The Interpretation and Use of S-Parameters in Lossy Lines

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Abstract

Although a fundamental parameter of transmission lines, the characteristic impedance is difficult to measure accurately. We suggest a method by which it may be easily determined from a measurement of the propagation constant. The method is based on a rigorous analysis from first principles using explicit and realistic approximations which include the effects of imperfect conductors. Results of numerical studies of lossy coaxial lines and of experiments with coplanar waveguides indicate that high accuracy is possible.

1. Introduction

Scattering parameters (S-parameters), while of physical significance in their own right, are not by themselves sufficient for a complete description of the electromagnetic fields in a transmission line. In fact, S-parameters fail to specify some of the fundamental parameters of interest for MMIC (monolithic microwave integrated circuit) design and device modeling—specifically, the load impedance relating the microwave voltage and current. In order to determine load impedance from S-parameters, we require knowledge of the characteristic impedance of the transmission line.

Often, the value of this characteristic impedance is difficult to ascertain. In the case of lossless coaxial lines, Z_0 is real and frequency-independent and may be calculated fairly accurately from dimensional properties. On the other hand, planar lines such as microstrip and coplanar waveguide are not so susceptible to analysis. Not only are the mathematical and numerical problems vastly more complicated, but the material coefficients, such as the dielectric constant of the substrate and the conductivity of the metals, are not typically known to great accuracy. Furthermore, in contrast to coax, planar lines are typically highly lossy and therefore possess a complex characteristic impedance which varies greatly with frequency [1]. For these reasons, Z_0 is difficult to compute accurately.

Two approaches have been taken to deal with this problem. The first is the use of the LRM calibration method [2], which replaces the line standard of TRL

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with a resistive "match" standard. The major drawback of this method is that the standard is complicated and difficult to characterize. The only available characterization method is a DC resistance measurement. Clearly, several different loads, each with a DC impedance of exactly 50 Ω, may respond differently to a microwave field and result in different calibrations. Furthermore, their reflection coefficients need not be real; as a result, load impedances outside the Smith chart may be measured for passive devices. For these reasons, LRM is unsatisfactory for precision calibrations.

Another approach to the problem of dispersive transmission lines is the *measurement* of Z_0. In conventional measurements, the line is connected to a device with supposedly known reference impedance. Such a procedure is closely related to the LRM method and has similar drawbacks. A more promising approach was presented in [3], which argues that Z_0 can be determined from a measurement of the propagation constant and knowledge of the free-space capacitance. Although the idea is interesting, the equations are unsupported, and the results erroneously predict that $\mathrm{Re}(Z_0)$ falls, rather than rises, at the lower frequencies. Our analysis shows that [3] neglects the conductor loss, which is typically the dominant effect in planar lines.

This paper will demonstrate the possibility of an indirect, approximate measurement of the characteristic impedance of transmission lines through measurement of the propagation constant. The appropriate equations are not those given by [3] but instead are the result of a rigorous analysis. We demonstrate that, in many practical cases, the method is extremely precise. In particular, we demonstrate the validity of the approach with numerical studies of lossy coaxial cable and also with measurements performed on coplanar waveguide. Both sets of results demonstrate the high accuracy with which Z_0 can be determined.

2. Theory

In a uniform transmission line, the normalized transverse electric and magnetic fields of a single mode propagating in the $+z$ direction will be denoted by $\boldsymbol{e}e^{-\gamma z}$ and $\boldsymbol{h}e^{-\gamma z}$, respectively. Here $\boldsymbol{e}$ and $\boldsymbol{h}$ are independent of z, and γ is the propagation constant. The complex power carried by the forward-propagating mode is given by the integral over the transverse cross section S as

$$p_o \equiv \int_S \boldsymbol{e} \times \boldsymbol{h}^* \cdot \boldsymbol{z}\, dS \,, \tag{1}$$

where $\boldsymbol{z}$ is the longitudinal unit vector. Define the microwave voltage v_o by the integral

$$v_o = -\int_{path} \boldsymbol{e} \cdot d\boldsymbol{l} \tag{2}$$

over some specified path. Now specify the characteristic impedance

$$Z_o \equiv |v_o|^2 / p_o^* . \tag{3}$$

Note that its phase is equal to that of $1/p_o^*$, but p_o is independent of the phase of $\boldsymbol{e}$ and $\boldsymbol{h}$. Hence the phase of Z_o is not arbitrary but unique. The magnitude of Z_o depends on the path used to define v_o.

We can write the ratio and product of γ and Z_o in the forms

$$\frac{\gamma}{Z_o} = j\omega C + G \tag{4}$$

and

$$\gamma Z_o = j\omega L + R , \tag{5}$$

where the circuit parameters C, G, L, and R are real. Equations (4) and (5) are identical to those derived from the circuit theory description of a transmission line with distributed shunt admittance $j\omega C + G$ and series impedance $j\omega L + R$. The circuit parameters can be expressed quite generally in terms of the fields [4] by

$$C \equiv \frac{1}{|v_o|^2}\left[\int_S \varepsilon' |\boldsymbol{e}|^2 \, dS - \int_S \mu' |h_z|^2 \, dS\right] , \tag{6}$$

$$L \equiv \frac{1}{|i_o|^2}\left[\int_S \mu' |\boldsymbol{h}|^2 \, dS - \int_S \varepsilon' |e_z|^2 \, dS\right] , \tag{7}$$

$$G \equiv \frac{\omega}{|v_o|^2}\left[\int_S \varepsilon'' |\boldsymbol{e}|^2 \, dS + \int_S \mu'' |h_z|^2 \, dS\right] , \tag{8}$$

and

$$R \equiv \frac{\omega}{|i_o|^2}\left[\int_S \mu'' |\boldsymbol{h}|^2 \, dS + \int_S \varepsilon'' |e_z|^2 \, dS\right] , \tag{9}$$

where $\varepsilon \equiv \varepsilon' - j\varepsilon''$ and $\mu \equiv \mu' - j\mu''$.

In principle, either equation (4) or (5) can be used to compute Z_o from a measurement of γ. However, the circuit parameters are not all known and their behavior can be quite complicated. We have analyzed the problem to determine which parameters are easiest to predict. One conclusion is that L depends strongly on the metal conductivity and frequency due to the internal inductance of the conductors. Likewise, R depends critically on the conductivity and is not negligible at the low frequencies. By contrast, in typical quasi-TEM lines, C is virtually equal to its perfectly conducting value C_o from DC up to very high frequencies, and G is negligible as long as the substrate is lossless. Hence, if we ignore G and assume that C is equal to C_o, equation (4) has the potential to provide an accurate means of determining Z_o based on a measurement of γ.

3. Examples

The validity of this approach was confirmed in several ways. Here we report on two of them. First, a numerical study of lossy 2.4 mm coaxial cable was undertaken. The resistivity was taken to be 2 μΩ-cm, approximately that of copper. The squares in figure 1 compare the estimate of Z_0, as computed from equation (4) with the assumptions that $G = 0$ and $C = C_o$, to the actual Z_0. The two values agree to within $2 \cdot 10^{-10}$ over the entire band. By contrast, the circles in Figure 1 illustrate the analogous comparison using equation (5) with the assumptions that $R = 0$ and $L = L_o$, in effect the assumptions of [3]. Although the agreement here is acceptable (0.1%) at the high end of the band, it is entirely incorrect at the low end and is always worse than the estimate based on equation (4) by many orders of magnitude.

We also considered experimental evidence from measurements of coplanar waveguide (CPW). Figure 2 shows the reflection coefficient of a small resistor terminating a length of CPW. The match is fairly good at the high end of the band, and the load looks resistive there. On the other hand, the curve begins to deviate below about 5 GHz. Assuming that this deviation is caused not by a variation of the load impedance but by a change in Z_0, we computed Z_0 using equation (4) and used the result to impedance-transform the data of Figure 2. The transformed data, shown in Figure 3, demonstrate that Z_0 as determined from equation (4) is indeed consistent with the lumped element experiment.

4. References

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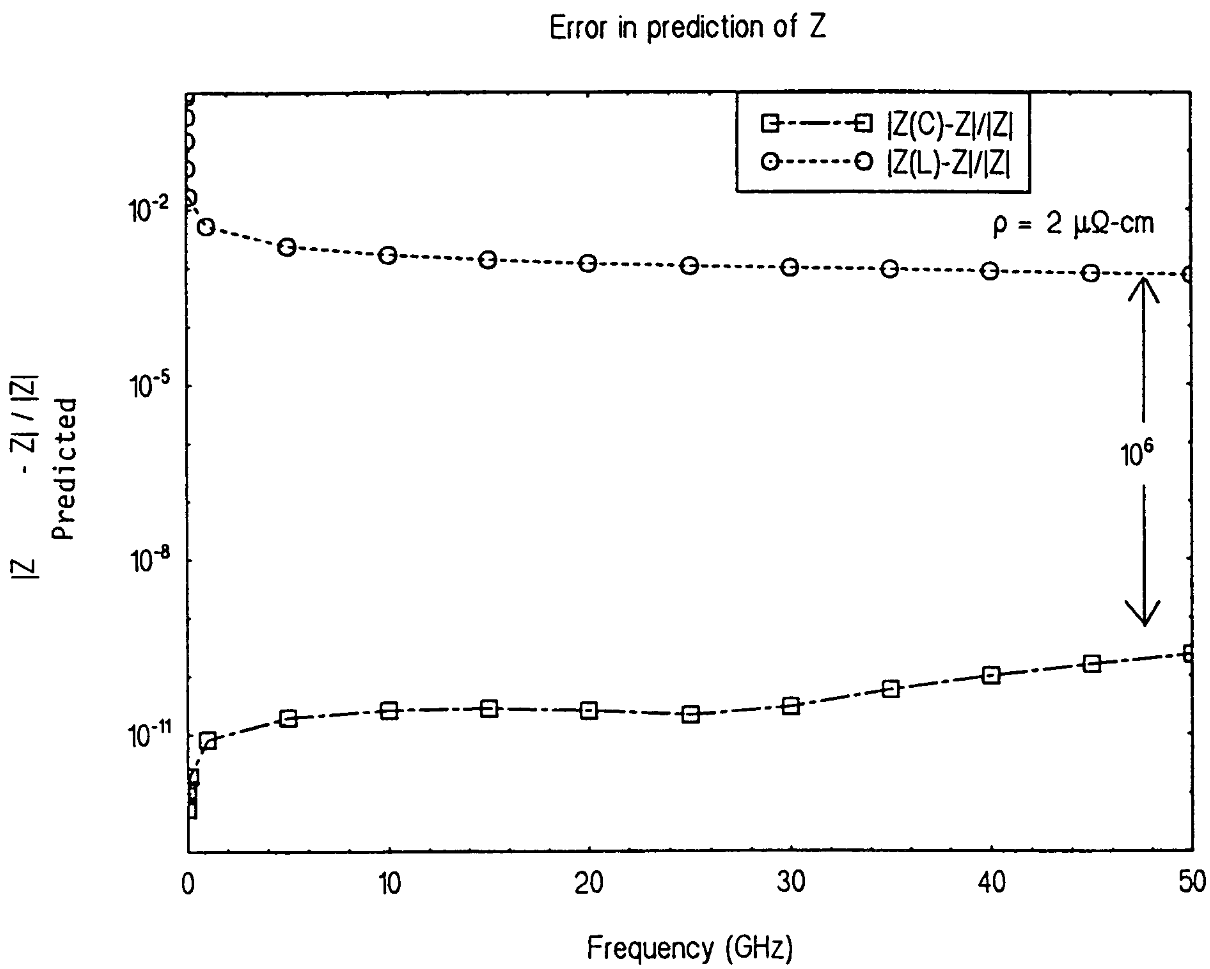


Error in prediction of Z
|Z(C)-Z|/|Z|
|Z(L)-Z|/|Z|
ρ = 2 μΩ-cm
10^6
10^-2
10^-5
10^-8
10^-11
|Z Predicted - Z| / |Z|
0
10
20
30
40
50
Frequency (GHz)

Figure 1

Lumped load reflection coefficient in CPW

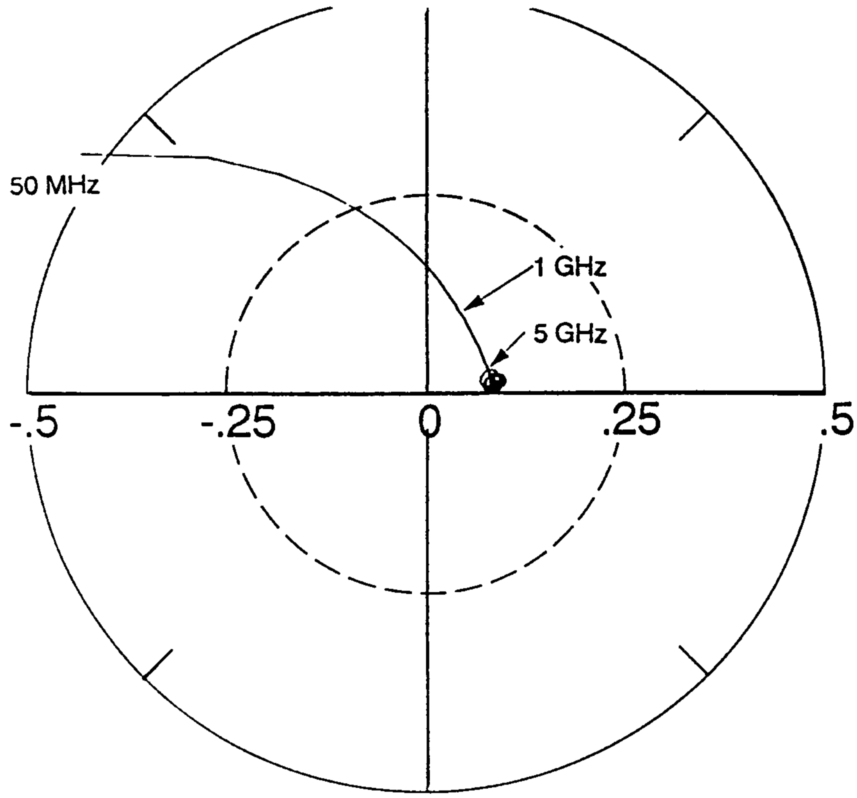


Figure 2

Corrected with proposed technique

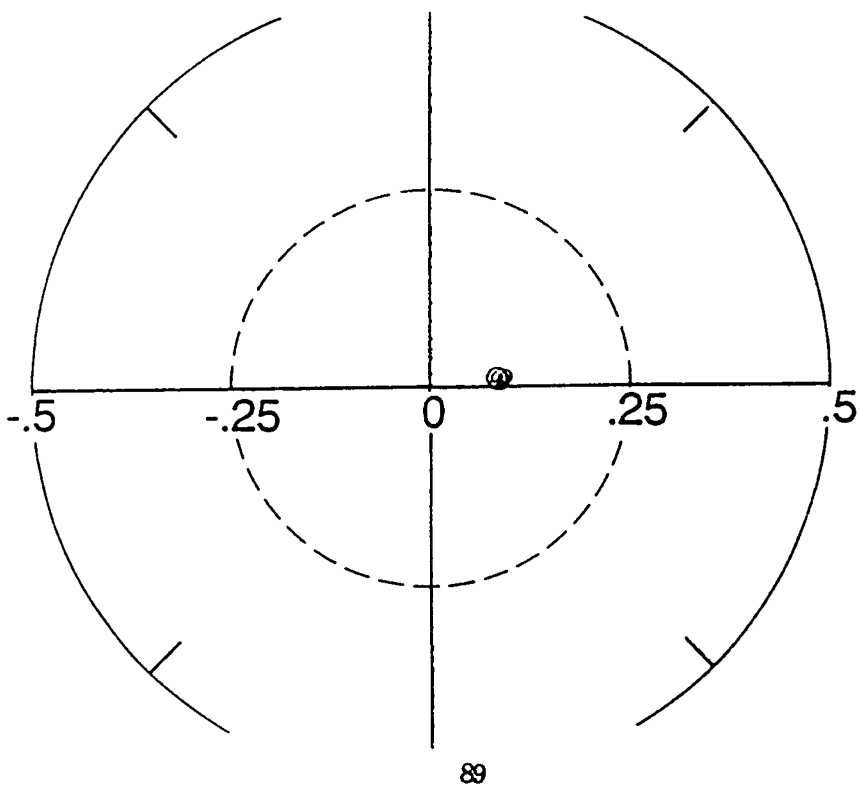
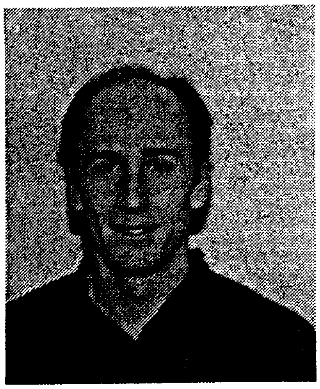


Figure 3



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