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Generation of optical coherent-state superpositions by number-resolved photon subtraction from the squeezed vacuum

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We have created heralded coherent-state superpositions (CSSs) by subtracting up to three photons from a pulse of squeezed vacuum light. To produce such CSSs at a sufficient rate, we used our high-efficiency photon-number-resolving transition edge sensor to detect the subtracted photons. This experiment is enabled by and utilizes the full photon-number-resolving capabilities of this detector. The CSS produced by three-photon subtraction had a mean-photon number of $2.75^{+0.06}_{-0.24}$ and a fidelity of $0.59^{+0.04}_{-0.14}$ with an ideal CSS. This confirms that subtracting more photons results in higher-amplitude CSSs.

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A coherent state of the electromagnetic field is often considered the most classical-like pure state, but a superposition of two coherent states with opposite phases has interesting quantum features. For example, coherent-state superpositions (CSSs) can be exploited for performing quantum information tasks and high-precision measurements. CSSs are also of fundamental interest: When they contain many photons, they are superpositions of macroscopically distinguishable states often called Schrödinger cat states. Schrödinger's Gedanken experiment of 1935 described a cat apparently held in a superposition of alive and dead states [1], but many researchers now use Schrödinger cat to refer to a quantum state that is a superposition of two highly distinguishable classical states such as a CSS of high amplitude or mean number of photons [2]. CSSs have been prepared in traveling optical modes with a mean of up to 2.0 optical photons by heralding [3–7]. With sufficiently high-quality and well-characterized CSSs, one can, in principle, quantum compute using simple linear optical components and homodyne measurements [8]. Less ambitiously, they can serve as flying qubits for quantum communication. In addition to potentially simple processing, advantages of CSSs in traveling optical modes include fast linear manipulations, transport over large distances, robustness if loss is controlled, and simple conversion to entangled optical states, all at room temperature.

The CSSs that we discuss here are superpositions of two coherent states $|\pm\alpha\rangle$ of a single mode of light, where $+\alpha$ and $-\alpha$ are the states' complex mode amplitudes. Our experiments aim to prepare two special instances of these CSSs: the odd and even CSSs defined as the superpositions $|-\alpha\rangle \pm |\alpha\rangle$ (unnormalized). These are distinguished by having only even (+) or odd (−) numbers of photons. For $|\alpha| \gg 1$, the states' mean number of photons $\langle n\rangle$ is approximately $|\alpha|^2$. Two quality measures for experimental CSSs are the fidelity of the created state with the nearest ideal CSS and the magnitude of the amplitude of this ideal CSS. There are two reasons to aim for large-amplitude CSSs. The first is that to be useful for super-resolution metrology, the probability $p_0 = 1 - \exp(-2|\alpha|^2)$ with which the superposed coherent states can be distinguished must be close to one. To achieve $p_0 > 0.99$, requires $|\alpha| > 1.52$. The second is that a minimum size estimated as $|\alpha| > 1.2$ is required for fault-tolerant quantum computing [9]. Because operation close to the lower bound is unrealistic due to excessive resource requirements, we are motivated to produce bigger CSSs. Similarly, high fidelity is required to avoid excessive overheads for eliminating unwanted errors due to deviations from an ideal CSS. The highest fidelity CSS achieved so far has $|\alpha| = 1.1$ and a fidelity $F = 0.76$ [7], while the largest has an effective size of $|\alpha| = 1.4$ and fidelity $F = 0.60$ [7]. We have created CSSs with amplitudes and fidelities of $|\alpha| = 1.76^{+0.02}_{-0.19}$ and $F = 0.59^{+0.04}_{-0.14}$, and $|\alpha| = 1.32^{+0.01}_{-0.02}$ and $F = 0.522^{+0.004}_{-0.010}$. Unlike the experiment reported in Ref. [7], our CSSs are generated in pulsed rather than continuous-wave mode. Pulsed operation is required for many applications to avoid the effects of light in neighboring modes in subsequent manipulations and measurements of the states.

To create the CSSs, we used the photon subtraction scheme depicted in Fig. 1. A squeezed vacuum state is prepared and sent through a weakly reflecting beam splitter. Reflected photons that are detected herald an approximate CSS in the transmitted beam. Because higher-amplitude and fidelity CSSs can be created by heralding on detecting multiple photons at once [10,11], we used a photon-number-resolving transition edge sensor (TES) [12,13] for subtracting two or three photons. The TES used in our experiment has an efficiency of 85% ± 2% and can resolve up to ten photons. This enabled obtaining higher-amplitude CSS at practical rates. We also subtracted one and two photons using avalanche photodiodes (APDs) for comparison.

For the experiments, we used a cavity-dumped 861.8-nm laser with transform-limited pulses of 140 fs (typical), pulse energies of 40 nJ, and a repetition frequency of 548 kHz. A fraction of each pulse with $> 10^9$ photons was used as the local oscillator (LO) in the homodyne detector. The rest pumped a temperature-tuned 150-μm $KNbO_3$ crystal to generate a second-harmonic pump pulse (efficiency 25%) for the OPA shown in Fig. 1. The OPA consists of a temperature-tuned

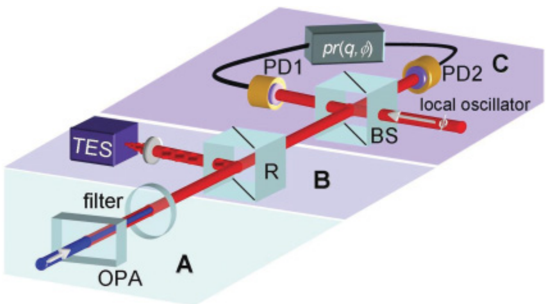

FIG. 1. (Color online) Scheme for optical CSS creation. An up-converted laser pulse enters an optical parametric amplifier (OPA) to create a squeezed vacuum state in section (A). After spectral filtering, this state is sent to a weakly reflecting beam splitter R in (B). Reflected photons that are detected herald a CSS emerging from R into (C). Its quadratures are measured by homodyne detection in (C).

200-μm long $KNbO_3$ crystal. We determined that the squeezed vacuum state generated can be modeled as a pure squeezed state with minimum quadrature variance $V_0 = -6.8$ dB subjected to a loss of $\gamma_s = 0.36$. We define the squeezing purity as $\eta_s = 1 - \gamma_s$. We used a variable beam splitter (R in Fig. 1) made with a half-wave plate and a polarizing beam splitter and configured to send from 2.5% (one-photon subtraction) to 20% (three-photon subtraction) of the light to the photon subtraction arm. Photons in this arm were spectrally filtered by a fiber Bragg grating with a bandwidth of 1.5 nm in a polarization-based circulator before being coupled to the photon detector or counter. The other arm of the variable beam splitter delivers the heralded CSS to a conventional homodyne detector for measuring the quadrature at the phase of the LO. The CSS temporal shape is significantly different from that of the original pump due to the large mismatch in group velocity in our $KNbO_3$ crystals. To compensate, we expanded the temporal width of the LO by up to a factor of 2 with a pulse-shaping setup [14]. The phase of the LO was adjusted by a piezomounted mirror displaced at a frequency of 2.75 Hz with a sawtooth profile to obtain a complete phase-space measurement of the CSS. Further technical details are in Ref. [15].

We reconstructed the states produced by photon subtraction immediately after the subtracting beam splitter by maximum-likelihood quantum state estimation as discussed in Ref. [16]. For this purpose, we considered the homodyne measurement setup including all of its losses such as those associated with the initial beam splitter and imperfect spatial mode matching the LO as a monolithic lossy quadrature measurement. This requires knowing the loss γ_h, which we experimentally determined to be $\gamma_h = 15\% \pm 2\%$. The uncertainty in γ_h propagates to an uncertainty in the reported CSS parameters. In particular, the fidelities differ by up to ± 0.02 if the boundary values for γ_h are used. However, the main uncertainty in our state reconstructions is due to finite sample statistics. We estimated this statistical uncertainty by parametric-bootstrap resampling [17]. We report inferred values, such as fidelities in the form $F^{U-F}_{-(F-L)}$, where F is the fidelity of the maximum-likelihood estimate from the experiment's data, U is the 84th percentile of the fidelities of the states estimated from resampled data sets, and L is the 16th percentile. We obtained 100 resampled data sets for one- and two-photon subtraction and 1000 for three-photon subtraction. There is a significant bias toward more mixed states in the resampling procedure, and the amount of bias increases with the purity of the state from which samples are generated. We did not correct for this bias in our reconstruction of the states, but note that it suggests that the true fidelities are above the reported ones.

TABLE I. Results for the one-, two-, and three-photon subtraction experiments. $W_{\min}$ and $\langle n\rangle$ are the minimum value and the mean-photon number of the reconstructed state, respectively. F is the fidelity of the reconstructed state compared to a theoretical CSS with amplitude $|\alpha|$.

	$W_{\min}$	$\langle n\rangle$	F	$\|\alpha\|$
One-photon experiment:				
APD	$-0.041^{+0.009}_{-0.001}$	$1.96^{+0.05}_{-0.04}$	$0.522^{+0.004}_{-0.010}$	$1.32^{+0.01}_{-0.02}$
Ref. [3]	-0.13 ± 0.01		0.70	0.89
Two-photon experiments:				
APDs	$-0.018^{+0.002}_{-0.002}$	$2.34^{+0.06}_{-0.05}$	$0.523^{+0.022}_{-0.014}$	$1.30^{+0.04}_{-0.02}$
TES	$-0.010^{+0.001}_{-0.001}$	$1.89^{+0.05}_{-0.06}$	$0.531^{+0.017}_{-0.018}$	$1.16^{+0.04}_{-0.04}$
Ref. [7]			0.60	1.4
Three-photon experiment:				
TES	$-0.116^{+0.073}_{-0.019}$	$2.75^{+0.06}_{-0.24}$	$0.59^{+0.04}_{-0.14}$	$1.76^{+0.02}_{-0.19}$

The reconstructed states have well-defined average photon numbers $\langle n\rangle$. The reported amplitudes are those of the nearest even or odd CSS, which is found by maximizing the fidelity with respect to the reconstructed state. The reported fidelities are these maximized ones. Table I summarizes our results.

Figure 2 shows the reconstructed Wigner function for a one-photon-subtracted state heralded by an APD. The quantum character of this state can be identified by its negativity near the origin of the Wigner function, whose minimum has a value of $W_{\min} = -0.041^{+0.009}_{-0.001}$. The state's fidelity is $F = 0.522^{+0.004}_{-0.010}$

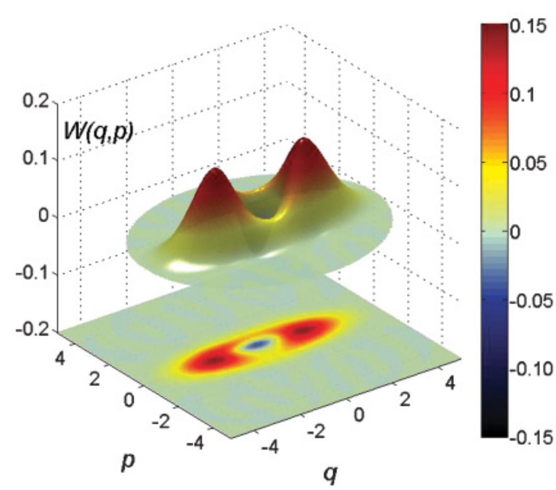

FIG. 2. (Color online) Maximum-likelihood estimate of an odd CSS generated by one-photon subtraction from a squeezed vacuum. The graph shows the unitless Wigner function value $W(q,p)$ as a function of the unitless quadratures of the electromagnetic field.

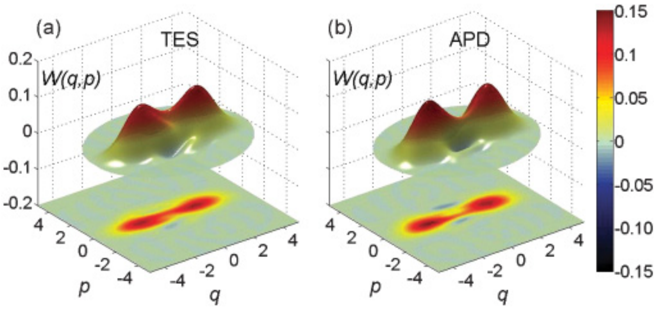

FIG. 3. (Color online) Wigner functions of the maximum-likelihood estimates of even CSS created by two-photon subtraction and heralded with (a) one transition edge sensor and (b) two multiplexed APDs.

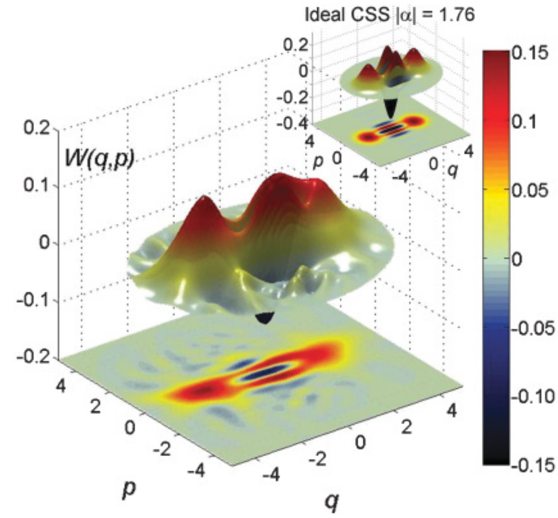

FIG. 4. (Color online) Maximum-likelihood estimate of an odd CSS after the subtraction of three photons from a squeezed vacuum. The reconstructed state has a fidelity of $F = 0.592^{+0.036}_{-0.142}$ with a CSS of amplitude $|\alpha| = 1.76^{+0.02}_{-0.19}$. Inset, Wigner function of an ideal odd CSS with $|\alpha| = 1.76$.

with respect to an odd CSS with $|\alpha| = 1.32^{+0.01}_{-0.02}$. This fidelity is higher than the maximum fidelity of $F = 0.487$ that any coherent state can have with the $|\alpha| = 1.32$ odd CSS. (Note that this is also the highest fidelity that any mixture of coherent states can have. The maximum fidelity of a coherent state with a CSS depends on the CSS's $|\alpha|$ and whether the CSS is even or odd. As $|\alpha|$ increases, this fidelity approaches 0.5 from above for even CSSs but from below for odd CSSs.) The amplitude of the CSS is notably larger than the $|\alpha| = 0.89$, $F = 0.70$, $W_{\min} = -0.13$ state described in Ref. [3]. The lower fidelity is primarily due to a lower squeezing purity η_s in our experiment.

We obtained even CSSs by two-photon subtraction. We performed two experiments, the first used a TES, the second used two APDs at the two outputs of a 50:50 beam splitter. For the APDs, coincidence heralds the presence of two photons in the subtraction arm. The reconstructed states are shown in Fig. 3. The TES measurement yielded a smaller CSS ($|\alpha_{\mathrm{TES}}| = 1.16^{+0.04}_{-0.04}$) than the APD measurement ($|\alpha_{\mathrm{APD}}| = 1.30^{+0.04}_{-0.02}$). The fidelities are $F_{\mathrm{TES}} = 0.531^{+0.017}_{-0.018}$ and $F_{\mathrm{APD}} = 0.523^{+0.022}_{-0.014}$. For comparison, the maximum fidelity of coherent states with an $|\alpha| = 1.16$ ($|\alpha| = 1.30$) even CSS is 0.552 (0.522). Earlier studies [7] showed the continuous-wave generation of even CSSs with $|\alpha| = 1.41$ and $F = 0.60$.

The fidelity of the heralded CSSs is affected not only by low squeezing purity, but also by unwanted photons not matching the LO mode but still visible to the detectors. In addition to stray light (which can, in principle, be controlled), such photons come from temporally similar modes that are also squeezed in the OPA. When squeezed light is produced by down-conversion of a pulsed pump laser, multiple spatial-temporal modes may be squeezed, and none of these modes is guaranteed to match the mode of the LO [18]. These other modes have similar spectra to the LO mode and therefore cannot be conventionally filtered. Detections due to photons in these modes degrade the fidelity of the CSSs. We quantify the effect of unwanted photons with the modal purity ξ_n of n photon subtraction—the probability that, when the subtraction detector registers n photons, these n photons were from the mode matching the LO. To estimate the modal purities, we used a single-mode photon subtraction model to fit our data [15]. From this, we determined $\xi_{2,\mathrm{TES}} = 0.62$ and $\xi_{2,\mathrm{APD}} = 0.85$, compared to $\xi_1 = 0.91$ for the one-photon subtraction experiment. The reason for the lower modal purity of the TES experiment is its greater sensitivity to stray photons from the LO. With the APDs, we can gate the detections to reject slightly delayed LO photons arising from downstream reflections. The TES is slower, so such gating is not possible.

The main advantages of the TES are the greater efficiency and the ability to directly count photons. In the two-photon subtraction experiments, this higher efficiency resulted in improving the rate at which CSSs were heralded by a factor of 3.

Three-photon subtraction events are extremely rare in our experiment. Nevertheless, using the TES we were able to detect 1087 three-photon events over a period of approximately 60 h. With three multiplexed APDs, we would have collected only about 120 events. Figure 4 shows the odd CSS.To increase the three-photon event rate, we increased the reflectivity of the photon subtraction beam splitter to 20%, sacrificing the fidelity of the CSS. The reconstructed state shows a negative minimum of its Wigner function $W_{\min} = -0.116^{+0.073}_{-0.019}$ and a mean-photon number of $2.75^{+0.06}_{-0.24}$. The state has fidelity $F = 0.59^{+0.04}_{-0.14}$ with an ideal CSS of $|\alpha| = 1.76^{+0.02}_{-0.19}$. The estimated modal purity in this experiment is $\xi_3 = 0.84$. Thus, we observed the predicted increase in CSS amplitude for three-photon subtraction, but the increase in fidelity is not statistically significant.

In conclusion, we have measured heralded optical CSSs created by subtracting up to three photons from a squeezed vacuum state, using APDs for one- and two-photon subtraction and a TES for two and three. It was only by taking advantage of the high-efficiency and the direct photon-counting capability of the TES that we were able to successfully subtract three photons with a sufficiently high rate of CSS production. The CSSs produced were analyzed by homodyne measurement and maximum-likelihood state estimation. The quality of the CSSs can be improved by reducing the losses experienced by the squeezed vacuum state before reaching the photon-subtraction beam splitter. For multiphoton subtraction, however, it is crucial to reduce the presence of unfilterable photons in unwanted modes. A promising route that addresses both problems is to tailor the squeezing source to create squeezed light only in a single mode matched to the LO. This route is being pursued in the photon-pair generation community [19–21]. Based on our findings, we propose that the combination of pure vacuum squeezing and high-efficiency detectors with photon-number-resolving capabilities can yield high-rate, high-amplitude, and high-fidelity CSSs to support quantum information processing and metrology beyond the quantum limit.

Note added. Recently, the authors became aware of a similar measurement that made use of photon-number-resolving transition edge sensors [22].

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