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Applications of cross-correlation functions

T.V. Vorburger^{a,*}, J.-F. Song^a, W. Chu^a, L. Ma^a, S.H. Bui^b, A. Zheng^a, T.B. Renegar^a

^a National Institute of Standards and Technology, 100 Bureau Drive, Stop 8212, Gaithersburg, MD 20899, USA

^b Veeco Metrology Group, 2650 Elvira Road, Tucson, AZ 85706, USA

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ABSTRACT

We describe several examples where we use cross-correlation functions to quantify the similarity of 2D surface profiles or of 3D areal surface topography images. The applications have included (1) the manufacture of standard reference material (SRM) bullets and casings, (2) methods to assess whether bullets or casings have been fired by the same firearm, and (3) research to quantify similarities or differences between profiles of the same surface measured by different techniques or between a master surface and its replicas. The cross-correlation maximum is the functional parameter used to quantify similarity. A second parameter, called the relative profile (2D) difference or relative areal topography (3D) difference, may also be used to quantify differences and to recognize the ambiguous condition when two results have different vertical (z -) scales but identical shapes. Most of these examples have been applied in support of ballistics inspection methods in crime labs, but the methods are generally useful for estimating the accuracy of surface replication techniques or the ability of different surface topography instruments to measure the same surface and provide the same result. The instruments used in these studies were a stylus instrument and a Nipkow-disk type confocal microscope. Cross-correlation functions may also be used to assess differences resulting from the use of different filters to modify the same surface profile or topography image.

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1. Introduction

In the field of surface texture analysis, scores of statistical parameters have been developed by engineers and scientists aiming to quantify important functional properties of rough and wavy surfaces [1]. Of key importance to surface metrologists, as opposed to other kinds of scientists or engineers, is the property of similarity. When two different techniques are used to measure the same surface, they should arrive at the same result, or when a master surface is used to make replicas, the surfaces of the replicas should have the same topography as one another and the same topography as the master. Quantitative parameters are required to describe directly the similarity between two measured profiles or topography images. It may not be sufficient if two similar profiles yield nearly identical values of parameters, such as roughness average R_a or spacing of irregularities RS_m . Direct point-by-point comparison of profiles may be needed for many applications. Over the past decade, our work has increasingly turned toward the issue of similarity of nominally identical surface profiles and topography images. The problems range over:

- The manufacture and verification of multiple units of standard reference materials (SRMs), designed to have nominally identical surface topographies [2].
- Methods to assess whether bullets and casings have been fired by the same firearm [3].
- Research to quantify similarities or differences between profiles of the same surface measured with different techniques [4].
- Methods to determine how closely a set of replicas matches the master surface from which they are derived [5].
- Linewidth measurement using atomic force microscopy and an image-stitching approach [6].

For all these problems, we have successfully used the traditional analytical method of correlation functions and for some of them, we have introduced a new parameter we call the relative profile (or relative topography) difference D_s [7].

In this paper we define two parameters, the cross-correlation maximum and the relative difference, in Section 2, then briefly indicate the instruments we used in Section 3, discuss results for correlation of profiles and correlation of areal topography images in Sections 4 and 5, respectively, and discuss a proposed application for comparing profile filters in Section 6.

* Corresponding author. Tel.: +1 301 975 3493; fax: +1 301 869 0822.

E-mail address: tvv@nist.gov (T.V. Vorburger).

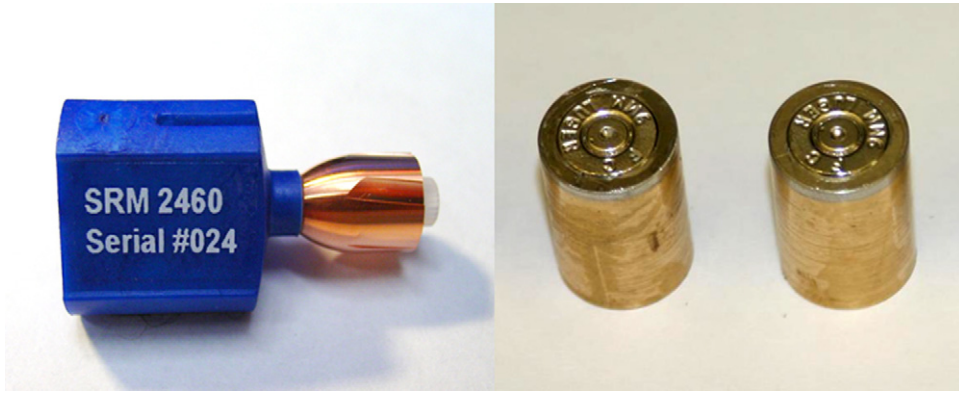


Fig. 1. Photos of a standard bullet (left) and two prototype standard casings (right).

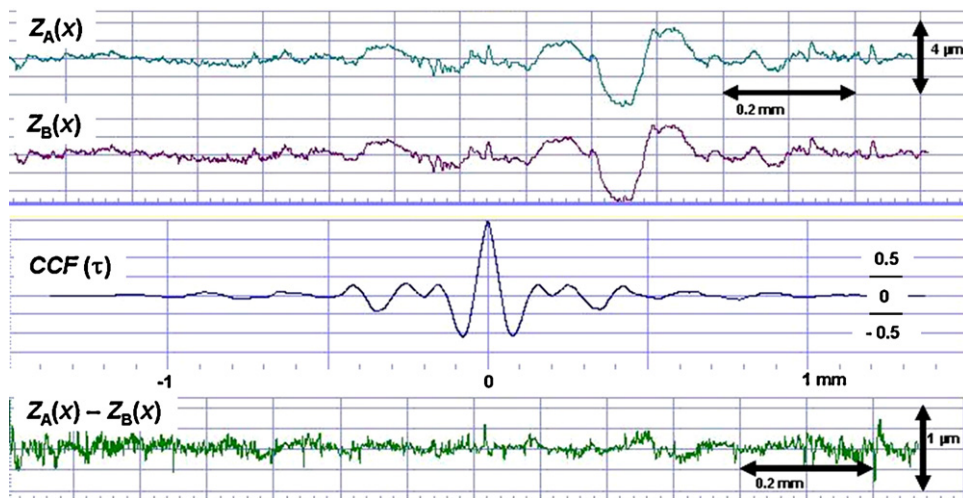


Fig. 2. Correlation results for LEA 5 of a standard bullet. The top profile is the original stylus profile of a fired bullet. The second profile is measured from LEA 5 of the standard by confocal microscopy. The third profile is the cross-correlation function of the two upper profiles, and the lowest profile is the difference profile at the position of optimum correlation. The calculated value of CCF_{max} is 99.2%.

2. Parameters for correlation

The cross-correlation function (CCF) between a pair of 2D profiles $Z_A(x)$ and $Z_B(x)$ may be defined by

$$CCF(A, B, \tau) = \lim_{L \rightarrow \infty} \left(\frac{1}{L} \int_{-L/2}^{L/2} Z_A(x) Z_B(x + \tau) dx \right) / [Rq(A) Rq(B)], \quad (1)$$

where τ is a shift distance variable between the two profiles and $Rq(A)$ and $Rq(B)$ are the values of root mean square (rms) roughness for the two profiles. If the two profiles are identical, then the value of the CCF as a function of τ has a clear maximum, which is equal to unity (100%) at the optimum shift distance $\tau = 0$. If the two profiles are similar but not identical, then the CCF has a maximum value which is less than unity, and the optimum shift distance may be different from zero. Therefore, the CCF is a useful function, and the CCF_{max} value is a useful parameter, for assessing quantitatively the similarity between two profiles. If instead of 2D profiles, we are dealing with a pair of areal topography images, a 3D version of the CCF function may be calculated by analogy with Eq. (1).

A complementary parameter to the CCF_{max} parameter is the relative profile difference D_s , given by the ratio:

$$D_s(A, B) = \frac{Rq^2(AB)}{Rq^2(A)}, \quad (2)$$

where $A-B$ stands for the difference profile $Z_A - Z_B$. If two profiles are identical, then the relative difference is zero. The relative difference

is useful to remove ambiguity when two profiles have identical shapes but different amplitudes. In that case, the CCF_{max} would be equal to unity, implying that the profiles are identical, but the D_s value would not be equal to zero, giving a clear indication that the profiles really are not identical.

3. Instruments

We used two types of instruments for this work:

- A stylus instrument [8,9], Model 120L Form Talysurf¹, having a laser interferometer transducer for readout along the z-axis, an encoder readout along the x-axis, and about a 1.6 μm stylus radius. The rms noise resolution along the z-axis is approximately 3.5 nm.
- A Nipkow-disk type confocal microscope [8,10], Nanofocus μSurf, used with a 20× microscope objective providing a field of view of about 0.8 mm × 0.8 mm. The rms vertical resolution due to noise and optical distortion is approximately 2.5 nm.

¹ Certain commercial equipment are identified in this paper to foster understanding. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the equipment identified are necessarily the best available for the purpose.

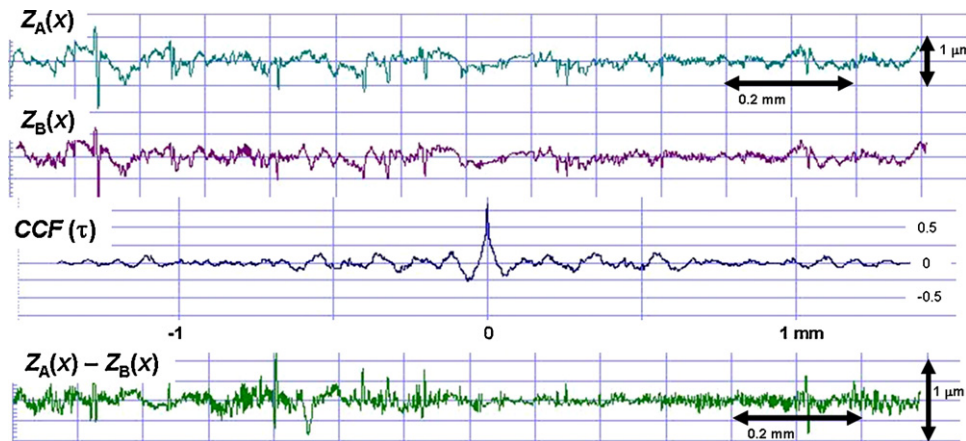


Fig. 3. Correlation results for LEA 6 of a standard bullet. The top profile is the original stylus profile of a fired bullet. The second profile is measured from LEA 6 of the standard by confocal microscopy. The third profile is the cross-correlation function of the two upper profiles, and the lowest profile is the difference profile at the position of optimum correlation. The calculated value of CCF_{\max} is 83.7%.

4. Correlation results involving master and replica profiles

We have developed a set of essentially identical standard bullets by numerically controlled diamond turning [11]. The land engraved areas (LEAs) of six fired master bullets were profiled with the stylus instrument and the six digitized profiles were converted into path instructions to produce six different LEAs on each of 20 replica bullets manufactured on the same setup. The replicas are used as standards for automated optical microscopes in crime labs. Fig. 1 shows one of the machined bullet standards. The corresponding surface roughness profiles on the replicas must be identical to one another and to the master. That means that the profile traced on LEA 1 of the master must have the same profile as LEA 1 on all the replicas, and so forth.

The replicas were tested with the stylus instrument and the agreement between the replica profiles and the six master profiles was extraordinary [12]. As a further test, we also measured the replicas with the confocal microscope and compared those measured profiles with the six profiles measured on the bullet masters with the stylus instrument. The distances across the LEAs were approximately 2 mm, so several of the $0.8 \text{ mm} \times 0.8 \text{ mm}$ topography images measured with the confocal microscope needed to be stitched together to make a profile spanning the width of the LEA. Then all profiles were bandwidth limited by applying a Gaussian filter [8] having a long cutoff (λ_c) of 0.25 mm and a Gaussian filter having a short cutoff (λ_s) of $2.5 \mu\text{m}$. Fig. 2 shows the profiles for LEA 5—where the highest correlation is observed between the replica, measured with confocal microscopy, and the master, measured with stylus profiling. The top graph shows the original profile, measured with the stylus instrument on the master bullet. The second graph shows the replica profile measured with the confocal microscope on the replica bullet. It is difficult to discern qualitatively any differences between them. The third graph shows the cross-correlation function of the two upper profiles. For quantitative measures of the similarity, the maximum value, CCF_{\max} is 99.2% at the optimum shift between the two profiles and D_s is 1.6%. In order to correct for a possible difference in the lateral scales of the two instruments, we also optimized the correlation value by manually adjusting the lateral magnification of the second profile before applying Eq. (1). The adjustment factor was 1.0032, nearly equal to unity, so the size of the lateral correction was very small.

Four of the five other LEAs show the same excellent agreement, yielding CCF_{\max} values of 99.1%, 97.5%, 98.1%, and 97.9%. All five profiles have rms roughness R_q between about 0.35 and $0.64 \mu\text{m}$. The CCF_{\max} for the profiles on LEA 6 (Fig. 3) is 83.7%, not as high as

the others – although the profiles still appear to be quite similar – mainly because R_q is smaller, about $0.15 \mu\text{m}$. At this z-scale, imperfections in the surfaces or the acquisitions begin to be apparent. The D_s value is 32.5%.

5. Areal correlation results

Standard casings have three-dimensional surface texture and are replicated from a master by electroforming [13,14]. Two prototypes are shown in Fig. 1. Thus far, the same master has been used to fabricate 136 replicas over several generations. The areal topography of these replicas is measured by confocal microscopy. Fig. 4 shows the correlation between breech face areas on two of these replicas. The upper two graphs show similar topography images having a $0.8 \text{ mm} \times 0.8 \text{ mm}$ field of view and consisting of 512×512 pixels. The lower three graphs show the filtered topography images along with the difference image between them. The Gaussian regression filter cutoff λ_c [15] was 0.25 mm. The middle image there shows the slight translational and rotational shift required for optimum registration of the two filtered images. The image on the lower right shows the difference between the two filtered images. The value of the areal CCF_{\max} is equal to 99.6%, and the value of D_s is 0.9%, indicating that the replication process has high reproducibility for features ranging smaller than about 0.25 mm. High correlation is also obtained for the firing pin impression [5], another important area where similarity between casings fired from the same firearm is important.

6. Correlation results testing the similarities of filters

Many different types of software are available to filter surface profiles and topography images and to calculate surface parameters. The accuracy of the results is an important issue for surface metrologists. In addition to accurate measurements, we need accurate software to analyze those measurements. Recently, national measurement institutes have created interactive websites [16–18] that allow comparison between parameters calculated by the user and the same parameters calculated at the Website, where the software has been tested and is considered to be accurate. In addition, comparisons of parameters calculated by different software packages have been published [19–21]. The comparisons show surprisingly good agreement for most parameters and surprising differences for a few others. The comparisons mainly focus on profiles already considered to be filtered and on the differences between calculated parameters. However, direct com-

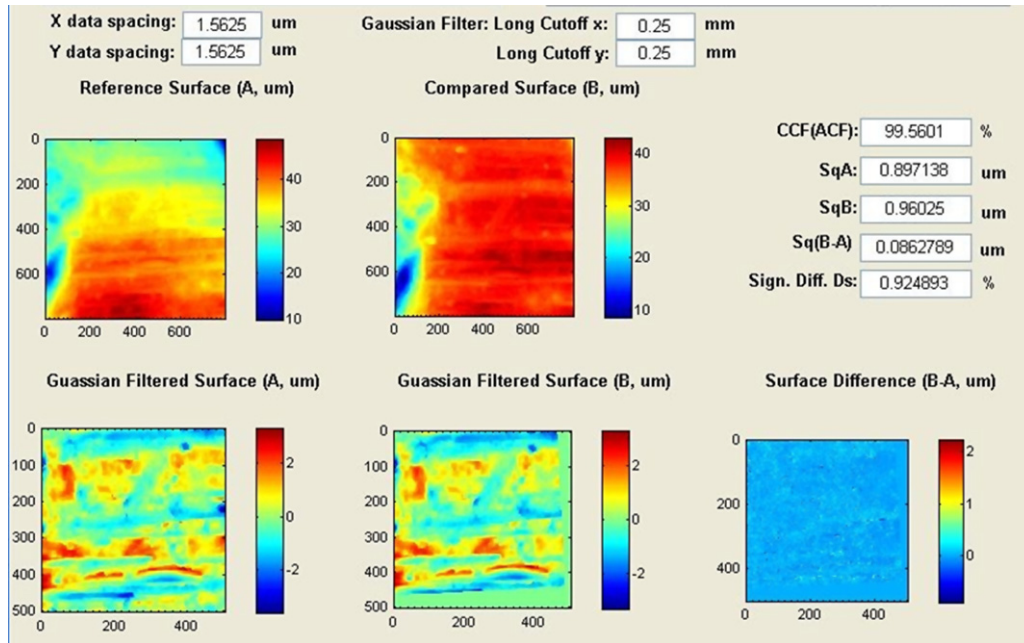


Fig. 4. Areal correlation of equivalent breech face areas on two prototype standard casings replicated from the same master. The top two images are topography images measured with a confocal microscope. Their topographies are similar, but they have slightly different color coding. The images directly below have been Gaussian filtered to remove long spatial wavelengths. To the right of them is the difference image. The value of CCF_{\max} is 99.6%.

parisons between the filters have not been pursued as widely. We propose here that cross-correlation between filtered profiles can be used to compare different filters and so understand the differences they generate.

As a preliminary test of this method, we have used the surface metrology algorithm testing service (SMATS) [16] at NIST to look for differences between profiles filtered by three different types of Gaussian long wave filters, a standard convolution, a fast Fourier transform [22], and a fast Gaussian approximation [23]. The profiles resulting from the three different filter examples agree extremely well, and the correlation is nearly perfect. Figs. 5 and 6 show, respectively, correlation results for two model profiles available from SMATS, a sinusoidal profile and a random white noise profile, the latter having no correlation between nearest neighbour points. The profiles are both 4 mm in length and contain 8000 points, and the λc filter cutoff used was 0.8 mm. The length of the convolution function was $\pm\lambda c$, which truncates the leading and trailing 0.8 mm lengths from the profiles, so the filtered profiles were only compared over the central length of 2.4 mm.

Fig. 5 shows the cross-correlation function comparing the sinusoidal profiles filtered by the convolution and the FFT Gaussian. The

cross-correlation does not show significant differences between the two profiles. The CCF function itself is highly periodic, having a value at $\tau = 0$ of 1.00... (100%) to at least 17 decimal places. However, the amplitudes of the maxima and minima increase slightly as the lateral position on the curve gets further from $\tau = 0$. This distortion may be due to the decreasing number of data points used in the calculation of CCF as the overlap between the profiles decreases, rather than due to any real differences between the filtered profiles. The value of D_s calculated at $\tau = 0$ is 2.57×10^{-31} , also indicating very close agreement between the two filtered profiles.

Fig. 6 compares the convolution filter with the fast Gaussian filter for the random profile. As expected the CCF shows only noise and no correlation between the filtered profiles except at $\tau = 0$. There the correlation is nearly perfect and the value of CCF_{\max} is 99.9999944%, surprisingly close to unity, even though the fast Gaussian is a regression-type approximation to the true Gaussian function. The value of D_s at $\tau = 0$ is 1.1×10^{-7} , indicating good agreement between the two filtered profiles. At the ends, the calculation of CCF depends on only a few points that overlap between the profiles and hence the results become unstable with values increasing and oscillating. At least for the sinusoidal and the white noise profiles, the CCF clearly indicates that the three filters are all highly accurate approximations of a true Gaussian filter.

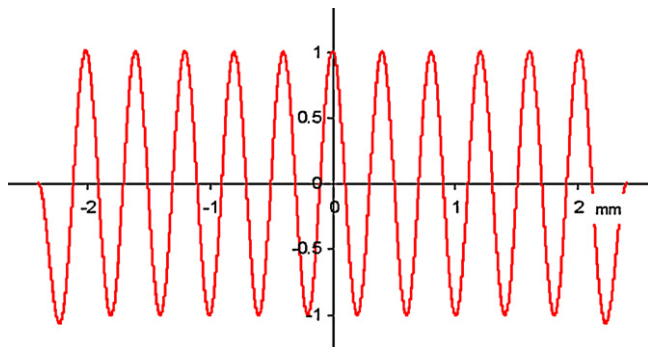


Fig. 5. Cross-correlation function between two profiles resulting from filtering a sinusoidal profile with a standard Gaussian convolution filter and with a fast Fourier transform Gaussian filter.

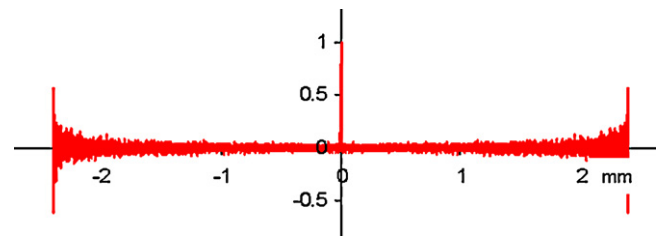


Fig. 6. Cross-correlation function between the profiles resulting from filtering a random, white noise profile with a standard Gaussian convolution filter and a fast Gaussian approximation.

7. Conclusions

There are many surface technology applications where similarity between surfaces is an important function that needs to be quantified. We have listed a few of those applications in this paper and described the use of cross-correlation functions and the parameters CCF_{\max} and relative difference D_s to quantify those similarities and differences. Using parameters that directly compare pairs of images or profiles is a more direct approach for testing similarity than comparing parameters of each, such as R_q , or even a few parameters together. Two profiles could have very similar sets of parameter values but be quite different in detail. We have not discussed the registration process itself, by which the optimum relative orientation for correlation is arrived at. The process of registration involves point-by-point comparison of two images or profiles and consumes more computing time than calculation of most parameters from individual profiles or images. This may become an important issue for topography images where six degrees of freedom determine the relative orientation between a pair of images. A number of techniques [24] have been developed for the image registration process. In addition, for both 2D and 3D registration problems, relative scale factors in all directions may be important, and internal nonlinearities may also need to be accounted for. There should be plenty of need for development of image registration and correlation techniques.

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