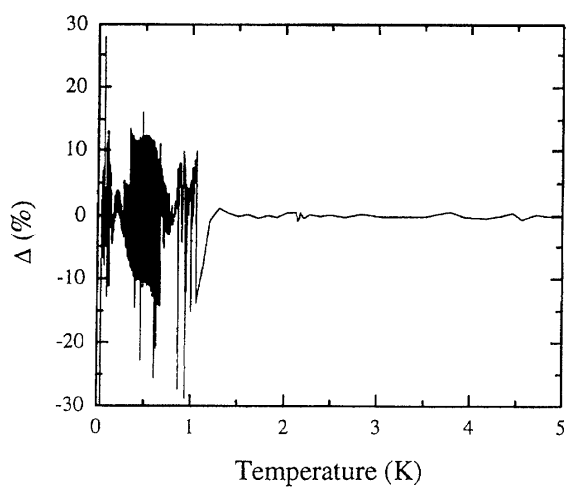

FIG. 10.4. The fractional deviation of values of the adopted database from the recommended values for the $\log_{10} \mu_-$ in liquid ^{4}He expressed in percent.

TABLE 10.1. Adopted database for the positive ion mobility

T_{90} (K)	μ_+ (m²/V·s)	Key	T_{90} (K)	μ_+ (m²/V·s)	Key	T_{90} (K)	μ_+ (m²/V·s)	Key	T_{90} (K)	μ_+ (m²/V·s)	Key
0.0255	2.110 00E+3	1	0.3953	3.800 00E+0	2	0.6550	4.450 00E−2	2	0.9788	8.090 00E+0	1
0.0421	1.730 00E+3	1	0.4020	4.970 00E+0	1	0.6599	6.240 00E−2	1	0.9988	7.150 00E−4	2
0.0459	1.380 00E+3	1	0.4063	3.360 00E+0	2	0.6648	5.830 00E−2	1	1.0026	6.440 00E−4	1
0.0599	1.320 00E+3	1	0.4108	4.010 00E+0	1	0.6781	2.970 00E−2	2	1.0218	5.600 00E−4	1
0.0656	1.220 00E+3	1	0.4124	2.950 00E+0	2	0.6784	4.590 00E−2	1	1.0509	4.800 00E−4	2
0.0736	1.050 00E+3	1	0.4214	2.630 00E+0	2	0.6788	3.960 00E−2	1	1.1350	2.570 00E−4	2
0.0836	1.100 00E+3	1	0.4216	4.180 00E+0	1	0.6845	4.230 00E−2	1	1.1991	1.660 00E−4	2
0.0850	9.130 00E+2	1	0.4304	2.380 00E+0	2	0.6910	3.220 00E−2	1	1.3003	9.160 00E−5	2
0.0922	9.450 00E+2	1	0.4326	2.860 00E+0	1	0.6947	3.500 00E−2	1	1.4015	5.480 00E−5	2
0.1037	8.460 00E+2	1	0.4404	2.050 00E+0	2	0.6972	3.360 00E−2	1	1.5036	3.530 00E−5	2
0.1052	7.750 00E+2	1	0.4502	2.170 00E+0	1	0.7031	2.030 00E−2	2	1.6057	2.440 00E−5	2
0.1103	7.700 00E+2	1	0.4515	1.750 00E+0	2	0.7038	2.970 00E−2	1	1.7119	1.800 00E−5	2
0.1250	5.960 00E+2	1	0.4525	3.510 00E+0	1	0.7108	2.700 00E−2	1	1.8161	1.390 00E−5	2
0.1259	6.510 00E+2	1	0.4525	3.610 00E+0	1	0.7154	2.180 00E−2	1	1.9172	1.090 00E−5	2
0.1259	5.710 00E+2	1	0.4525	2.830 00E+0	1	0.7213	2.250 00E−2	1	2.0203	8.420 00E−6	2
0.1479	4.210 00E+2	1	0.4625	1.460 00E+0	2	0.7292	1.380 00E−2	2	2.1215	6.220 00E−6	2
0.1505	4.100 00E+2	1	0.4731	1.680 00E+0	1	0.7318	1.650 00E−2	1	2.1365	5.860 00E−6	2
0.1560	3.950 00E+2	1	0.4735	1.230 00E+0	2	0.7330	1.930 00E−2	1	2.1516	5.550 00E−6	2
0.1636	2.850 00E+2	1	0.4856	1.010 00E+0	2	0.7402	1.690 00E−2	1	2.1676	5.170 00E−6	2
0.1688	3.060 00E+2	1	0.4893	1.150 00E+0	1	0.7474	1.540 00E−2	1	2.1738	5.040 00E−6	2
0.1740	2.140 00E+2	1	0.4996	8.050 00E−1	2	0.7505	1.250 00E−2	1	2.1788	4.960 00E−6	2
0.1790	2.070 00E+2	1	0.5073	9.810 00E−1	1	0.7591	1.310 00E−2	1	2.2046	4.720 00E−6	2
0.1865	1.560 00E+2	1	0.5116	6.420 00E−1	2	0.7593	8.860 00E−3	2	2.2647	4.520 00E−6	2
0.1865	1.820 00E−+2	1	0.5253	7.320 00E−1	1	0.7633	1.220 00E−2	1	2.3749	4.300 00E−6	2
0.1889	1.760 00E+2	1	0.5277	4.860 00E−1	2	0.7637	1.190 00E−2	1	2.4951	4.200 00E−6	2
0.1988	1.430 00E+2	1	0.5310	6.810 00E−1	1	0.7715	9.200 00E−3	1	2.6455	4.130 00E−6	2
0.2059	1.420 00E+2	1	0.5417	3.730 00E−1	2	0.7731	1.070 00E−2	1	2.8260	4.120 00E−6	2
0.2181	9.220 00E+1	1	0.5470	5.020 00E−1	1	0.7833	8.980 00E−3	1	3.0263	4.160 00E−6	2
0.2227	9.020 00E+1	1	0.5501	4.860 00E−1	1	0.7877	8.520 00E=3	1	3.2567	4.230 00E−6	2
0.2390	6.170 00E+1	1	0.5568	2.760 00E−1	2	0.7883	6.050 00E−3	2	3.5070	4.350 00E−2	2
0.2454	4.350 00E+1	1	0.5679	3.450 00E−1	1	0.7928	6.780 00E−3	1	3.7771	4.490 00E−2	2
0.2627	3.740 00E+1	1	0.5718	3.280 00E−1	1	0.8070	6.440 00E−3	1	3.9370	4.600 00E−2	2
0.2687	2.930 00E+1	1	0.5748	1.980 00E−1	2	0.8107	5.290 00E−3	1	4.1671	4.700 00E−2	2
0.2816	2.500 00E+1	1	0.5883	2.040 00E−1	1	0.8224	4.010 00E−3	2	4.3370	4.800 00E−2	2
0.3012	1.700 00E+1	1	0.5917	2.190 00E−1	1	0.8324	3.930 00E−3	1	4.4570	4.870 00E−2	2
0.3039	1.530 00E+1	1	0.5918	2.250 00E−1	1	0.8543	2.980 00E−3	1	4.5571	4.980 00E−2	2
0.3063	2.290 00E+1	1	0.5918	1.410 00E−1	2	0.8615	2.580 00E−3	2	4.7073	4.970 00E−2	2
0.3313	1.070 00E+1	1	0.6115	1.550 00E−1	1	0.8761	2.390 00E−3	1	4.8473	4.920 00E−6	2
0.3318	1.000 00E+1	1	0.6117	1.320 00E−1	1	0.8821	2.200 00E−3	1	4.9673	4.890 00E−6	2
0.3522	8.030 00E+0	1	0.6119	9.650 00E−2	2	0.8914	1.960 00E−3	1	5.0172	4.830 00E−6	2
0.3569	8.020 00E+0	1	0.6121	1.490 00E−2	1	0.8996	1.740 00E−3	2	5.0571	4.730 00E−6	2
0.3702	6.500 00E+0	1	0.6218	1.250 00E−2	1	0.9131	1.550 00E−3	1	5.0871	4.620 00E−6	2
0.3703	6.710 00E+0	1	0.6330	6.600 00E−2	2	0.9344	1.240 00E−3	1	5.1270	4.470 00E−6	2
0.3722	5.190 00E+0	2	0.6340	8.760 00E−2	1	0.9477	1.110 00E−3	2	5.1669	4.010 00E−6	2
0.3793	4.720 00E+0	2	0.6371	9.540 00E−2	1	0.9541	1.040 00E−3	1	5.1868	3.760 00E−6	2
0.3873	4.180 00E+0	2	0.6440	8.270 00E−2	1						
0.3917	5.020 00E+0	1	0.6508	7.480 00E−2	1						

TABLE 10.2. Knots and coefficients for the spline fit of $\log_{10} \mu_+$

Knots	Coefficients	Knots	Coefficients
$K(1) = 2.549\,509E-2$	$C(1) = 7.324\,336E-4$	$K(13) = 1.791\,246E+0$	$C(13) = -1.321\,435E-4$
$K(2) = 2.549\,509E-2$	$C(2) = 7.127\,592E-4$	$K(14) = 2.152\,366E+0$	$C(14) = -1.381\,473E-4$
$K(3) = 2.549\,509E-2$	$C(3) = 6.936\,548E-4$	$K(15) = 2.203\,504E+0$	$C(15) = -1.396\,425E-4$
$K(4) = 2.549\,509E-2$	$C(4) = 5.173\,557E-4$	$K(16) = 2.205\,589E+0$	$C(16) = -1.325\,568E-4$
$K(5) = 8.989\,802E-2$	$C(5) = 4.285\,367E-4$	$K(17) = 2.727\,358E+0$	$C(17) = -1.292\,173E-4$
$K(6) = 2.949\,869E-1$	$C(6) = 2.823\,892E-4$	$K(18) = 4.425\,492E+0$	$C(18) = -1.335\,656E-4$
$K(7) = 4.513\,886E-1$	$C(7) = 2.125\,491E-4$	$K(19) = 5.043\,800E+0$	$C(19) = -1.409\,701E-4$
$K(8) = 7.045\,304E-1$	$C(8) = 1.285\,926E-4$	$K(20) = 5.154\,401E+0$	$C(20) = -1.424\,813E-4$
$K(9) = 7.577\,476E-1$	$C(9) = 5.123\,347E-5$	$K(21) = 5.186\,798E+0$	
$K(10) = 7.944\,735E-1$	$C(10) = -3.728\,677E-5$	$K(22) = 5.186\,798E+0$	
$K(11) = 1.062\,780E+0$	$C(11) = -8.564\,176E-5$	$K(23) = 5.186\,798E+0$	
$K(12) = 1.394\,506E+0$	$C(12) = -1.067\,754E-4$	$K(24) = 5.186\,798E+0$	

TABLE 10.3. Adopted database for the negative ion mobility

T_{90} (K)	μ_- (m^2/V·s)	Key	T_{90} (K)	μ_- (m^2/V·s)	Key	T_{90} (K)	μ_- (m^2/V·s)	Key	T_{90} (K)	μ_- (m^2/V·s)	Key
0.0351	1.080 00E+3	1	0.1036	4.720 00E+0	1	0.2628	1.630 00E−1	1	0.3933	5.270 00E−2	1
0.0405	6.470 00E+2	1	0.1044	4.330 00E+0	1	0.2631	1.680 00E−1	1	0.3960	4.200 00E−2	1
0.0449	3.760 00E+2	1	0.1048	3.750 00E+0	1	0.2675	1.550 00E−1	1	0.3999	5.320 00E−2	1
0.0496	2.490 00E+2	1	0.1048	3.980 00E+0	1	0.2703	1.550 00E−1	1	0.4015	4.040 00E−2	1
0.0531	1.690 00E+2	1	0.1064	3.420 00E+0	1	0.2732	1.450 00E−1	1	0.4063	4.230 00E−2	2
0.0541	1.340 00E+2	1	0.1083	3.410 00E+0	1	0.2749	1.350 00E−1	2	0.4065	4.820 00E−2	1
0.0541	1.460 00E+2	1	0.1084	3.720 00E+0	1	0.2757	1.460 00E−1	1	0.4087	3.840 00E−2	1
0.0547	1.410 00E+2	1	0.1102	3.490 00E+0	1	0.2774	1.430 00E−1	1	0.4133	4.610 00E−2	1
0.0547	1.290 00E+2	1	0.1125	3.260 00E+0	1	0.2782	1.380 00E−1	1	0.4178	3.600 00E−2	1
0.0555	1.180 00E+2	1	0.1125	3.220 00E+0	1	0.2811	1.330 00E−1	1	0.4187	4.430 00E−2	1
0.0560	1.160 00E+2	1	0.1142	2.690 00E+0	1	0.2812	1.370 00E−1	1	0.4214	3.720 00E−2	2
0.0575	9.480 00E+1	1	0.1151	2.670 00E+0	1	0.2840	1.230 00E−1	2	0.4244	4.270 00E−2	1
0.0578	8.710 00E+1	1	0.1164	2.870 00E+0	1	0.2879	1.240 00E−1	1	0.4263	3.400 00E−2	1
0.0596	7.910 00E+1	1	0.1196	2.310 00E+0	1	0.2896	1.260 00E−1	1	0.4320	4.050 00E−2	1
0.0596	7.720 00E+1	1	0.1206	2.620 00E+0	1	0.2898	1.260 00E−1	1	0.4321	3.280 00E−2	1
0.0596	8.110 00E+1	1	0.1207	2.550 00E+0	1	0.2914	1.200 00E−1	1	0.4380	3.900 00E−2	1
0.0597	7.720 00E+1	1	0.1209	2.140 00E+0	1	0.2920	1.120 00E−1	2	0.4381	3.130 00E−2	1
0.0597	7.800 00E+1	1	0.1247	1.960 00E+0	1	0.2937	1.210 00E−1	1	0.4404	3.270 00E−2	2
0.0597	7.830 00E+1	1	0.1295	1.620 00E+0	1	0.2965	1.180 00E−1	1	0.4465	3.700 00E−2	1
0.0601	7.410 00E+1	1	0.1323	1.590 00E+0	1	0.2972	1.130 00E−1	1	0.4468	2.970 00E−2	1
0.0601	7.020 00E+1	1	0.1387	1.250 00E+0	1	0.2990	1.150 00E−1	1	0.4533	3.540 00E−2	1
0.0601	7.180 00E+1	1	0.1395	1.290 00E+0	1	0.3010	1.020 00E−1	2	0.4542	2.830 00E−2	1
0.0601	6.870 00E+1	1	0.1411	1.200 00E+0	1	0.3017	1.090 00E−1	1	0.4598	3.950 00E−2	1
0.0616	6.010 00E+1	1	0.1451	1.090 00E+0	1	0.3029	9.990 00E−2	1	0.4617	2.690 00E−2	1
0.0621	6.220 00E+1	1	0.1464	1.090 00E+0	1	0.3072	1.030 00E−1	1	0.4625	2.840 00E−2	2
0.0622	5.990 00E+1	1	0.1503	9.610 00E−1	1	0.3076	1.060 00E−1	1	0.4653	3.290 00E−2	1
0.0632	5.590 00E+1	1	0.1510	9.780 00E−1	1	0.3111	9.250 00E−2	2	0.4694	2.570 00E−2	1
0.0644	4.760 00E−1	1	0.1583	8.070 00E−1	1	0.3118	1.020 00E−1	1	0.4714	3.170 00E−2	1
0.0655	4.550 00E+1	1	0.1600	8.060 00E−1	1	0.3132	9.730 00E−2	1	0.4717	2.440 00E−2	1
0.0657	3.370 00E+1	1	0.1644	7.100 00E−1	1	0.3182	9.620 00E−2	1	0.4794	3.010 00E−2	1
0.0663	4.150 00E+1	1	0.1673	6.930 00E−1	1	0.3202	9.140 00E−2	1	0.4805	2.390 00E−2	1
0.0663	4.030 00E+1	1	0.1718	6.140 00E−1	1	0.3211	8.350 00E−2	2	0.4845	2.950 00E−2	1
0.0685	3.390 00E+1	1	0.1730	6.190 00E−1	1	0.3231	9.220 00E−2	1	0.4856	2.450 00E−2	2
0.0686	3.630 00E+1	1	0.1776	5.480 00E−1	1	0.3248	8.780 00E−2	1	0.4908	2.810 00E−2	1
0.0688	3.390 00E+1	1	0.1796	5.440 00E−1	1	0.3292	8.470 00E−2	1	0.4921	2.230 00E−2	1
0.0690	3.270 00E+1	1	0.1821	5.080 00E−1	1	0.3308	8.600 00E−2	1	0.4951	2.190 00E−2	1
0.0690	3.390 00E+1	1	0.1860	4.920 00E−1	1	0.3331	7.500 00E−2	2	0.4986	2.680 00E−2	1
0.0698	3.110 00E+1	1	0.1861	4.750 00E−1	1	0.3362	7.960 00E−2	1	0.4996	2.250 00E−2	2
0.0704	2.910 00E+1	1	0.1919	4.300 00E−1	1	0.3377	6.600 00E−2	1	0.5016	2.640 00E−2	1
0.0704	2.920 00E+1	1	0.1920	4.440 00E−1	1	0.3411	6.420 00E−2	1	0.5062	2.570 00E−2	1
0.0704	2.850 00E+1	1	0.1957	4.040 00E−1	1	0.3424	7.820 00E−2	1	0.5070	2.030 00E−2	1
0.0745	2.090 00E+1	1	0.1976	4.040 00E−1	1	0.3425	7.550 00E−2	1	0.5111	1.990 00E−2	1
0.0782	1.660 00E+1	1	0.2022	3.760 00E−1	1	0.3446	6.260 00E−2	1	0.5116	2.090 00E−2	2
0.0782	1.620 00E+1	1	0.2036	3.570 00E−1	1	0.3452	6.700 00E−2	2	0.5126	2.470 00E−2	1
0.0820	1.260 00E+1	1	0.2078	3.340 00E−1	1	0.3463	7.320 00E−2	1	0.5126	2.200 00E−2	1
0.0843	1.110 00E+1	1	0.2096	3.360 00E−1	1	0.3483	7.440 00E−2	1	0.5162	2.060 00E−2	1
0.0873	9.750 00E+0	1	0.2152	3.090 00E−1	1	0.3494	6.000 00E−2	1	0.5219	1.860 00E−2	1
0.0884	8.790 00E+0	1	0.2161	2.970 00E−1	1	0.3503	7.100 00E−2	1	0.5238	2.320 00E−2	1
0.0884	8.890 00E+0	1	0.2189	2.940 00E−1	1	0.3529	7.160 00E−2	1	0.5257	1.820 00E−2	1
0.0903	7.100 00E+0	1	0.2222	2.710 00E−1	1	0.3560	5.690 00E−2	1	0.5277	1.900 00E−2	2
0.0915	7.490 00E+0	1	0.2262	2.650 00E−1	1	0.3582	6.050 00E−2	2	0.5322	2.200 00E−2	1
0.0921	7.300 00E+0	1	0.2270	2.540 00E−1	1	0.3589	6.830 00E−2	1	0.5335	1.730 00E−2	1
0.0942	6.590 00E+0	1	0.2330	2.420 00E−1	1	0.3634	6.600 00E−2	1	0.5400	2.100 00E−2	1
0.0946	5.740 00E+0	1	0.2334	2.340 00E−1	1	0.3640	5.330 00E−2	1	0.5400	1.720 00E−2	1
0.0986	5.440 00E+0	1	0.2375	2.220 00E−1	1	0.3711	5.050 00E−2	1	0.5400	1.670 00E−2	1
0.0987	4.680 00E+0	1	0.2382	2.260 00E−1	1	0.3722	5.400 00E−2	2	0.5417	1.740 00E−2	2
0.0998	4.910 00E+0	1	0.2402	2.140 00E−1	1	0.3741	6.110 00E−2	1	0.5479	1.590 00E−2	1
0.1002	5.700 00E+0	1	0.2438	2.120 00E−1	1	0.3762	4.860 00E−2	1	0.5539	1.930 00E−2	1
0.1014	5.000 00E+0	1	0.2465	1.980 00E−1	1	0.3808	5.790 00E−2	1	0.5539	1.720 00E−2	1
0.1015	4.510 00E+0	1	0.2507	1.880 00E−1	1	0.3857	4.530 00E−2	1	0.5568	1.560 00E−2	2
0.1019	4.810 00E+0	1	0.2512	1.930 00E−1	1	0.3873	5.560 00E−2	1	0.5587	1.490 00E−2	1
0.1019	4.760 00E+0	1	0.2571	1.800 00E−1	1	0.3873	4.800 00E−2	2	0.5635	1.450 00E−2	1
0.1019	4.790 00E+0	1	0.2575	1.740 00E−1	1	0.3907	4.370 00E−2	1	0.5655	1.810 00E−2	1

TABLE 10.3. Adopted database for the negative ion mobility—Continued

T_{90} (K)	μ_- (m²/V·s)	Key	T_{90} (K)	μ_- (m²/V·s)	Key	T_{90} (K)	μ_- (m²/V·s)	Key	T_{90} (K)	μ_- (m²/V·s)	Key
0.5655	1.720 00E−2	1	0.6763	6.500 00E−3	1	0.8054	2.060 00E−3	1	1.4015	3.600 00E−5	2
0.5710	1.380 00E−2	1	0.6766	6.930 00E−3	1	0.8132	1.900 00E−3	1	1.5036	2.420 00E−5	2
0.5726	1.720 00E−2	1	0.6781	6.500 00E−3	2	0.8224	1.710 00E−3	2	1.6057	1.740 00E−5	2
0.5748	1.410 00E−2	2	0.6824	6.180 00E−3	1	0.8229	1.740 00E−3	1	1.7119	1.330 00E−5	2
0.5789	1.310 00E−2	1	0.6831	6.550 00E−3	1	0.8357	1.510 00E−3	1	1.8161	1.050 00E−5	2
0.5820	1.640 00E−2	1	0.6889	5.860 00E−3	1	0.8417	1.440 00E−3	1	1.9172	8.420 00E−6	2
0.5836	1.270 00E−2	1	0.6932	5.990 00E−3	1	0.8421	1.370 00E−3	1	2.0203	6.470 00E−6	2
0.5912	1.210 00E−2	1	0.6958	5.540 00E−3	1	0.8450	1.320 00E−3	1	2.1215	4.610 00E−6	2
0.5918	1.250 00E−2	2	0.7023	5.730 00E−3	1	0.8526	1.220 00E−3	1	2.1365	4.370 00E−6	2
0.5936	1.510 00E−2	1	0.7031	5.250 00E−3	2	0.8606	1.660 00E−3	1	2.1516	4.050 00E−6	2
0.6048	1.110 00E−2	1	0.7038	5.140 00E−3	1	0.8615	1.190 00E−3	2	2.1676	3.680 00E−6	2
0.6052	1.400 00E−2	1	0.7108	5.170 00E−3	1	0.8713	1.000 00E−3	1	2.1738	3.560 00E−6	2
0.6052	1.640 00E−2	1	0.7116	5.280 00E−3	1	0.8834	8.830 00E−4	1	2.1788	3.460 00E−6	2
0.6102	1.340 00E−2	1	0.7123	5.060 00E−3	1	0.8996	8.400 00E−4	2	2.2046	3.260 00E−6	2
0.6102	1.070 00E−2	1	0.7135	4.700 00E−3	1	0.9144	6.280 00E−4	1	2.2647	3.010 00E−6	2
0.6119	1.090 00E−2	2	0.7165	4.900 00E−3	1	0.9257	5.740 00E−4	1	2.3749	2.780 00E−2	2
0.6152	1.310 00E−2	1	0.7206	4.680 00E−3	1	0.9321	5.500 00E−4	1	2.4951	2.610 00E−6	2
0.6189	1.010 00E−2	1	0.7217	4.360 00E−3	1	0.9394	7.530 00E−4	1	2.6455	2.470 00E−6	2
0.6242	9.690 00E−3	1	0.7262	4.430 00E−3	1	0.9412	5.030 00E−4	1	2.8260	2.340 00E−6	2
0.6278	1.200 00E−2	1	0.7292	4.110 00E−3	2	0.9453	4.950 00E−4	1	3.0263	2.260 00E−6	2
0.6278	1.310 00E−2	1	0.7296	4.060 00E−3	1	0.9477	5.630 00E−4	2	3.2567	2.180 00E−6	2
0.6330	9.290 00E−3	2	0.7364	4.040 00E−3	1	0.9578	4.320 00E−4	1	3.5070	2.110 00E−6	2
0.6340	9.100 00E−3	1	0.7368	3.960 00E−3	1	0.9578	4.400 00E−4	1	3.7771	2.030 00E−6	2
0.6347	1.140 00E−2	1	0.7440	3.750 00E−3	1	0.9689	4.080 00E−4	1	3.9370	2.010 00E−6	2
0.6416	8.760 00E−3	1	0.7455	3.510 00E−3	1	0.9736	3.770 00E−4	1	4.1671	1.960 00E−6	2
0.6419	1.080 00E−2	1	0.7505	3.530 00E−3	1	0.9945	3.140 00E−4	1	4.3370	1.910 00E−6	2
0.6464	1.040 00E−2	1	0.7532	3.230 00E−3	1	0.9988	3.770 00E−4	2	4.4570	1.870 00E−6	2
0.6468	8.870 00E−3	1	0.7564	3.150 00E−3	1	1.0098	2.750 00E−4	1	4.5571	1.860 00E−6	2
0.6529	8.450 00E−3	1	0.7593	3.090 00E−3	2	1.0098	2.900 00E−4	1	4.7073	1.800 00E−6	2
0.6532	7.850 00E−3	1	0.7676	2.990 00E−3	1	1.0198	2.750 00E−4	1	4.8473	1.760 00E−6	2
0.6543	9.790 00E−3	1	0.7743	2.780 00E−3	1	1.0308	2.350 00E−4	1	4.9673	1.720 00E−6	2
0.6550	7.860 00E−3	2	0.7774	2.670 00E−3	1	1.0358	2.350 00E−4	1	5.0172	1.690 00E−6	2
0.6588	7.300 00E−3	1	0.7802	2.560 00E−3	1	1.0509	2.040 00E−4	1	5.0571	1.670 00E−6	2
0.6595	7.980 00E−3	1	0.7841	2.460 00E−3	1	1.0509	2.040 00E−4	1	5.0871	1.650 00E−6	2
0.6637	7.690 00E−3	1	0.7881	2.350 00E−3	1	1.0509	2.600 00E−4	2	5.1270	1.610 00E−6	2
0.6644	9.010 00E−3	1	0.7883	2.350 00E−3	2	1.1350	1.510 00E−4	2	5.1669	1.580 00E−6	2
0.6662	7.070 00E−3	1	0.7964	2.180 00E−3	1	1.1991	1.000 00E−5	2	5.1868	1.570 00E−6	2
0.6712	7.230 00E−3	1	0.7964	2.240 00E−3	1	1.3003	5.760 00E−5	2			

TABLE 10.4. Knots and coefficients for the spline fit of $\log_{10} \mu_-$

Knots	Coefficients
$K(1)=3.513\,060E-2$	$C(1)=7.033\,374E-4$
$K(2)=3.513\,060E-2$	$C(2)=6.736\,553E-4$
$K(3)=3.513\,060E-2$	$C(3)=5.158\,198E-4$
$K(4)=3.513\,060E-2$	$C(4)=4.508\,211E-4$
$K(5)=5.875\,667E-2$	$C(5)=4.002\,894E-4$
$K(6)=1.179\,363E-1$	$C(6)=2.552\,750E-4$
$K(7)=1.537\,502E-1$	$C(7)=2.496\,965E-4$
$K(8)=1.649\,872E-1$	$C(8)=1.466\,750E-4$
$K(9)=6.623\,561E-1$	$C(9)=1.088\,565E-4$
$K(10)=8.008\,330E-1$	$C(10)=4.892\,824E-5$
$K(11)=8.281\,207E-1$	$C(11)=1.064\,366E-5$
$K(12)=9.599\,097E-1$	$C(12)=-4.138\,096E-5$
$K(13)=1.152\,959$	$C(13)=-8.747\,069E-5$
$K(14)=1.351\,840$	$C(14)=-1.058\,640E-4$
$K(15)=1.607\,292$	$C(15)=-1.326\,369E-4$
$K(16)=2.021\,290$	$C(16)=-1.474\,489E-4$
$K(17)=2.176\,597$	$C(17)=-1.522\,085E-4$
$K(18)=2.186\,372$	$C(18)=-1.603\,564E-4$
$K(19)=2.189\,764$	$C(19)=-1.681\,873E-4$
$K(20)=2.374\,662$	$C(20)=-1.700\,160E-4$
$K(21)=3.014\,612$	$C(21)=-1.756\,753E-4$
$K(22)=4.660\,651$	$C(22)=-1.768\,276E-4$
$K(23)=4.838\,042$	$C(23)=-1.801\,619E-4$
$K(24)=5.105\,466$	$C(24)=-1.804\,100E-4$
$K(25)=5.186\,798$	
$K(26)=5.186\,798$	
$K(27)=5.186\,798$	
$K(28)=5.186\,798$	

TABLE 10.5. Recommended values of the positive ion mobility in liquid ^{4}He

T_{90} (K)	μ_+ (m^2/V·s)	T_{90} (K)	μ_+ (m^2/V·s)
0.1000	$8.577E+2$	2.3000	$4.447E-6$
0.1500	$3.902E+2$	2.3500	$4.349E-6$
0.2000	$1.403E+2$	2.4000	$4.274E-6$
0.2500	$4.723E+1$	2.4500	$4.218E-6$
0.3000	$1.765E+1$	2.5000	$4.178E-6$
0.3500	$8.127E+0$	2.5500	$4.151E-6$
0.4000	$4.202E+0$	2.6000	$4.135E-6$
0.4500	$2.146E+0$	2.6500	$4.127E-6$
0.5000	$9.786E-1$	2.7000	$4.124E-6$
0.5500	$4.069E-1$	2.7500	$4.125E-6$
0.6000	$1.627E-1$	2.8000	$4.128E-6$
0.6500	$6.605E-2$	2.8500	$4.133E-6$
0.7000	$2.870E-2$	2.9000	$4.140E-6$
0.7500	$1.359E-2$	2.9500	$4.148E-6$
0.8000	$6.271E-3$	3.0000	$4.158E-6$
0.8500	$3.148E-3$	3.0500	$4.170E-6$
0.9000	$1.756E-3$	3.1000	$4.184E-6$
0.9500	$1.066E-3$	3.1500	$4.198E-6$
1.0000	$6.906E-4$	3.2000	$4.215E-6$
1.0500	$4.676E-4$	3.2500	$4.233E-6$
1.1000	$3.247E-4$	3.3000	$4.252E-6$
1.1500	$2.299E-4$	3.3500	$4.272E-6$
1.2000	$1.659E-4$	3.4000	$4.293E-6$
1.2500	$1.221E-4$	3.4500	$4.316E-6$
1.3000	$9.174E-5$	3.5000	$4.339E-6$
1.3500	$7.033E-5$	3.5500	$4.364E-6$
1.4000	$5.505E-5$	3.6000	$4.389E-6$
1.4500	$4.398E-5$	3.6500	$4.416E-6$
1.5000	$3.583E-5$	3.7000	$4.443E-6$
1.5500	$2.971E-5$	3.7500	$4.470E-6$
1.6000	$2.503E-5$	3.8000	$4.499E-6$
1.6500	$2.139E-5$	3.8500	$4.528E-6$
1.7000	$1.852E-5$	3.9000	$4.557E-6$
1.7500	$1.622E-5$	3.9500	$4.586E-6$
1.8000	$1.434E-5$	4.0000	$4.616E-6$
1.8500	$1.277E-5$	4.0500	$4.646E-6$
1.9000	$1.140E-5$	4.1000	$4.677E-6$
1.9500	$1.014E-5$	4.1500	$4.707E-6$
2.0000	$8.945E-6$	4.2000	$4.737E-6$
2.0500	$7.787E-6$	4.2500	$4.767E-6$
2.1000	$6.653E-6$	4.3000	$4.797E-6$
2.1500	$5.550E-6$	4.3500	$4.826E-6$
2.1760	$5.025E-6$	4.4000	$4.855E-6$
2.1761	$5.024E-6$	4.4500	$4.883E-6$
2.1762	$5.022E-6$	4.5000	$4.910E-6$
2.1763	$5.020E-6$	4.5500	$4.934E-6$
2.1764	$5.019E-6$	4.6000	$4.954E-6$
2.1765	$5.017E-6$	4.6500	$4.969E-6$
2.1766	$5.015E-6$	4.7000	$4.978E-6$
2.1767	$5.013E-6$	4.7500	$4.979E-6$
2.1768	$5.012E-6$	4.8000	$4.970E-6$
2.1769	$5.010E-6$	4.8500	$4.951E-6$
2.1770	$5.008E-6$	4.9000	$4.921E-6$
2.2000	$4.733E-6$	4.9500	$4.878E-6$
2.2500	$4.571E-6$	5.0000	$4.822E-6$

TABLE 10.6. Recommended values of the negative ion mobility of liquid ^{4}He

T_{90} (K)	μ_- (m^2/V·s)	T_{90} (K)	μ_- (m^2/V·s)
0.1000	5.012E+0	2.2500	3.062E−6
0.1500	9.560E−1	2.3000	2.921E−6
0.2000	3.913E−1	2.3500	2.821E−6
0.2500	1.924E−1	2.4000	2.740E−6
0.3000	1.079E−1	2.4500	2.670E−6
0.3500	6.729E−2	2.5000	2.607E−6
0.4000	4.546E−2	2.5500	2.552E−6
0.4500	3.244E−2	2.6000	2.504E−6
0.5000	2.383E−2	2.6500	2.461E−6
0.5500	1.757E−2	2.7000	2.423E−6
0.6000	1.267E−2	2.7500	2.390E−6
0.6500	8.707E−3	2.8000	2.360E−6
0.7000	5.592E−3	2.8500	2.334E−6
0.7500	3.444E−3	2.9000	2.309E−6
0.8000	2.138E−3	2.9500	2.287E−6
0.8500	1.342E−3	3.0000	2.267E−6
0.9000	8.006E−4	3.0500	2.248E−6
0.9500	4.844E−4	3.1000	2.229E−6
1.0000	3.173E−4	3.1500	2.212E−6
1.0500	2.253E−4	3.2000	2.195E−6
1.1000	1.685E−4	3.2500	2.180E−6
1.1500	1.290E−4	3.3000	2.164E−6
1.2000	9.880E−5	3.3500	2.150E−6
1.2500	7.567E−5	3.4000	2.136E−6
1.3000	5.836E−5	3.4500	2.122E−6
1.3500	4.562E−5	3.5000	2.109E−6
1.4000	3.636E−5	3.5500	2.097E−6
1.4500	2.956E−5	3.6000	2.084E−6
1.5000	2.449E−5	3.6500	2.072E−6
1.5500	2.066E−5	3.7000	2.060E−6
1.6000	1.772E−5	3.7500	2.049E−6
1.6500	1.545E−5	3.8000	2.037E−6
1.7000	1.363E−5	3.8500	2.026E−6
1.7500	1.214E−5	3.9000	2.015E−6
1.8000	1.088E−5	3.9500	2.003E−6
1.8500	9.757E−6	4.0000	1.992E−6
1.9000	8.736E−6	4.0500	1.980E−6
1.9500	7.779E−6	4.1000	1.969E−6
2.0000	6.862E−6	4.1500	1.957E−6
2.0500	5.971E−6	4.2000	1.945E−6
2.1000	5.052E−6	4.2500	1.933E−6
2.1500	4.060E−6	4.3000	1.920E−6
2.1760	3.523E−6	4.3500	1.907E−6
2.1761	3.521E−6	4.4000	1.894E−6
2.1762	3.519E−6	4.4500	1.881E−6
2.1763	3.517E−6	4.5000	1.867E−6
2.1764	3.515E−6	4.5500	1.852E−6
2.1765	3.513E−6	4.6000	1.837E−6
2.1766	3.511E−6	4.6500	1.822E−6
2.1767	3.509E−6	4.7000	1.806E−6
2.1768	3.507E−6	4.7500	1.790E−6
2.1769	3.505E−6	4.8000	1.774E−6
2.1770	3.503E−6	4.8500	1.759E−6
2.2000	3.269E−6	4.9000	1.743E−6
2.2500	3.062E−6	4.9500	1.726E−6
2.3000	2.921E−6	5.0000	1.703E−6

10.1. Chronological Bibliography for Ion Mobilities

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11. Mutual Friction

This section is somewhat different from the others because there is a review article (Ref. 13) which already summarizes the data and presents a cubic spline fit.

Representative values of the dimensionless mutual friction coefficients B and B are contained in Table 11.3 (as compiled in Ref. 13) which are adequate for many purposes. We also provide values of α and α', which are defined as

$$\alpha = B\rho_n/2\rho, \quad \alpha' = B'\rho_n/2\rho.$$

However, experiments that have used B in their analyses have been conducted at a wide range of counterflow velocities and frequencies. For example, in counterflow turbulence experiments one uses B to convert from measurements of the attenuation of second sound (at the some frequency) as a function of heat flux to the vortex line density as a function of heat flux. Experimenters have used resonances varying from 4 Hz (Ref. 14) to greater than 23 kHz (Ref. 16) a range of 3 1/2 decades. Table 11.4 shows the corresponding range of B: at 1.9 K, B varies by more than 50% from 1 Hz to 10 kHz. The parameter B is also used to convert measurements of temperature gradient to line density. Here B is a function of the steady counterflow velocity. Experiments of interest to cryogenic engineers have been performed with heat fluxes as high as 20 W/cm^2, producing a considerable temperature difference over a length of order 1 cm (Ref. 15). The resulting counterflow velocity varies with position but is everywhere greater than 160 cm/s. In the other extreme, quantum turbulence experiments have been carried out with counterflow velocities as low as 0.1 cm/s. Table 11.5 shows that the change in B over the relevant velocity range can be of more than a factor of 2 at 1.9 K.

The theory of the frequency and velocity dependencies of mutual friction is contained in Ref. 13. Using this theory one can develop a method for obtaining the mutual friction parameters at arbitrary frequency or counterflow velocity. One implementation of this idea is referred to in Ref 16. Some results of this rather technical procedure are contained in Tables 11.2 and 11.3. New theoretical and experimental work on mutual friction may soon make this situation clearer and easier to use.

Table 11.3 has been converted to T_{90}, but we have not attempted to convert the mutual friction data of Tables 11.4 and 11.5 to T_{90} because the current limited accuracy of these coefficients leads to more uncertainty than does the temperature scale.

TABLE 11.1. Knots and coefficients for mutual friction parameter B. The spline returns $\log_{10} B$ vs $\log_{10} \varepsilon$, with $\varepsilon = 1 - T/T_\lambda$. For $T > 2.167$ K use the asymptotic expression $B = 0.47\varepsilon^{0.33}$

Knots	Coefficients
$K(1) = -5$	
$K(2) = -5$	
$K(3) = -5$	$C(1) = 1.319\ 281\ 444\ 33$
$K(4) = -5$	$C(2) = 1.124\ 527\ 078\ 01$
$K(5) = -2.5$	$C(3) = 0.639\ 314\ 792\ 565$
$K(6) = -2.0$	$C(4) = 0.313\ 383\ 532\ 495$
$K(7) = -0.8$	$C(5) = -0.162\ 687\ 403\ 543$
$K(8) = -0.387958059947$	$C(6) = 0.092\ 047\ 691\ 284$
$K(9) = -0.387958059947$	$C(7) = 0.188\ 452\ 616\ 588$
$K(10) = -0.387958059947$	
$K(11) = -0.387958059947$	

TABLE 11.2. Knots and coefficients for mutual friction parameter B'. The spline returns $\log_{10}(B' + 15)$ vs $\log_{10} \varepsilon$, with $\varepsilon = 1 - T/T_\lambda$. For $T > 2.134$ K use the asymptotic expression $B' = -0.34\varepsilon^{-0.33} + 1.01$

Knots	Coefficients
$K(1) = -5$	
$K(2) = -5$	
$K(3) = -5$	$C(1) = -8.472\ 180\ 325\ 26E-2$
$K(4) = -5$	$C(2) = 0.931\ 621\ 715\ 174$
$K(5) = -3.55$	$C(3) = 0.973\ 263\ 359\ 433$
$K(6) = -3.2$	$C(4) = 1.105\ 435\ 918\ 19$
$K(7) = -2.5$	$C(5) = 1.159\ 044\ 851\ 27$
$K(8) = -1.0$	$C(6) = 1.183\ 116\ 345\ 66$
$K(9) = -0.384\ 067\ 377\ 871$	$C(7) = 1.174\ 805\ 942\ 14$
$K(10) = -0.384\ 067\ 377\ 871$	$C(8) = 1.194\ 583\ 927\ 66$
$K(11) = -0.384\ 067\ 377\ 871$	
$K(12) = -0.384\ 067\ 377\ 871$	

TABLE 11.3. Recommended values of the mutual friction coefficients in helium II

T_{90} (K)	B	B'	α	α'
1.30	1.526	0.616	0.034	1.383E−02
1.35	1.466	0.535	0.042	1.543E−02
1.40	1.408	0.458	0.051	1.668E−02
1.45	1.351	0.385	0.061	1.746E−02
1.50	1.296	0.317	0.072	1.766E−02
1.55	1.243	0.255	0.084	1.721E−02
1.60	1.193	0.198	0.097	1.608E−02
1.65	1.144	0.149	0.111	1.437E−02
1.70	1.100	0.107	0.126	1.225E−02
1.75	1.059	0.075	0.142	1.003E−02
1.80	1.024	0.052	0.160	8.211E−03
1.85	0.996	0.041	0.181	7.438E−03
1.90	0.980	0.040	0.206	8.340E−03
1.95	0.981	0.045	0.236	1.079E−02
2.00	1.008	0.043	0.279	1.198E−02
2.02	1.031	0.037	0.302	1.097E−02
2.04	1.065	0.027	0.330	8.318E−03
2.06	1.115	0.009	0.366	3.018E−03
2.08	1.188	−0.019	0.414	−6.690E−03
2.10	1.298	−0.065	0.481	−2.412E−02
2.12	1.476	−0.143	0.581	−5.632E−02
2.14	1.790	−0.297	0.753	−1.249E−01
2.16	2.420	−0.683	1.097	−3.096E−01
2.162	2.515	−0.755	1.150	−3.453E−01
2.164	2.622	−0.842	1.210	−3.883E−01
2.166	2.747	−0.949	1.279	−4.416E−01
2.168	2.897	−1.085	1.362	−5.103E−01
2.170	3.154	−1.272	1.577	−6.358E−01
2.172	3.538	−1.549	1.769	−7.747E−01
2.174	4.227	−2.048	2.113	−1.024E+00
2.176	6.391	−3.613	3.195	−1.807E+00
2.1761	6.679	−3.821	3.339	−1.911E+00
2.1762	7.027	−4.074	3.514	−2.037E+00
2.1763	7.463	−4.389	3.732	−2.194E+00
2.1764	8.033	−4.801	4.017	−2.401E+00
2.1765	8.833	−5.380	4.417	−2.690E+00
2.1766	10.098	−6.295	5.049	−3.147E+00
2.1767	12.693	−8.172	6.347	−4.086E+00

TABLE 11.4. Mutual friction parameter B vs temperature for various second sound frequencies in the low amplitude limit (after Ref. 16)

T_{58} (K)	1 Hz	10 Hz	100 Hz	1000 Hz	10 000 Hz
1	1.507	1.509	1.511	1.513	1.515
1.1	1.549	1.557	1.564	1.572	1.579
1.2	1.520	1.538	1.556	1.574	1.592
1.3	1.431	1.464	1.499	1.534	1.571
1.4	1.268	1.318	1.370	1.427	1.487
1.5	1.098	1.159	1.227	1.302	1.387
1.6	0.958	1.026	1.104	1.194	1.300
1.7	0.855	0.926	1.011	1.112	1.236
1.8	0.788	0.863	0.953	1.063	1.203
1.9	0.760	0.836	0.929	1.045	1.194
2.0	0.788	0.863	0.954	1.067	1.210
2.1	1.106	1.197	1.304	1.432	1.588
2.11	1.192	1.287	1.398	1.531	1.691
2.12	1.299	1.398	1.514	1.651	1.815
2.13	1.436	1.541	1.661	1.802	1.967
2.14	1.623	1.732	1.856	1.998	2.164
2.15	1.902	2.014	2.140	2.282	2.442
2.16	2.415	2.528	2.652	2.787	2.933
2.17	4.600	4.698	4.798	4.899	5.002

TABLE 11.5. Mutual friction parameter B vs temperature for various vortex line-normal fluid relative velocities in the steady state limit (after Ref. 16)

T_{SS} (K)	0.1 (cm/s)	1 (cm/s)	10 (cm/s)	100 (cm/s)
1	1.503	1.507	1.511	1.515
1.1	1.539	1.553	1.568	1.583
1.2	1.501	1.536	1.572	1.609
1.3	1.406	1.472	1.543	1.619
1.4	1.244	1.343	1.456	1.585
1.5	1.080	1.205	1.360	1.556
1.6	0.949	1.092	1.285	1.555
1.7	0.856	1.012	1.238	1.590
1.8	0.798	0.967	1.227	1.672
1.9	0.778	0.956	1.239	1.756
2.0	0.812	0.990	1.268	1.763
2.1	1.139	1.350	1.656	2.140
2.11	1.226	1.445	1.760	2.247
2.12	1.334	1.563	1.885	2.369
2.13	1.473	1.711	2.037	2.511
2.14	1.662	1.906	2.231	2.681
2.15	1.942	2.190	2.506	2.914
2.16	2.455	2.700	2.990	3.332
2.17	4.635	4.835	5.039	5.243

11.1. Chronological Bibliography for Mutual Friction

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12. Viscosity and Kinematic Viscosity

Adopted Database

Author(s)	Key #	Method	Range (K)
Woods & Hollis Hallett	1	Couette viscometer	$0.78 \leq T \leq 2.079$
Tough *et al.*	2	vibrating wire	$1.52 \leq T \leq 2.16$
Goodwin	3	vibrating wire	$1.2 \leq T \leq 4.2$
Webeler & Allen	4	quartz crystal	$1.75 \leq T \leq 2.195$
Wang *et al.*	5	torsional oscillator	$1.8 \leq T \leq 4.4$

Comments and Key to Authors

(1) Reference 13.
(2) Reference 14.
(3) References 18, 22, and 23.
(4) Reference 21.
(5) Reference 28.
(6) The absolute accuracy of viscosity measurements is very hard to evaluate. The best guide is probably the deviation plot in Fig. 12.2.
(7) To convert Pa·s or kg/m·s to μP, multiply by 10^7, to

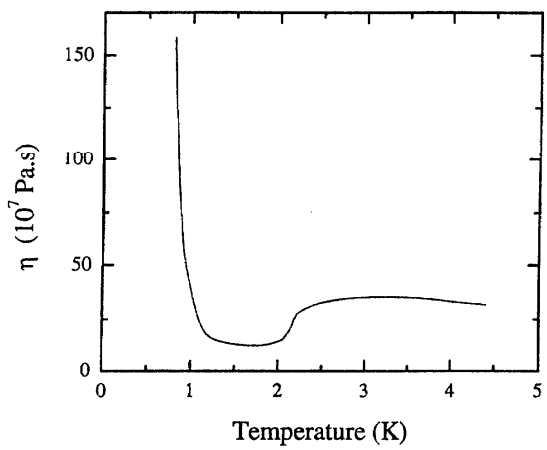

FIG. 12.1. The recommended values of the viscosity of liquid ^{4}He as a function of temperature at saturated vapor pressure.

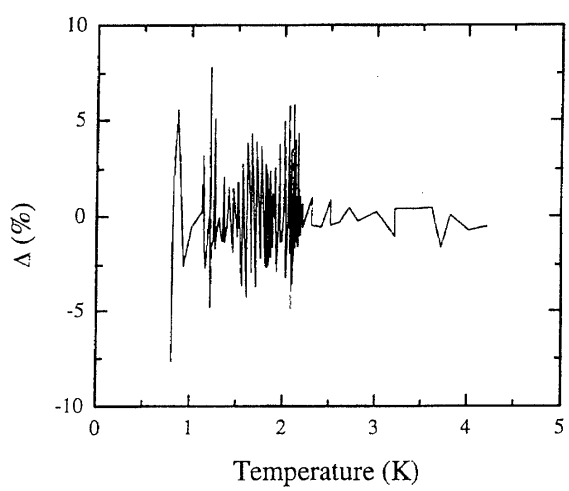

FIG. 12.2. The fractional deviation of values of the adopted database from the recommended values for the viscosity of liquid ^{4}He expressed in percent.

convert from m^2/s to cm^2/s, multiply by 10^4.

TABLE 12.1. Adopted database for the viscosity of liquid ^{4}He

T_{90} (K)	η (Pa·s)	Key	T_{90} (K)	η (Pa·s)	Key
0.7913	1.73E−05	1	1.9542	1.35E−06	3
0.8014	1.58E−05	1	1.9642	1.43E−06	2
0.8054	1.62E−05	1	1.9832	1.46E−06	1
0.8164	1.31E−05	1	1.9842	1.45E−06	2
0.8565	8.50E−06	1	1.9842	1.46E−06	5
0.9266	5.50E−06	1	2.0042	1.47E−06	2
1.0108	3.70E−06	1	2.0042	1.41E−06	3
1.122	2.18E−06	1	2.0243	1.53E−06	2
1.132	2.04E−06	1	2.0243	1.56E−06	5
1.142	2.06E−06	1	2.0443	1.57E−06	2
1.162	1.96E−06	1	2.0543	1.50E−06	3
1.2033	1.72E−06	2	2.0643	1.65E−06	2
1.2033	1.60E−06	3	2.0833	1.65E−06	1
1.2161	1.77E−06	1	2.0843	1.72E−06	2
1.2231	1.69E−06	2	2.0843	1.80E−06	4
1.2432	1.64E−06	2	2.0843	1.68E−06	5
1.2532	1.52E−06	3	2.0944	1.84E−06	4
1.2632	1.61E−06	2	2.1044	1.81E−06	2
1.2833	1.56E−06	2	2.1044	1.73E−06	3
1.3033	1.52E−06	2	2.1044	1.87E−06	4
1.3234	1.51E−06	2	2.1144	1.92E−06	4
1.3434	1.49E−06	2	2.1245	1.89E−06	2
1.3534	1.43E−06	3	2.1245	2.00E−06	5
1.3635	1.47E−06	2	2.1245	1.98E−06	4
1.3835	1.44E−06	2	2.1345	2.06E−06	4
1.4035	1.40E−06	2	2.1445	2.10E−06	2
1.4035	1.39E−06	3	2.1445	2.14E−06	4
1.4536	1.40E−06	2	2.1495	2.19E−06	4
1.4536	1.36E−06	3	2.1546	2.11E−06	3
1.5036	1.36E−06	2	2.1546	2.24E−06	4
1.5036	1.32E−06	3	2.1596	2.28E−06	4
1.5536	1.37E−06	2	2.1646	2.28E−06	2
1.5536	1.29E−06	3	2.1646	2.33E−06	4
1.6037	1.36E−06	2	2.1696	2.38E−06	4
1.6037	1.26E−06	3	2.1768	2.54E−06	5
1.6428	1.29E−06	1	2.1778	2.49E−06	4
1.6538	1.33E−06	2	2.1798	2.50E−06	4
1.6538	1.24E−06	3	2.1846	2.53E−06	4
1.7039	1.34E−06	2	2.1896	2.56E−06	4
1.7039	1.24E−06	3	2.1946	2.59E−06	4
1.754	1.32E−06	2	2.1996	2.62E−06	4
1.754	1.31E−06	4	2.2046	2.69E−06	5
1.754	1.25E−06	3	2.3047	2.94E−06	3
1.8041	1.33E−06	2	2.3047	2.98E−06	5
1.8041	1.30E−06	5	2.4049	3.16E−06	5
1.8041	1.26E−06	3	2.5052	3.24E−06	3
1.8231	1.28E−06	1	2.5052	3.28E−06	5
1.8241	1.34E−06	2	2.6054	3.36E−06	5
1.8441	1.34E−06	2	2.7057	3.40E−06	3
1.8441	1.30E−06	5	2.806	3.47E−06	5
1.8541	1.29E−06	3	3.0063	3.51E−06	5
1.8641	1.35E−06	2	3.2066	3.57E−06	3
1.8641	1.31E−06	5	3.2066	3.52E−06	5
1.8841	1.36E−06	2	3.4069	3.51E−06	5
1.8841	1.32E−06	5	3.607	3.47E−06	5
1.9042	1.34E−06	5	3.7071	3.51E−06	3
1.9042	1.32E−06	3	3.8071	3.41E−06	5
1.9042	1.35E−06	2	4.007	3.34E−06	5
1.9242	1.41E−06	2	4.2071	3.26E−06	3
1.9442	1.40E−06	2	4.2071	3.26E−06	5
1.9442	1.39E−06	5	4.407	3.16E−06	5

TABLE 12.2. Knots and coefficients for the spline fit of the viscosity liquid ^{4}He. The spline returns the viscosity in Pa s.

Knots	Coefficients
$K(1)=7.913\,364E-1$	$C(1)=1.730\,865E-5$
$K(2)=7.913\,364E-01$	$C(2)=6.577\,810E-6$
$K(3)=7.913\,364E-1$	$C(3)=4.956\,473E-6$
$K(4)=7.913\,364E-1$	$C(4)=1.862\,435E-6$
$K(5)=9.705\,100E-1$	$C(5)=1.452\,672E-6$
$K(6)=1.064\,730E+0$	$C(6)=1.308\,345E-6$
$K(7)=1.285\,930E+0$	$C(7)=1.273\,173E-6$
$K(8)=1.582\,100E+0$	$C(8)=1.338\,821E-6$
$K(9)=1.747\,010E+0$	$C(9)=1.613\,257E-6$
$K(10)=2.025\,680E+0$	$C(10)=1.956\,558E-6$
$K(11)=2.051\,740E+0$	$C(11)=2.296\,259E-6$
$K(12)=2.146\,961E+0$	$C(12)=2.514\,817E-6$
$K(13)=2.176\,800E+0$	$C(13)=2.487\,748E-6$
$K(14)=2.176\,800E+0$	$C(14)=2.715\,638E-6$
$K(15)=2.176\,800E+0$	$C(15)=3.125\,798E-6$
$K(16)=2.212\,906E+0$	$C(16)=3.487\,019E-6$
$K(17)=2.221\,800E+0$	$C(17)=3.564\,378E-6$
$K(18)=2.618\,000E+0$	$C(18)=3.486\,451E-6$
$K(19)=3.253\,700E+0$	$C(19)=3.270\,547E-6$
$K(20)=3.784\,200E+0$	$C(20)=3.226\,615E-6$
$K(21)=4.025\,400E+0$	$C(21)=3.160\,000E-6$
$K(22)=4.406\,982E+0$	
$K(23)=4.406\,982E+0$	
$K(24)=4.406\,982E+0$	
$K(25)=4.406\,982E+0$	

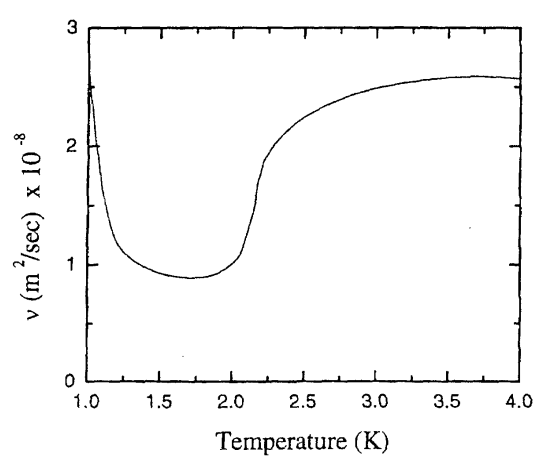

FIG. 12.3. The recommended values of the kinematic viscosity of liquid ^{4}He, $\nu=\eta/\rho$, as a function of temperature at the saturated vapor pressure.

TABLE 12.3. Recommended values of the viscosity of liquid ^{4}He at the saturated vapor pressure

T_{90} (K)	η (Pa·s)	T_{90} (K)	η (Pa·s)
0.8	1.582$E-5$	2.18	2.512$E-6$
0.85	9.537$E-6$	2.19	2.553$E-6$
0.9	6.288$E-6$	2.2	2.635$E-6$
0.95	4.806$E-6$	2.25	2.840$E-6$
1.0	3.873$E-6$	2.3	2.956$E-6$
1.05	3.028$E-6$	2.35	3.053$E-6$
1.1	2.391$E-6$	2.4	3.135$E-6$
1.15	1.980$E-6$	2.45	3.203$E-6$
1.2	1.736$E-6$	2.5	3.259$E-6$
1.25	1.604$E-6$	2.55	3.306$E-6$
1.3	1.527$E-6$	2.6	3.346$E-6$
1.35	1.466$E-6$	2.65	3.380$E-6$
1.4	1.416$E-6$	2.7	3.410$E-6$
1.45	1.377$E-6$	2.75	3.436$E-6$
1.5	1.346$E-6$	2.8	3.459$E-6$
1.55	1.323$E-6$	2.85	3.478$E-6$
1.6	1.306$E-6$	2.9	3.494$E-6$
1.65	1.295$E-6$	2.95	3.507$E-6$
1.7	1.290$E-6$	3.0	3.517$E-6$
1.75	1.290$E-6$	3.05	3.524$E-6$
1.8	1.298$E-6$	3.1	3.529$E-6$
1.85	1.316$E-6$	3.15	3.532$E-6$
1.9	1.347$E-6$	3.2	3.534$E-6$
1.95	1.397$E-6$	3.25	3.533$E-6$
2.0	1.468$E-6$	3.3	3.532$E-6$
2.05	1.569$E-6$	3.35	3.528$E-6$
2.1	1.803$E-6$	3.4	3.524$E-6$
2.11	1.868$E-6$	3.45	3.517$E-6$
2.12	1.936$E-6$	3.5	3.509$E-6$
2.13	2.008$E-6$	3.55	3.499$E-6$
2.14	2.083$E-6$	3.6	3.487$E-6$
2.15	2.161$E-6$	3.65	3.472$E-6$
2.16	2.252$E-6$	3.7	3.456$E-6$
2.17	2.385$E-6$	3.75	3.437$E-6$
2.171	2.402$E-6$	3.8	3.415$E-6$
2.172	2.419$E-6$	3.85	3.392$E-6$
2.173	2.438$E-6$	3.9	3.367$E-6$
2.174	2.457$E-6$	3.95	3.342$E-6$
2.175	2.477$E-6$	4.0	3.319$E-6$
2.176	2.498$E-6$	4.05	3.298$E-6$
2.1768	2.515$E-6$	4.1	3.279$E-6$
2.177	2.514$E-6$	4.15	3.261$E-6$
2.178	2.513$E-6$	4.2	3.244$E-6$
2.179	2.512$E-6$		

TABLE 12.4. Recommended values of the kinematic viscosity $\nu = \eta/\rho$ of liquid ^{4}He

T_{90} (K)	ν (m^2 s^{-1})	T_{90} (K)	ν (m^2 s^{-1})
0.80	1.090$E-7$	2.65	2.349$E-8$
0.85	6.572$E-8$	2.70	2.375$E-8$
0.90	4.333$E-8$	2.75	2.399$E-8$
0.95	3.311$E-8$	2.80	2.421$E-8$
1.00	2.669$E-8$	2.85	2.441$E-8$
1.05	2.086$E-8$	2.90	2.459$E-8$
1.10	1.648$E-8$	2.95	2.475$E-8$
1.15	1.364$E-8$	3.00	2.490$E-8$
1.20	1.197$E-8$	3.05	2.503$E-8$
1.25	1.106$E-8$	3.10	2.515$E-8$
1.30	1.052$E-8$	3.15	2.526$E-8$
1.35	1.010$E-8$	3.20	2.536$E-8$
1.40	9.756$E-9$	3.25	2.545$E-8$
1.45	9.484$E-9$	3.30	2.553$E-8$
1.50	9.273$E-9$	3.35	2.561$E-8$
1.55	9.114$E-9$	3.40	2.567$E-8$
1.60	8.997$E-9$	3.45	2.574$E-8$
1.65	8.918$E-9$	3.50	2.579$E-8$
1.70	8.878$E-9$	3.55	2.583$E-8$
1.75	8.880$E-9$	3.60	2.586$E-8$
1.80	8.929$E-9$	3.65	2.588$E-8$
1.85	9.048$E-9$	3.70	2.589$E-8$
1.90	9.263$E-9$	3.75	2.588$E-8$
1.95	9.598$E-9$	3.80	2.586$E-8$
2.00	1.008$E-8$	3.85	2.582$E-8$
2.05	1.077$E-8$	3.90	2.579$E-8$
2.10	1.237$E-8$	3.95	2.575$E-8$
2.15	1.481$E-8$	4.00	2.573$E-8$
2.20	1.804$E-8$	4.05	2.574$E-8$
2.25	1.945$E-8$	4.10	2.576$E-8$
2.30	2.027$E-8$	4.15	2.581$E-8$
2.35	2.097$E-8$	4.20	2.587$E-8$
2.40	2.157$E-8$	4.25	2.593$E-8$
2.45	2.207$E-8$	4.30	2.600$E-8$
2.50	2.250$E-8$	4.35	2.605$E-8$
2.55	2.287$E-8$	4.40	2.609$E-8$
2.60	2.320$E-8$		

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13. Dispersion Curve

Adopted Database

Author(s)	Key #	Method	Range (Å^{-1})
Svensson *et al.*	1	Neutron scattering	$0.3 \leq Q \leq 1.13$
Cowley & Woods	2	Neutron scattering	$0.2 \leq Q \leq 3.6$
Woods *et al.*	3	Neutron scattering	$1.9 \leq Q \leq 1.96$
Stirling *et al.*	4	Neutron scattering	$0.089 \leq Q \leq 0.8925$
Stirling	5	Neutron scattering	$1.88 \leq Q \leq 1.97$

Comments and Key to Authors

(1) Reference 10.
(2) Reference 3.
(3) Reference 7.
(4) Reference 9.
(5) Reference 13.
(6) The accuracy of neutron scattering measurements depends very much on the wave number range. It is probably best appreciated by inspecting the deviation plot in Fig. 13.3. The quantity E is a measure of the energy of an elementary excitation at a wave number Q. The wave number Q used here is expressed in reciprocal Å ($=10^8$).

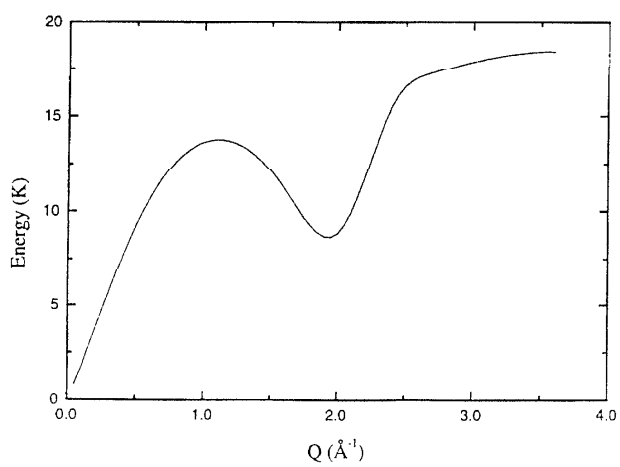

FIG. 13.1. Energy E in degrees Kelvin of elementary excitations in helium II as a function of wave number Q in reciprocal Å (Ref. 12).

TABLE 13.1. The adopted database used for the dispersion curve for liquid ^{4}He

Q(Å^{-1})	E/k (K)	Key	Q(Å^{-1})	E/k (K)	Key
0.089 40	1.613 09	4	1.100	13.8000	2
0.094 60	1.717 54	4	1.130	13.8200	1
0.1150	2.100 50	4	1.200	13.7500	2
0.1210	2.251 37	4	1.300	13.5000	2
0.1390	2.611 12	4	1.400	12.9500	2
0.1430	2.634 33	4	1.500	12.2000	2
0.1594	2.970 88	4	1.600	11.2000	2
0.1767	3.295 82	4	1.700	10.2500	2
0.1818	3.388 66	4	1.800	9.250 00	2
0.1938	3.632 36	4	1.880	8.694 00	5
0.1990	3.736 81	4	1.890	8.657 00	5
0.2000	3.700 00	2	1.900	8.700 00	2
0.2110	3.968 91	4	1.900	8.654 00	3
0.2162	4.084 96	4	1.900	8.634 00	5
0.2278	4.282 24	4	1.910	8.635 00	3
0.2329	4.386 69	4	1.910	8.616 00	5
0.2445	4.607 18	4	1.915	8.611 00	5
0.2495	4.711 63	4	1.920	8.626 00	3
0.2611	4.920 52	4	1.920	8.610 00	5
0.2776	5.233 85	4	1.925	8.606 00	5
0.2825	5.326 69	4	1.930	8.626 00	3
0.2938	5.523 98	4	1.930	8.606 00	5
0.2988	5.628 42	4	1.935	8.630 00	3
0.3000	5.570 00	1	1.935	8.612 00	5
0.3000	5.650 00	2	1.940	8.630 00	3
0.4000	7.400 00	2	1.940	8.609 00	5
0.4036	7.636 09	4	1.950	8.650 00	3
0.4082	7.717 32	4	1.950	8.633 00	5
0.4187	7.914 61	4	1.960	8.683 00	3
0.4232	7.995 84	4	1.960	8.672 00	5
0.4355	8.181 52	4	1.970	8.695 00	5
0.4498	8.378 81	4	2.000	8.950 00	2
0.4643	8.645 72	4	2.100	10.0000	2
0.4785	8.866 22	4	2.200	11.6500	2
0.4926	9.109 92	4	2.300	13.5500	2
0.5000	9.150 00	2	2.400	15.5000	2
0.5605	10.1544	4	2.500	16.4500	2
0.6000	10.7500	2	2.600	17.0000	2
0.6243	11.0015	4	2.700	17.3000	2
0.6965	11.8023	4	2.800	17.5000	2
0.7000	11.7500	2	2.900	17.7000	2
0.7649	12.4173	4	3.000	17.8500	2
0.8000	12.7200	1	3.100	18.0000	2
0.8000	12.6500	2	3.200	18.1500	2
0.8300	12.8815	4	3.300	18.3000	2
0.8925	13.2297	4	3.400	18.3500	2
0.9000	13.1500	2	3.500	18.4000	2
1.000	13.5500	2	3.600	18.4500	2

TABLE 13.2. Knots and coefficients for the spline fit of the dispersion curve of liquid ^{4}He

Knots	Coefficients
$K(1)=0.0894$	$C(1)=1.53895$
$K(2)=0.0894$	$C(2)=1.932$
$K(3)=0.0894$	$C(3)=4.8$
$K(4)=0.0894$	$C(4)=14.85$
$K(5)=0.15$	$C(5)=14.88$
$K(6)=0.510$	$C(6)=5.9384$
$K(7)=1.60$	$C(7)=16.5014$
$K(8)=2.023$	$C(8)=17.72455$
$K(9)=2.42$	$C(9)=18.43656$
$K(10)=2.665$	$C(10)=18.43545$
$K(11)=3.60$	
$K(12)=3.60$	
$K(13)=3.60$	
$K(14)=3.60$	

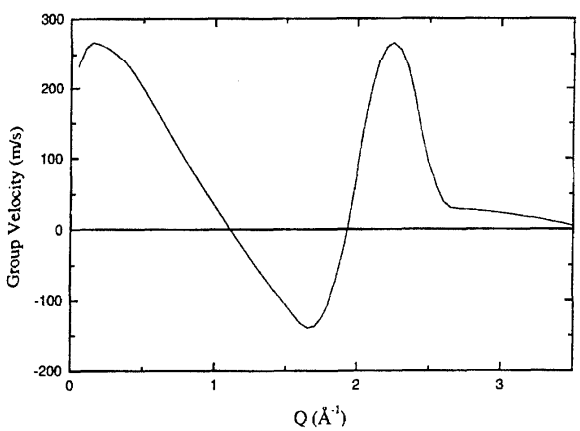

FIG. 13.2. Group velocity of elementary excitations in helium II as a function of wave number Q in reciprocal Å (Ref. 12).

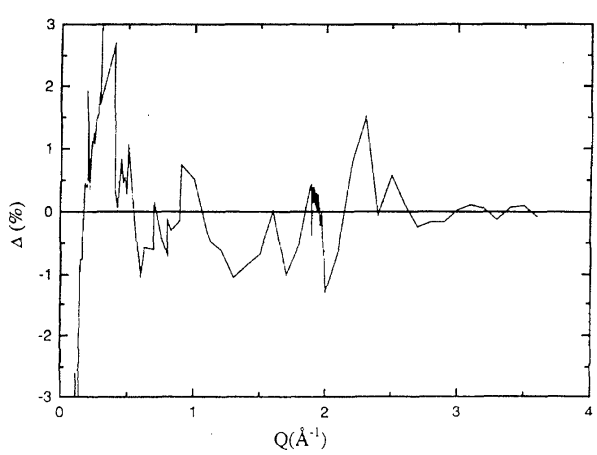

FIG. 13.3. The fractional deviation of values of the adopted database from the recommended values for the dispersion curve for liquid ^{4}He, expressed in percent.

TABLE 13.3. Recommended values of the dispersion curve of elementary excitations in liquid ^{4}He as a function of wave number Q

Q (Å^{-1})	E/k (K)	Q (Å^{-1})	E/k (K)
0.00	0.00[a]	1.85	8.866
0.05	0.804	1.90	8.667
0.10	1.747	1.95	8.644
0.15	2.757	2.00	8.833
0.20	3.772	2.05	9.271
0.25	4.772	2.10	9.941
0.30	5.749	2.15	10.784
0.35	6.694	2.20	11.742
0.40	7.598	2.25	12.751
0.45	8.452	2.30	13.753
0.50	9.249	2.35	14.687
0.55	9.979	2.40	15.492
0.60	10.641	2.45	16.111
0.65	11.237	2.50	16.544
0.70	11.767	2.55	16.832
0.75	12.232	2.60	17.019
0.80	12.633	2.65	17.147
0.85	12.971	2.70	17.258
0.90	13.248	2.75	17.366
0.95	13.464	2.80	17.471
1.00	13.62	2.85	17.573
1.05	13.717	2.90	17.671
1.10	13.757	2.95	17.765
1.15	13.74	3.00	17.855
1.20	13.667	3.05	17.94
1.25	13.54	3.10	18.019
1.30	13.359	3.15	18.093
1.35	13.125	3.20	18.161
1.40	12.839	3.25	18.222
1.45	12.503	3.30	18.276
1.50	12.118	3.35	18.323
1.55	11.684	3.40	18.363
1.60	11.202	3.45	18.394
1.65	10.68	3.50	18.417
1.70	10.148	3.55	18.431
1.75	9.644	3.60	18.435
1.80	9.204		

[a]The dispersion curve must be zero at zero wave number. Our spline returns $-2.485\,941E-4$ K, which is incorrect, but negligible.

13.1. Chronological Bibliography for Dispersion Curve

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14. Structure Factor

Adopted Database

Author(s)	Key #	Method	Range [Q (Å^{-1})]
Robkoff and Hallock	1	X-ray diffraction	$0.213<Q<5.129$
Svensson *et al.*	2	Neutron diffraction	$0.79<Q<6.66$

Comments and Key to Authors

(1) Reference 9.
(2) Reference 10.
(3) We adopted the theoretical value of the structure factor at zero wave number

$$S(0)=k\gamma T/mc^2=5.1x10^{-2},$$

where γ is the ratio of specific heats, m is the mass of the ^{4}He atom and c is the velocity of sound. The wave number Q used here is expressed in reciprocal Å ($=10^8$ cm).
(4) The authors of Refs. 9 and 10 are unable to explain the discrepancy in the two sets of data in the region of the peak. Our spline passes near the average of the two sets of data in the region of the peak. If a choice needs to be made, the x-ray data could be used for $Q<1$, and the neutron data for $Q>1$.

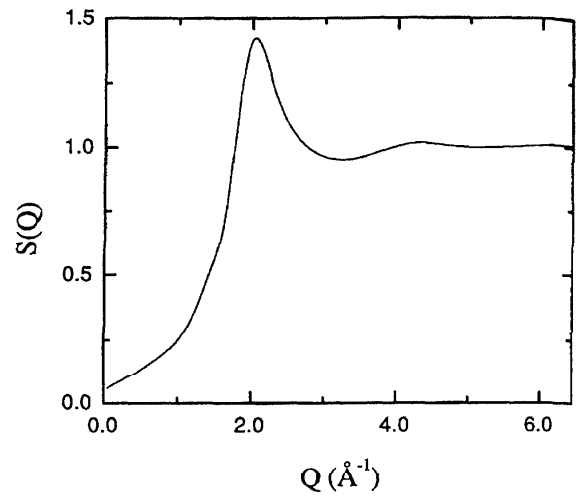

FIG. 14.1. The recommended values for the structure factor of liquid ^{4}He as a function of wave number.

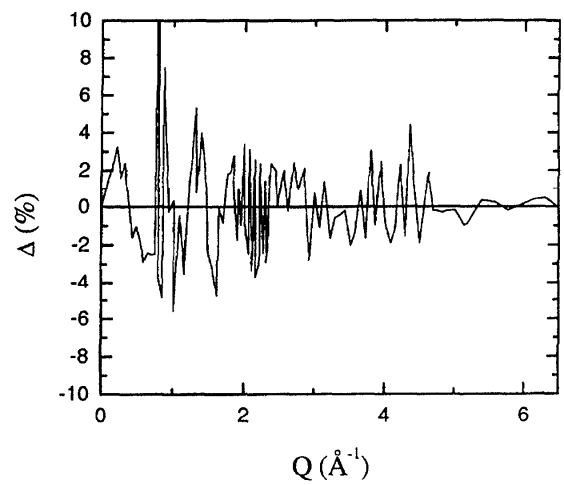

FIG. 14.2. The fractional deviation of values of the adopted database from the recommended values for the structure factor expressed in percent.

TABLE 14.1. Adopted database used for structure factor of liquid ^{4}He

Q (Å^{-1})	$S(Q)$	Key	Q (Å^{-1})	$S(Q)$	Key
1.000E+0	5.100 00E−2	3	2.315E+0	1.249 00E+0	1
2.130E−1	8.100 00E−2	1	2.370E+0	1.142 00E+0	2
2.670E−1	9.100 00E−2	1	2.450E+0	1.095 00E+0	2
3.200E−1	9.900 00E−2	1	2.468E+0	1.105 00E+0	1
3.730E−1	1.100 00E−1	1	2.560E+0	1.040 00E+0	2
4.270E−1	1.220 00E−1	1	2.620E+0	1.040 00E+0	1
4.800E−1	1.310 00E−1	1	2.700E+0	9.890 00E−1	2
5.330E−1	1.420 00E−1	1	2.771E+0	9.860 00E−1	1
5.860E−1	1.540 00E−1	1	2.850E+0	9.590 00E−1	2
6.400E−1	1.640 00E−1	1	2.921E+0	9.960 00E−1	1
6.930E−1	1.750 00E−1	1	3.000E+0	9.520 00E−1	2
7.460E−1	1.860 00E−1	1	3.068E+0	9.640 00E−1	1
7.900E−1	1.560 00E−1	2	3.140E+0	9.370 00E−1	2
7.990E−1	2.000 00E−1	1	3.217E+0	9.650 00E−1	1
8.520E−1	2.150 00E−1	1	3.290E+0	9.550 00E−1	2
8.600E−1	1.920 00E−1	2	3.364E+0	9.560 00E−1	1
9.400E−1	2.280 00E−1	2	3.430E+0	9.570 00E−1	2
1.010E+0	2.490 00E−1	2	3.509E+0	9.800 00E−1	1
1.011E+0	2.650 00E−1	1	3.570E+0	9.790 00E−1	2
1.090E+0	2.830 00E−1	2	3.653E+0	9.640 00E−1	1
1.169E+0	3.330 00E−1	1	3.720E+0	9.950 00E−1	2
1.170E+0	3.290 00E−1	2	3.795E+0	9.560 00E−1	1
1.240E+0	3.590 00E−1	2	3.860E+0	1.001 00E+0	2
1.320E+0	4.000 00E−1	2	3.936E+0	9.740.00E−1	1
1.328E+0	4.240 00E−1	1	4.000E+0	1.012 00E+0	2
1.400E+0	4.640 00E−1	2	4.075E+0	1.028 00E+0	1
1.470E+0	5.320 00E−1	2	4.140E+0	1.024 00E+0	2
1.485E+0	5.630 00E−1	1	4.213E+0	9.920 00E−1	1
1.550E+0	6.240 00E−1	2	4.280E+0	1.032 00E+0	2
1.620E+0	7.280 00E−1	2	4.349E+0	9.730 00E−1	1
1.642E+0	7.320 00E−1	1	4.410E+0	1.003 00E+0	2
1.700E+0	8.600 00E−1	2	4.484E+0	1.032 00E+0	1
1.770E+0	1.003 00E+0	2	4.550E+0	1.011 00E+0	2
1.799E−0	1.072 00E+0	1	4.617E+0	9.890 00E−1	1
1.850E+0	1.177 00E+0	2	4.690E+0	1.007 00E+0	2
1.851E+0	1.201 00E+0	1	4.748E+0	1.005 00E+0	1
1.903E+0	1.337 00E+0	1	4.820E+0	1.004 00E+0	2
1.920E+0	1.329 00E+0	2	4.877E+0	1.002 00E+0	1
1.955E+0	1.399 00E+0	1	4.990E+0	1.000 00E+0	2
2.000E+0	1.371 00E+0	2	5.004E+0	1.000 00E+0	1
2.006E+0	1.433 00E+0	1	5.129E+0	1.007 00E+0	1
2.058E+0	1.458 00E+0	1	5.180E+0	1.006 00E+0	2
2.070E+0	1.376 00E+0	2	5.380E+0	9.950 00E−1	2
2.109E+0	1.448 00E+0	1	5.570E+0	9.980 00E−1	2
2.150E+0	1.335 00E+0	2	5.760E+0	1.005 00E+0	2
2.161E+0	1.412 00E+0	1	5.940E+0	1.005 00E+0	2
2.212E+0	1.352 00E+0	1	6.130E+0	1.004 00E+0	2
2.220E+0	1.273 00E−0	2	6.310E+0	1.004 00E+0	2
2.264E+0	1.290 00E+0	1	6.480E+0	1.008 00E+0	2
2.300E+0	1.208 00E+0	2	6.660E+0	1.005 00E+0	2

TABLE 14.2. Knots and coefficients for the spline fit of structure factor

Knots	Coefficients
$K(1) = 0.000\,000E+00$	$C(1) = 5.098\,474E-02$
$K(2) = 0.000\,000E+00$	$C(2) = 8.713\,372E-02$
$K(3) = 0.000\,000E+00$	$C(3) = 1.381\,343E-01$
$K(4) = 0.000\,000E+00$	$C(4) = 2.211\,584E-01$
$K(5) = 7.467\,679E-01$	$C(5) = 3.034\,749E-01$
$K(6) = 8.843\,175E-01$	$C(6) = 5.022\,945E-01$
$K(7) = 1.171\,682E+00$	$C(7) = 7.050\,370E-01$
$K(8) = 1.483\,348E+00$	$C(8) = 1.644\,316E+00$
$K(9) = 1.615\,547E+00$	$C(9) = 1.146\,399E+00$
$K(10) = 1.954\,501E+00$	$C(10) = 8.365\,880E-01$
$K(11) = 2.269\,539E+00$	$C(11) = 1.045\,785E+00$
$K(12) = 2.738\,304E+00$	$C(12) = 9.733\,701E-01$
$K(13) = 4.266\,671E+00$	$C(13) = 1.020\,061E+00$
$K(14) = 4.686\,186E+00$	$C(14) = 1.004\,961E+00$
$K(15) = 6.660\,000E+00$	
$K(16) = 6.660\,000E+00$	
$K(17) = 6.660\,000E+00$	
$K(18) = 6.660\,000E+00$	

TABLE 14.3. Recommended values of the structure factor of liquid ^{4}He

Q (Å^{-1})	$S(Q)$	Q (Å^{-1})	$S(Q)$
0.000	5.098E−2	3.350	9.513E−1
0.050	5.834E−2	3.400	9.535E−1
0.100	6.588E−2	3.450	9.563E−1
0.150	7.362E−2	3.500	9.596E−1
0.200	8.154E−2	3.550	9.634E−1
0.250	8.967E−2	3.600	9.675E−1
0.300	9.799E−2	3.650	9.718E−1
0.350	1.065E−1	3.700	9.764E−1
0.400	1.152E−1	3.750	9.810E−1
0.450	1.242E−1	3.800	9.857E−1
0.500	1.333E−1	3.850	9.903E−1
0.550	1.427E−1	3.900	9.948E−1
0.600	1.522E−1	3.950	9.991E−1
0.650	1.620E−1	4.000	1.003
0.700	1.720E−1	4.050	1.007
0.750	1.822E−1	4.100	1.010
0.800	1.928E−1	4.150	1.012
0.850	2.040E−1	4.200	1.014
0.900	2.164E−1	4.250	1.016
0.950	2.303E−1	4.300	1.016
1.000	2.463E−1	4.350	1.016
1.050	2.648E−1	4.400	1.015
1.100	2.862E−1	4.450	1.014
1.150	3.109E−1	4.500	1.012
1.200	3.395E−1	4.550	1.010
1.250	3.716E−1	4.600	1.008
1.300	4.066E−1	4.650	1.006
1.350	4.438E−1	4.700	1.004
1.400	4.825E−1	4.750	1.003
1.450	5.219E−1	4.800	1.001
1.500	5.614E−1	4.850	1.000
1.550	6.049E−1	4.900	9.993E−1
1.600	6.633E−1	4.950	9.986E−1
1.650	7.466E−1	5.000	9.980E−1
1.700	8.529E−1	5.050	9.976E−1
1.750	9.721E−1	5.100	9.973E−1
1.800	1.094	5.150	9.972E−1
1.850	1.210	5.200	9.972E−1
1.900	1.308	5.250	9.973E−1
1.950	1.380	5.300	9.976E−1
2.000	1.418	5.350	9.979E−1
2.050	1.424	5.400	9.984E−1
2.100	1.405	5.450	9.989E−1
2.150	1.369	5.500	9.995E−1
2.200	1.323	5.550	1.000
2.250	1.273	5.600	1.001
2.300	1.226	5.650	1.001
2.350	1.184	5.700	1.002
2.400	1.148	5.750	1.003
2.450	1.116	5.800	1.004
2.500	1.089	5.850	1.004
2.550	1.065	5.900	1.005
2.600	1.045	5.950	1.006
2.650	1.028	6.000	1.006
2.700	1.013	6.050	1.007
2.750	1.000	6.100	1.008
2.800	9.889E−1	6.150	1.008
2.850	9.793E−1	6.200	1.008
2.900	9.712E−1	6.250	1.008
2.950	9.644E−1	6.300	1.009
3.000	9.590E−1	6.350	1.009
3.050	9.549E−1	6.400	1.008
3.100	9.519E−1	6.450	1.008
3.150	9.499E−1	6.500	1.008
3.200	9.490E−1	6.550	1.007
3.250	9.490E−1	6.600	1.006
3.300	9.498E−1		

14.1. Chronological Bibliography for Structure Factor

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15. Thermal Conductivity

Adopted Database

Author(s)	Key #	Range (K)
Ahlers	1	$2.172 \leq T \leq 2.176$
Behringer & Ahlers	2	$2.1728 \leq T \leq 4.209$
Tam & Ahlers	3	$2.172 \leq T \leq 2.542$
Dingus, Zhong & Meyer	4	$2.172 \leq T \leq 3.765$

Comments and Key to Authors

(1) Reference 10. Uncertainties as in No. (3). Systematic uncertainties increase below $T/T_\lambda - 1 < 10^{-6}$.
(2) Quoted in Ref. 17. Uncertainties: $T/T_\lambda - 1 > 10^{-5}$ roughly 0.1%. Otherwise uncertainties rise closer to T_λ and a formula given.
(3) Reference 20. Uncertainties: $T/T_\lambda - 1 < 10^{-5}$, $\pm 0.5\%$, otherwise $+0.1\%$.
(4) Reference 21, Table IV only. Uncertainty: $<5\%$.
(5) To convert from mW/cm·K to erg/cm·s·K multiply by 10^4.

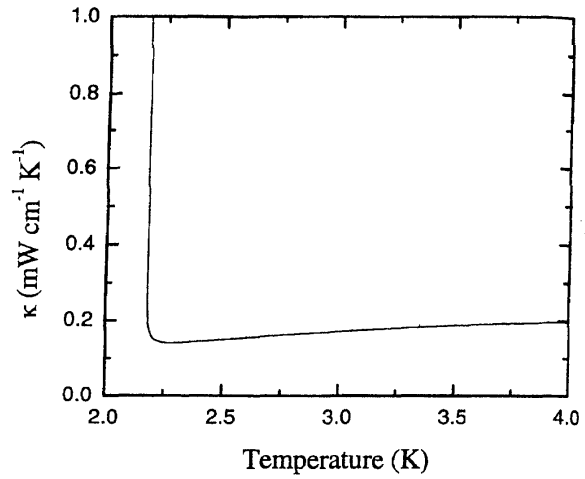

FIG. 15.1. The recommended values for the thermal conductivity of liquid ^{4}He as a function of temperature at the saturated vapor pressure.

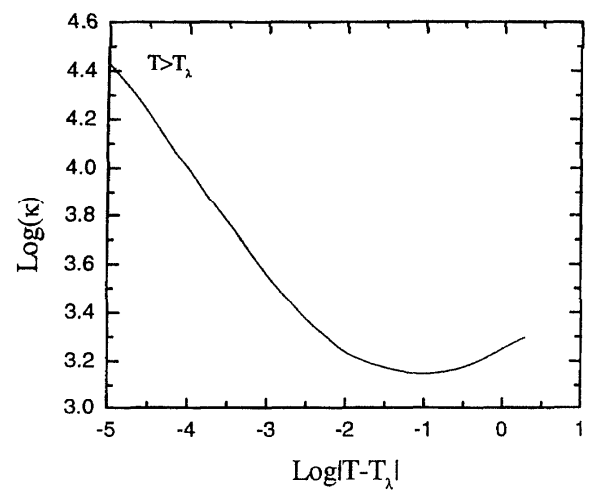

FIG. 15.2. Log plot of κ vs $|T-T_\lambda|$.

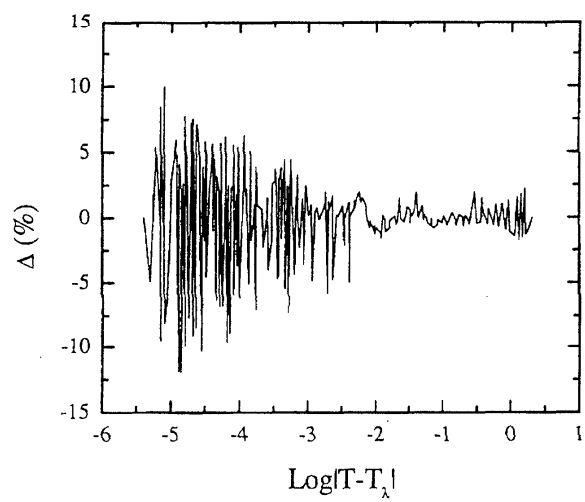

FIG. 15.3. Deviation of the database from the spline fit of thermal conductivity expressed in percent.

TABLE 15.1. Adopted database for the thermal conductivity of liquid ^{4}He

T_{90} (K)	κ (mW/cm·K)	Key	T_{90} (K)	κ (mW/cm·K)	Key	T_{90} (K)	κ (mW/cm·K)	Key	T_{90} (K)	κ (mW/cm·K)	Key
2.176 804	4.466 84E0	1	2.176 893	1.071 52E0	1	2.178 987	2.656 00E−1	3	2.219 484	1.459 52E−1	2
2.176 805	4.168 69E0	1	2.176 894	9.968 00E−1	3	2.179 120	2.584 00E−1	3	2.225 869	1.433 00E−1	4
2.176 806	3.404 00E0	4	2.176 896	9.670 00E−1	3	2.179 127	2.754 23E−1	1	2.227 626	1.444 34E−1	2
2.176 807	3.305 60E0	3	2.176 899	1.071 52E0	1	2.179 463	2.495 00E−1	4	2.228 390	1.437 00E−1	3
2.176 807	3.630 78E0	1	2.176 911	9.505 00E−1	4	2.179 606	2.452 00E−1	3	2.233 576	1.432 00E−1	3
2.176 807	3.028 50E0	3	2.176 917	8.782 00E−1	3	2.179 789	2.392 00E−1	4	2.235 797	1.434 30E−1	2
2.176 807	3.084 50E0	4	2.176 921	9.035 00E−1	3	2.179 993	2.329 00E−1	3	2.242 645	1.429 00E−1	4
2.176 808	3.097 80E0	3	2.176 925	8.853 90E−1	4	2.180 049	2.326 00E−1	4	2.242 667	1.421 00E−1	4
2.176 808	2.921 00E0	4	2.176 937	9.120 11E−1	1	2.180 067	2.312 00E−1	4	2.243 996	1.427 25E−1	2
2.176 808	2.764 10E0	3	2.176 943	8.069 00E−1	3	2.180 112	2.322 00E−1	4	2.248 106	1.426 20E−1	2
2.176 808	3.311 31E0	1	2.176 947	8.511 38E−1	1	2.180 147	2.318 00E−1	4	2.259 352	1.416 00E−1	3
2.176 809	3.019 95E0	1	2.176 960	8.039 00E−1	4	2.180 242	2.344 23E−1	1	2.259 664	1.411 00E−1	4
2.176 810	2.681 00E0	4	2.176 963	7.877 00E−1	3	2.180 253	2.293 00E−1	4	2.272 133	1.413 00E−1	3
2.176 810	2.627 70E0	3	2.176 973	8.317 64E−1	1	2.180 263	2.284 00E−1	3	2.276 905	1.406 00E−1	4
2.176 812	2.356 40E0	3	2.176 974	7.423 00E−1	3	2.180 330	2.278 00E−1	4	2.287 034	1.413 00E−1	3
2.176 812	2.396 00E0	4	2.176 978	7.554 00E−1	3	2.180 894	2.159 00E−1	3	2.289 631	1.418 14E−1	2
2.176 812	2.417 10E0	3	2.177 010	7.036 00E−1	4	2.180 918	2.160 00E−1	4	2.312 101	1.411 00E−1	4
2.176 813	2.754 23E0	1	2.177 022	7.079 46E−1	1	2.180 939	2.290 87E−1	1	2.327 997	1.423 00E−1	3
2.176 813	2.332 20E0	3	2.177 049	6.617 00E−1	4	2.181 006	2.165 76E−1	2	2.331 968	1.430 77E−1	2
2.176 814	2.321 10E0	4	2.177 050	6.500 00E−1	3	2.181 598	2.081 00E−1	4	2.331 969	1.428 70E−1	2
2.176 814	2.691 53E0	1	2.177 061	6.760 83E−1	1	2.181 708	2.055 00E−1	3	2.348 300	1.425 00E−1	4
2.176 815	2.256 10E0	3	2.177 093	6.309 57E−1	1	2.181 712	2.067 00E−1	3	2.369 424	1.436 00E−1	3
2.176 816	2.511 89E0	1	2.177 096	6.022 00E−1	4	2.182 109	2.008 00E−1	4	2.375 205	1.447 87E−1	2
2.176 816	2.100 00E0	3	2.177 126	5.778 00E−1	3	2.182 148	2.002 00E−1	3	2.385 618	1.441 00E−1	4
2.176 817	2.131 40E0	4	2.177 128	5.710 00E−1	3	2.182 532	1.957 00E−1	3	2.402 033	1.451 00E−1	3
2.176 818	2.344 23E0	1	2.177 145	5.707 00E−1	4	2.182 761	1.951 00E−1	4	2.419 273	1.464 97E−1	2
2.176 820	1.959 00E0	4	2.177 152	6.025 60E−1	1	2.183 106	1.914 00E−1	4	2.463 775	1.451 00E−1	4
2.176 820	1.935 70E0	3	2.177 169	5.888 44E−1	1	2.184 050	1.852 00E−1	3	2.479 577	1.484 00E−1	3
2.176 821	2.238 72E0	1	2.177 173	5.438 90E−1	4	2.184 066	1.864 00E−1	4	2.487 082	1.497 48E−1	2
2.176 821	1.891 50E0	3	2.177 199	5.236 00E−1	3	2.184 706	1.836 00E−1	3	2.533 625	1.519 43E−1	2
2.176 823	2.137 96E0	1	2.177 204	5.495 41E−1	1	2.184 832	1.832 27E−1	2	2.547 352	1.499 00E−1	4
2.176 823	1.892 20E0	4	2.177 230	5.118 00E−1	3	2.185 046	1.811 00E−1	4	2.562 433	1.522 00E−1	3
2.176 824	1.791 40E0	3	2.177 249	4.934 00E−1	3	2.186 237	1.766 00E−1	3	2.629 982	1.569 50E−1	2
2.176 826	1.753 10E0	3	2.177 264	5.370 32E−1	1	2.186 423	1.751 00E−1	4	2.638 230	1.556 00E−1	3
2.176 828	1.995 26E0	1	2.177 296	4.807 00E−1	3	2.186 423	1.763 00E−1	4	2.731 245	1.617 11E−1	2
2.176 829	1.695 80E0	4	2.177 305	4.805 50E−1	4	2.186 858	1.738 00E−1	3	2.731 339	1.618 12E−1	2
2.176 831	1.737 80E0	1	2.177 309	5.248 07E−1	1	2.188 800	1.691 00E−1	3	2.772 403	1.611 00E−1	3
2.176 832	1.590 30E0	3	2.177 333	4.897 79E−1	1	2.188 855	1.695 94E−1	2	2.838 136	1.666 13E−1	2
2.176 833	1.581 20E0	3	2.177 346	4.510 00E−1	3	2.189 593	1.652 00E−1	4	2.838 136	1.664 91E−1	2
2.176 833	1.737 80E0	1	2.177 441	4.570 88E−1	1	2.190 521	1.642 00E−1	3	2.920 833	1.670 00E−1	3
2.176 836	1.563 80E0	4	2.177 468	4.231 00E−1	3	2.190 657	1.655 00E−1	3	3.010 123	1.736 72E−1	2
2.176 841	1.419 10E0	3	2.177 498	4.135 00E−1	4	2.192 880	1.616 89E−1	2	3.085 844	1.725 00E−1	3
2.176 841	1.513 56E0	1	2.177 499	4.075 00E−1	3	2.192 923	1.608 00E−1	4	3.133 917	1.783 08E−1	2
2.176 842	1.418 60E0	3	2.177 500	4.178 00E−1	4	2.196 183	1.564 00E−1	4	3.300 888	1.841 15E−1	2
2.176 846	1.513 56E0	1	2.177 553	4.168 69E−1	1	2.196 908	1.567 15E−1	2	3.421 077	1.819 00E−1	3
2.176 846	1.374 00E0	4	2.177 634	3.845 83E−1	2	2.197 991	1.554 00E−1	3	3.483 569	1.896 01E−1	2
2.176 850	1.333 30E0	4	2.177 647	3.843 50E−1	4	2.199 452	1.515 00E−1	4	3.588 699	1.855 00E−1	3
2.176 851	1.445 44E0	1	2.177 649	3.823 00E−1	3	2.200 223	1.528 00E−1	3	3.643 302	1.928 95E−1	2
2.176 854	1.275 40E0	3	2.177 665	3.981 07E−1	1	2.200 336	1.533 00E−1	3	3.772 385	1.885 00E−1	3
2.176 854	1.239 90E0	3	2.177 726	3.641 00E−1	3	2.202 731	1.521 00E−1	4	3.772 385	1.885 00E−1	3
2.176 856	1.380 38E0	1	2.177 802	3.578 00E−1	3	2.203 281	1.519 39E−1	2	3.816 934	1.959 57E−1	2
2.176 863	1.190 50E0	4	2.177 824	3.562 00E−1	4	2.206 011	1.493 00E−1	4	3.909 782	1.972 74E−1	2
2.176 863	1.146 40E0	3	2.177 881	3.476 80E−1	4	2.206 011	1.495 00E−1	4	4.006 976	1.981 48E−1	2
2.176 866	1.318 26E0	1	2.177 918	3.404 00E−1	3	2.206 109	1.489 00E−1	3	4.109 041	1.988 52E−1	2
2.176 868	1.146 80E0	3	2.177 966	3.548 13E−1	1	2.209 292	1.481 00E−1	4	4.216 445	1.994 02E−1	2
2.176 872	1.258 93E0	1	2.178 076	3.238 00E−1	3	2.209 292	1.496 00E−1	4			
2.176 876	1.094 00E0	4	2.178 174	3.138 00E−1	3	2.209 381	1.483 00E−1	3			
2.176 881	1.067 30E0	3	2.178 264	3.090 30E−1	4	2.211 370	1.481 12E−1	2			
2.176 882	1.029 10E0	3	2.178 506	2.906 00E−1	3						
2.176 883	1.148 15E0	1	2.178 595	2.840 00E−1	4						
2.176 889	1.032 50E0	4	2.178 606	2.815 00E−1	3						
2.178 781	2.951 21E−1	1	2.214 110	1.467 00E−1	3						
2.178 813	2.730 00E−1	3	2.215 896	1.449 00E−1	4						
2.178 860	2.724 46E−1	2	2.216 593	1.440 00E−1	3						

TABLE 15.2. Knots and coefficients for the spline fit of thermal conductivity. This spline returns the conductivity in W/cm K

Knots	Coefficients
$K(1)=2.176\,804$	$C(1)=4.467\,094E+0$
$K(2)=2.176\,804$	$C(2)=4.079\,579E+0$
$K(3)=2.176\,804$	$C(3)=2.734\,695E+0$
$K(4)=2.176\,804$	$C(4)=1.932\,048E+0$
$K(5)=2.176\,806$	$C(5)=1.266\,490E+0$
$K(6)=2.176\,814$	$C(6)=7.907\,832E-1$
$K(7)=2.176\,841$	$C(7)=6.252\,596E-1$
$K(8)=2.176\,890$	$C(8)=4.624\,502E-1$
$K(9)=2.177\,066$	$C(9)=3.358\,313E-1$
$K(10)=2.177\,250$	$C(10)=2.422\,696E-1$
$K(11)=2.177\,640$	$C(11)=1.775\,809E-1$
$K(12)=2.178\,583$	$C(12)=1.549\,755E-1$
$K(13)=2.181\,366$	$C(13)=1.398\,263E-1$
$K(14)=2.190\,463$	$C(14)=1.416\,969E-1$
$K(15)=2.209\,490$	$C(15)=1.783\,677E-1$
$K(16)=2.290\,049$	$C(16)=1.926\,469E-1$
$K(17)=2.550\,721$	$C(17)=1.994\,140E-1$
$K(18)=4.216\,445$	
$K(19)=4.216\,445$	
$K(20)=4.216\,445$	
$K(21)=4.216\,445$	

TABLE 15.3. Recommended values of thermal conductivity in helium I

T_{90} (K)	κ (mW/cm·K)	T_{90} (K)	κ (mW/cm·K)
2.176 81	2.6979	2.75	0.1617
2.176 82	2.0756	2.80	0.1639
2.176 83	1.7337	2.85	0.1659
2.176 84	1.5163	2.90	0.1679
2.176 85	1.3609	2.95	0.1699
2.176 86	1.2451	3.00	0.1717
2.176 87	1.1600	3.05	0.1735
2.176 88	1.0968	3.10	0.1753
2.176 89	1.0463	3.15	0.1769
2.1769	1.0014	3.20	0.1785
2.1770	0.7287	3.25	0.1800
2.1775	0.4205	3.30	0.1815
2.1780	0.3337	3.35	0.1829
2.1785	0.2930	3.40	0.1843
2.1790	0.2678	3.45	0.1855
2.1795	0.2489	3.50	0.1868
2.1800	0.2354	3.55	0.1880
2.1850	0.1808	3.60	0.1891
2.1900	0.1647	3.65	0.1902
2.1950	0.1578	3.70	0.1912
2.20	0.1534	3.75	0.1922
2.25	0.1413	3.80	0.1931
2.30	0.1413	3.85	0.1940
2.35	0.1429	3.90	0.1949
2.40	0.1449	3.95	0.1957
2.45	0.1472	4.00	0.1965
2.50	0.1497	4.05	0.1972
2.55	0.1523	4.10	0.1979
2.60	0.1548	4.15	0.1986
2.65	0.1572	4.20	0.1992
2.70	0.1595		

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16. Latent Heat of Vaporization

Adopted Database

Author(s)	Key #	Range (K)
Van Dijk and Durieux	1	$1 \leq T \leq 4.2$
Berman and Poulter	2	$1.5 \leq T \leq 4.2$
Dana and Onnes	3	$2.2 \leq T \leq 5.18$
Ter Harmsel *et al.*	4	$2.2 \leq T \leq 4.9$
Theory	5	$0 \leq T \leq 0.9$

Comments and Key to Authors

(1) Reference 3. Discussion of data from Refs. 1 to 3.
(2) Reference 2. Uncertainties: $\pm 0.1\%$.
(3) Reference 1. Pioneering qualitative measurements.
(4) Reference 4. Uncertainties: Random $\pm 0.1\%$, systematic $\pm 0.03\%$.
(5) The latent heat at absolute zero is taken as $L_0 = 59.83$ J/mol. At low temperatures, $L = L_0 + \frac{5}{2}RT$ which was used to 0.9 K. Here, $R = 8.31451$ J/mol·K.

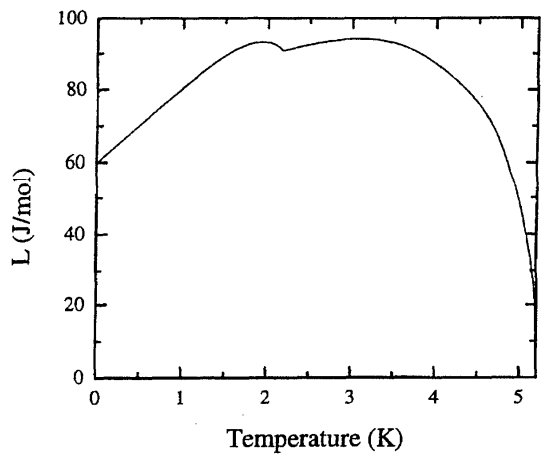

FIG. 16.1. The recommended values for the latent heat of vaporization of liquid ^{4}He as a function of temperature at the saturated vapor pressure.

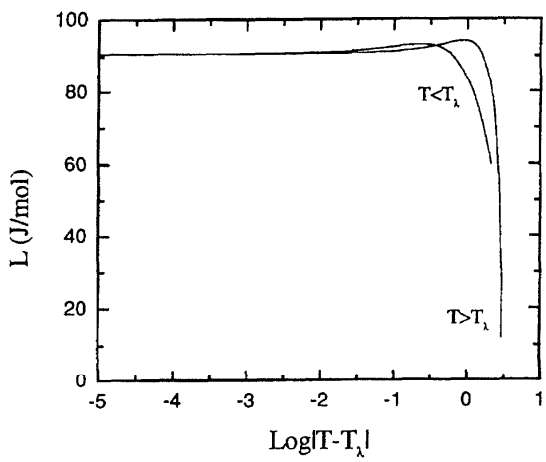

FIG. 16.2. Detail of the recommended values latent heat of vaporization about the lambda transition.

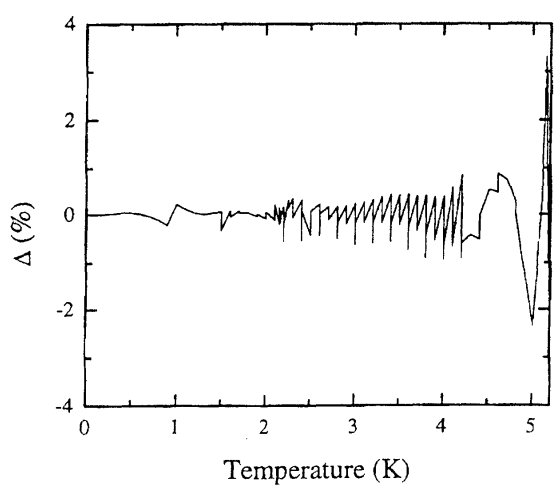

FIG. 16.3. The fractional deviation of values of the adopted database from the recommended values for the latent heat expressed in percent.

TABLE 16.1. Adopted database for latent heat of vaporization of liquid ^{4}He

T_{90} (K)	L (J/mol)	Key	T_{90} (K)	L (J/mol)	Key
0.0000	59.83	5	2.9061	93.81	1
0.1000	61.91	5	2.9061	93.81	2
0.2000	63.99	5	2.9061	94.16	4
0.3000	66.07	5	3.0063	93.91	1
0.4000	68.14	5	3.0063	93.90	2
0.5000	70.22	5	3.0063	94.70	3
0.6000	72.30	5	3.0063	94.29	4
0.7000	74.38	5	3.1064	93.90	1
0.8000	76.46	5	3.1064	93.90	2
0.9000	78.54	5	3.1064	94.30	4
1.0028	80.22	1	3.2066	93.75	1
1.1030	82.22	1	3.2066	93.78	2
1.2033	84.17	1	3.2066	94.60	3
1.3033	86.03	1	3.2066	94.19	4
1.4035	87.76	1	3.3067	93.44	1
1.5036	89.36	1	3.3067	93.50	2
1.5036	89.70	2	3.3067	93.95	4
1.6037	90.74	1	3.4069	92.99	1
1.6037	90.86	2	3.4069	93.06	2
1.7039	91.88	1	3.4069	93.90	3
1.7039	91.92	2	3.4069	93.56	4
1.8041	92.72	1	3.5070	92.42	1
1.8041	92.72	2	3.5070	92.46	2
1.9042	93.17	1	3.5070	92.99	4
1.9042	93.13	2	3.6070	91.64	1
2.0042	93.13	1	3.6070	91.67	2
2.0042	93.01	2	3.6070	92.70	3
2.1044	92.32	1	3.6070	92.27	4
2.1044	92.03	2	3.7071	90.71	1
2.1546	91.47	1	3.7071	90.71	2
2.1546	91.16	2	3.7071	91.37	4
2.1646	91.21	1	3.8071	89.53	1
2.1646	90.98	2	3.8071	89.55	2
2.1748	90.86	1	3.8071	90.70	3
2.1748	90.79	2	3.8071	90.26	4
2.1946	90.75	1	3.9070	88.17	1
2.1946	90.71	2	3.9070	88.22	2
2.2046	90.75	1	3.9070	88.96	4
2.2046	90.77	2	4.0070	86.56	1
2.2046	91.40	3	4.0070	86.62	2
2.2046	91.07	4	4.0070	87.70	3
2.2546	90.91	1	4.0070	87.44	4
2.2546	91.05	2	4.1071	84.59	1
2.3047	91.16	1	4.1071	84.76	2
2.3047	91.34	2	4.1071	85.62	4
2.3047	91.51	4	4.2071	83.55	4
2.4049	91.73	1	4.2071	82.34	1
2.4049	91.91	2	4.2071	82.46	2
2.4049	92.50	3	4.2071	83.80	3
2.4049	91.94	4	4.3070	81.17	4
2.5052	92.92	1	4.4070	78.80	3
2.5052	92.46	2	4.4070	78.41	4
2.5052	92.45	4	4.5071	75.27	4
2.6054	92.98	4	4.6072	72.00	3
2.6054	92.80	1	4.6072	71.69	4
2.6054	92.95	2	4.7073	67.58	4
2.6054	93.40	3	4.8073	62.50	3
2.7057	93.25	1	4.8073	62.65	4
2.7057	93.31	2	4.9073	56.39	4
2.7057	93.50	4	5.0072	48.10	3
2.8060	93.58	1	5.1070	36.00	3
2.8060	93.59	2	5.1569	26.80	3
2.8060	94.20	3	5.1868	16.00	3
2.8060	93.89	4	5.1958	0.00	0

TABLE 16.2. Knots and coefficients for the spline fit of the latent heat of vaporization of liquid ^{4}He

Knots	Coefficients
$K(1)=0.000\,000$	$C(1)=59.829\,83$
$K(2)=0.000\,000$	$C(2)=68.596\,42$
$K(3)=0.000\,000$	$C(3)=82.785\,76$
$K(4)=0.000\,000$	$C(4)=94.702\,15$
$K(5)=1.268\,000$	$C(5)=92.503\,50$
$K(6)=1.980\,630$	$C(6)=90.732\,36$
$K(7)=2.176\,800$	$C(7)=93.687\,27$
$K(8)=2.176\,800$	$C(8)=97.818\,99$
$K(9)=2.176\,800$	$C(9)=80.058\,77$
$K(10)=3.726\,500$	$C(10)=64.584\,54$
$K(11)=4.378\,900$	$C(11)=29.476\,41$
$K(12)=5.106\,127$	$C(12)=15.947\,83$
$K(13)=5.178\,168$	$C(13)=3.110\,047E-04$
$K(14)=5.195\,767$	
$K(15)=5.195\,767$	
$K(16)=5.195\,767$	
$K(17)=5.195\,767$	

TABLE 16.3. Recommended values of the latent heat of vaporization of liquid ^{4}He as a function of temperature at the saturated vapor pressure

T_{90} (K)	L (J/mol)	T_{90} (K)	L (J/mol)
0.00	59.83	2.35	91.71
0.05	60.87	2.40	91.98
0.10	61.91	2.45	92.24
0.15	62.95	2.50	92.50
0.20	64.00	2.55	92.74
0.25	65.04	2.60	92.97
0.30	66.08	2.65	93.18
0.35	67.13	2.70	93.38
0.40	68.17	2.75	93.56
0.45	69.21	2.80	93.71
0.50	70.24	2.85	93.85
0.55	71.28	2.90	93.96
0.60	72.31	2.95	94.05
0.65	73.33	3.00	94.11
0.70	74.35	3.05	94.14
0.75	75.37	3.10	94.14
0.80	76.38	3.15	94.11
0.85	77.38	3.20	94.05
0.90	78.37	3.25	93.94
0.95	79.36	3.30	93.80
1.00	80.33	3.35	93.63
1.05	81.30	3.40	93.41
1.10	82.26	3.45	93.14
1.15	83.21	3.50	92.84
1.20	84.14	3.55	92.48
1.25	85.06	3.60	92.08
1.30	85.97	3.65	91.63
1.35	86.87	3.70	91.13
1.40	87.73	3.75	90.58
1.45	88.56	3.80	89.97
1.50	89.35	3.85	89.31
1.55	90.09	3.90	88.59
1.60	90.77	3.95	87.82
1.65	91.38	4.00	87.00
1.70	91.91	4.05	86.13
1.75	92.36	4.10	85.20
1.80	92.72	4.15	84.22
1.85	92.98	4.20	83.19
1.90	93.13	4.25	82.11
1.95	93.16	4.30	80.98
2.00	93.07	4.35	79.80
2.05	92.80	4.40	78.57
2.10	92.27	4.45	77.27
2.18	90.75	4.50	75.86
2.18	90.75	4.55	74.32
2.18	90.75	4.60	72.59
2.18	90.75	4.65	70.64
2.18	90.74	4.70	68.44
2.18	90.74	4.75	65.94
2.18	90.74	4.80	63.11
2.18	90.74	4.85	59.90
2.18	90.73	4.90	56.28
2.18	90.73	4.95	52.22
2.18	90.73	5.00	47.67
2.20	90.87	5.05	42.59
2.25	91.15	5.10	36.95
2.30	91.43	5.15	29.34

16.1. Chronological Bibliography for Latent Heat

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17. Saturated Vapor Pressure

Between 0.65 and 5.0 K, T_{90} is defined in terms of the vapor pressure relations of ^{3}He and ^{4}He. In this range, the interpolating equation is of the form:

$$T_{90}=\sum_{i=0}^{9} A_i\left(\frac{\ln P-B}{C}\right)^i,$$

where P is the vapor pressure in Pa. For ^{4}He the values of the coefficients A_i and the constants B and C are given in Table 17.1. Although ITS-90 goes only down to 0.65 K, this equation is, in fact, valid to 0.5 K.

TABLE 17.1. Values of the coefficients in the equation for the saturated vapor pressure for liquid ^{4}He on the T_{90} scale

	^{4}He (1.25–2.1768 K)	^{4}He (2.1768–5.0 K)
A_0	1.392 408	3.146 631
A_1	0.527 153	1.357 655
A_2	0.166 756	0.413 923
A_3	0.050 988	0.091 159
A_4	0.026 514	0.016 349
A_5	0.001 975	0.001 826
A_6	−0.017 976	−0.004 325
A_7	0.005409	−0.004 973
A_8	0.013 259	0.0
A_9	0.0	0.0
B	5.6	10.3
C	2.9	1.9

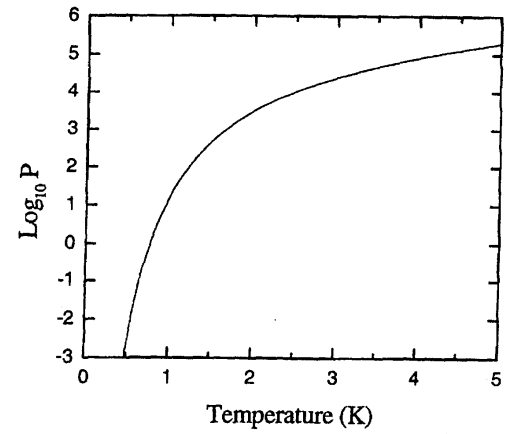

FIG. 17.2. Log_{10} of the saturated vapor pressure curve (in Pa) of liquid ^{4}He.

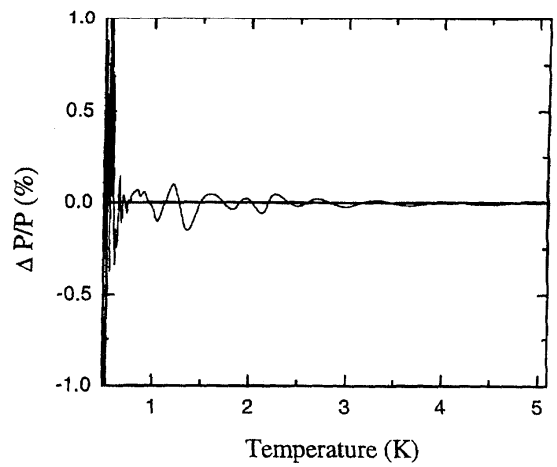

FIG. 17.3. The fractional deviation of values of the saturated vapor pressure of ^{4}He calculated with the spline from those calculated with the ITS-90 equations expressed as percent.

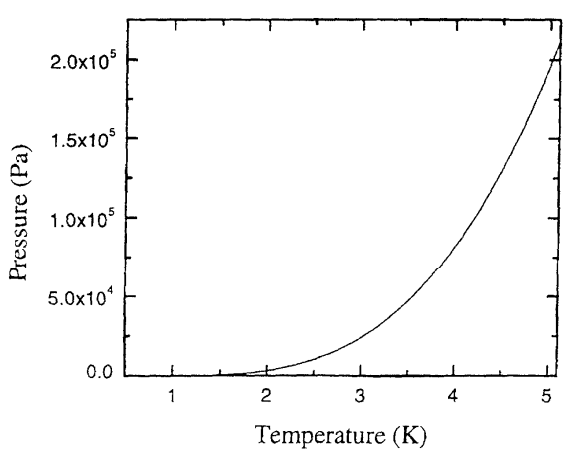

FIG. 17.1. The recommended values for the saturated vapor pressure (in Pa) of liquid ^{4}He calculated from the spline.

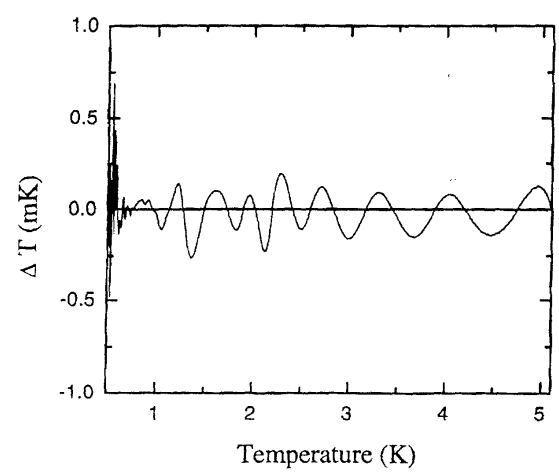

FIG. 17.4. Deviations in temperature ΔT corresponding to the deviations in pressure shown in Fig. 17.3.

Below 1.25 K, the simple equation

$$\ln(P)=i_0-\frac{L_0}{RT}+\frac{5}{2}\ln(T)$$

suffices to give P in terms of T or *vice versa*, where $L_0=59.83$ J/mol is the latent heat of evaporation at absolute zero, $i_0=\ln[(2\pi m)^{3/2}k^{5/2}h^{-3}]=12.2440$, and $R=8.314\,510$ J/mol·K.

Since the interpolating equation gives temperature as a function of pressure, we offer a spine which returns the pressure as a function of temperature. Table 17.2 gives the knots and coefficients.

TABLE 17.2. Knots and coefficients for the saturated vapor pressure of liquid ^{4}He on T_{90}

Knots	Coefficients	Knots	Coefficients
$K(1)=0.5$	$C(1)=0.001\,83797$	$K(18)=1.5$	$C(18)=1886.00$
$K(2)=0.5$	$C(2)=0.002\,50000$	$K(19)=1.7$	$C(19)=3194.00$
$K(3)=0.5$	$C(3)=0.003\,76000$	$K(20)=1.85$	$C(20)=5569.50$
$K(4)=0.5$	$C(4)=0.006\,24000$	$K(21)=2$	$C(21)=9279.10$
$K(5)=0.52$	$C(5)=0.012\,3600$	$K(22)=2.2$	$C(22)=15\,370.0$
$K(6)=0.53$	$C(6)=0.033\,0600$	$K(23)=2.5$	$C(23)=23\,480.0$
$K(7)=0.56$	$C(7)=0.093\,3900$	$K(24)=2.7$	$C(24)=37\,355.0$
$K(8)=0.6$	$C(8)=0.258\,560$	$K(25)=3$	$C(25)=57\,050.0$
$K(9)=0.65$	$C(9)=0.626\,900$	$K(26)=3.3$	$C(26)=87\,170.0$
$K(10)=0.7$	$C(10)=1.370\,25$	$K(27)=3.7$	$C(27)=132\,825$
$K(11)=0.75$	$C(11)=2.933\,05$	$K(28)=4.05$	$C(28)=179\,650$
$K(12)=0.8$	$C(12)=6.373\,18$	$K(29)=4.5$	$C(29)=211\,567$
$K(13)=0.85$	$C(13)=15.0042$	$K(30)=5.1$	
$K(14)=0.92$	$C(14)=38.0570$	$K(31)=5.1$	
$K(15)=1$	$C(15)=116.050$	$K(32)=5.1$	
$K(16)=1.1$	$C(16)=362.700$	$K(33)=5.1$	
$K(17)=1.25$	$C(17)=990.900$		

TABLE 17.3. Recommended values of the saturated vapor pressure of liquid ^{4}He

T_{90} (K)	P (Pa)	T_{90} (K)	P (Pa)
0.650	1.101$E-01$	2.900	2.063$E+04$
0.700	2.923$E-01$	2.950	2.229$E+04$
0.750	6.893$E-01$	3.000	2.405$E+04$
0.800	1.475$E+00$	3.050	2.589$E+04$
0.850	2.914$E+00$	3.100	2.783$E+04$
0.900	5.380$E+00$	3.150	2.987$E+04$
0.950	9.381$E+00$	3.200	3.201$E+04$
1.000	1.558$E+01$	3.250	3.425$E+04$
1.050	2.479$E+01$	3.300	3.659$E+04$
1.100	3.802$E+01$	3.350	3.904$E+04$
1.150	5.647$E+01$	3.400	4.160$E+04$
1.200	8.152$E+01$	3.450	4.426$E+04$
1.250	1.147$E+02$	3.500	4.705$E+04$
1.300	1.579$E+02$	3.550	4.994$E+04$
1.350	2.129$E+02$	3.600	5.296$E+04$
1.400	2.819$E+02$	3.650	5.609$E+04$
1.450	3.673$E+02$	3.700	5.935$E+04$
1.500	4.715$E+02$	3.750	6.273$E+04$
1.550	5.971$E+02$	3.800	6.625$E+04$
1.600	7.465$E+02$	3.850	6.989$E+04$
1.650	9.226$E+02$	3.900	7.366$E+04$
1.700	1.128$E+03$	3.950	7.757$E+04$
1.750	1.366$E+03$	4.000	8.162$E+04$
1.800	1.638$E+03$	4.050	8.580$E+04$
1.850	1.949$E+03$	4.100	9.013$E+04$
1.900	2.299$E+03$	4.150	9.461$E+04$
1.950	2.692$E+03$	4.200	9.923$E+04$
2.000	3.130$E+03$	4.250	1.040$E+05$
2.050	3.613$E+03$	4.300	1.089$E+05$
2.100	4.141$E+03$	4.350	1.140$E+05$
2.150	4.716$E+03$	4.400	1.193$E+05$
2.200	5.335$E+03$	4.450	1.247$E+05$
2.250	6.005$E+03$	4.500	1.303$E+05$
2.300	6.730$E+03$	4.550	1.360$E+05$
2.350	7.512$E+03$	4.600	1.419$E+05$
2.400	8.354$E+03$	4.650	1.480$E+05$
2.450	9.258$E+03$	4.700	1.543$E+05$
2.500	1.023$E+04$	4.750	1.608$E+05$
2.550	1.127$E+04$	4.800	1.674$E+05$
2.600	1.237$E+04$	4.850	1.743$E+05$
2.650	1.355$E+04$	4.900	1.813$E+05$
2.700	1.481$E+04$	4.950	1.886$E+05$
2.750	1.614$E+04$	5.000	1.960$E+05$
2.800	1.755$E+04$	5.050	2.037$E+05$
2.850	1.905$E+04$	5.100	2.116$E+05$

It is possible that one might want to know the vapor pressure on the T_{58} scale since the bulk of available data has been taken on T_{58}. Table 17.4 gives the knots and coefficients of a spline which returns the $\log_{10}$ of the saturated vapor pressure in μHg (the usual unit for T_{58}). μHg=0.1332 Pa.

17.1. Reference for T58

[1] C. F. Barenghi, R. J. Donnelly, and R. N. Hills, J. Low Temp. Phys. **51**, 319 (1983).

TABLE 17.4. Knots and coefficients of a spline which returns the $\log_{10}$ of the saturated vapor pressure on the T58 scale

Knots	Coefficients
$K(1)=1.000\,000$	$C(1)=2.079\,205$
$K(2)=1.000\,000$	$C(2)=2.290\,418$
$K(3)=1.000\,000$	$C(3)=2.641\,180$
$K(4)=1.000\,000$	$C(4)=3.067\,643$
$K(5)=1.151\,055$	$C(5)=3.434\,872$
$K(6)=1.287\,657$	$C(6)=3.744\,602$
$K(7)=1.432\,021$	$C(7)=4.016\,736$
$K(8)=1.598\,327$	$C(8)=4.269\,063$
$K(9)=1.740\,550$	$C(9)=4.453\,011$
$K(10)=1.899\,594$	$C(10)=4.558\,079$
$K(11)=2.101\,790$	$C(11)=4.624\,146$
$K(12)=2.176\,014$	$C(12)=4.760\,414$
$K(13)=2.183\,220$	$C(13)=4.923\,719$
$K(14)=2.288\,467$	$C(14)=5.078\,931$
$K(15)=2.579\,378$	$C(15)=5.188\,290$
$K(16)=2.740\,483$	$C(16)=5.287\,068$
$K(17)=2.889\,599$	$C(17)=5.379\,486$
$K(18)=3.040\,574$	$C(18)=5.463\,746$
$K(19)=3.189\,656$	$C(19)=5.543\,383$
$K(20)=3.340\,566$	$C(20)=5.618\,267$
$K(21)=3.479\,662$	$C(21)=5.689\,276$
$K(22)=3.630\,567$	$C(22)=5.756\,536$
$K(23)=3.779\,660$	$C(23)=5.820\,032$
$K(24)=3.920\,276$	$C(24)=5.861\,069$
$K(25)=4.070\,000$	$C(25)=5.880\,814$
$K(26)=4.215\,000$	
$K(27)=4.215\,000$	
$K(28)=4.215\,000$	
$K(29)=4.215\,000$	

18. Thermal Diffusivity of Helium I

The thermal diffusivity is defined as

$$D_T=\kappa/\rho C_{ps}.$$

The values of density have been given in Sec. 1 and of thermal conductivity have been given in Sec. 15. The specific heat at constant pressure needed in the Bénard convection experiments in helium I is a somewhat unusual quantity. The heating of the cell is done with the cold end thermally anchored to the main bath at the saturated vapor pressure. The liquid on the warm end expands at constant pressure, and therefore the value of specific heat must be evaluated at each temperature at the vapor pressure. We denote this quantity as C_{ps} and define it as

$$C_{ps}=C_s+TV_m\left(\frac{\partial P}{\partial T}\right)_s\alpha,$$

where V_m is the molar volume of helium, $\partial P/\partial T$ is the first derivative of the saturated vapor pressure (Sec. 17), and the thermal expansion coefficient α and specific heat have been discussed in Secs. 1 and 7. Therefore we simply give a table of recommended values for reference purposes. To convert from m^2/s to cm^2/s multiply by 10^4.

Experimental determinations of the diffusivity have been made by measuring relaxation times in a thermal conductivity apparatus and the associated relaxation time to reach steady state conditions. See M. Dingus, F. Zhong and H. Meyer, J. Low Temp. Phys. **65**, 185 (1986). The most recent and best, done with several cell spacings is shown in Fig. 19 in D. Murphy and H. Meyer, J. Low Temp. Phys. **99**, 745 (1995) and again in Fig. 12 of D. Murphy and H. Meyer, J. Low Temp. Phys **107**, 175 (1997).

TABLE 18.1. Table of recommended values of the thermal diffusivity of helium I at the saturated vapor pressure

T_{90} (K)	C_{ps} (J/mol·K)	D_T (m^2/s)
2.178	32.629	$2.801E-8$
2.180	27.100	$2.374E-8$
2.185	22.204	$2.231E-8$
2.190	19.399	$2.326E-8$
2.20	16.738	$2.510E-8$
2.25	12.301	$3.150E-8$
2.30	10.807	$3.588E-8$
2.35	10.041	$3.912E-8$
2.40	9.554	$4.176E-8$
2.45	9.272	$4.380E-8$
2.50	9.137	$4.528E-8$
2.55	9.106	$4.630E-8$
2.60	9.139	$4.699E-8$
2.65	9.200	$4.751E-8$
2.70	9.280	$4.791E-8$
2.75	9.378	$4.819E-8$
2.80	9.494	$4.836E-8$
2.85	9.627	$4.843E-8$
2.90	9.778	$4.839E-8$
2.95	9.947	$4.826E-8$
3.00	10.132	$4.804E-8$
3.05	10.334	$4.774E-8$
3.10	10.553	$4.737E-8$
3.15	10.789	$4.693E-8$
3.20	11.041	$4.643E-8$
3.25	11.309	$4.589E-8$
3.30	11.594	$4.530E-8$
3.35	11.894	$4.467E-8$
3.40	12.210	$4.401E-8$
3.45	12.542	$4.333E-8$
3.50	12.888	$4.263E-8$
3.55	13.250	$4.192E-8$
3.60	13.625	$4.120E-8$
3.65	14.015	$4.048E-8$
3.70	14.420	$3.976E-8$
3.75	14.841	$3.903E-8$
3.80	15.281	$3.830E-8$
3.85	15.743	$3.756E-8$
3.90	16.227	$3.681E-8$
3.95	16.737	$3.606E-8$
4.00	17.277	$3.529E-8$
4.05	17.848	$3.452E-8$
4.10	18.456	$3.373E-8$
4.15	19.106	$3.293E-8$
4.20	19.801	$3.211E-8$

19. Prandtl Number of Helium I

The Prandtl number is defined as

$$P_r = \nu / D_T .$$

Since the kinematic viscosity and thermal diffusivity have been given in Secs. 12 and 18, we simply list recommended values for helium I.

TABLE 19.1. Table of recommended values of the Prandtl number for helium I at the saturated vapor pressure

T_{90} (K)	P_r	T_{90} (K)	P_r
2.178	0.6139	3.15	0.5382
2.180	0.7244	3.20	0.5461
2.185	0.7746	3.25	0.5545
2.190	0.7511	3.30	0.5636
2.20	0.7184	3.35	0.5732
2.25	0.6175	3.40	0.5834
2.30	0.5649	3.45	0.5940
2.35	0.5361	3.50	0.6049
2.40	0.5164	3.55	0.6162
2.45	0.5039	3.60	0.6276
2.50	0.4969	3.65	0.6393
2.55	0.4940	3.70	0.6511
2.60	0.4936	3.75	0.6630
2.65	0.4944	3.80	0.6751
2.70	0.4958	3.85	0.6875
2.75	0.4979	3.90	0.7004
2.80	0.5007	3.95	0.7142
2.85	0.5041	4.00	0.7291
2.90	0.5082	4.05	0.7456
2.95	0.5130	4.10	0.7639
3.00	0.5183	4.15	0.7839
3.05	0.5243	4.20	0.8056
3.10	0.5309		

20. Displacement Length and Vortex Core Parameter

The displacement and healing lengths and vortex core parameter of helium II are discussed in the appendix in a review article by Barenghi, Donnelly and Vinen.[1] We give their spline fit and recommended values to the displacement length δ and recommended values of the core parameter a.

TABLE 20.1. Knots and coefficients for the displacement length δ in helium II as a function of $\log_{10}(1-T/T_\lambda)$. The displacement length is given by $10^{-8}\times 10^{S}$ where S is the value returned by the spline. The core parameter is 0.45 times the displacement length

Knots	Coefficients
$K(1)=-8.00$	$C(1)=5.565\,787$
$K(2)=-8.00$	$C(2)=4.059\,918$
$K(3)=-8.00$	$C(3)=2.634\,489$
$K(4)=-8.00$	$C(4)=0.720\,2562$
$K(5)=-1.55$	$C(5)=0.525\,8717$
$K(6)=-0.700$	$C(6)=0.320\,7715$
$K(7)=-0.530$	$C(7)=0.326\,4315$
$K(8)=0$	
$K(9)=0$	
$K(10)=0$	
$K(11)=0$	

TABLE 20.2. Table of recommended values of the displacement length δ of helium II

T_{90} (K)	δ (cm)
1.30	$3.072E-8$
1.35	$3.183E-8$
1.40	$3.303E-8$
1.45	$3.432E-8$
1.50	$3.568E-8$
1.55	$3.710E-8$
1.60	$3.855E-8$
1.65	$4.010E-8$
1.70	$4.181E-8$
1.75	$4.386E-8$
1.80	$4.649E-8$
1.85	$4.998E-8$
1.90	$5.478E-8$
1.95	$6.171E-8$
2.00	$7.247E-8$
2.02	$7.862E-8$
2.04	$8.647E-8$
2.06	$9.682E-8$
2.08	$1.111E-7$
2.10	$1.320E-7$
2.12	$1.654E-7$
2.14	$2.281E-7$
2.16	$4.040E-7$
2.162	$4.426E-7$
2.164	$4.912E-7$
2.166	$5.546E-7$
2.168	$6.416E-7$
2.170	$7.701E-7$
2.172	$9.837E-7$
2.174	$1.432E-6$
2.176	$3.373E-6$
2.1761	$3.691E-6$
2.1762	$4.095E-6$
2.1763	$4.630E-6$
2.1764	$5.377E-6$
2.1765	$6.517E-6$
2.1766	$8.536E-6$
2.1767	$1.350E-5$

20.1. Reference for Displacement Length and Vortex Core Parameter

[1] C. F. Barenghi, R. J. Donnelly, and W. F. Vinen, J. Low Temp. Phys. **52**, 189 (1983).

21. Nomenclature

Symbol or expression	Physical quantity	Unit symbol or value
m	^{4}He mass	6.646×10^{-24} g
M	Atomic weight of ^{4}He	4.0026 g/mol
k	Boltzmann's constant	$1.380\,658\times10^{-23}$ J/K
h	Planck's constant	$6.626\,075\,5\times10^{-34}$ J·s
$\hbar$	Planck's constant ($\div 2\pi$)	$1.054\,572\,66\times10^{-34}$ J·s
p	Pressure (Pascal)	Pa
V	Potential difference (volt)	V
T	Temperature (kelvin)	K
SVP	Saturated vapor pressure	
T_λ	The lambda transition at SVP	K
helium I (He I)	Liquid ^{4}He, $T > T_\lambda$	
helium II (He II)	Liquid ^{4}He, $T<T_\lambda$	
ρ, ρ_n, ρ_s	Density	g/cm^3
Q	Wave number of excitation	Å^{-1}(1 Å^{-1} = 10^{10} m^{-1})
$S(Q)$	Structure factor	
S	Entropy	J/g·K
$\mu_1, \mu_2, \mu_3, \mu_4$	Velocity of sound	m/s
α	Coefficient of thermal expansion at SVP	K^{-1}
L	Latent heat	J/mol
H	Enthalpy	J/mol
C_λ	Heat capacity at saturation pressure	J/mol·K
α_M	Molar polarizability	0.123 296 cm^3/mol
σ	Surface tension	N/m
η	Viscosity	Pa·s
$\nu = \eta/\rho$	Kinematic viscosity	m^2/s
D_T	Thermal diffusivity	m^2/s
$\mathrm{Pr} = \nu/D_T$	Prandtl number	
μ_+	Mobility of positive ions	m^2/V·s
μ_-	Mobility of negative ions	m^2/V·s
ϵ	Dielectric constant	
B, B'	Coefficients of mutual friction	
κ	Thermal conductivity	mW/cm·K
δ	Displacement length	cm

22. Acknowledgments

These tables in their present form have been in preparation for some 30 years. Many students and visitors have had a part in preparing them. From my own group there have been (in addition to my coauthor) James Brooks, whose thesis made all this possible, Thomas Wagner, James Donnelly, Christa Laursen, Ling Lui, James Mulder, Robert Riegelmann and most recently, Steve Stalp, Steve Hall and Rose Lowe-Webb. Professor Paul Roberts contributed by working out the theory of extracting thermodynamic properties when energy levels determined by neutron scattering are temperature dependent, and Dr. R. N. Hills introduced us to the advantages of cubic splines and wrote the first programs to generate and evaluate splines, which we still use today. Ron also helped develop the present tables format. Ling Lui began the enormous task of converting these tables to T_{90} as her master's thesis. This thesis explains the technical details of her method of converting to T_{90}. In doing so we had to consult a number of experts on temperature scale: Marty Durieux, Ralph Hudson, Michael Moldover, Richard Rusby and Clayton Swenson. When preliminary issues of these tables have been circulated, we have received many suggestions and new measurements. These have come from Guenter Ahlers, Dennis Greywall, Horst Meyer, Jay Maynard and Isadore Rudnick. This research was supported by the National Institute of Standards and Technology, Office of Standard Reference Data, under Grant No. NBS788, by the National Science Foundation through the years under a number of grants entitled "The Physics of Fluids" and currently, by the National Science Foundation under Grant No. DMR-9529609.

23. Appendix

23.1. Programs for Spline Evaluation

The evaluation routine for the splines used in this publication is discussed in the appendix to the paper by Donnelly, Donnelly and Hills [J. Low Temp. Phys. **44**, 471 (1981)]. A FORTRAN version was given there, and is reproduced below for convenience. In addition HP BASIC, VISUAL BASIC, and HP48G versions are also included.

23.1.1. Fortran Subroutine

N = Number of internal knots
A = Array of coefficients
Q = Array of knots
X = Independent variable value
Y = Return value of spline at point X

```
IFLAG=   Error flag: should be set to 0 before calling sub-
         routine
      SUBROUTINE EVAL (N, A, Q, X, Y, IFLAG)
      INTEGER I, J, K, JP3, JPL, IFLAG, N
      REAL A(15), D(15), Q(20), X, Y
      IF(X.LT.Q(4).OR.X.GT.Q(N+5)) GO TO 10
      I=0
      J=N+1
20    K=(I+J)/2
      IF(J-I.LE.1) GO TO 30
      IF(X.GE.Q(K+4)) GO TO 40
      J=K
      GO TO 20
40    I=K
      GO TO 20
30    JP3=J+3
      DO 50 I=J, JP3
50    D(I)=A(I)
      DO 60 K=1, 3
      JPL=JP3-K
      DO 60 I=J, JPL
60    D(I)=((X-Q(I+K))*D(I+1)+(Q(I+4)-X)*D(I))/
      (Q(I+4)-Q(I+K))
      Y=D(J)
      IFLAG=1
      GO TO 80
10    Y=0.0
      PRINT 70
70    FORMAT("X-VALUE OUT OF RANGE")
      IFLAG=2
80    RETURN
      END
```

23.1.2. Hewlett–Packard "Rocky Mountain" Basic Subroutine

```
Ncap7=   Number of knots, internal and external
K(*)=Array of knots
C(*)=Array of coefficients
X=   Value of independent variable
S=   Return value of spline at point X
Sd=  First derivative of spline at point X
Sdd=Second derivative of spline at point X
Ifail=Error flag

10    END
20    SUB Eval(Ncap7,K(*),C(*),X,S,Sd,Sdd.Ifail)
30    OPTION BASE 1
40    INTEGER J,J1,L
50    Ifail=0
60    IF (X>=K(4)) AND (X<K(Ncap7-3)) THEN
70       J1=0
80       J=Ncap7-7
90       LOOP
100         L=(J1+J)/2
110         EXIT IF J-J1<=1
120         IF X>=K(L+4) THEN
130            J1=L
140         ELSE
150            J=L
160         END IF
170      END LOOP
180      K1=K(J+1)
190      K2=K(J+2)
200      K3=K(J+3)
210      K4=K(J+4)
220      K5=K(J+5)
230      K6=K(J+6)
240      E2=X-K2
250      E3=X-K3
260      E4=K4-X
270      E5=K5-X
280      !
290      C11=((X-K1)*C(J+1)+E4*C(J))/(K4-K1)
300      Cd11=(C(J+1)-C(J))/(K4-K1)
310      C21=(E2*C(J+2)+E5*C(J+1))/(K5-K2)
320      Cd21=(C(J+2)-C(J+1))/(K5-K2)
330      C31=(E3*C(J+3)+(K6-X)*C(J+2))/(K6-K3)
340      Cd31=(C(J+3)-C(J+2))/(K6-K3)
350      C12=(E2*C21+E4*C11)/(K4-K2)
360      Cd12=(C21+E2*Cd21-C11+E4*Cd11)/
         (K4-K2)
370      Cdd12=2*(Cd21-Cd11)/(K4-K2)
380      C22=(E3*C31+E5*C21)/(K5-K3)
390      Cd22=(C31+E3*Cd31-C21+E5*Cd21)/
         (K5-K3)
400      Cdd22=2*(Cd31-Cd21)/(K5-K3)
410      S=(E3*C22+E4*C12)/(K4-K3)
420      Sd=(E3*Cd22+C22+E4*Cd12-C12)/
         (K4-K3)
430      Sdd=(E3*Cdd22+2*Cd22+E4*Cdd12
         -2*Cd12)/(K4-K3)
440   ELSE
450      S=0
460      Ifail=1
470      PRINT "ERROR DETECTED EVAL"
480      PRINT "Ncap7=";Ncap7
490      PRINT "X   =";X
500      PRINT "K(1)=";K(1)
510      PRINT "C(1)=";C(1)
520   END IF
530   SUBEND
```

23.1.3. C Subroutine

```
ncap7=Number of knots, internal and external
k=   Pointer to array of knots
c=   Pointer to array of coefficients
x=   Value of independent variable
first=  Address of variable to hold first derivative
second=   Address of variable to hold second derivative
This function returns the value of the spline at point X
#include <math.h>
double eval(ncap7,k,c,x,first,second) /* 3/09/88*/
int ncap7;
double k[],c[],x;
```

```
double*first,*second;

  int j,j1r,L,ifail;
  double s,k1r,k2,k3,k4,k5,k6,e2,e3,e4,e5,c11,cd11,c21,
  cd21,cd31,c12,cd12;
  double cdd12,c22,cd22,cdd22,c31,cdd21,sd,sdd;
  ifail=0;
  if ((x>=k[3]) && (x<=k[ncap7-3]))
  {
    j1r=-1;
    j=ncap7-7;
    L=(j1r+j)/2;
    while ((j-j1r)>1)
    {
      if (x>=k[L+4])
        j1r=L;
      else
        j=L;
      L=(j1r+j)/2;
    }
    k1r=k[j+1];
    k2=k[j+2];
    k3=k[j+3];
    k4=k[j+4];
    k5=k[j+5];
    k6=k[j+6];
    e2=x-k2;
    e3=x-k3;
    e4=k4-x;
    e5=k5-x;
    c11=((x-k1r)*c[j+1]+e4*c[j])/(k4-k1r);
    cd11=(c[j+1]-c[j])/(k4-k1r);
    c21=(e2*c[j+2]+e5*c[j+1])/(k5-k2);
    cd21=(c[j+2]-c[j+1])/(k5-k2);
    c31=(e3*c[j+3]+(k6-x)*c[j+2])/(k6-k3);
    cd31=(c[j+3]-c[j+2])/(k6-k3);
    c12=(e2*c21+e4*c11)/(k4-k2);
    cd12=(c21+e2*cd21-c11+e4*cd11)/(k4-k2);
    cdd12=2*(cd21-cd11)/(k4-k2);
    c22=(e3*c31+e5*c21)/(k5-k3);
    cd22=(c31+e3*cd31-c21+e5*cd21)/(k5-k3);
    cdd22=2*(cd31-cd21)/(k4-k3);
    s=(e3*c22+e4*c12)/(k4-k3);
    *first=(e3*cd22+c22+e4*cd12-c12)/(k4-k3);
    *second=(e3*cdd22+2*cd22+e4*cdd12-2*cd12)/
    (k4-k3);
  }
  else
  {
    s=0;
    ifail=1;
    printf ("Error detected in eval....\n");
    printf ("ncap7=%d\n",ncap7);
    printf ("x    =%lf\n",x);
  }
  return (s);
}
```

23.1.4. HP48G Series Calculator Subroutine

```
≪SWAP DUP SIZE OBJ→DROP 4-2/
 →n
  ≪SWAP
  →c t
   ≪
   IF 't≥c(n+4)AND c(2*n+1)≥=t'NOT
   THEN
    CLLCD "ERROR IN EVAL, RANGE!" 1 DISP 1
FREEZE
   ELSE
    0 'J1'STO 'n-3'→NUM 'J' STO
    WHILE
     'J-J1>1'
    REPEAT
     '(J1+J)/2'→NUM 'L' STO
     IF 't≥c(L+4+n)'
     THEN
      L 'J1' STO
     ELSE
      L 'J' STO
     END
    END
    1 6
    FOR x 'c(J + x + n)'→NUM NEXT
    →k1 k2 k3 k4 k5 k6
     ≪t k2-t k3-k4 t-k5 t-
      →e2 e3 e4 e5
      ≪'((t-k1)*c(J+1)+e4*c(J))/(k4-k1)'→NUM
       '(c(J+1)-c(J))/(k4-k1)'→NUM
       '(e2*c(J+2)+e5*c(J+1))/(k5-k2)'→NUM
       '(c(J+2)-c(J+1))/(k5-k2)'→NUM
       '(e3*c(J+3)+(k6-t)*c(J+2))/(k6-k3)'→NUM
       '(c(J+3)-c(J+2))/(k6-k3)'→NUM
       →c11 cd11 c21 cd21 c31 cd31
      ≪
       '(e2*c21+e4*c11)/(k4-k2)'→NUM
       '(c21+e2*cd21-c11+e4*cd11)/(k4-k2)'→NUM
       '2*(cd21-cd11)/(k4-k2)'→NUM
       '(e3*c31+e5*c21)/(k5-k3)'→NUM
       '(c31+e3*cd31-c21+e5*cd21)/(k5-k3)'→NUM
       '2*(cd31-cd21)/(k5-k3)'→NUM
       →c12 cd12 cdd12 c22 cd22 cdd22
       ≪ '(e3*c22+e4*c12)/(k4-k3)'→NUM 'S' STO
       '(e3*cd22+c22+e4*cd12-c12)/(k4-k3)'
       →NUM 'Sd' STO
       '(e3*cdd22+2*cd22+e4*cdd12-2*cd12)/
       (k4-k3)' →NUM
       'Sdd' STO
       { J J1 L } PURGE
       ≫
      ≫
     ≫
    ≫
  END
```

```
  ≫
 ≫
≫
```

23.1.5. Visual Basic Routine

```
'Visual Basic routine for EVAL( ). Knots and Coefficients
are
'read from a file as records.
'the (') sign marks a VB comment
Public Function Eval(NumKnots As Integer, ThisT As
Double) As Double
Dim I, J, K, JP3, JPL, IFLAG, N As Integer 'translated from
F77
        N=NumKnots-8 'number of internal knots
Dim This V As Double
ReDim D(1 To NumKnots) As Double 'the D(15) is the
Fortran code
'reality check (e.g. temperature range)...
If (ThisT<KnotVal(4)) Or (ThisT>KnotVal(N+5)) Then
        MsgBox "The Value of Temperature you entered is
ouside the fitted range. Will abort
          Eval...", 64 '(MsgBox...64) is one line
        Exit Function
End If
'temperature is in valid range ... do the work
'Again, this is translated from F77
I=0
J=N+1
StartOver: 'this is label 20 in original code
K=(I+J)/2 'label: 20
If J-I<=1 Then
        JP3-J+3
        For I=J To JP3
              D(I)=CoeffVal(I)
        Next I
        For K=1 To 3
              JPL=JP3-K
              For I=J To JPL

D(I)=((ThisT-KnotVal(I+K))*D(I+1)+(KnotVal(I+4)-Th

D(I))/(KnotVal(I+4)-KnotVal(I+K)) 'rest of above line
              Next I
        Next K
        ThisV=D(J)
        IFLAG=1
        Eval=ThisV 'return the value of spline at ThisT
        Exit Function
        End If
If (ThisT>=KnotVal(K+4)) Then
        I=K
        GoTo StartOver
End If
        J=K
        GoTo StartOver
End Function
```