$\bar{4}$-quasi-phase-matched interactions in GaAs microdisk cavities

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We describe quasi-phase-matched nonlinear interactions in materials that possess $\bar{4}$ symmetry, such as GaAs. $\bar{4}$ symmetry together with curved propagation geometries produce quasi phase matching (QPM) without need of external domain inversions. To improve efficiency of nonlinear optical mixing, $\bar{4}$-QPM can be combined with resonant microcavities such as whispering-gallery-mode microdisks. All interacting waves must be resonant with the cavity, which leads to stringent tuning requirements. We show tuning behavior for second-harmonic generation in a GaAs microdisk cavity and estimate that, with milliwatt-level external pumping, approximately 0.1% power-conversion efficiency can be obtained with negligible higher-order nonlinear effects.

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Quasi phase matching (QPM) is an important technique for obtaining efficient nonlinear optical mixing in materials such as $LiNbO_3$ and GaAs. It is achieved by periodically modulating the nonlinear susceptibility of a material [1,2]. QPM has enabled the use of nonlinear materials lacking birefringence (such as GaAs and related zincblende crystals) for efficient frequency conversion. For instance, QPM has been shown in GaAs using periodic domain inversions [3–5] and by Fresnel phase matching [6]. Phase matching has also been achieved in GaAs waveguides using form birefringence [7] and modal phase matching [8]. Recently, quasi-phase-matched frequency conversion using whispering-gallery modes of an AlGaAs circular microresonator has been proposed [9,10]. In this Letter, we clarify the physical mechanism for QPM in disk-shaped geometries. We distinguish between QPM in materials, like GaAs, that have $\bar{4}$ symmetry ($\bar{4}$-QPM) and effects of the resonant cavity. The cavity improves conversion efficiency while imposing constraints on the nonlinear process. We study the impact of these constraints for the example of second-harmonic generation (SHG) in a GaAs microdisk cavity and note that results can be readily extended to three-frequency processes in other crystals possessing $\bar{4}$ symmetry.

Conventional quasi phase matching utilizes periodic domain inversions, as depicted in Fig. 1(a). QPM can also be achieved using the $\bar{4}$ symmetry element. The $\langle 001\rangle$ axes in crystals like GaAs have $\bar{4}$ symmetry, which implies that a 90° rotation about $\langle 001\rangle$ is equivalent to an inversion. Consider waves propagating in a square GaAs crystal with [001] surface-normal, as sketched in Fig. 1(b). The crystal environment of the waves effectively rotates by 90° four times, which is equivalent to four domain inversions (neglecting phase shifts from reflection). The same effect is present in other bent or curved geometries including circular-shaped crystals [Fig. 1(c)]. QPM can be achieved in these geometries without external domain inversions. $\bar{4}$-QPM is possible in all nonlinear crystals with $\bar{4}$ symmetry, which include those of the $\bar{4}3m$ (GaAs, GaP, ZnSe, etc.), $\bar{4}2m$ (KH_2PO_4, chalcopyrites, etc.), and $\bar{4}$ crystal classes.

$\bar{4}$-QPM does not require a resonant cavity, but the geometry naturally lends itself to incorporation of a cavity. The optical modes in a cavity can have high quality factors (Q), which produce high circulating powers and can significantly increase overall conversion efficiency. For example, quasi-phase-matched SHG in a periodically poled $LiNbO_3$ (PPLN) disk resonator was demonstrated with up to 50% external power-conversion efficiency [11].

The fields in a microdisk can be calculated using Maxwell's equations [9,12]. Let us consider SHG using a TE-polarized fundamental (with magnetic field, H_1^z, and electric field in the plane of the disk) and TM-polarized second-harmonic (SH) (with electric field, E_2^z, orthogonal to the disk). Using the coordinates defined in Fig. 1, the fields can be expressed as

$$H_1^z(r,\phi,z,t) = A_1(\phi,t)\Psi_1(r,z)e^{i(\omega_1 t - m_1\phi)},$$

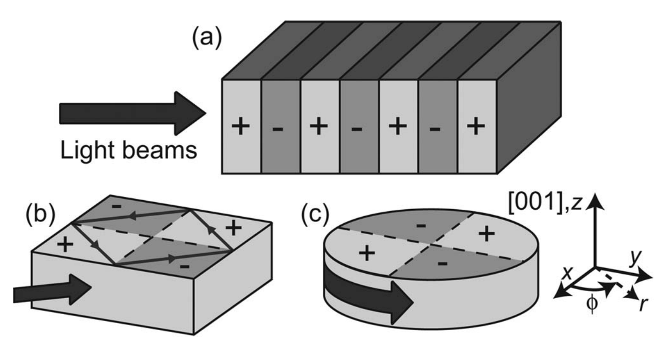

Fig. 1. Propagation geometries for (a) conventional quasi-phase-matched GaAs, and $\bar{4}$-QPM in (b) square-shaped and (c) circular-shaped GaAs crystals.

$$E_2^z(r,\phi,z,t) = A_2(\phi,t)\Psi_2(r,z)e^{i(\omega_2 t - m_2\phi)}, \qquad (1)$$

where $A_i(\phi,t)$ is the slowly varying amplitude at frequency ω_i normalized such that $|A_i|^2$ is the energy in the resonator [13]. The azimuthal number, m_i, is an integer for a resonant mode, and $\Psi_i(r,z)$ is the mode profile, determined according to [12]. Neglecting loss, the change in the SH amplitude in a GaAs microdisk is [9]

$$\frac{\partial A_2}{\partial \phi} = iA_1{}^2[K_+ e^{i(\Delta m+2)\phi} + K_- e^{i(\Delta m-2)\phi}], \qquad (2)$$

where $\Delta m = m_2 - 2m_1$, and K_+ and K_- are the coupling coefficients calculated from the nonlinear coefficient and the modal overlap integrals. When $\Delta m = +2$ or -2, QPM is achieved.

The $(\Delta m = \pm 2)$ QPM condition can be related to the intuitive model shown in Fig. 1(c). Consider a large GaAs disk where all waves propagate at radius R_0. The magnitude of the wavevector is $k_i = m_i / R_0$ [10]. Setting the phase mismatch, $\Delta k = k_2 - 2k_1$, equal to $\pm 2\pi/(\pi R_0)$ yields $\Delta m = \pm 2$.

QPM can often be achieved using higher-order spatial Fourier components of the modulation in the nonlinear coefficient [2]. The effective nonlinear coefficient in a square geometry [Fig. 1(b)] has a square-wave dependence versus distance (similar to that in conventional QPM), which implies that higher-order QPM is possible. However, Eq. (2) implies that the effective modulation in a disk geometry has only two spatial Fourier components (corresponding to $e^{2i\phi}$ and $e^{-2i\phi}$), so higher-order QPM is not available when using $\bar{4}$-QPM in a disk.

Efficient frequency conversion using $\bar{4}$-QPM in a microdisk cavity requires that (1) all fields are resonant in the cavity, (2) energy conservation is satisfied ($\omega_2 = 2\omega_1$ for SHG), and (3) the QPM condition is met ($\Delta m = \pm 2$). One can readily identify cavity resonances whose azimuthal numbers are related by $\Delta m = \pm 2$, but there is no guarantee that the generated wave at $2\omega_1$ exactly matches a resonance.

To explore implications of these three constraints, we considered SHG in a GaAs microdisk using $\lambda_1 \approx 1.9\ \mu$m. The fundamental can be produced by a narrow-linewidth optical parametric oscillator. Both waves can be coupled into and out of the microdisk using a tapered optical fiber. In a disk with radius $\approx 2.5\ \mu$m and thickness ≈ 180 nm, we identified TE-polarized ($\lambda_1 \approx 1.9\ \mu$m, $m_1 = 14$), and TM-polarized ($\lambda_2 \approx 0.95\ \mu$m, $m_2 = 30$) modes where $\Delta m = 2$. Both fields are in the lowest-order vertical modes, while the fundamental has one radial antinode and the SH has two. Figure 2 plots the tuning of the microcavity modes as disk thickness, radius, and temperature are each varied. From Fig. 2(a), there is only one thickness where energy conservation is satisfied. Similarly, there is only one location in Figs. 2(b) and 2(c) where $2\lambda_2 = \lambda_1$. The observation that each tuning plot in Fig. 2 produces one crossing point differs from previously reported results [14]. Typical GaAs microdisks can have Q in excess of 10^4 [15]. If $Q \approx 10^4$, the linewidth at 1.9 μm wavelength is $\Delta\lambda \approx 0.19$ nm, so that the thickness and radius in our example must be controlled to $\Delta t \approx 0.3$ nm and $\Delta R \approx 2$ nm, respectively. Hence, fabrication requirements for the GaAs microdisks are stringent. Higher Q values lead to even tighter fabrication tolerances. We note that, for this example, the resonance linewidths are narrower than the nonlinear-mixing acceptance bandwidths.

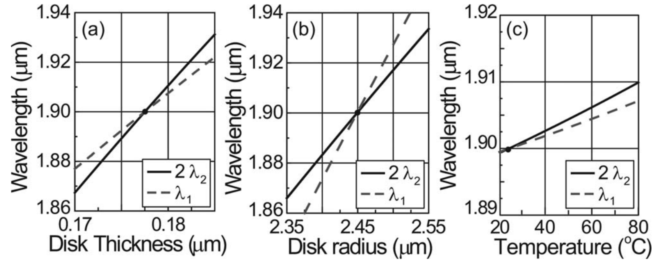

Fig. 2. $2\lambda_2$ and λ_1 resonances for a GaAs microdisk as a function of (a) thickness, (b) radius, and (c) temperature. In each graph, one parameter is varied while the others are fixed to $t = 0.177\ \mu$m, $R = 2.45\ \mu$m, or $T = 23°$C.

We can locate other microdisk geometries that support doubly resonant, quasi-phase-matched SHG. The fundamental wavelength, disk thickness, and radius must be adjusted together to satisfy the three constraints, which can be summarized by the tuning map in Fig. 3. Adjusting temperature produces fine tuning, as seen by the shift caused by $\Delta T = 50°$C. Also, incrementing m_1 (and changing m_2 to keep $\Delta m = 2$) produces other families of solutions. Other maps like Fig. 3 can be found for $\Delta m = -2$, and with mixing of other mode profiles (i.e., different numbers of vertical and radial antinodes).

Figure 3 can be used to find geometries for doubly resonant SHG. However, fine tuning will be required because of uncertainties in the dispersion relation [16], fabricated dimensions, and in the model [12] used to predict the resonant wavelengths. The disk thicknesses are typically fixed by the initial GaAs growth, while the disk radii can be varied by fabricating an array of microdisk sizes. Additional tuning can be obtained using temperature or through use of digital etching [17].

One major difference between $\bar{4}$-QPM in a GaAs microdisk and QPM in a PPLN disk is the QPM con-

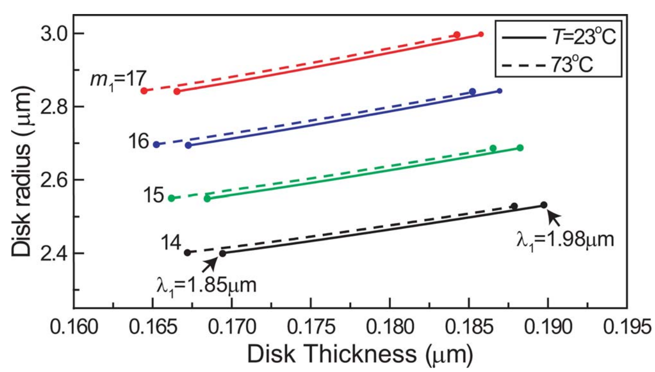

Fig. 3. (Color online) Map of microdisk geometries supporting doubly resonant, $\bar{4}$-QPM SHG with $\Delta m = 2$. Solid (dashed) lines represent $T = 23°$C (73°C). The wavelength range is limited to 1.85 μm $\leqslant \lambda_1 \leqslant$ 1.98 μm.

straint; in the former, QPM is a function of the disk size, while in the latter, QPM depends on the periodic poling rather than disk size. Required diameters for GaAs microdisks are around 2 μm to 10 μm for doubling $\lambda_1 \approx 2$ μm, whereas PPLN disks can be much larger ($\approx$1 mm diameters [11]). Thus, the mode spacings are much larger in GaAs disks that in PPLN disks. Combined with the fact that modes in the PPLN disk need not be related by $\Delta m = \pm 2$, finding disk geometries suitable for multiply resonant frequency conversion will be more difficult in a $\bar{4}$-QPM GaAs microdisk than in a PPLN disk cavity.

The conversion efficiency for SHG in a GaAs microdisk can be calculated using coupled-mode theory [13,14]. The coupled-mode equations are

$$\frac{\partial a_1}{\partial t} = i\omega_1 a_1 - \left(\frac{1}{\tau_1^0} + \frac{1}{\tau_1^c}\right) a_1 + s_1 \sqrt{\frac{2}{\tau_1^c}},$$

$$\frac{\partial a_2}{\partial t} = i\omega_2 a_2 - \left(\frac{1}{\tau_2^0} + \frac{1}{\tau_2^c}\right) a_2 + s^{NL}, \qquad (3)$$

where a_i are related to A_i in Eq. (2) by $a_i = A_i \exp(i\omega_i t)$. τ_i^0 and τ_i^c are the intrinsic and external-coupling photon lifetimes, respectively. Each τ_i is related to the corresponding quality factor by $Q_i = \omega_i \tau_i / 2$. The total quality factor (Q_i^t) can be determined from the coupling (Q_i^c) and intrinsic (Q_i^0) quality factors using $1/Q_i^t = 1/Q_i^c + 1/Q_i^0$. The fundamental is coupled to an external pump given by $|s_1|^2 = P_1^{\text{in}}$ and is assumed to remain undepleted. The nonlinear source, s^{NL}, can be calculated from Eq. (2) using

$$s^{NL} = i a_1^2 \delta\omega_{\text{FSR},2} K_{\pm}, \qquad (4)$$

where $K_{\pm} = K_{+}(K_{-})$ for $\Delta m = -2$ (+2). $\delta\omega_{\text{FSR},2}$ is the free-spectral range (FSR) at ω_2 in units of angular frequency. The FSR is related to the group velocity of the modes [18], which enters the expressions through the energy normalization for A_i [13]. Assuming steady state, Eqs. (3) and (4) yield

$$P_2^{\text{out}} = \frac{2}{\tau_2^c}|a_2|^2 = \frac{4Q_2^c}{\omega_2(1 + Q_2^c/Q_2^0)^2}\left(\frac{4Q_1^c}{\omega_1(1 + Q_1^c/Q_1^0)^2} P_1^{\text{in}} \delta\omega_{\text{FSR},2} |K_{\pm}|\right)^2. \qquad (5)$$

We calculated the SHG conversion efficiency in a GaAs microdisk for the process described in Fig. 2. Taking $Q_1^0 = Q_1^c = Q_2^0 = Q_2^c = 2 \times 10^4$ and $P_1^{\text{in}} = 1$ mW, the predicted output power is $P_2^{\text{out}} = 1.1$ μW, and the conversion efficiency is $\eta_2 = P_2^{\text{out}}/P_1^{\text{in}} = 0.11\%$. In comparison, a GaAs sample of length $2\pi R \approx 15$ μm with confocal focusing produces $\eta_2 \approx 10^{-6}\%$ for $P_1^{\text{in}} = 1$ mW. Inside the microdisk, the circulating fundamental and SH powers are 130 mW and 43 μW, respectively. High circulating intensities may produce higher-order nonlinearities, but we estimate, for these wavelengths and disk size, that self-phase modulation requires 3.2 W of circulating fundamental power, and two-photon absorption of the SH requires 2.5 W of circulating SH power.

We have described a QPM method available in materials with $\bar{4}$ symmetry. $\bar{4}$-QPM can be readily integrated with resonant cavities. We give three constraints necessary for combining $\bar{4}$-QPM with microdisk cavities and explored experimental parameters for observing SHG in GaAs microdisks. Using 1 mW of fundamental power, SHG can be observed without higher-order nonlinear effects. Quasi-phase-matched nonlinear mixing in microdisk structures will have interesting applications in development of miniature sources for spectroscopy and sensing and in generation of entangled photons for quantum information.

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