

# Sustainable Design of Reinforced Concrete Structures through CO<sub>2</sub> Emission Optimization

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## ABSTRACT

Efforts are being made to achieve more efficient operation of buildings with a view to reducing the construction industry's contribution to energy consumption and greenhouse gas emissions. That contribution also includes the energy embodied in structures, that is, the energy consumed in the processes of extracting, manufacturing, transporting and installing construction materials (including recycled materials) and elements. In particular, in spite of the use of additives such as fly ash, reinforced concrete structures, which are large consumers of cement, are responsible for a sizable proportion of worldwide carbon emissions. These emissions can be reduced significantly through the more efficient use of both concrete and steel that can be achieved by optimization. Modern optimization tools are now available that make it possible to perform efficiently large volumes of calculations applicable to a wide variety of structural engineering problems. This study presents an optimization approach developed with a view to allowing decision-makers to balance sustainability and economic objectives. To illustrate this approach a reinforced concrete frame under gravity and lateral loads is considered in this paper. It was found

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that, depending upon the parameter values used in the calculations, the design optimized with respect to the CO<sub>2</sub> footprint yields a CO<sub>2</sub> footprint lower by 5% to 10% than the design optimized with respect to cost. The reduction can be smaller for low-structures and other structures with predominantly tension-controlled members. However, for structures whose members predominantly experience large compressive forces, such as high-rise buildings, the reduction may be more significant. This may also be true of certain pretensioned or post-tensioned concrete members. Additional research aimed at ascertaining the extent to which this is the case is warranted.

Keywords: Carbon emissions; cost optimization; CO<sub>2</sub> footprint optimization; greenhouse gas emissions; reinforced concrete; optimization

## **INTRODUCTION**

Worldwide, buildings are responsible for between 25% and 40% of total energy use (IEA 2005). According to studies carried out by the Organization for Economic Cooperation and Development (OECD), the residential and commercial building sectors are responsible for approximately 30% of primary energy consumed and of greenhouse gas emissions in OECD countries (OECD 2003).

Most efforts to reduce carbon dioxide (CO<sub>2</sub>) emissions during a given building's service life are focused on reducing the energy required to operate and maintain it (i.e., the operating energy). Measures that significantly reduce operating energy have been implemented by design professionals and the building industry (WBCSD 2008). Some of these measures, like solar roofing, are fairly radical. Others, for example reducing the energy consumption of refrigerators, are more incremental in nature. However, operating energy is only one part of the total energy

consumed by buildings. Indeed, raw material acquisition, transport, processing (manufacturing), distribution, and construction represent embodied energy. Provided that a cradle to grave system boundary is employed (Goggins et al. 2010), the calculation of embodied energy also accounts for energy used for demolition (Yohanis and Norton 2002).

The quantification of embodied energy and CO<sub>2</sub> footprint for any particular building material is an inexact science and requires a “long view” look at the entire manufacturing and utilization process (using, e.g., Life Cycle Assessment (LCA), see Goggins et al. 2010). Nevertheless, reasonable estimates of the embodied energy and CO<sub>2</sub> footprint of most common construction materials have been compiled (e.g., Alcorn, 2003; Reddy and Jagadish, 2003; CTBUH, 2009; Hammond and Jones, 2008), and will be used in this paper.

The embodied energy of building materials, including concrete, can account for a fairly significant share of the total energy use of a country. Estimates suggest that 10% of the total energy consumption in the United Kingdom and Ireland is embodied in materials (UNDP 2007). The embodied energy’s share of total life-cycle energy was estimated to vary from country to country, with estimates varying can be as low 5% and as high as 40% (Sartori and Hestnes 2007). These percentages are likely to increase as the amount of operating energy decreases (Yohanis and Norton, 2002). The energy embodied in reinforced concrete structures contributes a non-negligible part -- as much as 5 to 10 % -- of that share.

For materials used in typical concrete mixes the embodied energy and CO<sub>2</sub> footprint values per unit volume are relatively low. However, because concrete is the most widely used material in construction, their total values in reinforced concrete structures are significant. Also, unlike steel, concrete is typically not recycled for direct reuse in most structures.

For reinforced concrete structures, embodied energy or CO<sub>2</sub> footprint reduction can be achieved not only by the use of novel building materials, such as low-carbon cements and clinker substitutes (Davidovits 1993; Gartner 2004; WBCSD-IEA 2009), and recycling (Thormark 2002), but also through the reduction of the CO<sub>2</sub> footprint through the optimization of reinforced concrete (RC) structural designs. In current practice, structural designs are typically optimized for total cost or total weight. From the view point of sustainability, however, optimized designs for embodied energy or CO<sub>2</sub> footprint are desirable as well. We emphasize that the CO<sub>2</sub> footprint reduction considered in this paper concerns only the RC structure, and that the CO<sub>2</sub> footprint embodied in the RC used in a building is only a fraction of the overall CO<sub>2</sub> footprint embodied in that building. Nevertheless, the reduction of the footprint embodied in the RC is a useful contribution to the reduction of the overall footprint.

Recent work has served to demonstrate the beginning interest in considering environmental factors in the optimization of RC structures. Paya-Zaforteza et al. (2009) used an approximate optimization method based on simulated annealing to minimize two objective functions, namely (1) the total CO<sub>2</sub> emissions embodied in the structure, and (2) the total structural cost. The design variables included the type of concrete and steel reinforcement for the columns and beams of each floor, the dimensions of the cross sections of the columns and beams, and the details of the longitudinal and shear reinforcement in the columns and beams. The methodology was applied to six typical building frames with up to 4 bays and up to 8 floors. The authors considered the objective functions one at a time and found that the optimum structure from the point of view of minimum emissions is only marginally (2.8%) more expensive than the optimum structure from the point of view of minimum cost.

Villalba et al. (2010) carried out a similar study for cantilever earth-retaining walls with heights from 4 m to 6 m and again found that optimum structure from the point of view of minimum embedded CO<sub>2</sub> emissions is only marginally (1.4%) more expensive than the optimum structure from the point of view of minimum cost. Interestingly, the authors found that walls optimized for cost require on average approximately 5% more concrete than walls optimized for embedded CO<sub>2</sub> emissions, although the latter require on average approximately 2% more steel. Furthermore, the concrete grade is larger in the case of the emissions-optimized walls.

Yeo and Gabbai (2011) investigated the implications, from the point of view of cost, of optimizing a simple reinforced concrete structural member (a rectangular beam of fixed moment and shear strengths) such that embodied energy is minimized. The results indicated that optimization of structural member design for embodied energy results in decreases on the order of 10% in embodied energy at the expense of an increase on the order of 5% in cost relative to a cost-optimized member. The exact reduction in embodied energy depends significantly on the value of the cost ratio of steel reinforcement to concrete, where that ratio must take into account not only the material costs of the concrete and steel, but also construction costs such as the placement costs of concrete and the installation costs of reinforcement. Also, results show that the minimum embodied energy section has a smaller volume of concrete and a larger amount of reinforcement in comparison to the minimum cost section. These findings confirmed those of Villalba et al. (2010). To ensure that ductility is adequate for design purposes in spite of the increase in the amount of steel, the constraints in the optimization procedure include a constraint with respect to the strain in the reinforcing bars.

The main objective of this paper is to apply an optimization method based on mathematical expressions of constraints and objective functions to a simple case study – a frame

structure under gravity and lateral load – to explore the implications, from the point of view of cost, of using the total CO<sub>2</sub> footprint as the objective function to be minimized. The structure considered in the case study is a simplified model that mimics essential features of an actual frame. For comparison, the implications from the point of view of CO<sub>2</sub> footprint are also examined for the case in which the total cost is used as the objective function. For each case, the role of the ratio of the cost of steel to that of concrete on the conclusions is also examined. The research is a first step toward developing more elaborate optimization procedures, based on more than one objective function, to be used as tools for making optimal decisions entailing societal costs of carbon emissions.

## **CASE STUDY: DESCRIPTION, AND OPTIMIZATION METHODOLOGY**

### ***Problem Description***

The study considers a reinforced concrete single frame (height  $H = 4.7$  m and length  $L = 12$  m) consisting of one beam and two columns (Fig. 1). It is assumed that (i) the column has a square section with dimension  $h_c$ , (ii) the beam has height  $h_b$ , and (iii) the beam width is  $b_b = h_c$ . (The latter assumption is adopted for the sake of simplification; in practice the width of the beam is less than the width of the columns to avoid reinforcement interference.) The structure is assumed to be subjected to gravity loading uniformly applied to the beam, and wind-induced lateral loading applied at height  $H$  as a concentrated load. Based on the provisions of ASCE 7-10 Standard (ASCE 2010), and denoting the tributary width of the structure by  $B$ , the gravity load consists of dead load and live load, estimated to be  $q_D = 2B$  [kN/m<sup>2</sup>] and  $q_L = 4B$  [kN/m<sup>2</sup>], respectively. The wind loads, induced by wind speeds with a 700-year mean recurrence interval (MRI) and a 50-year MRI were assumed to be  $W_{700\text{-yr}} = 1.33BH$  [kN/m<sup>2</sup>] and  $W_{50\text{-yr}} = 0.814BH$

[kN/m<sup>2</sup>], respectively. In addition to those loads, loads due to self-weight of the members were taken into account. Three load combinations are employed:

$$\text{LC1: } 1.2D + 1.6L_l \quad (1.a)$$

$$\text{LC2: } 1.2D + L_l + W \quad (1.b)$$

$$\text{LC3: } 0.9D + W \quad (1.c)$$

where  $D$  is dead load,  $L_l$  is live load, and  $W$  is wind load. The corresponding ultimate design loads, i.e.,  $P_u$  (axial force),  $M_u$  (bending moment), and  $V_u$  (shear force), acting on the critical sections of the beam and columns are summarized in Table 1, where  $q_1$ ,  $q_2$ , and  $q_3$  denote uniformly distributed ultimate gravity loads corresponding to the load combinations (LCs) specified in Eq. (1.a), (1.b), and (1.c), respectively. The design of the frame structure for strength and serviceability is based on the ACI 318-11M Code (ACI 2011). In addition to the loads listed in Table 1, the cases were considered where the columns were subjected to additional axial compression forces (i)  $P = 3000$  kN, and (ii)  $P = 6000$  kN. (These forces are, respectively, about 40% and 80% of the full compression strength of the concrete  $f'_c A_g$ .) This was done with a view to assessing the effect of hypothetical gravity loads due to additional floors in multistory buildings.

The objective of this study is to use optimization methods to determine feasible designs that minimize cost on the one hand and CO<sub>2</sub> footprint on the other, and to provide an insight into trade-offs between cost and energy optimization in structural design.

### ***Design Variables and Parameters***

The design variables are: the beam height  $h_b$ , the column height  $h_c$ ; the total area of the longitudinal reinforcement  $A_{sb1}$  for the mid-section and  $A_{sb2}$  for the end-section of the beam; the total area of the axial reinforcement  $A_{sc}$  of the column; the spacing  $s_{b1}$  and  $s_{b2}$  of the shear

reinforcement for the mid-span and end span of the beam; and the spacing  $s_{c1}$  and  $s_{c2}$  for the mid-span and end span of the column, the area of each reinforcement bar provided for shear resistance being  $A_{vb} = 201 \text{ mm}^2$ , corresponding to a #5 reinforcing bar. The length of the end span is assumed to be  $L_{vb} = L/4$  for each end of the beam and  $L_{vc} = H/4$  for the columns, while the length of the mid span is  $L/2$  for the beam and  $H/2$  for the columns. For numerical convenience all nine variables are treated as continuous variables. The design parameters, defined as constants during the optimization process, are listed in Table 2, and represent in our opinion common values used in RC practice.

### Objective Functions

The objective functions corresponding to minimization of cost and CO<sub>2</sub> footprint are

$$f_1 = C^c \left\{ 2h_c^2 H + h_c h_b L + \left( \rho_s \frac{R_C}{100} - 1 \right) \left[ \begin{aligned} &0.7LA_{sb1} + LA_{sb2} + \frac{2L_{vb}A_{vb}}{s_b}(2h_b + h_c) \\ &+ \frac{(L - 2L_{vb})A_{vb}}{s_{\max,b}}(2h_b + h_c) + (2.2H - h_b)A_{sc} \\ &+ \frac{4L_{vc}A_{vc}}{s_c}(4h_c - 4d'_b) + \frac{2(H - 2L_{vc})A_{vc}}{s_{\max,c}}(4h_c - 4d'_b) \end{aligned} \right] \right\} \quad (2)$$

$$f_2 = E^c \left\{ 2h_c^2 H + h_c h_b L + \left( \rho_s \frac{R_{CO2}}{100} - 1 \right) \left[ \begin{aligned} &0.7LA_{sb1} + LA_{sb2} + \frac{2L_{vb}A_{vb}}{s_b}(2h_b + h_c) \\ &+ \frac{(L - 2L_{vb})A_{vb}}{s_{\max,b}}(2h_b + h_c) + (2.2H - h_b)A_{sc} \\ &+ \frac{4L_{vc}A_{vc}}{s_c}(4h_c - 4d'_b) + \frac{2(H - 2L_{vc})A_{vc}}{s_{\max,c}}(4h_c - 4d'_b) \end{aligned} \right] \right\} \quad (3)$$

where  $C^c$  and  $E^c$  are, respectively, the cost and the CO<sub>2</sub> footprint of concrete per cubic meter,  $R_C$  is the ratio of the cost of steel per 100 kg to the cost of concrete per cubic meter,  $R_{CO2}$  is the ratio of the CO<sub>2</sub> footprint of 100 kg of reinforcement steel to the CO<sub>2</sub> footprint of concrete per cubic



meter, and  $\rho_s$  is the specific mass of steel. The first and second terms in the braces (curly brackets) of Eqs. 2 and 3 are the gross volume of the concrete in the columns and the beam. The expressions between square parentheses in Eqs. 2 and 3 are the volume  $V_s$  of steel in the columns and the beam (see Fig. 1 for details). In Eqs. 2 and 3 the cost of steel in the structure is calculated as a product of the volume of steel  $V_s$  and the cost of steel per volume ( $C^c \rho_s R_C / 100$ ), while the CO<sub>2</sub> footprint of steel is calculated as a product of  $V_s$  and the CO<sub>2</sub> footprint of steel per volume ( $E^c \rho_s R_{CO2} / 100$ ). The product of the term -1 in the expression between parentheses by the expression between the square brackets (i.e., the volume of steel  $V_s$ ) changes the gross volume into the net volume of concrete.

Estimates of the CO<sub>2</sub> footprints and the costs of construction materials can vary with time and location (Alcorn 2003; Guerra et al. 2011; Paya-Zaforteza et al. 2009; Sahab et al. 2005). The values employed in this study are summarized in Table 3.

### ***Formulation of Optimization Problem and Solution Method***

Constraints for this optimization problem can be divided into two parts: constraints for serviceability and for strength. The constraints for serviceability are: maximum allowable vertical deflection ( $L/240$ , Eq. 4) and maximum allowable horizontal deflection ( $H/400$ , Eq. 5):

$$\frac{qL^4}{48E_c I_b} \left( \frac{5}{8} - \frac{1}{\beta e + 2} \right) - \frac{L}{240} \leq 0 \quad (4)$$

$$\frac{W_{50\text{-yr}} H^3}{24E_c I_c} \left( \frac{4\beta + 6e}{\beta + 6e} \right) - \frac{H}{400} \leq 0 \quad (5)$$

where  $I_b$  and  $I_c$  are the moments of inertia, respectively, for beam and column,  $E_c$  is the modulus of elasticity of concrete,  $W_{50\text{-yr}}$  is the wind-induced lateral load for MRI = 50 years as previously defined.

The constraints for the strength of the beam include flexural strengths at mid-span and at the ends of the member (Eq. 6), minimum and maximum requirements for flexural reinforcement (Eqs. 7 and 8), shear strength at the ends (Eq. 9), and minimum and maximum requirements for shear reinforcement (Eqs. 10, 11 and 12):

$$M_u - \phi_b M_n \leq 0 \quad (6)$$

$$\max\left(0.25\sqrt{f'_c}, 1.4\right) \frac{b_b d_b}{f_y} - A_{sb} \leq 0 \quad (7)$$

$$A_{sb} f_y - \frac{3}{7} 0.85 f'_c \beta_1 (h_b - d'_b) b_b \leq 0 \quad (8)$$

$$V_u - \phi_v 0.17 \lambda \sqrt{f'_c} b_b (h_b - d'_b) - \phi_v \frac{A_{vb} f_y d_b}{s_b} \leq 0 \quad (9)$$

$$s_b - \frac{A_{vb} f_y}{\max\left(\frac{\sqrt{f'_c}}{16}, \frac{1}{3}\right) h_b} \leq 0 \quad (10)$$

$$s_b \leq \begin{cases} \min\left(\frac{d_b}{2}, 600 \text{ mm}\right) & \text{for } V_s < 0.33 \sqrt{f'_c} b_b d_b \\ \min\left(\frac{d_b}{4}, 300 \text{ mm}\right) & \text{for } V_s \geq 0.33 \sqrt{f'_c} b_b d_b \end{cases} \quad (11)$$

$$\frac{A_{vb} f_y d_b}{s_b} - 4 \lambda \frac{\sqrt{f'_c} b_b d_b}{6} \leq 0 \quad (12)$$

where  $b_b$  is the beam width,  $d_b$  is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement of the beam (defined as the difference between the height  $d_b$  and the concrete cover  $d'_b$ ),  $A_{sb}$  is the flexural reinforcement, and  $\beta_1$  is the factor relating depth of equivalent rectangular compression stress block to neutral axis depth. Additional variables in the above equations have been defined in previous sections or in Table 2.

The constraints for strength of the columns are functions of combined axial forces and moments (Eq. 13), minimum and maximum requirements for axial reinforcement (Eqs. 14 and 15), shear strength (Eq. 16), and minimum and maximum requirements for ties (Eqs. 17, 18, and 19).

$$f(M_u, P_u, \phi_b M_n, \phi_c P_n) \leq 0 \quad (13)$$

$$0.01h_c^2 - A_{sc} \leq 0 \quad (14)$$

$$A_{sc} - 0.08h_c^2 \leq 0 \quad (15)$$

$$V_u - \phi_v 0.17\lambda\sqrt{f'_c} \left( 1 + \frac{P_u}{14h_c^2} \right) h_c (h_c - d'_c) - \frac{\phi_v A_{vc} f_y (h_c - d'_c)}{s_c} \leq 0 \quad (16)$$

$$s_c - \frac{A_{vc} f_y}{\max \left( \frac{\sqrt{f'_c}}{16}, \frac{1}{3} \right) h_c} \leq 0 \quad (17)$$

$$s_c - \min(h_b, 48d_{vc}) \leq 0 \quad (18)$$

$$\frac{A_{vc} f_y d_c}{s_c} - 4\lambda \frac{\sqrt{f'_c} h_c d_c}{6} \leq 0 \quad (19)$$

where  $h_c$  is the width of the square column,  $d'_c$  is the concrete cover,  $d_c$  is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement of the column,  $A_{sc}$  is the area of axial reinforcement of the column,  $s_c$  is the spacing of the shear reinforcement with area of  $A_{vc}$ , and  $d_{vc}$  is the diameter of the shear reinforcement. Equation 13 represents the analytical expression of the reinforced concrete eccentricity-dependent axial-force/bending moment interaction equation. All constraints pertaining to RC member design for serviceability and strength, that is, Eqs. (4) to (19), are based on the ASCE 7-10 Standards (ASCE 2010) and the ACI 318-11M Code (ACI 2011).

In this study the constrained nonlinear optimization solver “fmincon” from MATLAB was used. Since this solver is not guaranteed to compute the global optimum of the problem it is useful to implement the procedure multiple times, selecting for each implementation a random starting point defined by a set of values of the design variables. This avoids obtaining local minima satisfying all the constraints and increases the chances of obtaining the global minimum of the problem.

## RESULTS

It is assumed that the cost and CO<sub>2</sub> footprint of concrete are specified (Table 3). The corresponding values for steel are defined for four ratios  $R_C$  ( $R_C = 0.6, 0.8, 1.0, \text{ and } 1.2$ ) and three ratios  $R_{CO2}$  ( $R_{CO2} = 0.068, 0.078, 0.088$ ). The choice of  $R_{CO2}$  ratios was based on estimates of the CO<sub>2</sub> footprint of recycled steel (approximately 35 kg of CO<sub>2</sub> per 100 kg of steel) and the CO<sub>2</sub> footprint of concrete (approximately 400 kg to 500 kg of CO<sub>2</sub> per m<sup>3</sup> of concrete; for example,  $35/450 = 0.078$ ) (Alcorn 2003). The ratio between the cost of the cost-optimized frame and the cost of the CO<sub>2</sub>-optimized frame is denoted by  $r_{cost}$ , and the ratio between the CO<sub>2</sub>

footprint of the cost-optimized frame and the CO<sub>2</sub> footprint of the CO<sub>2</sub>-optimized frame is denoted by  $r_{co2}$ .

It was indicated earlier that three gravity loadings for the columns were considered: (i) loadings due to the self-weight of the frame, (ii) loadings due to case (i) and an additional load  $P = 3000$  kN, and (iii) loadings due to case (i) and an additional load  $P = 6000$  kN. The three cases corresponded to three qualitatively different interaction equation diagrams, corresponding to the case of relatively large, medium, and small eccentricity of the axial force. For case (i) the calculations showed that the difference between the CO<sub>2</sub> footprints inherent in the cost-optimized and CO<sub>2</sub>-optimized designs was less than 2%. Therefore the focus in this study is on cases (ii) and (iii).

#### ***Dependence upon $R_C$ and $R_{CO2}$ of difference in costs and CO<sub>2</sub> footprints***

First, an investigation was performed into the dependence of design, optimized for cost or CO<sub>2</sub> footprint, on the variation of the relative cost between concrete and steel. Figure 2 shows the dependence upon  $R_C$  of the difference in costs,  $r_{cost}$  (in percentages of totals for the frame), between the cost-optimized frame and the CO<sub>2</sub>-optimized frame, and of the difference in CO<sub>2</sub> footprints,  $r_{co2}$ , between the cost-optimized frame and the CO<sub>2</sub>-optimized frame, for  $P = 3000$  kN and  $P = 6000$  kN. The higher value of  $R_C$  corresponds to an increase in the cost of steel, the cost of concrete being fixed. Note that the differences between the CO<sub>2</sub> footprint of the cost-optimized and the CO<sub>2</sub>-optimized frame generally increase as  $P$  increases. This suggests that the optimization is more effective in reducing the frame's CO<sub>2</sub> footprint if the members are subjected to large compressive forces; in particular, the potential of optimization from this point of view would be stronger for high-rise than for low-rise RC structures.

The dependence of the results upon the assumed values of the concrete and the steel footprint is represented in Fig. 3, which shows that as the ratio  $R_{CO_2}$  increases (i.e., as the CO<sub>2</sub> footprint of steel is larger) the advantage of optimizing with respect to the CO<sub>2</sub> footprint decreases.

#### ***Dependence upon concrete compressive strength of difference in costs and CO<sub>2</sub> footprints***

In addition, an investigation was performed into the effects of concrete compression strength on the optimization effectiveness (Figure 4). As the concrete strength increases, the difference  $r_{cost}$  between the costs of the cost-optimized and the CO<sub>2</sub>-optimized frames decreases. In addition, the difference  $r_{co2}$  between the CO<sub>2</sub> footprints of the cost-optimized and the CO<sub>2</sub>-optimized frames slightly increases. Thus, for higher strength concrete, the CO<sub>2</sub> optimization is more effective, that is, it results in (a) a smaller increase in cost and (b) a larger reduction in CO<sub>2</sub> footprint, with respect to the cost optimization

#### ***Contributions of concrete and reinforcement to costs and CO<sub>2</sub> footprints***

Also considered were the contributions of the concrete and steel to the cost and to the CO<sub>2</sub>-footprint different for cost-optimized and CO<sub>2</sub>-optimized frames. Figure 5 represents the contribution of concrete and steel in the columns and beam to the total cost and CO<sub>2</sub> footprint for (a) the cost-optimized and (b) the CO<sub>2</sub>-optimized frame, for  $R_C = 0.8$ ,  $R_{CO_2} = 0.078$ ,  $f'_c = 40$  MPa, and  $P = 6000$  kN. Figures 5a and 5b show that for the cost-optimized frame the contribution of concrete to the total cost is greater than for the CO<sub>2</sub>-optimized frame, while the opposite is true of the contribution of steel. Figures 5b and 5d show that the contribution of steel to the total CO<sub>2</sub> footprint is greater for the CO<sub>2</sub>-optimized frame than for the cost-optimized frame, while the opposite is true of the concrete. Figures 5b and 5d also show that most of the contribution to the total CO<sub>2</sub> footprint is due to the concrete, rather than to the steel. Therefore,

the statement that concrete has a lower CO<sub>2</sub> footprint than steel, as has been claimed, in e.g., Struble and Godfrey (2004) and Ashley and Lemay (2008), is only valid for the footprint of concrete and new steel per unit volume; however, that statement is not applicable to the footprint inherent in the concrete and reinforcing steel used in reinforced concrete structures.

## **CONCLUSIONS**

An exploratory study was presented with the goal of assessing the potential of optimizing reinforced concrete design for sustainability with respect to CO<sub>2</sub> emissions. The optimization with respect to the CO<sub>2</sub> footprint results in an increase in the relative amount of steel within the members' cross sections; however, the requisite ductility is assured via constraints specified in the optimization process. The reduction of the CO<sub>2</sub> footprint achieved by optimizing the design to achieve minimum carbon emissions, as opposed to optimizing the design to achieve minimum cost, is of the order of 5% to 15%, depending upon the parameter values being assumed. That reduction can be smaller for low-rise structures and other structures with predominantly tension-controlled members. However, for structures whose members experience predominantly large compressive forces, such as high-rise buildings, the reduction can be significant; this may also be true of certain prestressed or poststressed concrete members. Additional research aimed at ascertaining the extent to which this is the case is warranted.

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Table 1. Internal forces on critical sections of members

Section	Case	$P_u$	$M_u$	$V_u$
beam mid	1	-	$\frac{q_1 L^2 (3\beta e + 2)}{24(\beta e + 2)}$	-
beam end	1	-	$-\frac{q_1 L^2}{6(\beta e + 2)}$	$-\frac{q_1 L}{2}$
	2	-	$-\frac{q_2 L^2}{6(\beta e + 2)} - \frac{WH}{2} \frac{3e}{\beta + 6e}$	$-\frac{q_2 L}{2} - \frac{3WH}{L} \frac{e}{\beta + 6e}$
column end	1	$\frac{q_3 L}{2} - \frac{3WH}{L} \frac{e}{\beta + 6e}$	$-\frac{q_3 L^2}{6(\beta e + 2)} + \frac{WH}{2} \frac{3e}{\beta + 6e}$	-
	2	$\frac{q_1 L}{2}$	$-\frac{q_1 L^2}{6(\beta e + 2)}$	-
	3	$\frac{q_2 L}{2} + \frac{3WH}{L} \frac{e}{\beta + 6e}$	$-\frac{q_2 L^2}{6(\beta e + 2)} - \frac{WH}{2} \frac{3e}{\beta + 6e}$	-
	4	$\frac{q_1 L}{2} + 1.2\rho_c h_c^2 H$	$-\frac{q_1 L^2}{12(\beta e + 2)}$	$\frac{q_1 L^2}{4H(\beta e + 2)}$
	5	$\frac{q_2 L}{2} + 1.2\rho_c h_c^2 H + \frac{3WH}{L} \frac{e}{\beta + 6e}$	$\frac{q_2 L^2}{12(\beta e + 2)} + \frac{WH}{2} \frac{\beta + 3e}{\beta + 6e}$	$\frac{q_2 L^2}{4H(\beta e + 2)} + \frac{W}{2}$

Note:  $\beta$  is defined as the ratio of inertia moment of the column to inertia moment of the beam,

and  $e$  as  $H/L$ .

Table 2. Design parameters and corresponding values

Parameter	Value
Concrete compressive strength	$f'_c = 40 \text{ MPa (5.8 ksi)}$
Reinforcement yield strength	$f_y = 420 \text{ MPa (60 ksi)}$
Modulus of elasticity of steel	$E_c = 2 \times 10^5 \text{ MPa (29,000 ksi)}$
Specific mass of concrete	$\rho_c = 2400 \text{ kg/m}^3 \text{ (150 pcf)}$
Specific mass of steel	$\rho_s = 7850 \text{ kg/m}^3 \text{ (490 pcf)}$
Lightweight concrete factor	$\lambda = 1 \text{ (for normal weight)}$
Strength reduction factor for shear	$\phi_v = 0.75$
Strength reduction factor for flexure	$0.817 \leq \phi_b \leq 0.9$
Strength reduction factor for axial force	$0.65 \leq \phi_s \leq 0.9$
Maximum useable compression strain in the concrete	$\varepsilon_{cu} = 0.003$
Tributary width	$B = 7 \text{ m}$
Concrete cover (includes radius of fictitious bar having area $A_s$ )	$d' = 65 \text{ mm}$
Area of shear reinforcement (#5)	$A_{vb} = A_{vc} = 201 \text{ mm}^2$
Diameter of shear reinforcement (#5)	$d_{vc} = 15.875 \text{ mm}$

Table 3. CO<sub>2</sub> footprint and cost of concrete and reinforcing steel

Material	CO <sub>2</sub> footprint (Alcorn 2003)	Cost
Concrete ( $f'_c = 30$ MPa)	376 [CO <sub>2</sub> kg/m <sup>3</sup> ]	130 [US \$/m <sup>3</sup> ]
Concrete ( $f'_c = 40$ MPa)	452 [CO <sub>2</sub> kg/m <sup>3</sup> ]	135 [US \$/m <sup>3</sup> ]
Steel, recycled ( $f_y = 420$ MPa)	35.2 [CO <sub>2</sub> kg/100 kg]	108 [US \$/100kg]

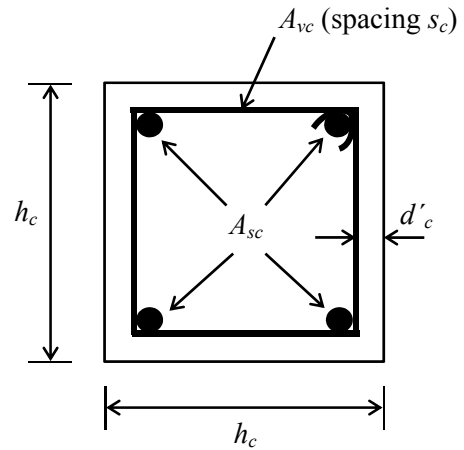
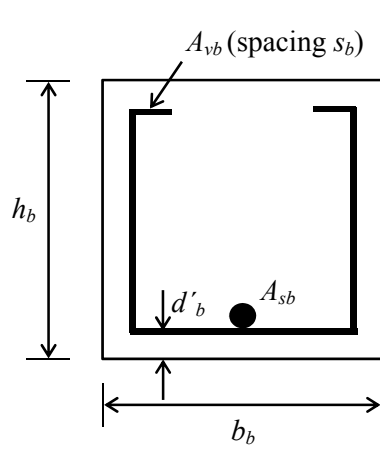
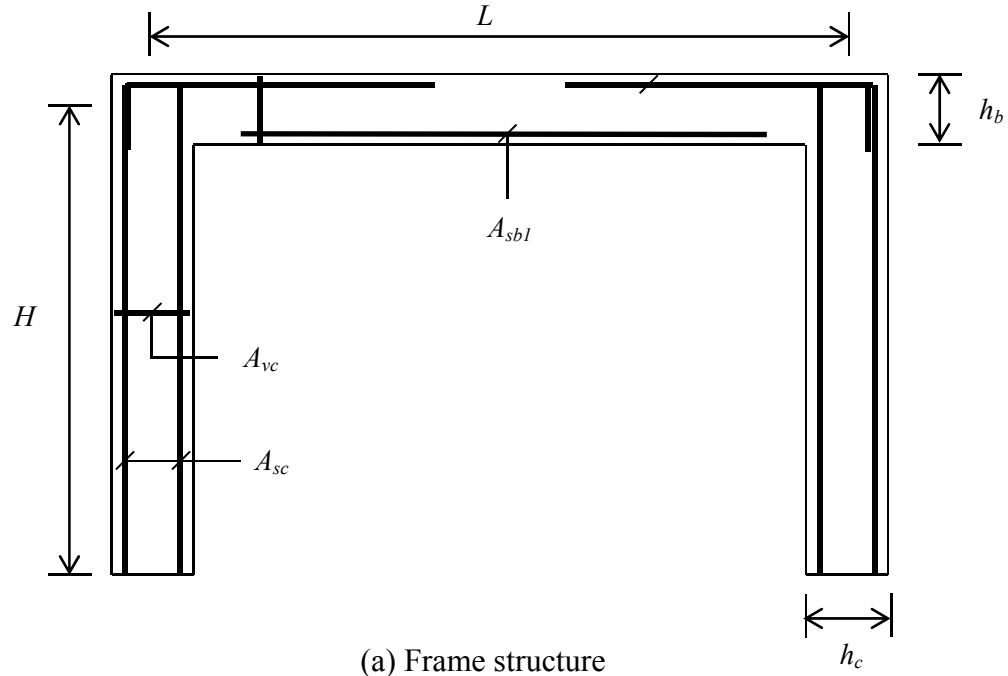


Fig. 1. Schematic of a frame structure and section details.

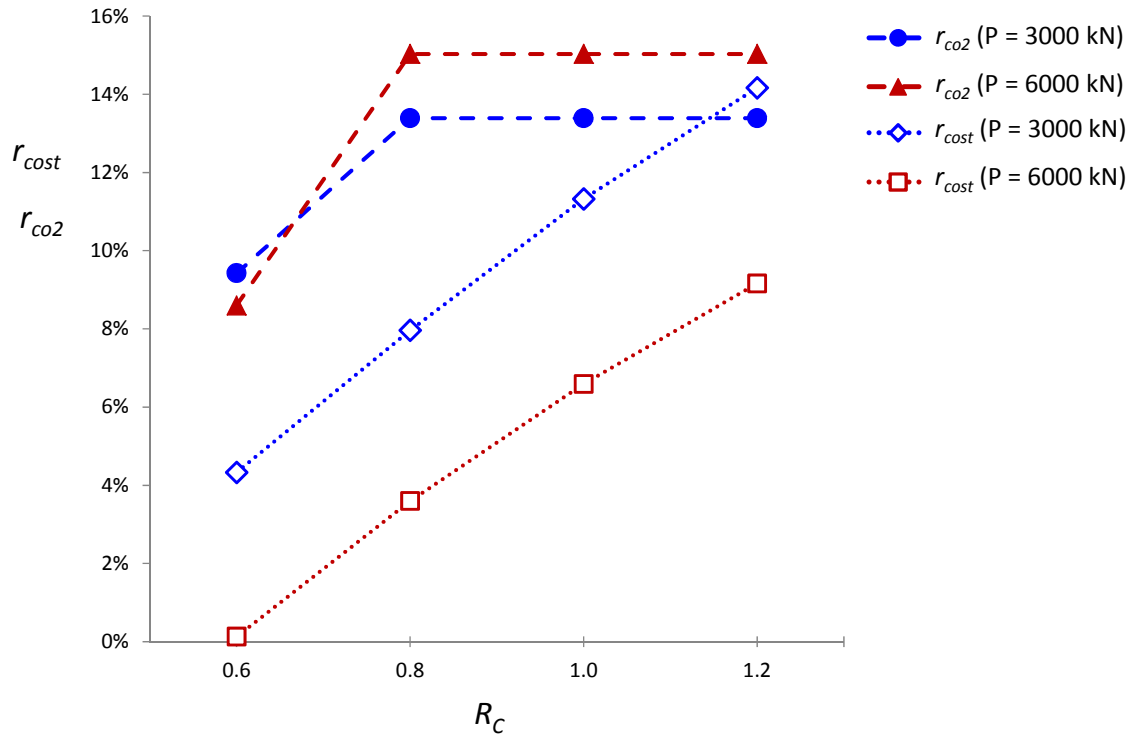


Fig. 2. Dependence upon  $R_C$  of difference in costs and CO<sub>2</sub> footprints (in percentages of totals for the frame) between cost-optimized frame and CO<sub>2</sub>-optimized frame, for  $R_{CO2} = 0.078$  and  $f'_c = 40$  MPa.

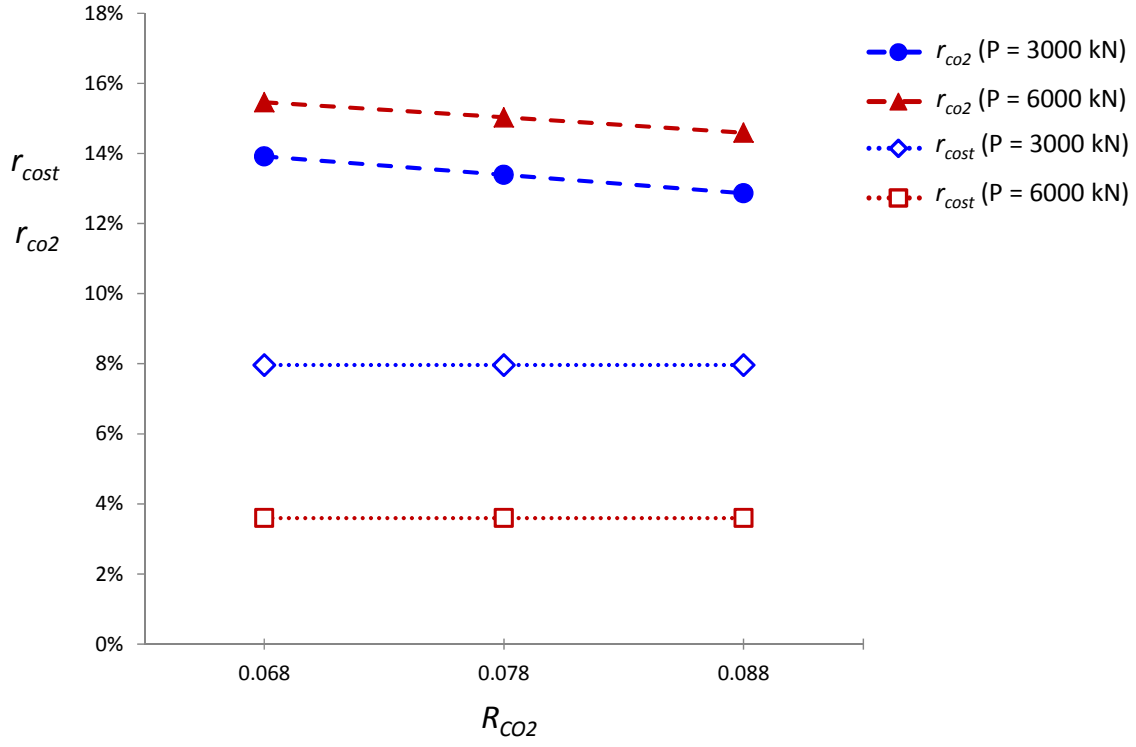


Fig. 3. Dependence upon  $R_{CO2}$  of difference in costs and CO<sub>2</sub> footprints (in percentages of totals for the frame) between cost-optimized frame and CO<sub>2</sub>-optimized frame, for  $R_C = 0.8$  and  $f_c = 40$  MPa.

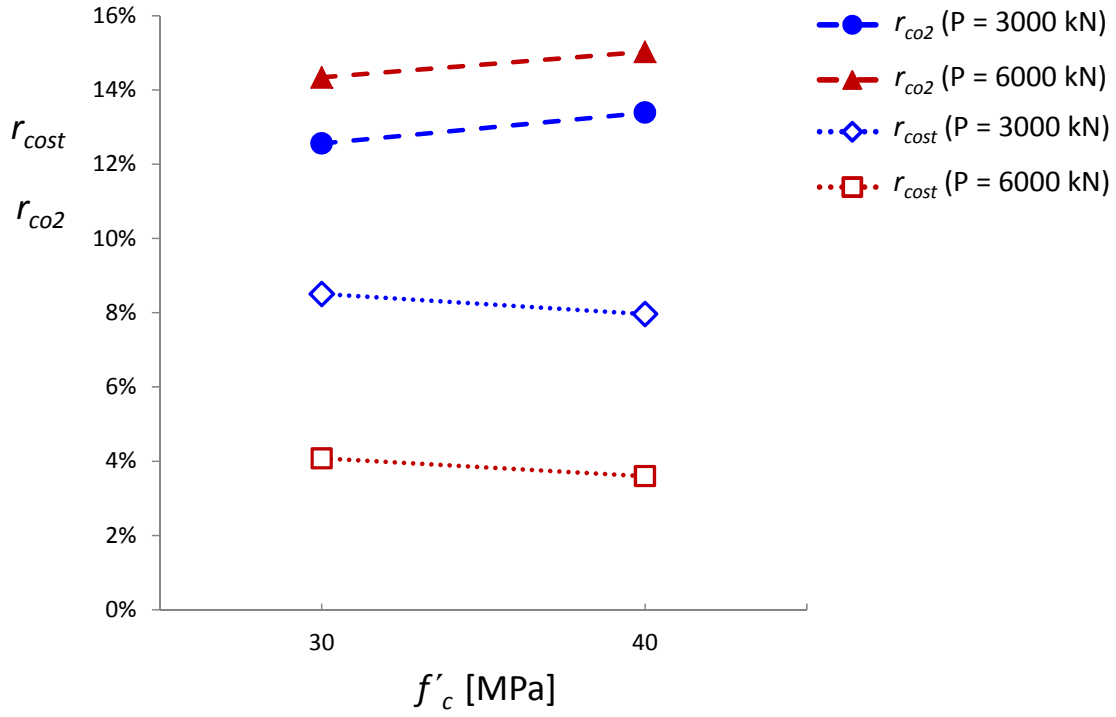
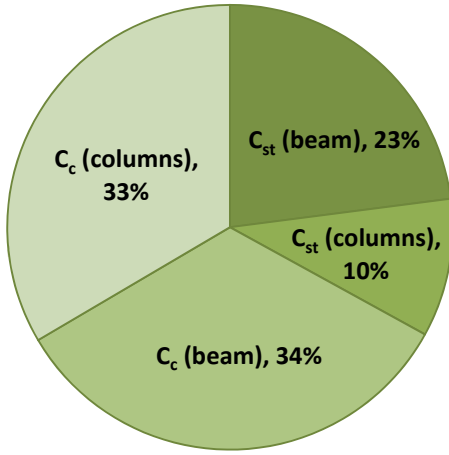
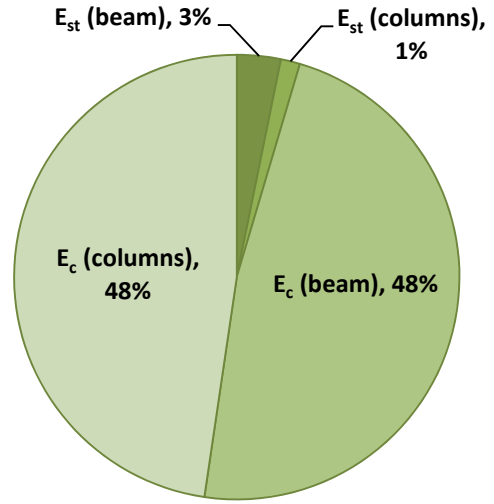


Fig. 4. Dependence upon  $f'_c$  of difference in costs and CO<sub>2</sub> footprints (in percentages of totals for the frame) between cost-optimized frame and CO<sub>2</sub>-optimized frame, for  $R_C = 0.8$  and  $R_{CO_2} = 0.078$ .

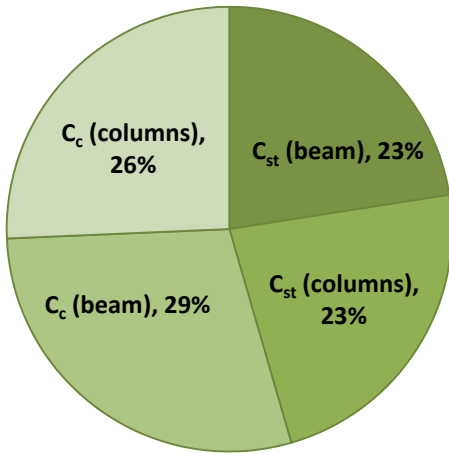




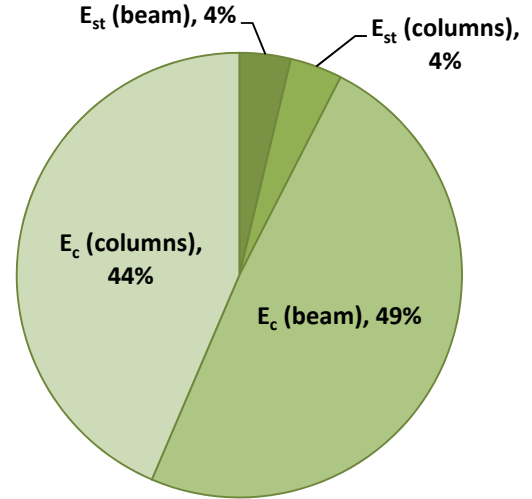
(a) Cost ratio  
for cost-optimized frame



(b) CO<sub>2</sub> footprint ratio  
for cost-optimized frame



(c) Cost ratio  
for CO<sub>2</sub>-optimized frame



(d) CO<sub>2</sub> footprint ratio  
for CO<sub>2</sub>-optimized frame

Fig. 5. Contributions of concrete and reinforcement to the total costs and total CO<sub>2</sub> footprints (notations:  $C_c$  = cost of concrete;  $C_{st}$  = cost of reinforcing steel;  $E_c$  = CO<sub>2</sub> footprint of concrete;  $E_{st}$  = CO<sub>2</sub> footprint of reinforcing steel), for  $R_C = 0.8$ ,  $R_{CO_2} = 0.078$ ,  $f'_c = 40$  MPa, and  $P = 6000$  kN.