Distributed Coverage Optimization in a Network of Static and Mobile Sensors

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Abstract—This paper proposes efficient schemes to increase sensing coverage in a network composed of both mobile and static sensors. The proposed deployment techniques properly assign a virtual weight to every point in the sensing field, based on the information received from the other sensors regarding their sensing radii, and the location of the static ones. The multiplicatively weighted Voronoi (MW-Voronoi) diagram is used to discover the coverage holes corresponding to different mobile sensors with different sensing ranges. According to the proposed strategies, the mobile sensors move out of the area covered by static sensors, to a point from where it can cover the coverage holes of the static sensors. As a result, under the proposed strategies coverage holes in the network are reduced. Simulation results are provided to demonstrate the effectiveness of the strategies developed in this paper.

I. INTRODUCTION

Recent advances in microelectromechanical systems (MEMS) technology have made it possible to fabricate small energy-efficient mobile sensors for both military and civilian applications. Some of the emerging applications of cooperative mobile sensor networks include biomedical engineering, tracking vehicles and environmental monitoring, to name only a few [1], [2], [3]. In a practical sensor deployment strategy, a number of important constraints need to be taken into account. Such constraints include limited sensing and communication ranges, limited energy, and limited information exchange between the sensors [4], [5]. Furthermore, the initial location of the sensors in the field may not be known *a priori* in many practical applications [6].

The problem of sensor network coverage for a group of mobile sensors following a prescribed trajectory is investigated in [7]. In [8], gradient descent algorithms are proposed for a class of utility functions to increase network coverage. A multi-objective sensor deployment and power assignment algorithm is proposed in [9], where the underlying optimization problem is decomposed into a number of scalar singleobjective subproblems, which are to be solved simultaneously. In [10], an algorithm is proposed to monitor an environmental boundary with mobile agents, where the boundary is optimally approximated by a polygon. Distributed control laws are provided in [11] for the disk-covering and sphere-packing problems, using non-smooth gradient flows. In [12] the problem of locating a finite number of sensor for detecting an event in a given region is investigated such that the maximum probability of non-detection is minimized. A decentralized, adaptive control law is described in [13] to place a network of mobile robots optimally for sensing in their environment. In [14], distributed control laws are presented to achieve convex equi-partition configuration in mobile sensor networks. Different deployment strategies are subsequently introduced to increase sensing coverage. The problem of covering an environment with a network of robots with different sensor footprints is investigated in [15]. An efficient procedure is introduced in [16] to move the sensors in such a way that the maximum error variance and extended prediction variance are minimized.

In the present work, new distributed sensor deployment strategies are introduced for a network consisting of both static and mobile sensors. The multiplicatively weighted Voronoi (MW-Voronoi) diagram is utilized to find the coverage holes, where the weight assigned to each mobile sensor is proportional to its sensing radius [17]. In the proposed strategies, namely, farthest weighted vertex (FWV) and Max-area, every static sensor broadcasts its sensing radius and location to all mobile sensors. Each mobile sensor subsequently assigns a proper virtual weight to every point in the field based on the received information. The algorithms are then performed iteratively to compute the target location for each mobile sensor.

The plan of the rest of the paper is as follows. In Section II, some preliminary material concerning the Voronoi diagram and MW-Voronoi diagrams as well as their important properties are briefly discussed. The problem is defined in Section III, where some important notations and assumptions are also presented. The proposed deployment algorithms are introduced in Section IV, as the main contribution of the paper. Simulations are given in Section V, which demonstrate the efficacy of the proposed deployment strategies. The paper concludes with a summary in Section VI.

II. BACKGROUND

Let **S** be a set of *n* distinct weighted nodes in the plane denoted by $(S_1, w_1), (S_2, w_2), \ldots, (S_n, w_n)$, where $w_i > 0$ is the weighting factor associated with S_i , for any $i \in \mathbf{n} :=$ $\{1, 2, \ldots, n\}$. It is desired now to partition the plane into *n* regions such that:

• Each region contains only one node, and

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 the nearest node, in the sense of weighted distance, to any point inside a region is the node assigned to that region.

The diagram obtained by the partitioning described above is called the *multiplicatively weighted Voronoi* (MW-Voronoi) diagram [18]. Analogous to conventional Voronoi diagram, the mathematical characterization of each region obtained by this type of partitioning is as follows:

$$\Pi_i = \left\{ Q \in \mathbb{R}^2 \mid w_j d(Q, S_i) \le w_i d(Q, S_j), \forall j \in \mathbf{n} - \{i\} \right\}$$
(1)

for any $i \in \mathbf{n}$, where $d(Q, S_i)$ is the Euclidean distance between Q and S_i . According to (1), any point Q in the *i*-th MW-Voronoi region Π_i has the following property:

$$\frac{d(Q,S_i)}{d(Q,S_j)} \le \frac{w_i}{w_j}, \quad \forall i \in \mathbf{n}, \ \forall j \in \mathbf{n} - \{i\}$$
(2)

Definition 1. A pair of nodes whose MW-Voronoi regions share one boundary curve are referred to as *neighbors*.

Definition 2. The Apollonian circle of the segment AB is the locus of all points E such that $\frac{AE}{BE} = k$ [19]. This circle is denoted by $\Omega_{AB,k}$.

To construct the *i*-th MW-Voronoi region, the Apollonian circles of the *i*-th node and other nodes need to be drown first. In other words, the Apollonian circle $\Omega_{S_iS_j}, \frac{w_i}{w_j}$ is obtained for every $S_j \in \mathbf{n} - \{i\}$. The smallest region created by these circles which contains the *i*-th node is, in fact, the *i*-th MW-Voronoi region.

The MW-Voronoi diagram is used to develop sensor deployment strategies in this paper. Each sensor has a sensing area which is a circle whose size can be different for distinct sensors. Represent each sensor in the field as a node with a weight equal to its sensing radius, and sketch the MW-Voronoi diagram. From the characterization of the MW-Voronoi regions provided in (1), it is straightforward to show that if a sensor cannot detect a phenomenon in its corresponding region, other sensors cannot detect it either. This means that in order to find the coverage holes in the network, it would suffice to compare the MW-Voronoi region of each node with its local coverage area.

III. PROBLEM FORMULATION

Consider a group of n mobile and m static sensors randomly distributed in a field, and assume that the sensors have different sensing ranges, which are circles centered at the position of the sensors. It is desired that the mobile sensors change their location in a proper distributed manner such that the total covered area (by both mobile and static sensors) increases.

Represent each mobile sensor in the field as a node and sketch the corresponding MW-Voronoi regions for all mobile sensors as described in the previous section, to cover the entire sensing field. Recall from the characterization of the MW-Voronoi diagram that the nearest sensor to any point inside a MW-Voronoi region (in the sense of weighted distance) is the one inside it. Hence, if a mobile sensor cannot detect a certain point inside its corresponding region, that point cannot be detected by any other mobile sensor in the field either. Hence, in order to identify the coverage holes (i.e. the uncovered points in the field), it suffices that each mobile sensor checks its own MW-Voronoi region to find the points it cannot cover.

In the remainder of this paper, \mathcal{V} denotes the MW-Voronoi diagram constructed based on the position and sensing radii of the mobile sensors only.

Assumption 1. Since the number of sensors in a mobile sensor network is typically large [20], it is assumed that the graph representing sensors' communication topology is connected [21]. As a result, each mobile sensor can obtain the information of other sensors, and then calculate its MW-Voronoi region accurately.

Definition 3. Consider the mobile sensor S_i with the sensing radius r_i and the corresponding MW-Voronoi region Π_i in $\mathcal{V}, i \in \mathbf{n}$. Let Q be an arbitrary point inside Π_i . The intersection of the region Π_i and a circle of radius r_i centered at Q is referred to as the *i*-th coverage area w.r.t. Q. Note that this area can be covered by any mobile or static sensor. Part of the *i*-th coverage area w.r.t. Q which is not covered by any static sensor is referred to as the *i*-th dynamic coverage area w.r.t. Q, and is denoted by $\lambda_{\Pi_i}^Q$. The *i*-th dynamic coverage area w.r.t. the location P_i of the sensor S_i is called the dynamic local coverage area of that sensor. Also, the total covered area is denoted by ψ , and the part of ψ which is not covered by any static sensor will be referred to as the *total dynamic coverage* area. Let this area be denoted by λ .

Definition 4. Consider an arbitrary point Q inside the MW-Voronoi region Π_i , $i \in \mathbf{n}$. The region inside Π_i which is not covered by any static sensor and lies outside the *i*-th coverage area w.r.t. Q referred to as the *i*-th coverage hole w.r.t. Q, and is denoted by $\theta_{\Pi_i}^Q$. The *i*-th coverage hole w.r.t. the location P_i of the sensor S_i is called the *local coverage* hole of that sensor. Also, the union of all local coverage holes in the sensing field is referred to as the *total coverage* hole, and is denoted by θ . From the properties of the MW-Voronoi diagram it is straightforward to verify that $\theta = \sum_{i=1}^n \theta_{\Pi_i}^{P_i}$.

IV. DEPLOYMENT PROTOCOLS

In this section, two efficient deployment strategies are presented for a distributed sensor network. First, every static sensor broadcasts its sensing radius and location to mobile sensors, and then each mobile sensor assigns a proper weight $\varphi(q)$ to every point in the field based on the received information. For a point q, the weight $\varphi(q)$ is a positive constant if and only if this point cannot be covered by any static sensor in the field. Otherwise, it is a negative amount whose absolute value depends on: (i) the number of static sensors that can cover q, and (ii) the distance between q and such static sensors. More precisely:

$$\varphi(q) = \begin{cases} -\sum_{i \in \mathbf{k}_q} f(q, \dot{r_i}, \dot{S_i}) & \text{if } q \text{ is covered by} \\ & \text{some static sensors,} \\ C & \text{otherwise} \end{cases}$$

where C is a positive constant, \dot{S}_i and \dot{r}_i are the position and radius of the *i*-th static sensor, respectively, and \mathbf{k}_q is the set of all static sensors that cover the point q. Furthermore, $f(q, \dot{r}_i, \dot{S}_i)$ is an appropriate decreasing function of $d(q, \dot{S}_i)$ over $[0, \dot{r}_i]$ (e.g., a candidate example, $f = \dot{r}_i - d(q, \dot{S}_i)$). The following definition will prove useful in the presentation of the proposed algorithms.

Definition 5. Consider the mobile sensor S_i with the sensing radius r_i and the corresponding MW-Voronoi region Π_i , $i \in$ **n**, and let X be an arbitrary point inside Π_i . The integral of the weight function $\varphi(.)$ over the intersection of the region Π_i and a circle of radius r_i centered at X, denoted by $C(X, r_i)$, is referred to as the *i*-th weighted coverage w.r.t. X. The mathematical characterization of the *i*-th weighted coverage w.r.t. X is as follows:

$$\beta_{\Pi_i}^X = \int_{\Pi_i \cap C(X, r_i)} \varphi(q) dq \tag{4}$$

The weighted coverage w.r.t. the location P_i of the mobile sensor S_i is called the *local weighted coverage* of that sensor.

Once the weights are assigned to all points in the field, the proposed algorithms are performed iteratively. At each iteration, every mobile sensor first broadcasts its location and sensing radius to other mobile sensors, and then constructs its MW-Voronoi region based on the similar information it receives from other mobile sensors. Then, every mobile sensor finds its destination point in its MW-Voronoi region according to the deployment strategy of each algorithm (introduced later). Once the new target location \dot{P}_i is calculated, both the weighted coverage and dynamic coverage area w.r.t. this location, i.e. $\beta_{\Pi_i}^{\dot{P}_i}$ and $\lambda_{\Pi_i}^{\dot{P}_i}$, are obtained. If this weighted coverage is greater than the previous local weighted coverage and also dynamic coverage area is increased, i.e. $\beta_{\Pi_i}^{\dot{P}_i} > \beta_{\Pi_i}^{P_i}$ and $\lambda_{\Pi_i}^{\acute{P}_i} > \lambda_{\Pi_i}^{P_i}$, then the mobile sensor moves to the new destination; otherwise, it remains in its current position. Finally, when none of the sensors' weighted coverage or dynamic coverage area in its corresponding MW-Voronoi region would be increased by a certain level, there is no need to continue the iterations. In order to terminate the algorithm in finite time, a proper coverage improvement threshold ϵ is defined such that if the increase in the dynamic local coverage area by none of the mobile sensors exceeds ϵ in an iteration, then the algorithm terminates. Note that the algorithms introduced in this paper are different only in the techniques used to find the destination point for each sensor. It can be shown that the total coverage increases under the proposed algorithms, and also the algorithms converge in finite time.

The details of the proposed strategies will be presented in the next two subsections.

A. Farthest Weighted Vertex (FWV) Strategy

In this strategy, if all vertices of the *i*-th region have negative weight (i.e., all vertices can be covered by at least one static sensor), then S_i moves toward the vertex with minimum

absolute value, up to the point from which that vertex is covered. If, on the other hand, there are one or more vertices with positive weights, then S_i moves toward the farthest one, denoted by $V_{i,fwv}$. Again, it continues moving up to the point from which it can cover that vertex. If the *i*-th region does not have any vertices, then S_i does not move and remains in its current position.

Fig. 1 shows an operational example of the FWV Algorithm. In this example, 45 mobile sensors are randomly placed in a $50m \times 50m$ flat space: 25 with a sensing radius of 3m, 10 with a sensing radius of 2.5m, 5 with a sensing radius of 3.5m, and 5 with a sensing radius of 4.5m. There are also 3 static sensors with the sensing range of 8m, 9m and 10m. The communication range of the mobile and static sensors are assumed to be 20m and 40m, respectively. In this figure, three snapshots are provided, and in each one the local coverage of both mobile sensors (yellow filled circles) and static sensors (green filled circles) are depicted. The MW-Voronoi diagram \mathcal{V} is also depicted in the figure. The initial coverage in this setup is 58.29% (first snapshot), but after the first round it increases to 68.57% (second snapshot), and finally it reaches 80.22% (third snapshot). It can be observed from Fig. 1(c) that in the final round the mobile sensors are located out of the area covered by static sensors, and that the points they cover are not fully covered by static sensors.

B. Max-area Strategy

The Max-area is a MW-Voronoi-based coverage optimization approach which aims to locally maximize the weighted coverage of each sensor inside its own region [22]. Given an MW-Voronoi region and a disk-shaped sensing pattern of a sensor, Max-area strategy finds a point inside the region which if the sensor moves there, then the intersection of the weighted area of the region and the sensing disk is maximized. In the special case, if the radius of the sensing disk is sufficiently large, then the solution to this problem is the center of the smallest enclosing circle of the region. In addition to the small sensing radius, if the field is uniformly weighted, then the optimum point is the center of largest inscribed ball inside the region, which is known as the *Chebychev center of the region*.

In general, finding the optimum point inside the MW-Voronoi region is not straightforward, and an iterative nonlinear optimization approach may be used to find it. Such an algorithm considers the intersection area noted above as an objective function, and uses the gradient of this objective function to determine the moving direction for the sensor (the objective function is guaranteed to increase if the sensor moves in that direction). In this optimization problem, the set of constraints is characterized by the boundaries of the region, and the gradient is computed iteratively to assess the proximity of the optimum point.

V. SIMULATION RESULTS

The two algorithms proposed in the previous section are applied to a flat space of size $50m \times 50m$. It is assumed that there are 3 static sensors with the sensing radii of 8m, 9m and

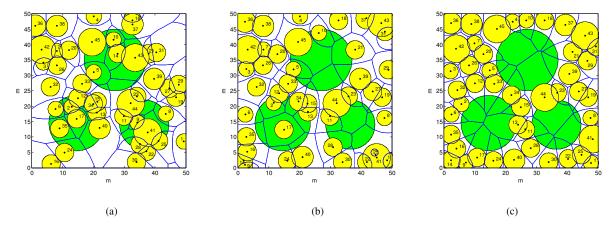


Fig. 1: Snapshots of the execution of the FWV strategy for a network of 45 nonidentical sensors with random initial distribution. (a) Initial coverage; (b) coverage after the first round, and (c) final coverage.

10m in the field. Assume also that a number of mobile sensors are randomly placed in the field. The communication range of the mobile and static sensors are assumed to be 20m and 40m, respectively. In each simulation, the algorithm terminates when none of the mobile sensors' dynamic coverage area in its corresponding MW-Voronoi region increases by more than 0.1m^2 or none of the sensors' weighted coverage increases if it makes another move. The results presented in this example for field coverage are all the average values obtained by using 20 random initial locations for the sensors.

Assume first there are 27 sensors: 15 with a sensing radius of 3m, 6 with a sensing radius of 2.5m, 3 with a sensing radius of 3.5m, and 3 with a sensing radius of 4.5m. The coverage factor (defined as the ratio of the covered area to the overall area) of the sensor network in each round is depicted in Fig. 2 for the two algorithms proposed in this paper. As it can be seen from this figure, although the FWV strategy outperforms the Max-area strategy in the first few rounds, their final coverage is approximately the same.

It is desired now to compare the performance of the two algorithms in terms of the number of mobile sensors n. To this end, consider three more setups: n=18, 36 and 45, in addition to the previous setup. Let changes in the number of identical mobile sensors in the new setups be proportional to the changes in the total number of mobile sensors (e.g., for the case of n=18 there will be 10 mobile sensors with a sensing radius of 3.5m, and 2 with a sensing radius of 4.5m). In Fig. 3, the final coverage of the algorithms is depicted for different number of sensors. It can be observed from this figure that the final coverage of both algorithms are approximately the same for various setups.

Another important means of assessing the performance of sensor deployment algorithms is the time it takes to reach the desired coverage level. This time depends on the number of rounds it takes for the sensors to provide a prescribed coverage level, as well as the sensor deployment time in each

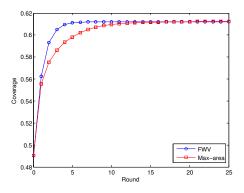


Fig. 2: Network coverage per round for 27 mobile sensors.

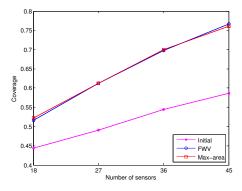


Fig. 3: Network coverage for different number of sensors using the proposed algorithms.

round. Thus, to compare the proposed methods in terms of deployment speed in reaching the desired coverage level, the stopping round and also the time duration of each round should be taken into consideration. As it can be seen from Fig 4, the number of rounds (required to meet a certain termination condition) is larger in the Max-area strategy than that in the FWV strategy. In addition, the sensor deployment time in each round for the Max-area strategy is larger than that for the FWV strategy. Therefore, the FWV algorithm is a good candidate

for field coverage as far as the deployment time is concerned.

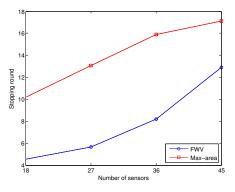


Fig. 4: The number of rounds required to reach the termination conditions for different number of sensors using the proposed algorithms.

Another important factor in the performance evaluation of different algorithms is the energy consumption of the sensors, which is directly related to the moving distance of the sensors. It can be observed from Fig. 5 that the average moving distance of the Max-area strategy is smaller than that in the FWV strategy considerably. Hence, the Max-area algorithm is a better candidate for field coverage as far as the sensors' energy consumption is concerned.

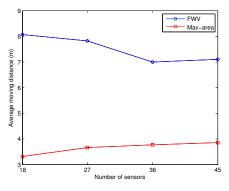


Fig. 5: The average distance each mobile sensor travels for different number of sensors, using the proposed algorithms.

VI. CONCLUSIONS

Two sensor deployment strategies are introduced in this work to increase the sensing coverage in a network of mobile and static sensors. The problem is addressed in the most general case, where the sensing radii of different sensors are not the same. A multiplicatively weighted Voronoi (MW-Voronoi) diagram is then employed to develop two distributed deployment algorithms. According to the proposed algorithms, each mobile sensor assigns a proper weight to every point in the field, based on the information it receives from static sensors. The mobile sensors then move iteratively to proper locations out of the covered area of static sensors, in such a way that coverage holes of the network are reduced. Simulations are presented to compare the performance of the proposed approaches for different number of sensors in the network. It is shown that the Max-area strategy outperforms the other method, as far as the energy consumption is concerned. On the other hand, the FWV strategy is more efficient in terms of fast time response.

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