An Innovative Approach for Controlling Hysteretic Structures Using Static Output Feedback H_∞ Algorithm and Stochastic Linearization Techniques

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ABSTRACT

For control design, hysteretic structures present a challenge due to changes in the structural parameters during large seismic events. The conventional approach is to linearize the structure at the initial equilibrium state, thus ignoring the hysteretic characteristics of the structure when computing the gain matrices. This study extends the static output feedback H_∞ algorithm, which was developed previously for linear systems, to nonlinear structures using a newly developed procedure that uses stochastic equivalent linearization. In this procedure, the hysteretic parameters are linearized assuming that the ground motion is a white noise filtered with the Kanai-Tajimi power spectral density, and the control algorithm is designed using the linearized system of equations. The effectiveness of this procedure is demonstrated by simulation results of a hysteretic single-degree-of-freedom structure subjected to earthquake ground motion.

INTRODUCTION

Robust control is important for civil engineering structures, since the stiffness, damping, and environmental excitations involve considerable uncertainties. A controlled system is said to possess robust stability if the system remains stable when its parameters vary within certain expected limits. A controlled system is said to possess robust performance if it can satisfy performance requirements, such as steady state tracking and a certain level of disturbance attenuation. The problem of designing controllers that satisfy both robust stability and performance requirements is called robust control. Robust control of structures for seismic applications using H_∞ algorithms has received wide attention in recent years (Schmitendorf et al., 1994, Kose et al., 1996, and Chase and Smith, 1996).

Under strong earthquakes, many structural members and connections will yield, and the response will become hysteretic. When deformations enter into the inelastic range, the structure is in a critical stage and the control system is most needed. Also, for hybrid control systems that incorporate passive dampers or isolation devices with active or semi-active controllers, the passive devices often exhibit a nonlinear hysteretic behavior. Consequently, active and semi-active control systems must be able to deal with hysteretic structures and operate effectively in the nonlinear range of the response. Since the stiffness of the structure and consequently the system matrix change during hysteretic response, it is not always possible to find the optimum controller that will satisfy the control criteria. The procedure that has been suggested in the past is to linearize the structure at the initial equilibrium state. This procedure, however, ignores the hysteretic characteristics of the structure, the extent of nonlinear deformations, and the intensity of the expected ground motion.

Robust H_∞ algorithms with static output feedback, which have been developed previously for linear structures, are extended in this study to nonlinear structures. Static output feedback uses the measured output from a limited number of sensors installed at strategic locations about the structure so as to minimize the number of sensors. This method can be very effective for controlling hysteretic structures since it does not require measurement of the evolutionary Bouc-Wen hysteretic variable, which cannot be measured. The objectives of this study are to extend the H_∞ control algorithms to hysteretic structures, and to present and investigate the effectiveness of a newly developed procedure that uses stochastic equivalent linearization to compute the control gain matrix. In this procedure, the hysteretic parameters are linearized by assuming the ground motion to be a filtered white noise using the Kanai-Tajimi power spectral density.

H_∞ ALGORITHM FOR LINEAR AND HYSTERETIC STRUCTURES

Using the state space representation, the equation of motion of a linear *n*-degree of freedom structure subjected to a disturbance or ground motion, $W = \ddot{x}_g$, and controlled with *m* controllers using a static output feedback strategy, is given by:

$$\dot{X} = AX + BU + GW \tag{1}$$

$$Z = H_1 X + H_2 U \tag{2}$$

$$Y = \Theta X \tag{3}$$

$$U = KY = K\Theta X \tag{4}$$

where X is the 2*n*-dimensional state vector which includes displacements and velocities, *U* is the *m*-dimensional control input vector, *Z* is the *r*-dimensional controlled output vector ($r \leq 2n + m$), and *Y* is the *p*-dimensional measured output vector ($p \leq 2n$). A is the system matrix; *B* and *G* are the influence matrices for the control input and disturbance, respectively; H_1 and H_2 are weighting matrices selected to impose different penalties on certain states and/or controls; *K* is the control gain matrix, and Θ is a mapping matrix that determines the locations of sensors for static output feedback control. When $\Theta = I$, where *I* is the identity matrix, a full-state feedback control is achieved.

The objective of the H_∞ control algorithm is to determine the *m* x *p* gain matrix, *K,* such that: 1) the closed loop system defined by Eqs. 1 through 4 remains stable (robust stability) and 2) the H_∞ norm of the transfer function, T_{wz}, from the disturbance input, W, to the controlled output, Z, remains less than a certain level of disturbance attenuation, $\gamma > 0$ (robust performance). A possible algorithm for computing the static output control law has been presented in Kose et al. (1996) and is summarized in Sadek et al. (2002).

For hysteretic structures, the motion is described by the following system of differential equations defined in the physical coordinate system:

$$M\ddot{x} + C\dot{x} + K_{el}x + K_{in}\nu = DU - m\ddot{x}_g \tag{5}$$

where M, C, K_{el}, and K_{in} are the mass, damping, elastic stiffness, and inelastic stiffness matrices, respectively; m is the mass vector, and D defines the locations of the control forces. x is the inter-story drift vector and ν is the evolutionary hysteretic n-dimensional vector whose i-th component is represented by the Bouc-Wen model (Wen, 1976) as:

$$\dot{\nu}_i = D_{yi}^{-1}[a_i\dot{x}_i - \beta_i\,|\dot{x}_i|\,|\nu_i|^{\eta_i - 1}\,\nu_i - \lambda_i\dot{x}_i\,|\nu_i|^{\eta_i}] \qquad i = 1, 2, \ldots, n \tag{6}$$

This model permits the simulation of a large number of hysteretic shapes by varying the four parameters η_i, a_i, β_i, and λ_i. D_{yi} is the yield deformation.

The state space representation of Eqs. 5 and 6 is given by:

$$\dot{X}_1 = g(X_1) + B_1U + G_1\ddot{x}_g \tag{7}$$

where $X_1 = [\{x\} \quad \{\dot{x}\} \quad \{\nu\}]^T$ is the augmented $3n$-dimensional state vector, B_1 and G_1 are influence matrices, and $g(X_1) = [\{\dot{x}\} \quad -\overline{M}^{-1}(\overline{C}\dot{x} + \overline{K}_{el}x + \overline{K}_{in}\nu) \quad \{\dot{\nu}\}]^T$ is a $3n$-dimensional vector that is a nonlinear function of the elements of X_1.

For hysteretic structures, the control forces will be computed as $U = KY_1$, where Y_1 is the p-dimensional measured output vector ($p \le 3n$). As for the case of linear systems, $Y_1 = \Theta_1 X_1$ where Θ_1 is the (p x $3n$) mapping matrix. The use of static output feedback control for hysteretic structures is very practical, since it not only requires a smaller number of sensors, but it permits ignoring the measurement of the non-measurable evolutionary variable, ν. The measurement of this variable can be ignored by assigning zeros to the corresponding elements in the Θ_1 matrix. The effectiveness of this strategy will be demonstrated using numerical simulations.

Since the stiffness of the structure and, consequently, the system matrix change during hysteretic response, it is not always possible to find the optimum gain matrix that will satisfy the control criteria. The procedure that has been suggested in the past is to linearize the structure at the initial equilibrium state. This procedure has been used extensively for hysteretic structures with a variety of algorithms, such as the linear quadratic regulator and the sliding mode control algorithms. The linearization of Eq. 7 for hysteretic structures at the initial equilibrium stage ($X_1 = 0$) leads to the linear state space equation, Eq. 1. Hence, the same H_∞ algorithm described earlier for linear structures is used for the linearized model. The resulting algorithm is not optimal, since the evolution of the hysteretic variable and the ranges of inelastic excursions the structure will experience are not taken into account in the computation of the gain matrix. The next section presents a stochastic equivalent linearization procedure to deal with structural nonlinearities in the development of the control gain matrix. This procedure can be easily implemented in other linear control algorithms, such as the linear quadratic regulator.

STOCHASTIC EQUIVALENT LINEARIZATION

This procedure is based on the work of Wen (1980) who presented the method of stochastic equivalent linearization for hysteretic structures under random excitation. The theoretical formulation in Wen (1980) for this approach was performed on a single-degree-of-freedom (SDOF) hysteretic structure, and is presented herein for multi-degree-of-freedom (MDOF) structures.

Open Loop Case (No Control): Consider an uncontrolled MDOF hysteretic structure governed by the following system of differential equations of motion:

$$M\ddot{x} + C\dot{x} + K_{el}x + K_{in}\nu = -m\ddot{x}_g \tag{8}$$

The stochastic linearization procedure consists of replacing Eq. 6 with the following equivalent linear equation:

$$\dot{\nu} = -K_e\nu - C_e\dot{x} \tag{9}$$

where K_e and C_e are diagonal matrices containing the linearization coefficients k_{ei} and c_{ei}, which are numerical coefficients describing the normalized hysteretic characteristics of the structure undergoing nonlinear motion and do not represent physical stiffness or damping. They are determined by minimizing the expected value of the mean square error between Eqs. 6 and 9. Wen (1980) and Chang et al. (1986), assuming that the excitation is a zero-mean stationary Gaussian process, derived the equations for estimating K_e and C_e in terms of the standard deviations and expected values of $\dot{x}$ and ν.

In this study, the ground motion is considered to be a stationary excitation that can be modeled as a white noise with constant spectral density, S_0, filtered through the Kanai-Tajimi filter such that the power spectral density is given by:

$$G(\omega) = \frac{1 + 4\xi_g{}^2(\omega/\omega_g)^2}{\left[1 - (\omega/\omega_g)^2\right]^2 + (2\xi_g\omega/\omega_g)^2} S_0 \tag{10}$$

where the Kanai-Tajimi parameters ξ_g and ω_g represent ground damping and frequency, respectively. In this case, the ground motion, $\ddot{x}_g$, in Eq. 8 is given by:

$$\ddot{x}_g = \omega_g{}^2\varphi + 2\zeta_g\omega_g\dot{\varphi} \quad \text{and} \quad \ddot{\varphi} + 2\zeta_g\omega_g\dot{\varphi} + \omega_g{}^2\varphi = n(t) \tag{11}$$

where *n(t)* is the white noise excitation. Eqs. 11 can be combined with Eqs. 8 and 9 to give the following state space representation of size *3n+2*:

$$\dot{\Phi} = L\Phi + F \tag{12}$$

where the state vector $\Phi = \left[\{x\} \quad \{\dot{x}\} \quad \{\nu\} \quad \phi \quad \dot{\phi}\right]^T$, $F_i = 0$ except $F_{3n+2} = n(t)$, and *L* is an augmented system matrix that is found in Sadek et al. (2002).

The covariance matrix of Φ is *V*, from which all the standard deviations and expected values required for computing the linearized coefficients, k_{ei} and c_{ei} can be extracted. It has been shown (Wen, 1980) that *V* satisfies the following Lyapunov matrix equation:

$$LV + VL^T + \overline{F} = 0 \tag{13}$$

where $\overline{F}_{ij} = 0$ except $\overline{F}_{(3n+2),(3n+2)} = 2\pi S_0$.

Since K_e and C_e in matrix L depend on the elements of *V*, an iterative procedure is required to solve Eq. 13.

Closed Loop Case (Controlled): For the closed loop case, Eq. 12 must be modified to account for the effect of the applied control force ($U = KY_1 = K\Theta_1 X_1$), therefore:

$$\dot{\Phi} = (L + \overline{B}K\Theta_1 T)\Phi + F \tag{14}$$

where the *3n+2* x *m* matrix $\overline{B} = \left\lfloor [0] \quad [M^{-1}D] \quad [0] \quad \{0\} \quad \{0\} \right\rfloor$ and the *3n* x *3n+2* matrix $T = [I_{(3nx3n)} \quad 0_{(3nx2)}]$. In this case, the Lyapunov matrix equation takes the form:

$$(L + \overline{B}K\Theta_1 T)V + V(L + \overline{B}K\Theta_1 T)^T + \overline{F} = 0 \tag{15}$$

Since *K* depends on the selection of the coefficients K_e and C_e for the closed loop case, an iterative procedure is required. This procedure may be summarized as follows: (1) Assume K_e and C_e as recommended for the open loop case. (2) Compute the H_∞ static output feedback gain matrix, *K*, for the linearized structure with the assumed K_e and C_e. (3) Solve the Lyapunov equation, Eq. 15, to compute *V*. Compute new values for K_e and C_e based on the computed *V*. (4) Iterate on steps 2 and 3 until convergence is achieved. Using this procedure, a few iterations are generally enough to reach convergence.

NUMERICAL EXAMPLE

In this example, a hysteretic SDOF structure with a mass of 10^3 kg, pre-yield stiffness of 157.9 x 10^3 N/m, and damping coefficient of 1256.6 Ns/m is considered. Prior to yielding, the period of the structure is 0.5 s and the damping ratio is 5 %. The hysteretic parameters are: D_y=0.0005 m, a=1, $\beta = \lambda$=0.5, α=0.1, and η=1. The input excitation is the S69E component of the Taft Lincoln School Tunnel, Kern County earthquake, 1952; scaled to a peak ground acceleration of 0.2*g*. The Kanai-Tajimi parameters for this ground motion are ξ_g=0.32 and ω_g=18.46 rad/s. For this peak ground acceleration, S_0=55 x 10^{-4} m^2/s^3.

The structure was analyzed using different control algorithms and strategies, and the results are presented in Table 1. The peak relative displacement and absolute acceleration responses are presented in column 1 of the table for the uncontrolled structure. The structure was analyzed using the H_∞ algorithm with the conventional approach (linearization at the initial equilibrium state) and with a full-state control strategy. The H_∞ control parameters used in all analyses are listed in Sadek et al. (2002). The peak responses, shown in column 2, show a substantial reduction in the displacement response with the cost of somewhat higher acceleration due to the application of the control forces.

The structure was then analyzed using the H_∞ algorithm with the stochastic equivalent linearization procedure described in this paper and with a full-state control strategy. The results of this analysis are presented in column 3. Comparing columns 2 and 3, the effectiveness of the method of stochastic equivalent linearization is demonstrated. This procedure resulted in better reductions in both the displacement and acceleration responses when compared with the conventional approach.

The fourth column of the table presents the response using the stochastic linearization procedure with static output feedback. The control parameters are the same as the previous case (column 3) with the exception that the matrix Θ_1 is adjusted to ignore measuring the hysteretic variable, ν. Comparing columns 4 and 3, it can be seen that with a slightly smaller control force, the static output feedback controller resulted in almost the same response as the full-state controller.

Table 1. Response of the structure with and without control

Control algorithm	Uncontrolled	H_∞, conventional	H_∞, stochastic linearization	H_∞, stochastic linearization
Control strategy		Full-state	Full-state	Static output
Peak control force (N)	0	650	650	642
x_{max} (cm)	4.35	2.43	2.23	2.24
$\ddot{x}^a_{max}$ (m/s^2)	0.81	1.15	1.04	1.03

CONCLUSIONS

This study shows that H_∞ control algorithms can be applied to hysteretic structures and that the newly developed procedure using stochastic equivalent linearization to compute the control gain matrix is effective. The effectiveness of the stochastic equivalent linearization over the conventional approach for dealing with structural nonlinearities (linearization at the initial equilibrium state) was demonstrated using numerical simulations of a hysteretic single-degree-of-freedom structure under earthquake ground motion. The success of this new approach is due to the inclusion of the linearized differential equation form of the Bouc-Wen evolutionary variable in the state space representation of the structure used by the H_∞ controller. This paper also shows the effectiveness of using a static output feedback strategy for controlling hysteretic structures. Numerical simulations indicated that for this case, the control effectiveness is not lost when the evolutionary hysteretic variable is not measured. The procedure presented in this paper can be easily implemented in other linear control algorithms, such as the linear quadratic regulator.

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