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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course contents in specified fields. No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455.

Resource Letter QHE-1: The integral and fractional quantum Hall effects

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This Resource Letter provides a guide to the literature on the integral and fractional quantum Hall effects. The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of somewhat more specialized nature; and the letter A indicates rather specialized or advanced material. An asterisk (*) indicates articles that are especially useful or interesting; a double asterisk (**) indicates those articles to be included in an accompanying reprint book.

I. INTRODUCTION

The quantum Hall effect has led to improving our understanding of the physics of condensed matter, a material-independent electrical resistance standard, a reevaluation of the fundamental physical constants, even more stringent tests of quantum electrodynamics, and new insight regarding fractional quantization of quantum mechanical observables. All this has transpired within a decade; in fact, most of the drama occurred within only 3 years.

The foundations for the field were laid out much earlier. J. R. Schrieffer first predicted in 1957 that interesting things would happen in narrow inversion layers at high electric fields. Under such conditions, the motion of carriers perpendicular to the interface would be restricted to discrete energy levels and an effectively two-dimensional conducting gas would exist. This was subsequently confirmed by Fowler *et al.* in 1966 who showed that a quantized two-dimensional electron gas could be established in the inversion layer of silicon MOSFET (metal-oxide-semiconductor field-effect transistor) structures. In 1975, Ando *et al.* predicted that the Hall conductivity of such two-dimensional systems would have material-independent quantized values each time the Fermi level passed between Landau levels, but it was not clear how these quantum levels would be experimentally identifiable. In 1976, Kawaji and Wakabayashi discovered that the diagonal component of the conductivity tensor of these electron gases dramatically vanished over a range of magnetic fields. Remarkably flat plateaus in the Hall resistivity were published without comment by Englert and von Klitzing in 1978.

Studies in this field suddenly intensified when the spectacular material independence of these levels was demonstrated in the Nobel Prize-winning work of von Klitzing. Von Klitzing accurately measured the resistance plateaus and discovered that they were quantized to within at least 10 ppm according to the simple formula $R_H = h/ie^2$, where h is Planck's constant, e is the electronic charge, and i is an integer. The formula predicted by Ando *et al.* for the Hall conductivity is essentially equivalent. $\sigma_H = ie^2/h$ when $\sigma_{xx} = 0$.

Closely associated with the quantum Hall effect is the concept of Anderson localization. It was originally thought that a two-dimensional electron gas in the limit of zero temperature would not be conducting if a characteristic "localization length" were less than the sample size (strong localization). All states were expected to have localized wavefunctions unable to conduct in two dimensions. These ideas had to be revised. It was found that the effect of a magnetic field was to allow the existence of narrow bands of extended states at the centers of each Landau level.

It was subsequently shown by Laughlin, using a simple, but elegant, gauge-invariance argument, that the peculiar nature of the quantization of a two-dimensional electron gas allows a normally geometry-dependent property, the Hall resistance, to take on sample-independent values.

Within 3 years yet another fundamentally different quantization of the Hall resistance was discovered when

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Tsui *et al.* extended measurements to higher magnetic fields and lower temperatures using samples with greater mobility. They found that the Hall conductance could also be quantized in precise *fractional* multiples of e^2/h. Whereas superficially this "fractional quantum Hall effect" might seem to be merely a simple extension of the original discovery, Laughlin showed that it is in fact based on a completely different physical foundation. The fractional effect arises in low disorder samples where the correlation caused by Coulomb repulsion dominates over the disorder, and localization effects play a secondary role.

These discoveries have led to an explosion of research. In selecting more than 300 important papers for citation in this Resource Letter, we have given careful consideration to over 2000 papers. The reader who wishes to probe deeper into the subject can obtain other relevant papers as well as newer work by a citation search on the key papers listed here in each topic or by going to the references in the reviews. A simple data-base selection of all papers with the keywords "quant..." and "Hall" separated by up to two connecting words yields over 500 quantum Hall effect papers.

There is a clear separation possible between research papers on the original quantum Hall effect, now called the integer quantum Hall effect, and those of the fractional quantum Hall effect; this Resource Letter is organized accordingly. We have generally included parallel discoveries in both the original silicon–MOSFET-based system and the newer GaAs heterostructure system.

II. JOURNALS

We list here the journals in which 97% of the original articles have been published. They are listed roughly in order of their frequency, but *do not include* conference proceeding issues, which are covered in Sec. III. Thus the journal *Surface Science* is low on this list even though, as a result of its publishing the "Electronic Properties of Two-Dimensional Systems" conference proceedings, it has in fact published more relevant papers than any other journal:

Physical Review Letters
Physical Review B
Journal of the Physical Society of Japan
Soviet Physics—JETP
Journal of Physics C
Solid State Communications
Physics Letters A
Nuclear Physics B. Field Theory and Statistical Systems
Metrologia
Physics Status Solidi B
JETP Letters
Surface Science
Soviet Technical Physics Letters
Molecular Crystals and Liquid Crystals
Semiconductor Science and Technology
Journal of Applied Physics
Japan Journal of Applied Physics
Applied Physics Letters
Chinese Physics
Synthetic Metals
Journal of Physics D
IEEE Journal of Quantum Electronics
Soviet Physics—Semiconductors
Journal of Physics Letters
Journal of Vacuum Science and Technology

III. CONFERENCE PROCEEDINGS

A. EP2DS—International conference on electronic properties of two-dimensional electron systems

The study of two-dimensional gas physics became the subject of a series of biannual conferences entitled "Electronic Properties of Two-Dimensional Systems (EP2DS)" starting in 1975. The proceedings of these conferences have traditionally been published in special issues of the journal Surface Science. Although most of the major advances were published in Physical Review Letters, these conferences have continued to be the primary source for current research on the quantum Hall effect. More detailed papers have usually been subsequently published in the Physical Review B, the Journal of the Physical Society of Japan, and the Journal of Physics C.

Since these conferences have been so crucial to the quantum Hall effect, we will list all seven, even though the first three occurred before von Klitzing reported the dramatic precision of quantization of the Hall effect.

1. **"EP2DS-I,"** Surf. Sci. **58** (1976). (A)
2. **"EP2DS-II,"** Surf. Sci. **73** (1978). (A)
3. **"EP2DS-III,"** Surf. Sci. **98** (1–3) (1980). (A)
4. **"Proceedings of the 4th International Conference on Electronic Properties of 2-Dimensional Systems,"** Surf. Sci. **113** (1–3) (1982). (A)

*5. **"Proceedings of the 5th International Conference on Electronic Properties of 2-Dimensional Systems,"** Surf. Sci. **142** (1–3) (1984). (A)

*6. **"Electronic Properties of 2-Dimensional Systems. Proceedings of the Yamada Conference XIII."** Surf. Sci. **170** (1–2) (1986) (A)

*7. **"Electronic Properties of 2-Dimensional Systems—VII."** Surf. Sci. **196** (1–3) (1988). (A)

B. Other conference proceedings

8. **Anderson Localization. Proceedings of the Fourth Taniguchi International Symposium,** Sendai, Japan, 3–8 November 1981, edited by Y. Nagaoka and H. Fukuyama (Springer-Verlag, Berlin, 1981). (A)
9. **Symposium on Recent Topics in Semiconductor Physics in Commemoration of the Sixtieth Birthday of Professor Yasutada Uemura,** Tokyo, Japan, 29 March 1982, edited by H. Kamimura and Y. Toyozawa (World Scientific, Singapore, 1983). (A)
10. **Application of High Magnetic Fields in Semiconductor Physics. Proceedings of the International Symposium,** Grenoble, France, 13–17 September 1982, edited by G. Landwehr (Springer-Verlag, Berlin, 1983). (I)
11. **Two-Dimensional Systems. Heterostructures and Superlattices. Proceedings of the International Winter School,** Mautendorf, Austria, 26 February–2 March 1984, edited by G. Bauer, F. Kuchar, and H. Heinrich (Springer-Verlag, Berlin, 1984).

*12. **Two-Dimensional Systems: Physics and New Devices. Proceedings of the International Winter School,** Mautendorf, Austria 24–28 February 1986, edited by G. Bauer, F. Kucher, and H. Heinrich (Springer-Verlag, Berlin, 1986). (I)

13. **Localization, Interaction and Transport Phenomena. Proceedings of the International Conference,** Braunschweig, FRG, 23–28 August 1984, edited by B. Kramer, G. Bergmann, and Y. Bruynseraede (Springer-Verlag, Berlin, 1985). (A)
14. **Foundations of Quantum Mechanics: In the Light of New Technolo-**

gy, Proceedings of the 2nd International Symposium,** Tokyo, Japan 1–4 September 1986, edited by M. Namiki, Y. Ohnuki, Y. Murayama, and S. Nomura (Physical Society of Japan, Tokyo, 1987). (I)

15. **"The Physics of the Two-Dimensional Electron Gas."** Oostduinkerke, Belgium, 2–14 June 1986. NATO Advanced Study Institute Series B **157**, edited by J. T. Devreese and F. M. Peeters (Plenum, New York, 1987). (A)

16. **High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference,** Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987). (I)

*17. **Interfaces, Quantum Wells, and Superlattices,** Banff, Alberta, 16–29 August 1987. NATO Advanced Study Institute Series B **179**, edited by R. Taylor and C. R. Leavens (Plenum, London, 1988). (A)

C. Metrological and engineering conferences

The international electrical standards community holds a conference entitled the "Conference on Precision Electromagnetic Measurements" (CPEM) every 2 years. The proceedings have traditionally been published in the IEEE Transactions on Instrumentation and Measurement.

18. **CPEM '82**—IEEE Trans. Instrum. Meas. **IM-32** (1) (1983). (I)

19. **CPEM '84**—IEEE Trans. Instrum. Meas. **IM-34** (2) (1985). (I)

*20. **CPEM '86**—IEEE Trans. Instrum. Meas. **IM-36** (2) (1987). (I)

*21. **CPEM '88**—IEEE Trans. Instrum. Meas. **IM-38** (2) (1989). (I)

When quantum Hall work has appeared at other conferences, it is usually in the form of an introduction to scientists of related disciplines.

22. **Quantum Metrology and Fundamental Constants,** Erice, Sicily, Italy, 16–28 November 1981. NATO Advanced Study Institute Series B **98**, edited by P. H. Cutler and A. A. Lucas (Plenum, New York, 1983). (I)

*23. **Fundamental Constants in Physics and Metrology. 70th PTB—Symposium,** Metrologia 22(3) (1986). (I)

IV. BOOKS

At present there are only two books devoted to the quantum Hall effect, but they provide excellent coverage of both the theoretical and experimental situations.

*24. **The Quantum Hall Effect,** edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987), 419 pp. Both the integer and fractional quantum Hall effects are covered in this book. (A)

*25. **The Fractional Quantum Hall Effect: Properties of an Incompressible Quantum Fluid,** T. Chakraborty and P. Pietiläinen (Springer-Verlag, New York, 1988), Springer Series in Solid State Science, Vol. 85, 175 pp. This book gives a detailed review of the fractional quantum Hall effect. (A)

The following books covering semiconductor physics and technology are also useful.

26. **Physics of Semiconductor Devices,** S. M. Sze (Wiley, New York, 1981), 868 pp. (I)

27. **MOS Physics and Technology,** E. H. Nicollian and J. R. Brews (Wiley, New York, 1982), 906 pp. (I)

V. LANDMARK PAPERS

*28. **"Magneto-Oscillatory Conductance in Silicon Surfaces,"** A. B. Fowler, F. F. Fang, W. E. Howard, and P. J. Stiles, Phys. Rev. Lett. **16**, 901–3 (1966). This paper experimentally demonstrates the existence of a two-dimensional electron gas in silicon MOSFETs. (I)

29. **"Theory of Hall Effect in a Two-Dimensional Electron System," T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. **39**, 279–88 (1975). These authors apply a self-consistent Born approximation and show that there is a correction to σ_{xy} that is proportional to σ_{xx}. They also find that when the Fermi level is located within a dilute concentration of impurity levels between two Landau levels that the impurity states do not contribute to the Hall current, and that the remaining states in the Landau level carry the same Hall current as the unperturbed Landau level. (A)

*30. **"Quantum Galvanomagnetic Properties of n-Type Inversion Layers on Si (100) MOSFET,"** S. Kawaji and J. Wakabayashi, Surf. Sci. **58**, 238–45 (1976). These authors were the first to suggest that immobile electrons are involved in the transport properties of the two-dimensional electron gas in silicon inversion layers because $\sigma_{xx} \cong 0$ over a range of carrier densities. (I)

*31. **"Analysis of ρ_{xx} Minima in Surface Quantum Oscillations on (100) n-Type Silicon Inversion Layers,"** Th. Englert and K. von Klitzing, Surf. Sci. 73, 70–80 (1978). This is the first paper to show flat plateaus of ρ_{xy} vs carrier density (gate voltage) for silicon MOSFETs. (I)

32. **"New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance," K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494–7 (1980). This Nobel prize-winning paper showed the astonishingly accurate quantization of the Hall resistance. (I)

33. **"Quantized Hall Resistance and the Measurement of the Fine-Structure Constant," R. E. Prange, Phys. Rev. B **23**, 4802–5 (1981). This is an exact solution for localization and σ_{xy} quantization due to single delta-function impurities. Mobile electrons passing near impurity sites are found to carry an extra dissipationless current that exactly compensates for the immobility of localized electrons. (A)

34. **"Quantized Hall Conductivity in 2 Dimensions," R. B. Laughlin, Phys. Rev. B **23**, 5632–3 (1981). This is an elegant and general theoretical argument using gauge invariance and the existence of a mobility gap to explain the accurate quantization of the Hall resistance. (I)

35. **"Two-Dimensional Magnetotransport in the Extreme Quantum Limit," D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559–62 (1982). This is the first observation of the fractional quantum Hall effect. They observed a plateau with a filling factor of 1/3 in very high-mobility GaAs/AlGaAs heterojunctions. (A)

36. **"Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations," R. B. Laughlin, Phys. Rev. Lett. **50**, 1395–8 (1983). This is the original paper that provided the now generally accepted explanation of the fractional quantum Hall effect. (A)

VI. CURRENT RESEARCH TOPICS

The quantum Hall effect literature divides rather cleanly into reviews, integral quantum Hall effect papers, and fractional quantum Hall effect papers. Theoretical papers are concerned with why the quantization works so well and what corrections might occur for finite currents, temperatures, measurement frequencies, and sample geometries. The study of Anderson localization is an important aspect of theoretical papers on the integer effect. Theoretical papers on the fractional effect generally focus on the nature of the correlated ground state and properties of low-lying excited states. The experimental papers test the quantization against experimentally tractable variables.

A. Reviews

1. Background information

*37. **"Electronic Properties of Two-Dimensional Systems,"** T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437–672 (1982). This comprehensive review provides a complete background for the understanding of the entire subject of two-dimensional electron gas physics. It contains approximately 2000 references. (I)

*38. **"Disordered Electronic Systems,"** P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287–337 (1985). A review of Anderson localization with 330 references. (A)

2. *Reviews of the integer quantum Hall effect*

*39. **"Experimental Aspects and Metrological Applications,"** by M. E. Cage, Ref. 24, pp. 37–68. (I)

40. **"Effects of Imperfections and Disorder,"** by R. E. Prange, in Ref. 24, pp. 69–99. (I)

41. **"Topological Considerations,"** by D. J. Thouless, Ref. 24, pp. 101–16. (A)

42. **"Field Theory, Scaling and the Localization Problem,"** by A. M. M. Pruisken, Ref. 24, pp. 117–73. (A)

43. **"Quantized Hall Effect and Localization,"** T. Ando, in *Symposium on Recent Topics in Semiconductor Physics in Commemoration of the Sixtieth Birthday of Professor Yasutada Uemura*, edited by H. Kamimura and Y. Toyozawa (World Scientific, Singapore, 1983), pp. 72–104. A theoretical discussion, based on computer simulations, is presented that shows how Anderson localization is related to the quantum Hall effect. (I)

44. **"The Discovery of the Quantum Hall Effect," G. Landwehr, Metrologia **22**, 118–27 (1986). A detailed historical account of the quantum Hall effect discovery. (E)

45. **"The Quantized Hall Effect," K. von Klitzing, Rev. Mod. Phys. **58**, 519–31 (1986). This is von Klitzing's Nobel prize lecture. (I)

46. **"Localization in Landau Levels of 2D Systems and the Quantum Hall Effect,"** T. Ando, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 2–10. (I)

*47. **"Integral Quantum Hall Effect for Nonspecialists,"** D. R. Yennie, Rev. Mod. Phys. **59**, 781–824 (1987). This paper is written by an elementary particle theorist and is intended as an introductory review for nonspecialists. (I)

3. *Reviews of the fractional quantum Hall effect*

The book given as Ref. 25 is dedicated to the fractional quantum Hall effect. Other reviews are:

*48. **"Experimental Aspects,"** by A. E. Chang, Ref. 24, pp. 175–232. (I)

49. **"Elementary Theory: The Incompressible Quantum Fluid,"** by R. B. Laughlin, Ref. 24, pp. 233–301. (A)

50. **"The Hierarchy of Fractional States and Numerical Studies,"** by F. D. M. Haldane, Ref. 24, pp. 303–52. (A)

51. **"Collective Excitations,"** by S. M. Girvin, Ref. 24, pp. 353–80. (A)

52. **"The Fractional Quantum Hall Effect," D. C. Tsui and H. L. Störmer, IEEE J. Quantum Electron. **QE-22**, 1711–19 (1986). An excellent, readable review. (I)

*53. **"Strange Quantum Numbers in Condensed Matter and Field Theory,"** J. R. Schrieffer, *The Lesson of Quantum Theory. Niels Bohr Centenary Symposium*, Copenhagen, 3–7 October 1985, edited by J. de Boer, E. Dal, and D. Ulfbeck (North-Holland, Amsterdam, 1986). This paper is a brief survey of situations in condensed matter physics and in high-energy physics in which fractional quantum numbers appear. (A)

*54. **"Fractional Quantization of the Hall Effect,"** R. B. Laughlin, *Two-Dimensional Systems. Heterostructures and Superlattices. Proceedings of the International Winter School*, edited by G. Bauer, F. Kuchar, and H. Heinrich (Springer-Verlag, Berlin, 1984), pp. 279–87. (A)

*55. **"Recent Advances in the Study of the Fractional Quantum Hall Effect,"** R. J. Nicholas, R. G. Clark, A. Usher, J. R. Mallett, A. M. Suckling, J. J. Harris, and C. T. Foxon, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 146–55. (I)

4. *Reviews of both the integer and fractional quantum Hall effect*

*56. **"Summary, Omissions, and Unanswered Questions,"** by S. M. Girvin, Ref. 24, pp. 381–399. This summarizes the status of the QHE field and introduces a proposal for a Landau–Ginzberg theory of the quantum Hall effect. (A)

57. **"Theory of the Quantized Hall Conductance,"** B. I. Halperin, Helv. Phys. Acta **56**, 75–102 (1983). This is an excellent early review of both the integer and fractional quantum Hall effects and contains an interpretation of the physical meaning of the Laughlin wavefunction. (A)

*58. **"The Quantized Hall Effect,"** H. L. Stormer and D. C. Tsui, Science **220**, 1241–6 (1983). (E)

*59. **"The Quantized Hall Effect,"** B. I. Halperin, Sci. Am. **254**, 52–60 (1986). (E)

*60. **"The Quantum Hall Effect,"** A. MacKinnon, Sci. Prog. **70**, 521–37 (1986). A well-illustrated exposition of the basic principles. (E)

*61. **"Quantised Hall Effect,"** H. Aoki. Rep. Prog. Phys. **50**, 655–730 (1987). An excellent, clear, and detailed review paper. (I)

B. Integer quantum Hall effect research papers

First we list the papers relating directly to the precision of quantization of the Hall resistance and the establishment of the integral quantum Hall effect as a general property of two-dimensional electrons, independent of the system.

1. *Experimental papers checking the quantization of* R_H

The initial discovery paper by von Klitzing, Dorda, and Pepper, which used a silicon MOSFET device, has already been identified in Sec. V as Ref. 32. Subsequent papers in this section are:

62. **"Resistance Standard Using Quantization of the Hall Resistance of GaAs-Al$_x$Ga$_{1-x}$As Heterostructures,"** D. C. Tsui and A. C. Gossard, Appl. Phys. Lett. **38**, 550–2 (1981). The first observation of the quantized Hall resistance in a GaAs/AlGaAs heterojunction. (I)

*63. **"Quantized Hall Resistivity in Si-MOSFETs Measured at Liquid ^{3}He Temperatures,"** K. Yoshihiro, J. Kinoshita, K. Inagaki, C. Yamanouchi, J. Moriyama, and S. Kawaji, J. Phys. Soc. Jpn. **51**, 5–6 (1982). The first measurements at ^{3}He temperatures. The $i = 4$ plateau is shown to have double the resistance of the $i = 8$ plateau to within 0.18 ppm, and triple the resistance of the $i = 12$ plateau to within 0.28 ppm. (I)

*64. **"Determination of the Fine-Structure Constant Using GaAs-Al$_x$Ga$_{1-x}$As Heterostructures,"** D. C. Tsui, A. C. Gossard, B. F. Field, M. E. Cage, and R. F. Dzuiba, Phys. Rev. Lett. **48**, 3–6 (1982). The first high-accuracy (0.1 ppm) measurement of the quantized Hall resistance. GaAs heterojunctions were used. (I)

*65. **"High Precision Measurements of the Hall Effect for Silicon MOS Inversion Layers in Strong Magnetic Fields,"** K. Yoshihiro, J. Kinoshita, K. Inagaki, C. Yamanouchi, J. Moriyama, and S. Kawaji, Surf. Sci. **113**, 16–21 (1982). The first high-accuracy (0.7 ppm) measurement of the quantized Hall resistance in silicon MOSFETs. (I)

66. **"High Precision Measurements of the Quantized Hall Resistance at the PTB,"** L. Bliek, E. Braun, F. Melchert, P. Warnecke, W. Schlapp, G. Weimann, K. Ploog, G. Ebert, and G. E. Dorda, IEEE Trans. Instrum. Meas. **IM-34** 304–5 (1985). The sample independence of the quantized Hall resistance in silicon MOSFETs and GaAs heterojunctions was verified to 0.03 ppm. (I)

67. **"Precise Quantized Hall Resistance Measurements in GaAs/Al$_x$Ga$_{1-x}$As and In$_x$Ga$_{1-x}$As/InP Heterostructures,"** F. Delahaye, D. Dominguez, F. Alexandre, J. P. Andre, J. P. Hirtz, and M. Razeghi, Metrologia **22**, 103–10 (1986). The sample independence between different types of heterojunctions and different quantum states of GaAs heterojunctions is demonstrated to an accuracy of 0.022 ppm. (I)

68. **"The Relationship Among the SI Ohm, the Ohm at NPL, and the Quantized Hall Resistance," A. Hartland, R. G. Jones, B. P. Kibble, and D. J. Legg, IEEE Trans. Instrum. Meas. **IM-36**, 208–13 (1987). This paper is representative of the use of this effect as an absolute, or SI, resistance standard. (I)

2. Quantization theorems

These theoretical papers deal directly with the basis of the quantization of the Hall resistance. The landmark papers of Ando *et al.*, Prange, and Laughlin have already been identified in Sec. V as Refs. 29, 33, and 34, respectively. Listed below are additional papers that explain other aspects of the quantization theory.

69. **"Explanation of Quantized-Hall-Resistance Plateaus in Heterojunction Inversion Layers,"** G. A. Baraff and D. C. Tsui, Phys. Rev. B **24**, 2274–7 (1981). This is an explanation that does not assume localized electrons, but instead invokes tunneling between the donor electron reservoir and the two-dimensional electron gas to maintain the plateau widths. (I)
70. **"Quantized Hall Conductance, Current-Carrying Edge States, and the Existence of Extended States in a Two-Dimensional Disordered Potential,"** B. I. Halperin, Phys. Rev. B **25**, 2185–90 (1982). This paper explores and strengthens Laughlin's original quantization argument. (A)

*71. **"Conduction in a Strong Field in Two-Dimensions: The Quantum Hall Effect,"** R. E. Prange and R. Joynt, Phys. Rev. B **25**, 2943–6 (1982). Semiclassical considerations are discussed for impurity scattering in a magnetic field. (A)

72. **"Theory of Quantized Hall Conductivity in Two Dimensions," P. Streda, J. Phys. C **15**, L717–21 (1982). This is an important formulation of the quantization theorem in terms of the derivative of the total particle number with respect to the magnetic field. (A)

73. **"Quantized Hall Conductance in a Relativistic Two-Dimensional Electron Gas,"** A. H. MacDonald, Phys. Rev. B **28**, 2235–6 (1983). Demonstrates that there are no relativistic corrections to the quantized Hall resistance. (A)

*74. **"Conditions for the Quantum Hall Effect,"** R. Joynt and R. E. Prange, Phys. Rev. B **29**, 3303–17 (1984). These authors explore quantization and localization due to potentials of various length scales. (A)

75. **"A New Approach to the Quantum Hall Effect,"** R. Woltjer, R. Eppenga, J. Mooren, C. E. Timmering, and J. P. André, Europhys. Lett. **2**, 149–55 (1986). Macroscopic inhomogeneities are shown to mimic the effects of localized electrons in producing plateaus. (I)

*76. **"Universality of Quantum Hall Effect: Topological Invariant and Observable,"** H. Aoki and T. Ando, Phys. Rev. Lett. **57**, 3093–6 (1986). The authors show that the Hall conductance averaged over a suitable set of boundary conditions is exactly quantized even for small systems. (A)

77. **"Perturbational Calculation of the Quantum Hall Conductivity,"** S. Hikami, Prog. Theor. Phys. **77**, 602–21 (1987). Quantization of the Hall conductance is derived by a summation of a series of diagrams in perturbation theory. (A)
78. **"Edge States, Transmission Matrices, and the Hall Resistance,"** P. Streda, J. Kucera, and A. H. MacDonald, Phys. Rev. Lett. **59**, 1973–5 (1987). This paper gives a generalization of the Landauer formulation of one-dimensional transport for the case of the quantum Hall effect and derives corrections to the quantization of σ_{xy} due to the presence of finite σ_{xx}. (A)

3. Numerical simulations

These papers present numerical simulations of the behavior of the two-dimensional electron gas in the presence of a variety of potentials. Most consider the scaling function β, which determines the way the conductivity varies as the limit of infinite-size systems is approached.

79. **"Magnetotransport in Two Dimensions: Some Numerical Results,"** A. MacKinnon, L. Schweitzer, and B. Kramer, Surf. Sci. **142**, 189–95 (1984). (A)
80. **"Electron Localization in a Two-Dimensional System in Strong Magnetic Fields. I: Case of Short-Range Scatterers,"** T. Ando, J. Phys. Soc. Jpn. **52**, 1740–9 (1983). (A)
81. **"Electron Localization in a Two-Dimensional System in Strong Magnetic Fields. II. Long-Range Scatterers and Response Functions,"** T. Ando, J. Phys. Soc. Jpn. **53**, 3101–11 (1984). (A)
82. **"Electron Localization in a Two-Dimensional System in Strong Magnetic Fields. III. Impurity-Concentration Dependence and Level-Mixing Effects,"** T. Ando, J. Phys. Soc. Jpn. **53**, 3126–35 (1984). (A)
83. **"Universal Scaling Relation of Conductivities in Quantized Landau Levels,"** T. Ando, Surf. Sci. **170**, 243–8 (1986). (A)
84. **"Critical Localization and Low-Temperature Transport in Two-Dimensional Landau Quantization,"** H. Aoki and T. Ando, Surf. Sci. **170**, 249–55 (1986). (A)
85. **"Localization in Strong Magnetic Fields and Quantum Hall Effect,"** T. Ando, Prog. Theor. Phys. Suppl. **84**, 69–96 (1985). (A)

*86. **"Scaling Functions in Quantum Hall Effect,"** T. Ando, J. Phys. Soc. Jpn. **55**, 3199–209 (1986). (A)

4. Theories of localization

These papers examine the way in which the localized eigenfunctions become extended as the energy eigenvalue approaches the center of the Landau level. They have been separated according to whether or not they use field-theoretic techniques.

Nonfield theories include:

87. **"Self-Consistent Treatment of Dynamical Diffusion Coefficient of Two-Dimensional Random Electron System under Strong Magnetic Fields."** Y. Ono, J. Phys. Soc. Jpn. **51**, 3544–52 (1982). (A)

*88. **"Borel-Padé Analysis for the Two-Dimensional Electron in a Random Potential Under a Strong Magnetic Field,"** S. Hikami, Phys. Rev. B **29**, 3726–9 (1984). This is a perturbation calculation of σ_{xx} at the centers of Landau levels. (A)

89. **"Critical Localization in Two-Dimensional Landau Quantization,"** H. Aoki and T. Ando, Phys. Rev. Lett. **54**, 831–4 (1985). (A)
90. **"Some Unresolved Questions in the Theory of Localization,"** P. W. Anderson, in *Localization, Interaction, and Transport Phenomena. Proceedings of the International Conference*, edited by B. Kramer, G. Bergmann, and Y. Bruynseraede (Springer-Verlag, Berlin, 1985), pp. 12–17. (A)

*91. **"Localization Length and Inverse Participation Ratio of Two-Dimensional Electron in the Quantized Hall Effect,"** S. Hikami, Prog. Theor. Phys. **76**, 1210–21 (1986). (A)

*92. **"Quantum Hall Effect: From the Winding Number to the Flow Diagram,"** H. Aoki and T. Ando, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986. edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 45–8. (A)

Field theories include:

*93. **"Electron Delocalization by a Magnetic Field in Two Dimensions,"** H. Levine, S. B. Libby, and A. M. M. Pruisken, Phys. Rev. Lett. **51**, 1915–18 (1983). These authors found a topological term in the action of the field theory which describes localization. This magnetic field induced term allows the existence of extended states in the center of each Landau level. Further details are provided in the next four papers. (A)

94. **"On Localization in the Theory of the Quantized Hall Effect: A Two-Dimensional Realization of the θ-Vacuum,"** A. M. M. Pruisken, Nucl. Phys. B, Field Theor. Stat. Syst. **B235(FS11)**, 277–98 (1984). (A)

*95. **"Theory of the Quantized Hall Effect. I,"** H. Levine, S. B. Libby, and A. M. M. Pruisken, Nucl. Phys. B. Field Theor. Stat. Syst. **B240(FS12)**, 30–48 (1984). (A)

*96. **"Theory of the Quantized Hall Effect. II,"** H. Levine, S. B. Libby, and A. M. M. Pruisken, Nucl. Phys. B. Field Theor. Stat. Syst. **B240(FS12)**, 49–70 (1984). (A)

*97. **"Theory of the Quantized Hall Effect. III,"** H. Levine, S. B. Libby, and A. M. M. Pruisken, Nucl. Phys. B. Field Theor. Stat. Syst. **B240(FS12)**, 71–90 (1984). (A)

*98. **"The Quantum Hall Effects, σ-models at $\theta = \pi$ and Quantum Spin Chains,"** I. Affleck, Nucl. Phys. B. Field Theor. Stat. Syst. **B257(FS14)**, 397–406 (1985). In this paper, the field theory of localization is related to the field theory of quantum spin chains. (A)

*99. **"Exact Critical Exponents for Quantum Spin Chains Non-Linear σ-models at θ = π and the Quantum Hall Effect,"** I. Affleck, Nucl. Phys. B. Field Theor. Stat. Syst. **B265(FS15)**, 409–47 (1986). (A)

100. **"Localization and the Integral Quantum Hall Effect,"** A. M. M. Pruisken, in *Localization, Interaction, and Transport Phenomena. Proceedings of the International Conference*, edited by B. Kramer, G. Bergmann, and Y. Bruynseraede (Springer-Verlag, Berlin, 1985), pp. 188–207. (A)

101. **"Quasiparticles in the Theory of the Integral Quantum Hall Effect (I),"** A. M. M. Pruisken, Nucl. Phys. B. Field Theor. Stat. Syst. **B285(FS19)**, 719–59 (1987). (A)

102. **"Quasi Particles in the Theory of the Integral Quantum Hall Effect (II). Renormalization of the Hall Conductance or Instanton Angle Theta,"** A. M. M. Pruisken, Nucl. Phys. B. Field Theor. Stat. Syst. **B290(FS20)**, 61–86 (1987). (A)

103. **"Universal Singularities in the Integral Quantum Hall Effect,"** A. M. M. Pruisken, Phys. Rev. Lett. **61**, 1297–1300 (1988). A scaling theory of the quantum Hall effect is used to explain the observed temperature and magnetic field dependences of the magnetoresistance. (A)

5. Periodic potential theories

These papers consider the effect of a periodic potential on a two-dimensional electron gas. Such periodicity predicts interesting effects yet to be experimentally observed. For example, the quantized values of the Hall conductance are not simple monotonically increasing functions of the number of filled bands. There is a periodic potential associated with the atomic lattice, but the length scale under consideration here is much larger. It is the length associated with the area containing one quantum of flux (the magnetic length), which is typically 50 to 100 lattice constants.

*104. **"Quantized Hall Conductance in a Two-Dimensional Periodic Potential,"** D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405–8 (1982). This is a powerful analysis of conductance presenting topological arguments proving that σ_{xy} is quantized whenever the chemical potential is in the gap between bands. (A)

105. **"Holonomy, the Quantum Adiabatic Theorem, and Berry's Phase,"** Barry Simon, Phys. Rev. Lett. **51**, 2167–70 (1983). This author expresses the results of the above paper in more formal mathematical language. (A)

*106. **"Quantized Hall Effect in Two-Dimensional Periodic Potentials,"** D. J. Thouless, Phys. Rep. **110**, 279–91 (1984). This is a longer version of Ref. 104.

See also the book chapter by Thouless, Ref. 41.

6. The vanishing of ρ_{xx}

The landmark paper by Kawaji and Wakabayashi, Ref. 30, was the first in a series of experimental papers that examine the vanishing of the diagonal resistivity and show that the diagonal resistivity of these two-dimensional electron gas systems is orders of magnitude smaller than that of any other nonsuperconducting system. Other important papers are:

*107. **"Zero Resistance State and Origin of the Quantized Hall Effect in Two-Dimensional Electron Systems,"** H. L. Störmer, D. C. Tsui, and A. C. Gossard, Surf. Sci. **113**, 32–8 (1982). These authors observed an upper limit to ρ_{xx} of $5\times10^{-7}\ \Omega$ for a GaAs heterostructure at 1.23 K. (I)

108. **"Magnetization Measurements on a Two-Dimensional Electron System,"** T. Haavasoja, H. L. Störmer, D. J. Bishop, V. Narayanamurti, A. C. Gossard, and W. Wiegmann, Surf. Sci. **142**, 294–7 (1984). They observed an upper limit of $10^{-10}\ \Omega$ for a GaAs heterostructure at 0.4 K. The relaxation time for the decay of induced currents was 300 s. (I)

7. Measurements of temperature dependence at the plateau centers

The quantum Hall effect is only observed at temperatures below about 4K. The manner by which the diagonal resistivity vanishes and the plateaus form as the temperature is lowered provides information about the microscopic nature of the electronic states. The temperature dependence is also of crucial importance to the application of the quantum Hall effect as a resistance standard, or as a method of determining the fine structure constant.

First, the following papers deal with the temperature dependence of the diagonal resistivity:

*109. **"Quantized Hall Effect at Low Temperatures,"** M. A. Paalanen, D. C. Tsui, and A. C. Gossard, Phys. Rev. B **25**, 5566–9 (1982). These authors cooled GaAs heterostructures to 50 mK and observed spectacularly sharp stepwise transitions between quantized Hall resistance plateaus. (I)

*110. **"Hopping Conduction in the Landau Level Tails in GaAs-Al_x. GA_{1-x} As Heterostructures at Low Temperatures,"** G. Ebert, K. von Klitzing, C. Probst, E. Schuberth, K. Ploog, and C. Weimann, Solid State Commun. **45**, 625–8 (1983). Measurements made down to 10 mK are interpreted in terms of the variable-range hopping theory of Ono. (I)

111. **"Energy Dissipation Processes in the Quantum Hall Regime,"** K. von Klitzing, G. Ebert, N. Kleinmichel, H. Obloh, and G. Weimann, in *Proceedings of the 17th International Conference on the Physics of Semiconductors*, edited by J. D. Chadi and W. A. Harrison, 271–4 (1985). Data on the temperature dependence of ρ_{xx} vs T for GaAs heterostructures and Si MOSFETs are shown to lead to thermal activation energies that are much larger than expected. (I)

The temperature dependences of the slope, width, and midpoint value of the plateaus have been found to be very similar to that of the diagonal resistivity. In addition to Paalanen *et al.* given in Ref. 109, other key papers are:

*112. **"Quantized Hall and Transverse Resistivities in Silicon MOS *n*-Inversion Layers,"** K. Yoshihiro, J. Kinoshita, K. Inagaki, C. Yamanouchi, J. Moriyama, and S. Kawaji, Physica B–C **117–118**, 706–8 (1983). The first observation of the proportionality between $\Delta\rho_{xy}$ and ρ_{xx}. (I)

113. **"Analysis of Quantized Hall Resistance at Finite Temperatures,"** B. Tausendfreund and K. von Klitzing, Surf. Sci. **142**, 220–4 (1984). At higher temperatures, the Hall plateaus have a nonzero slope that these authors measured between 1.5 and 10 K and found to have an exponential change with inverse temperature. (I)

*114. **"Temperature Dependence of the Quantum Hall Resistance,"** M. E. Cage, B. F. Field, R. F. Dziuba, S. M. Girvin, A. C. Gossard, and D. C. Tsui, Phys. Rev. B **30**, 2285–8 (1984). Demonstrated that for GaAs heterostructures $\Delta\rho_{xy}$ is proportional to ρ_{xx} over at least four orders of magnitude change in ρ_{xx} between 1.2 and 4.2 K. It was also shown that even when Hall plateaus are exceedingly flat, the values of ρ_{xy} can deviate significantly from the zero- temperature value. (I)

The following two papers, and Ref. 103 examine the scaling relationship between the diagonal and Hall resistivities over the entire range of temperature and magnetic field.

115. **"Localization and Scaling in the Quantum Hall Regime,"** H. P. Wei, D. C. Tsui, and A. M. M. Pruisken, Phys. Rev. B **33**, 1488–91 (1986). Measurements on InGaAs/InP heterostructures from 0.5 K to 50 K of the temperature dependence of the entire parameter space of ρ_{xx} and ρ_{xy} were analyzed using a scaling theory. (I)

116. **"Experiments on Delocalization and Universality in the Integral Quantum Hall Effect,"** H. P. Wei, D. C. Tsui, M. A. Paalanen, and A. M. M. Pruisken, Phys. Rev. Lett. **61**, 1294–6 (1988). (I)

8. Current dependence and breakdown

The quantization only applies in the small current limit. Precisely how the diagonal resistivity and plateau values are at first only mildly degraded by increasing current and then how the effect abruptly breaks down are shown in the following papers. The more recent papers attempt to examine this question in precisely defined restricted geometries.

*117. **"Two-Dimensional Magneto-Quantum Transport on GaAs-Al$_x$Ga$_{1-x}$As Heterostructures Under Non-Ohmic Conditions,"** G. Ebert, K. von Klitzing, K. Ploog, and G. Weimann, J. Phys. C (GB) **16**, 5441–8 (1983). The first observation of the breakdown of ρ_{xx} with increasing current. (I)

118. **"Dissipation and Dynamic Nonlinear Behavior in the Quantum Hall Regime," M. E. Cage, R. F. Dziuba, B. F. Field, E. R. Williams, S. M. Girvin, A. C. Gossard, D. C. Tsui, and R. J. Wagner, Phys. Rev. Lett. **51**, 1374–7 (1983). Demonstrated that the breakdown occurs in spatially localized regions of GaAs heterostructures and that there is a transient switching among distinct dissipative states. (I)

119. **"Effect of the Current on the Plateau Width in the Quantum Hall Effect,"** V. N. Zavaritskiĭ and V. B. Anzin, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 249–50 (1983). Trans. in JETP Lett. **38**, 294–6 (1983). The plateau width was shown to be proportional to the current. (I)

*120. **"Breakdown of the Quantum Hall Effect due to Electron Heating,"** S. Komiyama, T. Takamasu, S. Hiyamizu, and S. Sasa, Solid State Commun. **54**, 479–84 (1985). A model is given for the hysteric nature of the breakdown based on superheated electrons. (I)

*121. **"Low-Voltage Breakdown of the Quantum Hall State in a Laterally Constricted Two-Dimensional Electron Gas,"** J. R. Kirtley, Z. Schlesinger, T. N. Theis, F. P. Milliken, S. L. Wright, and L. F. Palmateer, Phys. Rev. B **34**, 1384–7 (1986). This is the first observation of breakdown in samples only a few micrometers wide. The breakdown occurred at voltages corresponding to the cyclotron energy. (I)

*122. **"A New Quantum Effect in the Transverse Magnetoresistance of Two-Dimensional Conductors with a Narrow Constriction in the Conducting Channel,"** L. Bliek, G. Hein, V. Kose, J. Niemeyer, G. Weimann, and W. Schlapp, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 113–17. The breakdown current was surprisingly large for their narrow samples, and ρ_{xx} was observed to have quantized values for currents near breakdown. (I)

123. **"Critical Currents of the Quantum Hall Effect in the Mesoscopic Regime,"** P. G. N. de Vegvar, A. M. Chang, G. Timp, P. M. Mackiewich, J. E. Cunningham, R. Behringer, and R. E. Howard, Phys. Rev. B **36**, 9366 (1987). High-mobility GaAs/AlGaAs samples of 80–200 nm widths and 0.7- to 2-μm lengths were studied. (I)

The following five papers describe theories that explain the dissipation as being a result of Čerenkov radiation as the electrons travel faster than the velocity of sound in the semiconductor.

124. **"Critical Current of the Quantum Hall Regime,"** P. Středa, Phys. Status Solidi B **124**, K97–9 (1984). (I)

125. **"Critical Non-Dissipative Current of Quantum Hall Regime,"** P. Středa and K. von Klitzing, J. Phys. C **17**, L483–6 (1984). (I)

*126. **"Electron–Phonon Interactions and the Breakdown of the Dissipationless Quantum Hall Effect,"** O. Heinonen, P. L. Taylor, and S. M. Girvin, Phys. Rev. B **30**, 3016–19 (1984). (I)

*127. **"On the Phonon-Assisted Breakdown of the Dissipationless Quantum Hall Effect,"** L. Smrčka, J. Phys. C **18**, 2897–908 (1985). (I)

*128. **"Breakdown of the Quantum Hall Effect,"** P. Středa, J. Phys. C **19**, L155–9 (1986). (I)

See also the experimental paper by Komiyama *et al.*, Ref. 120, for an analysis of the final stages of breakdown.

129. **"Dissipative Transport in Transition Regions between Quantum Hall Plateaus,"** B. Shapiro, Phys. Rev. B **33**, 8447–57 (1986). (I)

*130. **"Size-Dependent Quantised Breakdown of the Dissipationless Quantum Hall Effect in Narrow Channels,"** L. Eaves and F. W. Sheard, Semicond. Sci. Technol. **1**, 346–9 (1986). This work analyzes the data of Bliek *et al.* (see previous section) on the breakdown in narrow constrictions by invoking quasielastic inter-Landau level scattering. It explains the high current densities, the magnetic field values of steps in ρ_{xx} and the quantized values of ρ_{xx}. (I)

9. *Quantum interference effects*

Quantum interference effects of conducting carriers in a quantum Hall sample can be observed at low temperatures in very narrow samples, or with high precision in ordinary samples. This quantum interference is observed as resistance irregularities (referred to as "fluctuations") observed as the magnetic field is swept. It is a manifestation of interference among individual paths of the carriers, and may ultimately limit the accuracy of resistance standards based on the quantum Hall effect.

131. **"Abrupt Disappearance of the Quantum Hall Effect Observed in Silicon *n*-Inversion Layers,"** J. Kinoshita, K. Inagaki, C. Yamanouchi, K. Yoshihiro, J. Wakabayashi, and S. Kawaji, in *Proceedings of the 2nd International Symposium: Foundations of Quantum Mechanics in the Light of New Technology*, Tokyo, 1–4 September 1986, edited by M. Namiki, Y. Ohnuki, Y. Murayama, and S. Nomura (Physical Society of Japan, Tokyo, 1987), pp. 150–4. These authors observe resistance fluctuations appearing in the plateaus of a 100-μm-wide silicon MOSFET sample as the temperature, measurement current, and carrier concentration are lowered. (A)

*132. **"Conductance Fluctuations with Variations of Electron Density in Inversion Layers of Silicon,"** J. Kinoshita, K. Inagaki, C. Yamanouchi, K. Yoshihiro, J. Wakabayashi, and S. Kawaji, in *Anderson Localization. Proceedings of the International Symposium*, Tokyo, Japan, 16–18 August 1987, edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1988), pp. 365–9. The authors expand on their work presented in Ref. 131. (A)

*133. **"Quantum Transport in an Electron-Wave Guide,"** G. Timp, A. M. Chang, P. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, Phys. Rev. Lett. **59**, 732–5 (1987). The quantum interference fluctuations are studied in very narrow samples of GaAs/AlGaAs heterostructures. (I)

134. **"Deviation of the Quantum Hall Effect from Exact Quantization in Narrow GaAs-Al$_x$Ga$_{1-x}$As Heterostructure Devices,"** A. M. Chang, G. Timp, T. Y. Chang, J. E. Cunningham, P. M. Mankiewich, R. E. Behringer, and R. E. Howard, Solid State Commun. **67**, 769–72 (1988). Deviations of 250 Ω were found for the $i = 4$ plateau in 0.2-μm wide samples at 50 mK. (I)

In addition to the interpretation given in Ref. 132, the following theoretical paper discusses this effect.

135. **"Quantum Hall Effect in Quasi One-Dimensional Systems: Resistance Fluctuations and Breakdown,"** J. K. Jain and S. A. Kivelson, Phys. Rev. Lett. **60**, 1542–5 (1988). (A)

10. *Current and potential distribution*

There has been considerable discussion concerning the shape of current flow and equipotential lines within homogeneous quantum Hall devices. The following series of experimental papers all clearly establish that the current flows *throughout* the width of the sample. Multiple voltage contacts are placed in a row or array across the sample in order to measure the flow pattern at small values of ρ_{xx}.

*136. **"Hall Potential Distribution in Quantum Hall Experiments,"** G. Ebert, K. von Klitzing, and G. Weimann, J. Phys. C **18**, L257–60 (1985). (I)

*137. **"Distribution of the Quantized Hall Potential in GaAs-Al$_x$Ga$_{1-x}$As Heterostructures,"** H. Z. Zheng, D. C. Tsui, and A. M. Chang, Phys. Rev. B **32**, 5506–9 (1985). (I)

138. **"Experimental Study of the Current Flow in the Quantum Hall Regime,"** Ch. Simon, B. B. Goldberg, F. F. Fang, M. K. Thomas, and S. Wright, Phys. Rev. B **33**, 1190–8 (1986). (I)

139. **"Equipotential Distribution in the Quantum Hall Effect in GaAs-AlGaAs Heterostructures,"** E. K. Sichel, M. L. Knowles, and H. H. Sample, J. Phys. C **19**, 5695–713 (1986). (I)

Theoretical work by Halperin, Středa, Woltjer *et al.*, and Středa *et al.* (listed earlier as Refs. 70, 72, 75, and 78, respectively) of the quantization theorem section should be consulted along with the following papers:

140. **"Hall Voltage Dependence on Inversion-Layer Geometry in the Quantum Hall-Effect Regime,"** R. W. Rendell and S. M. Girvin, Phys. Rev. B **23**, 6610–14 (1981). These authors give a classical electrodynamics model for the current distribution in an inversion layer. (I)

*141. **"Quantized Magnetoresistance in Two-Dimensional Electron Systems,"** F. F. Fang and P. J. Stiles, Phys. Rev. B **27**, 6487–8 (1983). This gives a clear exposition of why the source–drain resistance is ρ_{xy} rather than ρ_{xx}. (I)

142. **"Linear Response and the Quantum Hall Effect,"** T. McMullen, Phys. Rev. B **32**, 1415–18 (1985). The author discusses why electrostatic screening does not modify the Hall quantization. (I)

143. **"Quantum Hall Effect with Realistic Boundary Conditions,"** Qian Niu and D. J. Thouless, Phys. Rev. B **35**, 2188–97 (1987). These authors attempt a realistic calculation of how current is injected from electrodes into the Hall bar device. (I)

144. **"Electrical Potential and Current Distribution for the Quantized Hall Effect,"** B. Neudecker and K. H. Hoffmann, Solid State Commun. **62**, 135–9 (1987). The authors use classical electrodynamics to determine the current and potential distribution. (I)

For random potentials that are smooth, in the sense that they change by only a small amount over a distance equal to the cyclotron orbit, the electron motion can be represented semiclassically by $\mathbf{E}\times\mathbf{B}$ drift along contour lines of constant potential. In two dimensions there exists a unique value of the energy for contours that percolate across the entire sample. The following papers use this percolation model:

*145. **"On the Conductivity of Two Dimensional Electrons in a Strong Magnetic Field,"** S. V. Iordansky, Solid State Commun. **43**, 1–3 (1982). (I)

*146. **"Quantum Percolation and Quantization of Hall Resistance in Two-Dimensional Electron Gas,"** R. F. Kazarinov and Serge Luryi, Phys. Rev. B **25**, 7626–30 (1982). (I)

147. **"Theory of Quantized Hall Effect at Low Temperatures,"** Serge Luryi and R. F. Kazarinov, Phys. Rev. B **27**, 1386–89 (1983). (I)

148. **"Localization, Percolation, and the Quantum Hall Effect,"** S. A. Trugman, Phys. Rev. B **27**, 7539–46 (1983). (A)

11. Thermal transport effects (integer and fractional)

The experimental and theoretical situations remain unclear for the thermal conductivity and thermoelectric effect. It may be that phonon drag effects, which have not yet been dealt with theoretically, will turn out to be important. The bibliographic separation between integral and fractional is not very well defined for these effects. Therefore, all papers dealing with thermoelectric effects are placed in this section.

The following three papers generally verify the theoretical predictions that the thermopower is a universal function of the ratio of the thermal energy to the cyclotron energy in the absence of disorder, and also that its tensor components oscillate across the Hall plateaus, are exponentially small, and are thermally activated on the Hall plateaus.

Experimental papers:

*149. **"Thermopower Measurements on the Two-Dimensional Electron Gas of GaAs-Al$_x$Ga$_{1-x}$As Heterostructures,"** H. Obloh, K. von Klitzing, and K. Ploog, Surf. Sci. **142**, 236–40 (1984). (I)

150. **"Thermoelectric Effects in Silicon MOSFETs in High Magnetic Fields,"** R. P. Smith, H. Closs, and P. J. Stiles, Surf. Sci. **142**, 246–8 (1984). (I)

151. **"Thermoelectric Properties of a Two-Dimensional Electron Gas Exhibiting the Quantum Hall Effect,"** J. S. Davidson, E. D. Dahlberg, A. J. Valois, and G. Y. Robinson, Phys. B **33**, 2941–4 (1986). (I)

*152. **"Thermoelectric Properties of GaAs-Ga$_{1-x}$Al$_x$As Heterojunctions at High Magnetic Fields,"** R. Fletcher, J. C. Maan, K. Ploog, and G. Weimann, Phys. Rev. B **33**, 7122–33 (1986). Measurements of the thermoelectric properties of GaAs heterostructures were performed at fields up to 20 T. The thermopowers were much larger than predicted. (I)

*153. **"Phonon-Drag Effect in GaAs-Al$_x$Ga$_{1-x}$As Heterostructures at Very Low Temperatures,"** C. Ruf, H. Obloh, B. Junge, E. Gmelin, and K. Ploog, Phys. Rev. B **37**, 6377–80 (1988). Measurements were made at temperatures down to 200 mK and magnetic fields up to 7 T. (I)

*154. **"Density of States and Electron–Phonon Coupling in Two-Dimensional Electron Systems at High Magnetic Fields,"** J. P. Eisenstein, in *Interfaces, Quantum Wells, and Superlattices*, Banff, Alberta, Canada, 16–19 August 1987. NATO Advanced Science Institute Series B **179**, edited by C. R. Leavens and R. Taylor, (Plenum, New York, 1988), pp. 271–97. Magnetization and thermal conductance measurements are described and interpreted. (I)

*155. **"Quantum Oscillations in the Thermal Conductance of GaAs/AlGaAs Heterostructures,"** J. P. Eisenstein, A. C. Gossard, and V. Narayanamurti, Phys. Rev. Lett. **59**, 1341 (1987). A $T^{2.65}$ power law was found for the contribution of the two-dimensional electron gas to the thermal conductivity. (I)

Theoretical papers:

*156. **"Inversion Layer Thermopower in High Magnetic Field,"** S. M. Girvin and M. Jonson, J. Phys. C **15**, L1147–51 (1982). This paper, which neglects disorder, predicts oscillations in the thermopower with universal peak heights. (I)

*157. **"On the Theory of Thermomagnetic Transport and Its Application to Two-Dimensional Systems,"** P. Středa and H. Oji, Phys. Lett. A **102**, 201–3 (1984). The quantum and classical contributions to the thermoelectric transport coefficients are expressed as functions of thermodynamic quantities. (I)

*158. **"Universal Transport Properties of Inversion Layers in Quantizing Magnetic Field,"** P. Středa, Phys. Status Solidi B **125**, 849–60 (1984). Various transport coefficients, including thermopower, are calculated using the Kubo formula. (I)

*159. **"Thermoelectric Effect in a Weakly Disordered Inversion Layer Subject to a Quantizing Magnetic Field,"** M. Jonson and S. M. Girvin, Phys. Rev. B **29**, 1939–46 (1984). This paper also calculates the thermopower tensor using a modified Kubo formula. (I)

12. Specific heat and magnetization measurements (integer and fractional)

The measurement of thermodynamic properties such as specific heat and magnetization give information on the total density of states (both localized and nonlocalized) but are extremely difficult for a two-dimensional electron gas. As in the case of thermopower, we will make no attempt to separate integer effects from fractional effects. Besides the work of Eisenstein listed in Ref. 154, the following papers cover these important basic properties.

*160. **"Specific Heat of Two-Dimensional Electrons in GaAs-GaAlAs Multilayers,"** E. Gornik, R. Lassnig, G. Strasser, H. L. Störmer, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 1820–3 (1985). Measurements on both 172- and 94-layer samples suggested a density of states composed of Gaussian peaks superimposed upon a flat background. (I)

161. **"Density of States and de Haas–van Alphen Effect in Two-Dimensional Electron Systems," J. P. Eisenstein, H. L. Störmer, V. Narayamamurti, A. Y. Cho, A. C. Gossard, and C. W. Tu, Phys. Rev. Lett. **55**, 875–8 (1985). Evidence is found for a relatively constant density of states with a significant density between Landau levels from measurements on a 50-layer sample and also on a higher mobility single-layer sample. (I)

13. Magnetocapacitance measurements (integer and fractional)

Measurements as a function of magnetic field of the capacitance between the electron gas and a nearby metal gate electrode can be analyzed to give indirect information about the density of states.

162. **"Capacitance Observations of Landau Levels in Surface Quantization,"** M. Kaplit and J. N. Zemel, Phys. Rev. Lett. **21**, 212–5 (1968). These authors pioneered the method of using capacitance measurements to observe the two-dimensional electron gas in silicon inversion layers. (I)

163. **"Capacitance Measurements of a Quantized Two-Dimensional Electron Gas in the Regime of the Quantized Hall Effect,"** R. K. Goodall, R. J. Higgins, and J. P. Harrang, Phys. Rev. B **31**, 6597–608 (1985). These authors present a new model for the interpretation of capacitance data and conclude that it is probably impossible to determine directly the density of states of two-dimensional electron systems from capacitance measurements. (I)

*164. **"Fractional Quantization in ac Conductance of $Al_xGa_{1-x}As$ Capacitors,"** T. W. Hickmott, Phys. Rev. Lett. **57**, 751–4 (1986). Minima were observed in the ac conductance of GaAs heterostructure capacitors at fractional filling factors $\nu = \frac{1}{3}$ and $\frac{2}{3}$ at 100 kHz. (I)

*165. **"The Two-Dimensional Density of States at Fractional Filling Factors,"** T. P. Smith, III, W. I. Wang, and P. J. Stiles, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 173–7. Measurements of the magnetocapacitance of GaAs heterostructures for fractional filling factors $\nu = \frac{1}{3}$ and $\frac{2}{3}$ were made at 1 kHz and interpreted in terms of the density of states. (I)

166. **"Hysteresis Phenomena in Charging of Si MOSFET in Quantizing Magnetic Field,"** V. M. Pudalov and S. G. Semenchinsky, Solid State Commun. **51**, 713–7 (1984). Magnetocapacitance experiment measures the gate charging current as magnetic field or gate voltage is swept. Hysterisis in sweep direction is observed. (I)

14. ac *measurements (integer and fractional)*

Measurements made by both pulse and cw techniques of the resistance minima and plateaus of the quantum Hall effect have been carried out at frequencies up to the far-infrared. After some initial controversy, it now seems clear that the basic effect is not frequency dependent until the frequencies are so high that cyclotron resonance and optical transitions can be excited. Nevertheless, low-frequency ac measurements can provide a convenient contactless method of probing the electron gas.

Experimental papers:

167. **"The Frequency Effect and the Quantised Hall Resistance,"** M. Pepper and J. Wakabayashi, J. Phys. C **16**, L113–17 (1983). Measurements are reported on silicon inversion layers that indicate that the Hall resistance at frequencies up to 20 kHz is significantly different from that at dc. (I)

*168. **"Non-equilibrium Behavior of the Two-Dimensional Electron Gas in the Quantized Hall Resistance Regime of $GaAs/Al_{0.3}Ga_{0.7}As$,"** F. Kuchar, G. Bauer, G. Weimann, and H. Burkhard, Surf. Sci. **142**, 196–202 (1984). By using pulsed measurements of between 250-μs and 5-ms duration, these authors conclude that the quantum Hall effect is established within 100 μs of the application of the current. (I)

*169. **"Capacitively Coupled Measurements of the Quantized Hall Effect in Silicon Inversion Layers,"** T. P. Smith and P. J. Stiles, Solid State Commun. **52**, 941–3 (1984). This paper reports that if parasitic capacitances are carefully taken into account, then contactless measurements up to 100 kHz yield the same quantum Hall behavior as at dc. (I)

*170. **"Quantum Hall Measurements with Pulsed Electric Fields,"** R. Woltjer, J. Mooren, J. Wolter, and J. P. André, Solid State Commun. **53**, 331–3 (1985). This work establishes that the quantum Hall effect in GaAs heterostructures is established to within 0.5% even when probed by pulses of only 100-ns duration. (I)

171. **"The Role of Length Scale in the Competition between Localization and Interaction Effects in MBE-Grown Modulation Doped GaAs-GaAlAs Heterojunctions,"** C. McFadden, A. P. Long, H. W. Myron, M. Pepper, D. Andrews, and G. J. Davies, J. Vac. Sci. Technol. B **3**, 639–43 (1985). Measurements at frequencies up to 45 MHz are made that show disappearance of integral plateaus and enhancement of fractional state plateaus. These are interpreted as evidence for an effective delocalization of electrons. (I)

172. **"Frequency-Dependent Magnetoconductance Quantisation in 2D Systems—A Disorder Effect,"** T. G. Powell, R. Newbury, A. P. Long, C. McFadden, H. W. Myron, and M. Pepper, J. Phys. C **18**, L497–505 (1985). Measurements were reported on silicon MOSFETs at frequencies between 250 kHz and 32 MHz. (I)

*173. **"Microwave Hall Conductivity of the Two-Dimensional Electron Gas in $GaAs\text{-}Al_xGa_{1-x}As$,"** F. Kuchar, R. Meisels, G. Weimann, and W. Schlapp, Phys. Rev. B **33**, 2965–7 (1986). Microwave conductivity measurements were made at 33 GHz. They show nearly the same integer plateaus as measured at dc. The authors therefore conclude that there is no delocalization of electrons. (I)

174. **"Capacitively Coupled Measurements of the Magnetoconductivity Tensor in Heterostructures,"** T. P. Smith, M. Heiblum, and P. J. Stiles, in *Proceedings of the 17th International Conference on the Physics of Semiconductors*, edited by J. D. Chadi and W. A. Harrison (Springer-Verlag, New York, 1985), pp. 393–6. Measurements were made on GaAs heterostructures between 100 Hz and 100 kHz using capacitively coupled contacts. (I)

175. **"ac Conductivity of (AlGa)As-GaAs Heterostructures and Silicon MOSFETs in the Quantum Hall Regime,"** B. B. Goldberg, T. P. Smith, M. Heiblum, and P. J. Stiles, Surf. Sci. **170**, 180–6 (1986). By using both gated and ungated quantum Hall samples, these authors conclude that the apparent frequency dependence of the quantum Hall effect was an artifact of the gate capacitance. (I)

176. **"The Frequency Dependence of the Diagonal Conductivity of a 2DEG in GaAs Heterostructure in the Quantum Hall Regime,"** J. I. Lee, B. B. Goldberg, M. Heiblum, and P. J. Stiles, Solid State Commun. **64**, 447–50 (1987). Novel technique for high-frequency measurements. Found significant effects above 1 MHz. (I)

177. **"High-Frequency Conductivity in the Quantum Hall Effect Regime,"** F. Kuchar, R. Meisels, K. Y. Lim, P. Pichler, G. Weimann, and W. Schlapp, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, Germany, 1987), pp. 95–103. Deviations from the dc behavior of the quantum Hall effect in GaAs heterojunctions occur at frequencies of about 1000 GHz, which is about an order of magnitude smaller than the cyclotron frequency. (I)

Theoretical papers:

178. **"Theory of the ac Breakdown of the Quantum Hall Effect,"** R. Joynt, J. Phys. C **18**, L331–6 (1985). A characteristic frequency for deviations from the dc quantum Hall behavior is calculated in terms of a coherence length for the scattering potentials. (I)

179. **"The Microwave Hall Effect of a Two-Dimensional Electron Gas,"** R. Meisels and F. Kuchar, Z. Phys. B **67**, 443–7 (1987). Calculations of microwave coupling to the two-dimensional gas are presented for the geometry of their experimental work. (I)

15. Collective modes

The literature on cyclotron resonance in two-dimensional systems is too extensive and too loosely connected to the quantum Hall effect itself to be within the scope of this survey. We will simply list the following papers as a starting point for readers wishing to investigate further.

180. **"Mott exciton in a Quasi-Two-Dimensional Semiconductor in a Strong Magnetic Field,"** I. V. Lerner and Yu. E. Lozovik, Zh. Eksp. Teor. Fiz. **78**, 1167–75 (1978). Trans. in Sov. Phys. JETP **51**, 588–92 (1980). (A)

*181. **"Two-Dimensional Electrons in a Strong Magnetic Field,"** Yu. A. Bychkov, S. V. Iordanskii, and G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 152–5 (1981). Trans in JETP Lett. **33**, 143–6 (1981). (A)

*182. **"Electron Spin Resonance on GaAs-Al$_x$Ga$_{1-x}$As Heterostructures,"** D. Stein, K. von Klitzing, and G. Weimann, Phys. Rev. Lett. **51**, 130–3 (1983). This is the fundamental reference on spin resonance in these systems. (I)

183. **"Two-Dimensional Electron-Hole System in a Strong Magnetic Field,"** Yu. A. Bychkov and È. I. Rashba, Solid State Commun. **48**, 399–402 (1983). (A)

*184. **"Excitations from a Filled Landau Level in the Two-Dimensional Electron Gas,"** C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655–68 (1984). Systematic treatment of Coulomb effects in cyclotron resonance. (A)

185. **"Observation of Bulk and Edge Magnetoplasmons in a Two-Dimensional Electron Fluid,"** D. B. Mast, A. J. Dahm, and A. L. Fetter, Phys. Rev. Lett. **54**, 1706–9 (1985). Excitations in an electron gas on the surface of liquid ^{4}He. (I)

186. **"Dynamical Hall Effect in a Two-Dimensional Classical Plasma,"** D. C. Glattli, E. Y. Andrei, G. Deville, J. Poitrenaud, and F. I. B. Williams, Phys. Rev. Lett. **54**, 1710–13 (1985). Excitations in an electron gas on the surface of liquid ^{4}He. (A)

187. **"Magnetoplasmon Excitations from Partially Filled Landau Levels in Two Dimensions,"** A. H. MacDonald, H. C. A. Oji, and S. M. Girvin, Phys. Rev. Lett. **55**, 2208–11 (1985). First nonperturbative calculation of cyclotron excitations from a partially filled Landau level. (A)

188. **"Edge Magnetoplasmons under Conditions of the Quantum Hall Effect,"** V. A. Volkov, D. V. Galchenkov, L. A. Galchenkov, I. M. Grodnenskiĭ, O. R. Matov, and S. A. Mikhaĭlov, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 510–13 (1986). Trans. in JETP Lett. **44**, 655–9 (1986). (A)

*189. **"Dynamical Conductivity of the GaAs Two-Dimensional Electron Gas at Low Temperature and Carrier Density,"** Z. Schlesinger, W. I. Wang, and A. H. MacDonald, Phys. Rev. Lett. **58**, 73–6 (1987). This paper discusses the relation of the cyclotron resonance linewidth to the disorder and to Coulomb interactions. (A)

16. Photoconductivity measurements

The basic effect of photoexcitation of two-dimensional electron gases is to increase the carrier density and move the plateaus to higher magnetic fields. In high-purity samples, optically excited carriers can be very long lived.

190. **"Photoconductivity of Silicon Inversion Layers,"** Y. Shiraki, J. Phys. C **10**, 4539–44 (1977). A wavelength-independent photoconductivity was measured in silicon MOSFETs and was correlated with the Shubnikov–De Haas oscillations in ρ_{xx}. (I)

191. **"Magnetic Field Dependence of the Photoresponse of the Electron Inversion Layer on (100) Si,"** C. F. Lavine, R. J. Wagner, and D. C. Tsui, Surf. Sci. **113**, 112–17 (1982). These authors use laser pulses to show that the photostimulation heats only the two-dimensional electron gas when the laser frequency is near an inversion layer cyclotron resonance. (I)

192. **"Persistent Photoconductivity and the Quantized Hall Effect in In$_{0.53}$Ga$_{0.47}$As/InP Heterostructures,"** H. P. Wei, D. C. Tsui, and M. Razeghi, Appl. Phys. Lett. **45**, 666–8 (1984). The photogeneration of electron–hole pairs in high-mobility InGaAs/InP heterostructures and the trapping of the holes is shown to lead to a persistent photoconductivity. This increase in carrier concentration remained constant until the sample was heated above 40 K. (I)

193. **"Effect of Illumination on the Galvanomagnetic Characteristics of a 2D Electron Gas in a Strong Magnetic Field,"** V. B. Anzin, V. G. Veselago, V. N. Zavaritskiĭ, and A. M. Prokhorov, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 231–3 (1984). Trans. in JETP Lett. **40**, 1002–5 (1984). Persistent photoconductivity measurements were made on silicon MOSFETs showing relaxation times up to 10^4 s. (I)

194. **"Negative Photoconductivity of Two-Dimensional Holes in GaAs/AlGaAs Heterojunctions,"** M. J. Chou, D. C. Tsui, and G. Weimann, Appl. Phys. Lett. **47**, 609–11 (1985). Photogeneration of electrons are shown to decrease the density of carriers in a two-dimensional hole gas in GaAs heterostructures modulation doped with beryllium. (I)

195. **"Investigation of Parallel Conduction in GaAs/Al$_x$Ga$_{1-x}$As Modulation-Doped Structures in the Quantum Limit,"** M. A. Reed, W. P. Kirk, and P. S. Kobiela, IEEE J. Quantum Electron **QE-22**, 1753–9 (1986). These researchers generate alternate conduction paths by optically exciting deep donor traps in the AlGaAs layer in order to study systematically how parallel conduction affects quantum transport measurements of the two-dimensional electron gas. (I)

17. Measurements under hydrostatic pressure

Studies of the quantum Hall effect under hydrostatic pressure are interpreted in terms of the change of carrier density caused by the pressure.

196. **"Two-Dimensional Electron Transport Under Hydrostatic Pressure,"** J. L. Robert, J. M. Mercy, C. Bousquet, A. Raymond, J. C. Portal, G. Gregoris, and J. Beerens, in *Two-Dimensional Systems, Heterostructures and Superlattices. Proceedings of the International Winter School*, edited by G. Bauer, F. Kuchar, and H. Heinrich (Springer-Verlag, Berlin, 1984), pp. 252–61. Measurements were made on GaAs heterostructures up to a pressure of 13 kbar (1.3 GPa). A linear decrease in the electron density by a factor of 8 was observed and interpreted as being the result of a deepening of silicon impurity levels in the AlGaAs layer. (I)

197. **"Quantum Properties of the Two-Dimensional Electron Gas in the *n*-Inversion layers of InSb Grain Boundaries under High Pressure,"** R. Herrmann, W. Kraak, G. Nachtwei, and Th. Schurig. Phys. Status Solidi B **135**, 423–35 (1986). The carrier density was shown to decrease linearly by about a factor of 3 as the pressure was increased to 600 MPa. (I)

198. **"The Effect of Hydrostatic Pressure on a Ga$_{0.47}$In$_{0.53}$As/InP Heterojunction with Three Electric Sub-Bands,"** D. Gauthier, L. Dmowski, S. Ben Amor, R. Blondel, J. C. Portal, M. Razeghi, P. Maurel, F. Omnes, and M. Laviron, Semicond. Sci. Technol. **1**, 105–9 (1986). A decrease of 16% in carrier density was observed for pressures up to 15 kbar (1.5 Gpa). (I)

199. **"A Study of Parallel Conduction and the Quantum Hall Effect in GaInAs–AlInAs Heterojunctions Using Magnetotransport Measurements under Hydrostatic Pressure,"** G. Gregoris, J. Beerens, S. Ben Amor, L. Dmowski, J. C. Portal, D. L. Sivco, and A. Y. Cho, J. Phys. C **20**, 425–40 (1987). The application of pressure up to about 7 kbar is shown to suppress the effects of parallel conduction in this system and thereby allow the observation of quantized Hall plateaus. (I)

200. **"Magnetotransport Measurements under Hydrostatic Pressure in Two-Dimensional Electron and Electron–Hole Systems,"** G. Gregoris, J. Beerens, L. Dmovski, S. B. Amor, and J. C. Portal, in *Optical Properties of Narrow-Gap Low-Dimensional Structures*, St. Andrews, Scotland, 29 July–1 August 1986. Proceedings of a NATO Advanced Research Workshop, edited by C. M. Sotomayor Torres, J. C. Portal, J. C. Maan, and R. A. Strandling (Plenum, New York, 1987), pp. 337–48. (I)

201. **"High Pressure Transport Experiments in 2D Systems,"** J. L. Robert, A. Raymond, and C. Bousquet, in *Optical Properties of Narrow-Gap Low-Dimensional Structures*, St. Andrews, Scotland, 29 July–1 August 1986, Proceedings of a NATO Advanced Research Workshop, edited by C. M. Sotomayor Torres, J. C. Portal, J. C. Maan, and R. A. Strandling (Plenum, New York, 1987), pp. 313–24. (I)

18. Measurements in hole systems

The quantum Hall effect is independent of whether the carriers are electrons or holes. Besides the photoconductivity work of Chou *et al.*, previously identified as Ref. 194, the following papers explore the quantum Hall effect in hole–carrier systems.

202. **"Energy Structure and Quantized Hall Effect of Two-Dimensional Holes,"** H. L. Störmer, Z. Schlesinger, A. Chang, D. C. Tsui, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **51**, 126–9 (1983). A beryllium-doped GaAs heterostructure was studied and found to have an $i = 1$ plateau quantized to better than 100 ppm. (I)

203. **"Properties of a 2D Hole Gas at a Silicon Surface in Ultrastrong Magnetic Fields,"** G. M. Gusev, Z. D. Kvon, I. G. Neizvestnyi, V. N. Ovsyuk, and P. A. Cheremnykh, in Pis'ma Zh. Eksp. Teor. Fiz. **39**, 446–9 (1984). Trans. in JETP Lett. **39**, 541–3 (1984). The quantum Hall effect is observed in a two-dimensional hole gas in silicon MOSFETs. (I)

204. **"Quantum Hall Effect in a Two-Dimensional Electron-Hole Gas,"** E. E. Mendez, L. Esaki, and L. L. Chang, Phys. Rev. Lett. **55**, 2216–19 (1985). Measurements on GaSb/InAs are presented that show the simultaneous quantized Hall behavior of both electrons and holes. It is shown that the system can be treated as consisting of parallel electron and hole gases whose difference in density governs the Hall plateaus and filling factors. (I)

205. **"Transport Properties of *p*-Type GaAs–(GaAl)As Heterojunctions in High Magnetic Fields,"** G. Landwehr, in *High Magnetic Fields in Semiconductor Physics*, Proceedings of the International Conference, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 295–303. This paper is a review of two-dimensional hole gas systems in GaAs heterostructures. (I)

19. Effects of impurities and inhomogeneities

It is generally felt by theorists that the integral quantum Hall effect would be unobservable in a homogeneous impurity-free two-dimensional electron gas system. The plateaus and resistance minima would have no width. In fact, the paper by Woltjer *et al.*, Ref. 75, shows that inhomogeneities can lead to plateaus. Although systematic investigation of impurity and inhomogeneity effects is difficult, the following papers also contribute to our understanding.

206. **"Effects of Interface Charge on the Quantum Hall Effect,"** J. E. Furneaux and T. L. Reinecke, Phys. Rev. B **33**, 6897–908 (1986). Sodium ions controllably diffused into the SiO_2 surface layer of silicon MOSFETs were used as localized impurities, and were shown to increase linearly the quantized Hall plateau width and decrease the electron mobility. (I)

207. **"Quantized Hall Effect in Gated AlGaAs/GaAs Heterostructures: Localization as a Function of Number Density and Magnetic Field,"** L. X. He, K. P. Martin, R. J. Higgins, J. S. Brooks, P. R. Jay, and P. Delescluse, Solid State Commun. **59**, 691–5 (1986). (I)

208. **"Magnetic Field Dependence of Gate Voltage and Current in a GaAs Heterostructure in the Quantum Hall Regime,"** D. Weiss, V. Mosser, V. Gudmundsson, R. R. Gerhardts, and K. von Klitzing, Solid State Commun. **62**, 89–91 (1987). (I)

209. **"Effect of Repulsive and Attractive Scattering Centers on the Magnetotransport Properties of a Two-Dimensional Electron Gas,"** R. J. Haug, R. R. Gerhardts, K. von Klitzing, and K. Ploog, Phys. Rev. Lett. **59**, 1349–52 (1987). These authors systematically doped GaAs heterostructures with both donor (Si) and acceptor (Be) impurities and measured their effect on the quantum Hall plateaus and on ρ_{xx}. (I)

20. Noise measurements

The following papers have investigated the low-frequency noise spectra of quantum Hall effect samples.

210. **"Low Frequency Noise and the Quantised Hall Effect,"** B. W. Ricketts, J. Phys. D **18**, 885–92 (1985). (I)

211. **"Non-equilibrium Electrons in the Quantum Hall Effect Regime,"** K. Yoshihiro, J. Kinoshita, K. Inagaki, C. Yamanouchi, Y. Murayama, T. Endo, M. Koyanagi, J. Wakabayashi, and S. Kawaji, Surf. Sci. **170**, 193–201 (1986). (I)

212. **"Noise Due to Localized States in the Quantum Hall Regime,"** A. J. Kil, R. J. J. Zijlstra, P. M. Koenraad, J. A. Pals, and J. P. André, Solid State Commun. **60**, 831–4 (1986). (I)

21. Nonstandard 2-D systems

The universality of the quantum Hall effect is evident from the following sample of papers reporting its observation in systems other than the usual silicon MOSFETs or GaAs heterostructures. Some systems may have more favorable band structures for the metrological application of the quantum Hall effect at particularly low magnetic fields or for the accurate use of the odd integer steps.

Experimental papers:

213. **"The Quantum Hall Effect in Modulation Doped $In_{0.53}Ga_{0.47}$As-InP Heterojunctions,"** Y. Guldner, J. P. Hirtz, J. P. Vieren, P. Voisin, M. Voos, and M. Razeghi, J. Physique Lett. **43**, L613–16 (1982). (I)

214. **"Quantum Transport in GaInAs-AlInAs Heterojunctions, and the Influence of Intersubband Scattering,"** J. C. Portal, R. J. Nicholas, M. A. Brummell, A. Y. Cho, K. Y. Cheng, and T. P. Pearsall, Solid State Commun. **43**, 907–11 (1982). (I)

215. **"Two-Dimensional Magneto-Transport in a New Type of Heterostructure, InP/*n*-AlInAs,"** M. Inoue and S. Nakajima, Solid State Commun. **50**, 1023–5 (1984). (I)

216. **"Quantized Hall Effect in the *n*-Inversion Layer in InSb Grain Boundaries,"** R. Herrmann, W. Kraak, and M. Gliński, Phys. Status Solidi B **125**, K85–8 (1984). (I)

217. **"Quantum Hall Effect and Fermi Surface Instabilities in $(TMTSF)_2ClO_4$,"** M. Ribault, J. Cooper, D. Jérome, D. Mailly, A. Moradpour, and K. Bechgaard, J. Physique Lett. **45**, L935–41 (1984). This paper deals with a layered, anisotropic organic conducting system, $(TMTSF)_2X$, that exhibits the quantized Hall effect. (I)

218. **"First Observation of the Quantum Hall Effect in a $Ga_{0.47}In_{0.53}$As-InP Heterostructure with Three Electric Subbands,"** M. Razeghi, J. P. Duchemin, J. C. Portal, L. Dmowski, G. Remeni, R. J. Nicholas, and A. Briggs, Appl. Phys. Lett. **48**, 712–14 (1986). (I)

219. **"First Observation of the Two-Dimensional Properties of the Electron Gas in $Ga_{0.49}In_{0.51}$P/GaAs Heterojunctions Grown by Low Pressure Metalorganic Chemical Vapor Deposition,"** M. Razeghi, P. Maurel, F. Omnes, S. B. Armor, L. Dmowski, and J. C. Portal, Appl. Phys. Lett. **48**, 1267–9 (1986). (I)

220. **"Investigation of the Two-Dimensional Electron Gas in HgCdTe by Quantum Hall Effect Measurements,"** W. P. Kirk, P. S. Kobiela, R. A. Schiebel, and M. A. Reed, J. Vac. Sci. Technol. A **4**, 2132–6 (1986). (I)

221. **"Quantum Hall Effect in Bicrystals of p-$Hg_{0.75}Cd_{0.23}Mn_{0.02}$Te,"** G. Grabecki, T. Suski, T. Dietl, T. Skośkiewicz, and M. Gliński, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, Wurzburg, Germany, 18–22 August 1986, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 127–30. (I)

222. **"Quantization of the Hall Effect in an Anisotropic Three-Dimensional Electronic System,"** H. L. Störmer, J. P. Eisenstein, A. C. Gossard, W. Wiegmann, and K. Baldwin, Phys. Rev. Lett. **56**, 85–8 (1986). See also theoretical discussions by Ulloa and Kirczenow, Ref. 224, and by Tao and Campbell, Ref. 225. (I)

223. **"Magnetotransport in Field-Induced Spin Density Waves in Bechgaard Salts,"** Shu-Ren Chang and K. Maki, J. Low Temp. Phys. **67**, 91–102 (1987). (I)

Theoretical papers:

*224. **"Novel Surface States and the Quantum Hall Effect in an Anisotropic Three-Dimensional System,"** S. E. Ulloa and C. Kirczenow, Phys. Rev. Lett. **57**, 2991–4 (1986). This is an explanation of the small excitation gap seen by Störmer *et al.* in Ref. 222. (I)

225. **"Quantized Hall Effect in Quasi-3-Dimensional Systems,"** Tao Pang and C. E. Campbell, Phys. Rev. B **35**, 1459–60 (1987). The general criteria for the existence of the quantum Hall effect in quasi-three-dimensional systems is presented. (I)

*226. **"Quantized Hall Effect in the Field-Induced Density-Wave Phases of Low-Dimensionality Conductors,"** D. Poilblanc, G. Montambaux, M. Héritier, and P. Lederer, Phys. Rev. Lett. **58**, 270–3 (1987). (I)

227. **"Open Orbits and Generalized Quantum Hall Effect,"** M. Y. Azbel, P. Bak, and P. M. Chaikin, Phys. Rev. Lett. **59**, 926–9 (1987). Measurements in TMTSF compounds. (I)

22. Miscellaneous

228. **"Quantized Magnetoresistance in Multiply Connected Perimeters in Two-Dimensional Systems,"** F. F. Fang and P. J. Stiles, Phys. Rev. B **29**, 3749–51 (1984). Rational fractions of h/e^2 can be obtained by making multiple connections to a single quantized Hall resistance device. (I)

229. **"Experimental Search for Dynamic Current Oscillations in the Quantum Hall Effect,"** V. J. Goldman, S. E. Barrett, D. C. Tsui, and K. Alavi, Phys. Lett. A **123**, 311–12 (1987). No evidence was found for the equivalent of the ac Josephson effect in the quantum Hall effect regime. (I)

C. Fractional quantum Hall effect research papers

We first list the papers describing the discovery of the fractionally quantized level hierarchy and which established that this is a general effect of high-mobility two-dimensional electrons independent of system. These new levels are identified by a fractional quantum number ν.

1. Experimental discovery and determination of hierarchy

Following the landmark discovery paper of Tsui *et al.*, Ref. 35, the following papers explored the nature of these new quantum levels.

*230. **"Fractional Quantization of the Hall Effect,"** H. L. Störmer, A. Chang, D. C. Tsui, J. C. M. Hwang, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **50**, 1953–6 (1983). These authors observed levels with $\nu = 1/3, 2/3, 4/3, 2/5, 3/5, 4/5$ and $2/7$ in electron and hole gases in GaAs heterostructures. They suggested the existence of fractional quantization in multiple series (hierarchies). (I)

*231. **"Observation of a Fractional Quantum Number,"** D. C. Tsui, H. L. Störmer, J. C. M. Hwang, J. S. Brooks, and M. J. Naughton, Phys. Rev. B **28**, 2274–5 (1983). This paper showed that the Hall resistance was quantized in GaAs heterostructures to better than 100 ppm for the $\nu = 1/3$ level. Temperature dependence studies of the $\nu = 1/3$ and $2/3$ levels allowed the determination of activation energies. Also they concluded that the integral and fractional quantum Hall effects are very different in origin and are competing with each other. (I)

232. **"High Magnetic Field Studies of the Two-Dimensional Electron Gas in GaInAs-InP Superlattices,"** J. C. Portal, R. J. Nicholas, M. A. Brummel, M. Razeghi, and M. A. Poisson, Appl. Phys. Lett. **43**, 293–5 (1983). (I)

*233. **"Fractional Quantum Hall Effect in a Two-Dimensional Hole System,"** E. E. Mendez, W. I. Wang, L. L. Chang, and L. Esaki, Phys. Rev. B **30**, 1087–9 (1984). These authors reported the observation of a $\nu = 3/5$ plateau and evidence in ρ_{xx} for $\nu = 3/7, 6/5, 5/3, 4/3, 3/5$, and $2/5$ in GaAs heterostructure hole gases. (I)

*234. **"Higher-Order States in the Multiple-Series, Fractional, Quantum Hall Effect,"** A. M. Chang, P. Berglund, D. C. Tsui, H. L. Störmer, and J. C. M. Hwang, Phys. Rev. Lett. **53**, 997–1000 (1984). Quantization to 230 ppm for $\nu = 2/5$ and 1300 ppm for $\nu = 3/5$ are reported along with structure in ρ_{xx} that suggests additional levels at $\nu = 3/7, 4/7, 4/9$, and $5/9$. The temperature dependence of the $\nu = 2/5$ and $3/5$ levels was studied. (I)

235. **"Effect of Slow Interface States on the Quantum Hall Effect,"** D. A. Syphers, K. C. Woo, and P. J. Stiles, Phys. Rev. B **30**, 3557–60 (1984). A clear plateau with a filling factor of $\nu = 5/2$ was observed in a silicon MOSFET. (I)

*236. **"Fractional Quantum Hall Effect at Filling Factors up to $\nu = 3$,"** G. Ebert, K. von Klitzing, J. C. Maan, G. Remenyi, C. Probst, G. Weimann, and W. Schlapp, J. Phys. C **17**, L775–9 (1984). Measurements were presented showing that the $\nu = 4/3$ and $5/3$ plateaus in GaAs were quantized to 1000 ppm. The temperature dependence of these levels were also presented. (I)

*237. **"Anomalies in the Magnetotransport Properties of a Two-Dimensional Electron Gas in Silicon Metal-Oxide-Semiconductor Structures in High Magnetic Fields,"** I. V. Kukushkin, V. B. Timofeev, and P. A. Cheremnykh, Zh. Eksp. Teor. Fiz. **87**, 2223–33 (1984). Trans. in Sov. Phys. JETP **60**, 1285–90 (1984). The first observations of fractionally quantized Hall plateaus in silicon MOSFETs are reported. Features are reported for $\nu = 1/3, 2/3, 4/3, 5/3, 7/3, 8/3, 4/5$, and $6/5$. (I)

238. **"Odd and Even Fractionally Quantized States in GaAs-GaAlAs Heterojunctions,"** R. G. Clark, R. J. Nicholas, A. Usher, C. T. Foxon, and J. J. Harris, Surf. Sci. **170**, 141–7 (1986). Evidence of even fractions at $\nu = 1/4, 1/2$, and $3/4$. (I)

*239. **"Observation of an Even-Denominator Quantum Number in the Fractional Quantum Hall Effect,"** R. Willett, J. P. Eisenstein, H. L. Störmer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. **59**, 1776–9 (1987). The first indisputable observation of a fractional level with even denominator ($\nu = 5/2$) is reported. The Hall resistance was quantized to better than 5000 ppm. (I)

240. **"Experimental Determination of Fractional Charge e/q of Quasiparticle Excitations in the Fractional Quantum Hall Effect,"** R. G. Clark, J. R. Mallett, S. R. Haynes, J. J. Harris, and C. T. Foxon, Phys. Rev. Lett. **60**, 1747 (1988). (I)

241. **"Collapse of the Even-Denominator Fractional Quantum Hall Effect in Tilted Fields,"** J. P. Eisenstein, R. Willett, H. L. Störmer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. Lett. **61**, 997–1000 (1988). (I)

242. **"Quantitative Experimental Test for the Theoretical Gap Energies in the Fractional Quantum Hall Effect,"** R. L. Willett, H. L. Störmer, D. C. Tsui, A. C. Gossard, and J. H. English, Phys. Rev. B **37**, 8476–9 (1988). (I)

1. Experimental papers checking the quantization of R_H

The generally accepted theory of the fractional quantum Hall effect is based on the seminal 1983 paper by R. B Laughlin, Ref. 36. Laughlin developed a remarkable set of variational wavefunctions to describe the highly correlated ground state of interacting electrons in a fractionally filled Landau level. He demonstrated that this special ground state, into which the electrons could condense at filling factors of 1/3, 1/5, 1/7, etc., had the property of being incompressible, i.e., there was an excitation gap for charged excitations. Furthermore, he showed that the natural quasiparticle excitations above this ground state carried fractional charge. The landmark paper by Laughlin established a framework upon which the later papers expanded.

Papers dealing with the Laughlin–liquid ground state are the following:

243. **"Laughlin States in Higher Landau Levels,"** A. H. MacDonald, Phys. Rev. B **30**, 3550–3 (1984). (A)

244. **"Exact Results for the Fractional Quantum Hall Effect with General Interactions,"** S. A. Trugman and S. Kivelson, Phys. Rev. B **31**, 5280–4 (1985). (A)

*245. **"Off-Diagonal Long-Range Order, Oblique Confinement and the Fractional Quantum Hall Effect,"** S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. **58**, 1252–5 (1987). This paper exhibits a correlation function that measures the topological order in the fractional quantum Hall effect ground state. (A)

Papers dealing with quasiparticles and the hierarchy of fractional states are the following:

246. **"Fractional Quantization of the Hall Effect: A Hierarchy of Incompressible Quantum Fluid States," F. D. M. Haldane, Phys. Rev. Lett. **51**, 605–8 (1983). This fundamental paper invents the idea of the hierarchy and gives a boson representation of it. (A)

247. **"Remarks on the Laughlin Theory of the Fractionally Quantized Hall Effect,"** P. W. Anderson, Phys. Rev. B **28**, 2264–5 (1983). (A)

248. **"Particle-Hole Symmetry in the Anomalous Quantum Hall Effect,"** S. M. Girvin, Phys. Rev. B **29**, 6012–14 (1984). The author presents a fermion representation of the hierarchy. (A)

249. **"Primitive and Composite Ground States in the Fractional Quantum Hall Effect,"** R. B. Laughlin, Surf. Sci. **142**, 163–72 (1984). This is also a fermion representation of the hierarchy. (A)

250. **"Hierarchy of Plasmas for Fractional Quantum Hall States,"** A. H. MacDonald, G. C. Aers, and M. W. C. Dharma-wardana, Phys. Rev. B **31**, 5529–32 (1985). In this paper the ground-state energies in the hierarchy are found by the use of a hypernetted-chain calculation. (A)

*251. **"Quasiparticle States and the Fractional Quantum Hall Effect,"** A. H. MacDonald and S. M. Girvin, Phys. Rev. B **33**, 4414–17 (1986). This paper gives wavefunctions for quasiholes and quasielectrons whose properties can be exactly computed. (A)

252. **"Quasiparticle States in the Fractional Quantum Hall Effect,"** A. H. MacDonald and S. M. Girvin, Phys. Rev. B **34**, 5639–53 (1986). More detailed exposition of the above paper. (A)

*253. **"Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States,"** B. I. Halperin, Phys. Rev. Lett. **52**, 1583–6 (1984). A representation of the hierarchy by the use of fractional statistics is given. (A)

The following paper deals with a mathematical method useful in studying fractional quantization:

254. **"Formalism for the Quantum Hall Effect: Hilbert Space of Analytic Functions,"** S. M. Girvin and T. Jach, Phys. Rev. B **29**, 5617–25 (1984). This paper describes the projection of the Schrödinger equation into the lowest Landau level so that the neglect of mixing of higher Landau level states is automatic. (A)

3. Numerical simulations

*255. **"Ground State of Two-Dimensional Electrons in Strong Magnetic Fields and 1/3 Quantized Hall Effect,"** D. Yoshioka, B. I. Halperin, and P. A. Lee, Phys. Rev. Lett. **50**, 1219–22 (1983). This is the first numerical calculation of the fractional quantum Hall effect. The authors found the ground-state energy of a cluster of electrons as a function of filling factor and detected small downward cusps in the ground-state energy. (A)

*256. **"Ground State of the Two-Dimensional Charged Particles in a Strong Magnetic Field and the Fractional Quantum Hall Effect,"** D. Yoshioka, Phys. Rev. B. **29**, 6833–9 (1984). (I)

*257. **"Finite-Size Studies of the Incompressible State of the Fractionally Quantized Hall Effect and its Excitations,"** F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237–40 (1985). These calculations on a spherical system make a very important confirmation of the Laughlin picture. (I)

258. **"Monte Carlo Evaluation of Trial Wave Functions for the Fractional Quantized Hall Effect: Disk Geometry,"** R. Morf and B. I. Halperin, Phys. Rev. B **33**, 2221–46 (1986). (I)

*259. **"Configuration-Interaction Calculations on the Fractional Quantum Hall Effect,"** G. Fano, F. Ortolani, and E. Colombo, Phys. Rev. B **34**, 2670–80 (1986). This is similar to Haldane and Rezayi, but for larger systems. (I)

260. **"Excitation Spectrum of the Fractional Quantum Hall Effect: Two Component Fermion System,"** D. Yoshioka, J. Phys. Soc. Jpn. **55**, 3960–8 (1986). (I)

261. **"Electron and Quasiparticle Density of States in the Fractionally Quantized Hall Effect,"** E. H. Rezayi, Phys. Rev. B **35**, 3032–5 (1987). This author provides the spectrum of excitations for electrons and holes in the fractional quantum Hall effect. (A)

262. **"Fractional Quantum Hall Effect and Specific Heat of 2-D Electrons,"** D. Yoshioka, J. Phys. Soc. Jpn. **56**, 1301–4 (1987). (A)

4. Theories of fractional statistics

The paper listed above by Halperin, Ref. 253, was the first application of fractional statistics to the fractional quantum Hall effect. Another is:

*263. **"Fractional Statistics and the Quantum Hall Effect,"** D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722–3 (1984). This is a beautiful demonstration that fractional statistics and fractional charge of quasiparticles can be computed from Berry's phase. (M. V. Berry's phase is the phase change that occurs when a system is taken adiabatically around some path by changing a parameter and brought back to the original state.) (A)

5. Theories of collective excitations

In their paper on collective excitations in the integer quantum Hall effect, Kallin and Halperin, Ref. 184, made a remark suggesting that there could be an anologous situation in the fractional quantum Hall effect. Other papers are:

*264. **"Excitons in the Fractional Quantum Hall Effect,"** R. B. Laughlin, Physica **126B**, Pt. III, 254–9 (1984). This paper describes the behavior of bound pairs of quasiparticles. (A)

*265. **"Collective-Excitation Gap in the Fractional Quantum Hall Effect,"** S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581–3 (1985). The collective excitations in the FQHE are here described by analogy with phonon/roton modes in analogy with superfluid helium. (A)

266. **"New Gapless Modes in the Fractional Quantum Hall Effect of Multicomponent Fermions,"** M. Rasolt, F. Perrot, and A. H. MacDonald, Phys. Rev. Lett. **55**, 433–6 (1985). (A)

267. **"Magneto-roton Theory of Collective Excitations in the Fractional Quantum Hall Effect," S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys, Rev. B **33**, 2481–94 (1986). (A)

268. **"Collective Excitations of Fractional Hall States and Wigner Crystallization in Higher Landau Levels,"** A. H. MacDonald and S. M. Girvin, Phys. Rev. B **33**, 4009–13 (1986). (A)

269. **"Shear Modulus of Laughlin-Type Wave Functions,"** S. T. Chui, Phys. Rev. B **34**, 1409–11 (1986). This paper shows that the Laughlin liquid in some sense has a finite shear strength. (A)

270. **"Collective Excitations in the Fractional Quantum Hall Effect of a Multicomponent Fermion System,"** M. Rasolt and A. H. MacDonald, Phys. Rev. B **34**, 5530–9 (1986). (A)

271. **"Density–density Response Function in the Fractional Quantum Hall Effect,"** M. Saarela, Phys. Rev. B **35**, 854–6 (1987). This paper presents a Jastro correlated basis function approach to collective modes. (A)

272. **"Superlattice Magnetoroton Bands,"** M. C. A. Oji, A. H. MacDonald, and S. M. Girvin, Phys. Rev. Lett. **58**, 824–7 (1987). This paper discusses collective modes in the FQHE in multilayers. (A)

*273. **"Phonons, Rotons and Fractionally-Charged Vortices in the Quantum Hall Effect,"** S. M. Girvin, in *Interfaces, Quantum Wells, and Superlattices*, Banff, Alberta, Canada, 16–19 August 1987, NATO Advanced Science Institute Series B **179**, edited by C. R. Leavens and R. Taylor (Plenum, New York, 1988), pp. 319–31. (A)

6. Theories of ring exchange

*274. **"Cooperative-Ring-Exchange Theory of the Fractional Quantized Hall Effect,"** S. Kivelson, C. Kallin, D. P. Arovas, and J. R. Schrieffer, Phys. Rev. Lett. **56**, 873–6 (1986). This is the first attempt to explain the fractional quantum Hall effect in terms of cooperative ring exchanges. This situation is analogous to the ring exchanges in superfluid ^{4}He. (A)

*275. **"Cooperative Ring Exchange and the Fractional Quantized Hall Effect in an Incompressible Fluid,"** G. Baskaran, Phys. Rev. Lett. **56**, 2716–19 (1986). (A)

276. **"The Canonical Description and Bohr–Sommerfeld Quantization Condition for the Fractional Quantum Hall Effect System,"** Z. Su,

H. Pang, Y. Xie, and K. Chou, Phys. Lett. A **123**, 249–53 (1987). These authors make a careful analysis of phase factors in the ring exchange picture. (A)

*277. **"Cooperative Ring Exchange and the Fractional Quantum Hall Effect,"** S. Kivelson, C. Kallin, D. P. Arovas, and J. R. Schreiffer, Phys. Rev. B **36**, 1620–46 (1987). This is a longer version of their paper listed at the start of this section. (A)

278. **"Ring Exchange and the Fractional Quantum Hall Effect,"** D. J. Thouless and Qin Li, Phys. Rev. B **36**, 4581–3 (1987). This is a model calculation to check the sign of the ring exchange amplitude. (A)

*279. **"Generalized Cooperative-Ring-Exchange Theory of the Fractional Quantum Hall Effect,"** D. H. Lee, G. Baskaran, and S. Kivelson, Phys. Rev. Lett. **59**, 2467–70 (1987). These authors derive an approximation to Laughlin's wavefunction within the ring exchange picture. (A)

7. Theories of magnetization

280. **"Nature of the Magnetization of a Two-Dimensional Layer of Electrons Under Conditions of the Quantum Hall Effect,"** L. V. Iogansen, Pis'ma Zh. Tekh. Fiz. **12**, 636–40 (1986). Trans. in Sov. Tech. Phys. Lett. **12**, 262–3 (1986). (A)

8. Wigner crystallization

The two-dimensional electrons are apparently in a gaseous state in the integral quantum Hall effect and in a liquid state in the fractional quantum Hall effect. One therefore wonders if they can also exist in the solid state.

Experimental papers:

281. **"High Magnetic-Field Transport in a Dilute Two-Dimensional Electron Gas,"** E. E. Mendez, M. Heiblum, L. L. Chang, and L. Esaki, Phys. Rev. B **28**, 4886–8 (1983). The magnetoresistance exhibited no structural features for filling factors below 1/5 in GaAs/AlGaAs heterostructures, suggesting that a transition from a quantum liquid to a crystalline state may occur. (I)

282. **"Thermodynamic Measurement on the Melting of a Two-Dimensional Electron Solid,"** D. C. Glattli, E. Y. Andrei, and F. I. B. Williams, Phys. Rev. Lett. **60**, 420–3 (1988). The authors measure the specific heat of the solid phase of two-dimensional electrons on the surface of liquid ^{4}He. (I)

*283. **"Observation of a Magnetically Induced Wigner Solid,"** E. Y. Andrei, G. Deville, D. C. Glattli, F. I. B. Williams, E. Paris, and B. Etienne, Phys. Rev. Lett. **60**, 2765–8 (1988). A magnetic field induced liquid-to-solid phase of two-dimensional electrons in a GaAs/AlGaAs heterojunction at a filling factor of $\nu = 0.23 \pm 0.04$ is observed by detection of a gapless magnetophonon excitation branch with rf spectroscopy. (A)

284. **"Comment on Observation of a Magnetically Induced Wigner Solid,"** H. L. Störmer and R. L. Willett, Phys. Rev. Lett. **62**, 972 (1989). A comment on Ref. 283. (I)

285. **"Andrei *et al.* Reply,"** E. Y. Andrei, G. Deville, D. C. Glattli, F. I. B. Williams, E. Paris, and B. Etienne, Phys. Rev. Lett. **62**, 1926–27 (1989). The reply to Ref. 284. (I)

*286. **"Termination of the Series of Fractional Quantum Hall States at Small Filling Factors,"** R. L. Willett, H. L. Störmer, D. C. Tsui, L. N. Pfeiffer, K. W. West, and K. W. Baldwin, Phys. Rev. B **38**, 7881–4 (1988). Striking increases in ρ_{xx} and ρ_{xy} are observed below $\nu = 1/5$. This is consistent with the onset of Wigner crystallization in which the Wigner lattice is pinned to random disorder sites. (I)

The theoretical papers give an estimation of the critical filling factor ν_c, below which the Laughlin liquid gives way to the Wigner crystal:

*287. **"Liquid–Solid Transition and the Fractional Quantum-Hall Effect,"** P. K. Lam and S. M. Girvin, Phys. Rev. B **30**, 473–5 (1984). The authors predict that ν_c is slightly larger than 1/7. (A)

288. **"Crystallization of the Incompressible Quantum-Fluid State of a Two-Dimensional Electron Gas in a Strong Magnetic Field,"** D. Levesque, J. J. Weis, and A. H. MacDonald, Phys. Rev. B **30**, 1056–8 (1984). These authors predict that ν_c is less than 1/9. (A)

The following papers investigate the Wigner crystal for two-dimensional electrons in a high magnetic field:

289. **"Ground-State Energy of a Two-Dimensional Charge-Density-Wave State in a Strong Magnetic Field,"** D. Yoshioka and P. A. Lee, Phys. Rev B **27**, 4986–96 (1983). (A)

290. **"Static and Dynamic Properties of a Two-Dimensional Wigner Crystal in a Strong Magnetic Field,"** K. Maki and X. Zotos, Phys. Rev. B **28**, 4349–56 (1983). (A)

291. **"Computer Renormalization-Group Calculation for the Fractionally Quantized Hall Effect,"** S. T. Chui, Phys. Rev. B **32**, 1436–8 (1985). (A)

292. **"Solid versus Fluid, and the Interplay between Fluctuations, Correlations, and Exchange in the Fractional Quantized Hall Effect,"** S. T. Chui, T. M. Hakim, and K. B. Ma, Phys. Rev. B **33**, 7110–21 (1986). (A)

293. **"Many-Particle Exchanges in a Class of Correlated Gaussian Wave Functions for the Fractionally Quantized Hall Effect,"** S. T. Chui, Phys. Rev. B **34**, 1130–8 (1986). (A)

*294. **" 'Hall Crystal' versus Wigner Crystal,"** Z. Tešanović, F. Axel, and B. I. Halperin, Phys. Rev. B **39**, 8525–51 (1989). (A)

9. Measurements of temperature dependence

The original discovery paper by Tsui, Störmer, and Gossard of the fractional quantum Hall effect, Ref. 35, presented temperature-dependence data along with their plateau measurements. Other papers by Chang *et al.*, Ref. 230, and by Ebert *et al.*, Ref. 236, also contained temperature-dependence data. Also, Ref. 238 presents activation energies for the $\nu = 2/3$, 4/3, 5/3, 3/5, 7/5, and 4/7 levels. Additional temperature-dependence results are given in the following papers:

*295. **"Fractional Quantum Hall Effect at Low Temperatures,"** A. M. Chang, M. A. Paalanen, D. C. Tsui, H. L. Störmer, and J. C. M. Hwang, Phys. Rev. B **28**, 6133–6 (1983). Temperature-dependence measurements were made from 65 to 770 mK in GaAs heterostructures with the results interpreted in terms of activation energies. (I)

*296. **"Disorder and the Fractional Quantum Hall Effect,"** M. A. Paalanen, D. C. Tsui, A. C. Gossard, and J. C. M. Hwang. Solid State Commun. **50**, 841–4 (1984). These authors show that GaAs heterostructure samples with higher mobility have enhanced fractional and inhibited integer quantum Hall effects. (I)

*297. **"Activation Energies of the 1/3 and 2/3 Fractional Quantum Hall Effect in GaAs/Al$_x$Ga$_{1-x}$As Heterostructures,"** S. Kawaji, J. Wakabayashi, J. Yoshino, and H. Sakaki, J. Phys. Soc. Jpn. **53**, 1915–8 (1984). (I)

*298. **"Magnetic Field Dependence of Activation Energies in the Fractional Quantum Hall Effect,"** G. S. Boebinger, A. M. Chang, H. L. Störmer, and D. C. Tsui, Phys. Rev. Lett. **55**, 1606–9 (1985). These authors studied the temperature dependence of $\nu = 1/3$, 2/3, 4/3, 5/3, 2/5, and 3/5 levels, demonstrated that the $n/3$ levels were all governed by a single activation energy, and that the $n/5$ levels were all governed by a different single activation energy. In both cases the activation energies were much smaller than predicted by theory. (I)

299. **"Studies of the Fractional Quantum Hall Effect in a Silicon MOSFET,"** J. E. Furneaux, D. A. Syphers, J. S. Brooks, G. M. Schmiedeshoff, R. G. Wheeler, and P. J. Stiles, Surf. Sci. **170**, 154–9 (1986). Measurements were made on a silicon MOSFET to temperatures as low as 50 mK. (I)

*300. **"Energy Gaps and the Role of Disorder under Conditions of Fractional Quantization of the Hall Resistance in Silicon Metal-Oxide-Semiconductor Structures,"** I. V. Kukushkin and V. B. Timofeev, Zh. Eksp. Teor. Fiz. **89**, 1692–703 (1985). Trans. in Sov. Phys. JETP **62**, 976–82 (1985). Activation energies were presented for $\nu = 1/3$, 2/3, 4/3, and 4/5 levels in silicon MOSFETs. (I)

*301. **"Fractional Quantum Hall Effect of *p*-Type GaAs-(GaAl)As Heterostructures in the Millikelvin Range,"** G. Remény, G. Land-

wehr, W. Heuring, G. Weimann, and W. Schlapp, in *High Magnetic Fields in Semiconductor Physics. Proceedings of the International Conference*, 18–22 August 1986, Wurzburg, Germany, edited by G. Landwehr (Springer-Verlag, Berlin, 1987), pp. 166–72. Measurements of the temperature dependence of the $\nu = 2/3$ level were made down to 50 mK. (I)

*302. **"Second Activation Energy in the Fractional Quantum Hall Effect,"** J. Wakabayashi, S. Sudou, S. Kawaji, K. Hirakawa, and H. Sakaki, J. Phys. Soc. Jpn. **56**, 3005–8 (1987). Extensive measurements on a GaAs heterostructure for the $\nu = 2/3$ level suggested the existence of two activation energy gaps. (I)

*303. **"Activation Energies and Localization in the Fractional Quantum Hall Effect,"** G. S. Boebinger, H. L. Störmer, D. C. Tsui, A. M. Chang, J. C. M. Hwang, A. Y. Cho, C. W. Tu, and G. Weimann, Phys. Rev. B **36**, 7919–29 (1987). These authors present an extensive study of the temperature dependence of the magnetoconductivity of GaAs/AlGaAs heterostructures with mobilities ranging from 150 000 to 1 000 000 cm^2/Vs. Temperatures down to 30 mK and magnetic fields up to 30 T were used. (I)

304. **"Ultralow-Temperature Behavior of the $\nu = 5/2$ Fractional Quantum Hall Effect,"** P. L. Gammel, D. J. Bishop, J. P. Eisenstein, J. H. English, A. C. Gossard, R. Ruel, and H. L. Störmer, Phys. Rev. B **38**, 10128–30 (1988). (I)

10. Photoconductivity measurements

*305. **"Electron Concentration Dependent Fractional Quantization in a Two-Dimensional System,"** R. G. Clark, R. J. Nicholas, M. A. Brummel, A. Usher, S. Collocott, J. C. Portal, F. Alexandre, and J. M. Masson, Solid State Commun. **56**, 173–6 (1985). The electron concentration in a GaAs heterostructure was progressively altered by use of the persistent photoconductivity effect and the changes in the $\nu = 5/3$, 7/5, and 4/3 levels were measured. (I)

306. **"Microwave Photoresistivity of a Two-Dimensional Electron Gas and the Fractional Quantum Hall Effect,"** Y. Guldner, M. Voos, J. P. Vieren, J. P. Hirtz, and M. Heiblum, Phys. Rev. B **36**, 1266–8 (1987). (I)

11. Experimental studies of hole systems

Papers cited earlier by Mendez *et al.*, Ref. 233, and by Reményi *et al.*, Ref. 301, studied the fractional quantum Hall effect in hole systems.

Another paper is:

307. **"Quantized Hall Effect in Single Quantum Wells of InAs,"** E. E. Mendez, L. L. Chang, C. A. Chang, L. F. Alexander, and L. Esaki, Surf. Sci. **142**, 215–9 (1984). A double-layer system with coexisting electrons and holes. A plateau is observed at $\nu = 5/2$. (I)

12. Miscellaneous

Papers examining thermal transport effects, specific heat and magnetism, ac measurements for samples, and conditions where the fractional quantum Hall effect is observable are not easily distinguishable from integer quantum Hall effect studies. They have therefore been placed in Secs. VI B 11, 12, and 13, respectively.

*308. **"Fractional Quantum Hall Effect in Tilted Magnetic Fields,"** R. J. Haug, K. von Klitzing, R. J. Nicholas, J. C. Maan, and G. Weimann, Phys. Rev. B **36**, 4528–30 (1987). These authors report changes in activation energies when the sample is tilted relative to the magnetic field. They concluded that in a tilted field there appears to be a different behavior for "electronlike" and "holelike" fractional states. (I)

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