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Test Procedures for Electric Motors Under 10 CFR Part 431

Supersedes NIST Technical Note 1422

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Abstract

The procedures for electric motor efficiency testing promulgated by Part 431 of Chapter II of Title 10, Code of Federal Regulations, are discussed. The operating characteristics of the sampling plans for certification and enforcement testing are presented together with examples of the application of the sampling plans. The criteria for substantiation of an Alternative Energy Determination Method (AEDM) are also discussed.

Keywords

electric motor efficiency; electric motor testing; energy policy; operating characteristics; sampling plan

Ordering

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Test Procedures for Electric Motors Under 10 CFR Part 431

1 Introduction

This note provides analysis of the procedures for efficiency testing of polyphase electric motors promulgated by Part 431 of the Code of Federal Regulations [1]. It is intended to supplement the materials published in the *Federal Register* [1, 2], and to specifically address issues regarding the performance of these sampling plans in establishing conformance with the minimum nominal efficiencies mandated by the Energy Policy and Conservation Act of 1975 (EPCA), as amended [3]. Part 431 will be referred to herein as the Final Rule. In the context of the Final Rule, laboratory measurements of motor efficiency are used for three purposes: 1) certification of efficiency performance; 2) substantiation of an Alternative Efficiency Determination Method (AEDM); and 3) enforcement testing. This note discusses each of these topics in turn and thus supersedes NIST Technical Note 1422 [4], which dealt solely with enforcement testing.

The remainder of this document is organized as follows: Section 2 discusses the general objectives and constraints for testing under EPCA; Section 3 discusses the guidelines for motor efficiency labeling established by NEMA Standard MG 1-1993; Section 4 discusses the model assumptions used in calculating the operating characteristics; Section 5 discusses the operating characteristics of the sampling plan for certification testing; Section 6 discusses substantiation of an Alternate Energy Determination Method (AEDM); Section 7 discusses the operating characteristics of the Sampling Plan for Enforcement Testing. For the convenience of the reader, relevant portions of the Final Rule are provided in Appendix A.

2 General guidelines

We begin with a brief summary of the general objectives and guidelines for testing under the EPCA legislation. A general statement of purpose of the EPCA legislation is provided in 42 U.S.C. 6312(a):

> It is the purpose of this part to improve the efficiency of electric motors and pumps and certain other industrial equipment in order to conserve the energy resources of the Nation.

To this end, EPCA establishes energy efficiency standards, i.e., a set of minimum nominal efficiencies, for certain general purpose electric motors.

The EPCA legislation establishes that a program of systematic testing be used to demonstrate that energy efficiency standards are met. The objectives and limitations of testing under EPCA are stated in 42 U.S.C. 6314(a)(2):

> Test procedures prescribed in accordance with this section shall be reasonably designed to produce test results which reflect energy efficiency, energy use, and estimated operating costs of a type of industrial equipment (or class thereof) during a representative average use cycle (as determined by the Secretary), and shall not be unduly burdensome to conduct.

The EPCA legislation, in 42 U.S.C. 6314(d)(1), further requires that the represented energy efficiency be based on product testing:

> ... no manufacturer, distributor, retailer or private labeler may make any representation—
>
> (A) in writing (including any representation on a label), or
>
> (B) in any broadcast advertisement,
>
> respecting the energy consumption of such equipment or cost of energy consumed by such equipment, unless such equipment has been tested in accordance with such test procedure and such representation fairly discloses the results of such testing.

To re-cap, the purposes of EPCA are met provided: 1) the *average* energy efficiency of each covered product is not less than the applicable EPCA efficiency standard, and 2) the *average* energy efficiency of each covered product is not less than the represented energy efficiency. Compliance with EPCA energy efficiency standards and with the represented energy efficiency is demonstrated by a program of systematic testing. EPCA stipulates that testing should not be unduly burdensome to conduct. Thus the two key criteria for the evaluation of a sampling plan established under EPCA are: 1) the assurance provided by that plan that the average performance of that product meets or exceeds the EPCA standard efficiency and the represented efficiency, and 2) the burden placed on industry by testing under the plan.

3 Industry practice

Industry guidelines for efficiency labeling are established by NEMA Standard MG 1-1993 [5]. Table 12-8 of this standard establishes a set of "Nominal Efficiencies" that are to be used for purposes of labeling and a "Minimum Efficiency" that is associated with each Nominal Efficiency. The NEMA Nominal Efficiencies as well as the EPCA nominal efficiencies are listed in Table 1. It should be noted that the EPCA standard values were adapted from the NEMA standard. Under §12.58.2 of the NEMA Standard, two conditions must be satisfied for a motor to be labeled with a given Nominal Efficiency:

1. " ... the Nominal Efficiency ... shall be not greater than the average efficiency of a large population of motors of the same design."
2. "The full-load efficiency ... shall be not less than the minimum value ... associated with the nominal value. ... "

Under the Final Rule, measurement of motor efficiencies is based on two standards: 1) IEEE Standard 112-1996 [6] and 2) CSA-390 [7]. The reader is referred to these standards and to the Final Rule for a discussion of efficiency measurements for electric motors. When under these standards as directed by the Final Rule the determination of efficiency is based on two measured quantities: 1) a measurement of the total losses, and 2) measurement of the output power. The NEMA Standard establishes efficiency levels that differ by increments of approximately 10% of the full-load losses. The Minimum Efficiencies correspond to approximately 120% of the rated loss.

4 Methods of analysis

The sampling plans established by the Final Rule are examined here by means of model calculations to predict their operating characteristics, where the operating characteristics of a sampling are the estimated probabilities of being found in compliance when testing a specified population of motors. Calculation of the operating characteristics of these sampling plans relies on numerical methods of approximation. A discussion of these algorithms can be found in separate reports [4, 8]. Detailed information regarding the distribution of motor efficiencies is required to model the operating characteristics. Following methods used by the NEMA, Motors and Generators Section [9], we assume that motor efficiencies are normally distributed. The distribution of efficiencies of units of a basic model is thus characterized by two parameters: the true mean, μ, and the standard deviation, σ, of the population.

The operating characteristics are given below in terms of the total losses. This method of presentation has the advantage that the full range of the Nominal Efficiencies can be displayed in a single plot. For this presentation, we define the Loss Fraction, LF, by the ratio,

$$LF = \frac{100 - RE}{100 - TE} \times 100, \tag{1}$$

where RE is the rated efficiency and TE is the true efficiency of the population. So stated, the loss fraction is a percentage of the rated full-load losses such that 100% corresponds to the case where the true efficiency of the population and the rated efficiency are equal.

The open-ended nature of testing under the Final Rule introduces a complication. Testing under the Final Rule is open-ended in that the number of motors tested is not fixed from the outset of testing: In the case of certification testing, a manufacturer could test as few as five motors, but may test any arbitrarily large number of motors; likewise the Sampling Plan for Enforcement Testing specifies an initial sample of five but allows testing of up to 20 motors. This scenario is difficult to characterize statistically, and we have chosen to treat testing in the approximation that the sample size is fixed from the outset.

5 Compliance certification

The full text of the sampling plan for compliance certification may be found in Appendix A of this report. To emphasize the salient features of the sampling plan, we paraphrase these criteria as follows:

Compliance with a rated efficiency is demonstrated provided:

(A) The average full-load efficiency of a sample of not fewer than five motors is not less than the value given by the following expression,

$$\frac{100}{1 + 1.05\left(\frac{100}{RE} - 1\right)}, \tag{2}$$

where RE is the rated efficiency, and

Table 1: The NEMA Nominal and Minimum Efficiencies. The EPCA nominal efficiencies, i.e., motor efficiencies that are specified explicitly as minimum nominal efficiencies, are shown in bold. This table is adapted from Table 12-8 of NEMA Standard MG 1-1993 [5].

Nominal Efficiency	Minimum Efficiency	Nominal Efficiency	Minimum Efficiency
99.0	98.8	**94.1**	93.0
98.9	98.7	**93.6**	92.4
98.8	98.6	**93.0**	91.7
98.7	98.5	**92.4**	91.0
98.6	98.4	**91.7**	90.2
98.5	98.2	**91.0**	89.5
98.4	98.0	**90.2**	88.5
98.2	97.8	**89.5**	87.5
98.0	97.6	**88.5**	86.5
97.8	97.4	**87.5**	85.5
97.6	97.1	**86.5**	84.0
97.4	96.8	**85.5**	82.5
97.1	96.5	**84.0**	81.5
96.8	96.2	**82.5**	80.0
96.5	95.8	**81.5**	78.5
96.2	95.4	80.0	77.0
95.8	95.0	**78.5**	75.5
95.4	94.5	77.0	74.0
95.0	94.1	**75.5**	72.0
94.5	93.6		

(B) No individual motor in the sample shall have full-load efficiency less than the value given by the following expression,

$$\frac{100}{1 + 1.15\left(\frac{100}{RE} - 1\right)}, \tag{3}$$

where RE is the rated efficiency.

The operating characteristics of this sampling plan are shown in Figure 1. The data shown depict the outcome when testing a sample of five motors. The contours shown in the figure correspond to equal probabilities of demonstrating compliance. For example, if the average efficiency of a basic model is equal to the rated efficiency, i.e., $LF = 100\%$, and the standard deviation of the total losses is approximately 6% of the total losses, then the probability of demonstrating compliance with the rated efficiency is approximately 95%. The risk of a false determination of non-compliance is thus approximately 5%, in this case.

5.1 Examples of compliance testing

Several hypothetical cases of testing under this sampling plan are presented. Assume for this discussion that a basic model is being tested to demonstrate compliance with a rated efficiency of 89.5%. The condition on the sample mean is obtained by using Eq. 2, which, for a rated efficiency of 89.5, yields 89.0. The condition on the sample minimum efficiency is obtained by using Eq. 3, which, for a rated efficiency of 89.5, yields 88.1.

A demonstration of compliance: Five motors are selected at random from a representative population of motors and tested. The results of testing yield efficiencies of 89.9, 89.2, 89.0, 89.3, and 89.4. The mean efficiency of the sample and the sample minimum efficiency are thus 89.4 and 89.0, respectively. Since the sample mean is greater than 89.0 and the least efficient motor has an efficiency that is greater than 88.1, the criteria for a demonstration of compliance are satisfied and the manufacturer may represent the basic model to be 89.5% efficient.

Non-compliance due to a low mean: Five motors are selected at random from a representative population of motors and tested. The test results yield efficiencies of 88.9, 88.8, 88.6, 89.0, and 89.1. The mean efficiency for the sample and the sample minimum efficiency are thus 88.9 and 88.6, respectively. Since the sample mean is less than 89.0, the

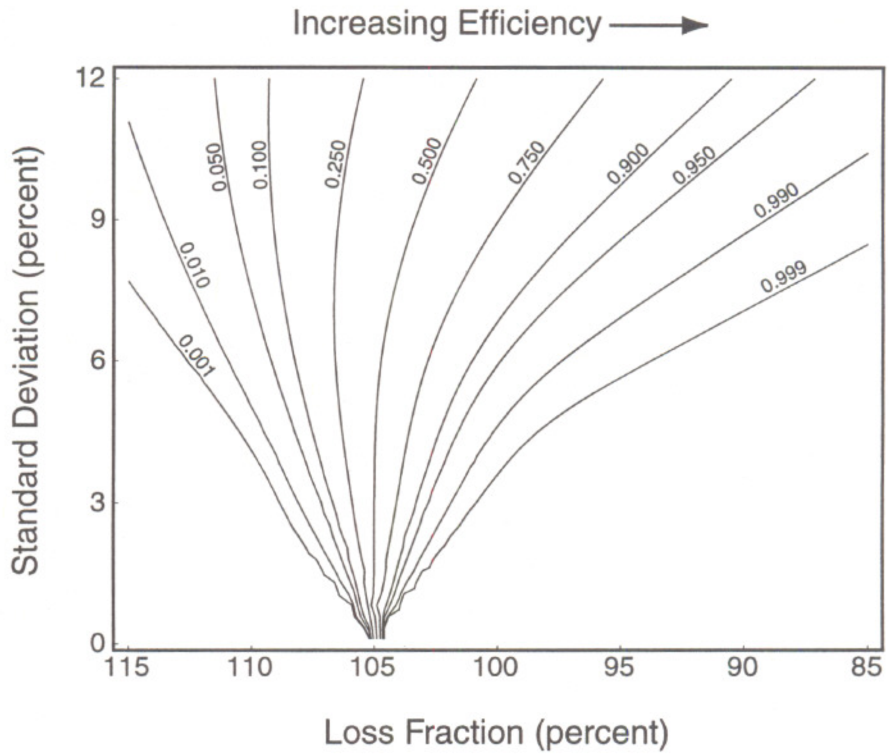

Figure 1: The operating characteristics of the sampling plan for certification of compliance. The contours indicate the probability of demonstrating compliance, e.g., the 0.900 contour corresponds to a 90% likelihood of demonstrating compliance while testing under the sampling plan. The model calculations shown are for a sample of five.

criteria for a demonstration of compliance are not satisfied; the manufacturer may not represent the basic model to have a rated efficiency of 89.5. The manufacturer may elect to conduct further testing; however the initial test results must be included when computing the new sample mean and sample minimum. Assume that the manufacturer elects to test two additional motors and that the new test results yield efficiencies of 89.3 and 89.5. Since the sample mean for the entire sample of 7 motors is equal to 89.0 and no unit in the sample tested below the minimum efficiency, the basic model is determined to be in compliance with a rated efficiency of 89.5.

Non-compliance due to a low minimum: Five motors are selected at random from a representative population of motors and tested. The test results yield efficiencies of 89.9, 89.2, 88.0, 89.3, 89.4, and 89.9. The mean efficiency for the sample and the sample minimum efficiency are thus 89.2 and 88.0, respectively. Since the sample minimum is less than 88.1 the basic model is not in compliance; the manufacturer may not represent the basic model to have a rated efficiency of 89.5. Since the 88.0 test result may not be excluded, further testing will serve no purpose and testing is at an end. Compliance with a rated efficiency of 89.5 has not been demonstrated.

Table 2: t coefficients for specified confidence.[a]

t	Statistical Confidence	Probability of Exceeding E
3	99.7	0.003
2.56	99	0.010
2	95.5	0.045
1.96	95	0.050
1.64	90	0.100

[a]Adapted from ASTM Std E 122-89

6 AEDM substantiation

Per Subpart B §431.24(a)(3) of the Final Rule, laboratory efficiency measurements may also provide the basis for substantiation of an Alternative Efficiency Determination Method (AEDM); i.e., a mathematical model based on engineering or statistical analysis, computer simulation or modeling, or other analytic evaluation of performance data that may be used to assign motor efficiency.

To emphasize the salient features of the Final Rule we paraphrase §431.24(a)(3) as follows:

An AEDM is substantiated provided:

1. At least five different basic models are tested according to the procedures established for certification testing, and

2. The total power loss calculated by the AEDM is within the interval of ±10% of the mean total power loss determined from the actual testing for each of the basic models tested.

The scenario described, in which testing is required to conform to a predetermined precision is addressed by ASTM Standard E 122-89 [10]. This standard is based on the t statistic and establishes the sample size required to determine a two-sided confidence interval on the estimate of the mean. The following discussion provides an estimate of the number of tests needed to support the ±10% precision required by the Final Rule.

Following the ASTM standard, the sample size n needed to support a tolerance E is given by the following expression:

$$n = \left(\frac{t\sigma}{E}\right)^2, \tag{4}$$

where σ is the standard deviation, t is a coefficient that corresponds to the desired statistical confidence. Values of t for commonly specified statistical confidences are presented in Table 2. Since a high statistical confidence is desired, we set the coefficient t to three. The error tolerance, E, is 10% of the total loss, i.e.,

$$E = 0.10(100 - RE). \tag{5}$$

If we further assume, as discussed in section 3, that a 20% tolerance in total loss corresponds to three standard deviations, i.e.,

$$3\sigma = 0.20(100 - RE), \tag{6}$$

we may arrive at the following result,

$$n = \left[\frac{0.20(100 - RE)}{0.10(100 - RE)}\right]^2, \tag{7}$$

and conclude that no fewer than four motors should be tested. The Final Rule, in establishing a minimum sample of five for compliance testing, appears to be consistent with this result.

7 Enforcement testing

Enforcement testing is likely to be required only in the circumstance that all other means of resolving conflicting interpretations of test results and/or labeled motor efficiencies have been exhausted. Further, as it may be necessary to enter results obtained by testing under the Sampling Plan for Enforcement Testing into evidence, the Sampling Plan for Enforcement Testing is based on well-established statistical methods and follows clearly delineated procedures. The Sampling Plan for Enforcement Testing is based on a procedure, which is due to C. Stein [11, 12], for obtaining a confidence interval on a mean.

Since the Sampling Plan for Enforcement Testing may recommend that certain adverse actions be taken against a manufacturer—e.g., relabeling, the cessation of the sale and distribution of certain basic models, and the assessment of fines—the risk of a false determination of noncompliance should be small. The sampling plan is based on a 97.5% statistical confidence, thus the risk to a manufacturer of a false determination of noncompliance is no greater than 2.5%.

The legislation is supported by ensuring that the mean efficiency of each basic model is not less than the EPCA nominal efficiency and the rated efficiency. This objective may be satisfied by demonstrating that the mean efficiency obtained by tests conducted on a random sample of motors exceeds a lower control limit. The Sampling Plan for Enforcement Testing estimates the true mean full-load efficiency of the basic model and the confidence that this estimate exceeds a lower control limit. The Sampling Plan for Enforcement Testing assumes that the true mean full-load efficiency, the standard deviation of the motor efficiencies, and the distribution of motor efficiencies are not known.

The Sampling Plan for Enforcement Testing has been adapted from that provided for appliance testing under Part 430 [13]. This sampling plan is based on the t-test. The t-test is well suited to this application as it is well known to be insensitive to departures from the assumption of normally distributed data: The t-test is a test on a mean, i.e., an average of independent values obtained by a random sample. In general, sums of arbitrary, independent random values tend to have a distribution that is *almost* normal. Hence, the t-test is not strongly influenced by the exact form of the underlying distribution.

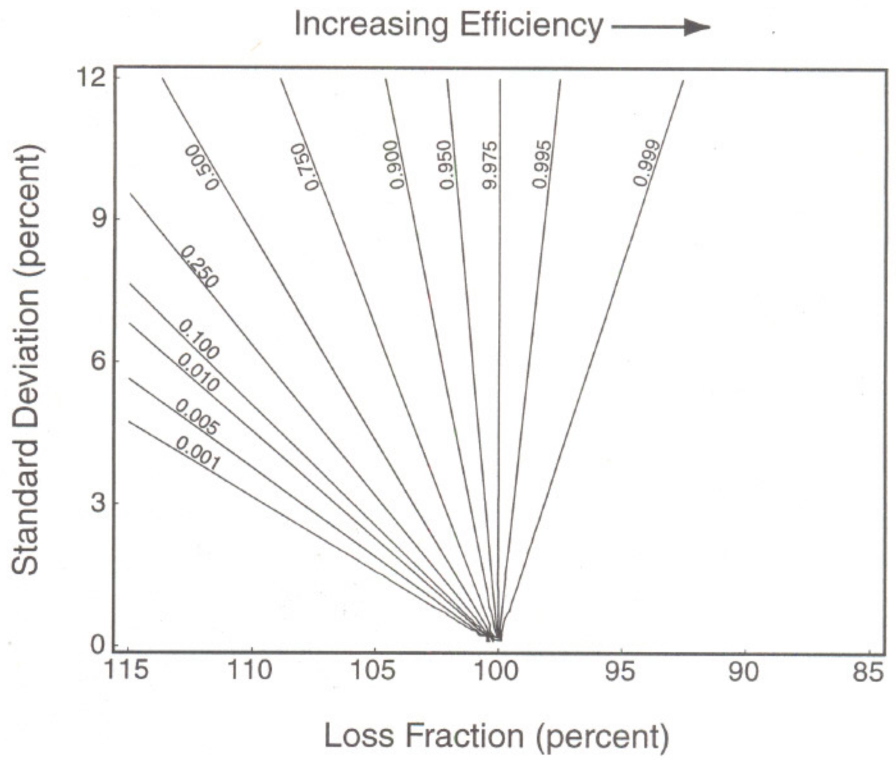

Figure 2: The operating characteristics of the Sampling Plan for Enforcement Testing. Model calculations are for samples of five and a maximum of 20. The contours indicate the probability of demonstrating compliance, e.g., the 0.90 contour corresponds to a 90% likelihood of demonstrating compliance.

7.1 Method

The best estimate of the true mean efficiency that may be obtained by tests conducted on a random sample is the mean efficiency of that sample,

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \tag{8}$$

where X_i is the measured full-load efficiency of unit i, and n is the number of units tested. The uncertainty of this estimate depends on two factors: 1) the size of the sample, i.e., the number of motors tested, and 2) the underlying variability in the entire population of motors. The sample standard deviation,

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}, \tag{9}$$

is one measure of the variability of the motor efficiencies. The standard error in the mean,

$$SE(\bar{X}) = \frac{S}{\sqrt{n}}, \tag{10}$$

provides an estimate of the standard deviation of the mean efficiency as determined by tests conducted on samples of n units. If we assume that the efficiencies of the entire population of motors are normally distributed about the true mean full-load efficiency, μ, then the ratio,

$$t = \frac{\mu - \bar{X}}{SE(\bar{X})}, \tag{11}$$

is distributed according to a probability density function that is known in statistics literature as the t-distribution. The values of t associated with commonly specified percentiles are readily available and are included in many references on statistics [14].

Establishing a lower control limit. Equation (11) provides an expression for the mean of the sample:

$$\bar{X} = \mu - tSE(\bar{X}). \tag{12}$$

We may assume, by hypothesis, that the units to be tested are drawn from a population of motors for which the mean full-load efficiency is equal to or greater than the rated efficiency RE. If t is the 97.5th percentile of the t-distribution appropriate to the sample size, then the probability of obtaining a mean efficiency,

$$\bar{X} \geq RE - tSE(\bar{X}), \tag{13}$$

is not less than 97.5%, which recommends the lower control limit,

$$LCL = RE - tSE(\bar{X}). \tag{14}$$

To apply this method, a random sample of motors is tested and the mean and standard error in the mean are calculated. Based on the size of the sample and the confidence desired, the appropriate t-value is selected and the lower control limit calculated. For example, 97.5% confidence and a sample size of five units recommends a t-value of 2.776. Provided the mean efficiency obtained from the random sample is not less than the lower control limit, as defined by Eq. (14), we may assert with a confidence not less than 97.5% that the true mean efficiency of the entire population is not less than the rated efficiency and thus that the basic model is in compliance.

In any statistical test there is some probability of incorrectly concluding noncompliance. By design, the probability that the mean efficiency for a random sample drawn from this population of motors would fall below the lower control limit, hence, the risk of incorrectly concluding that the basic model is in noncompliance, is not greater than 2.5%.

There is some probability that the estimate of the standard deviation and, therefore, that the standard error in the mean is large and that the lower control limit may be set, by chance, to an exceptionally large value. To avoid this circumstance, it is sufficient to establish a tolerance for the standard error in the mean, $SE(\bar{X})$. The tolerance for the standard error should be chosen to be appropriate for the size and type of motor being tested and to be supported across the industry.

By definition, the efficiency as a percentage can be expressed as,

$$\mu = \frac{P_{out}}{P_{in}} \times 100, \tag{15}$$

where P_{in} and P_{out} are the input and output power, respectively. Following the convention used by NEMA [5], the minimum efficiency is calculated at constant output power, thus

$$\begin{aligned} \mu_{min} &= \frac{P_{out}}{P_{in} + 0.20(P_{in} - P_{out})} \times 100 \\ &= \frac{\mu}{120 - 0.20\mu} \times 100, \end{aligned} \tag{16}$$

which is again expressed as a percentage. The lower control limit must then satisfy two conditions:

$$\begin{aligned} LCL &= RE - tSE(\bar{X}) \text{ and} & (17)\\ &\geq \frac{RE}{120 - 0.20RE} \times 100. & (18) \end{aligned}$$

The second condition is obtained from Eq. (16) by setting the efficiency equal to the RE.

Discussion. By design, the tolerances for the motor efficiency specified by the Final Rule are closely associated with the NEMA guidelines for motor efficiency labeling, and are thus likely to follow quality control practices used by industry. This has several potential advantages: 1) industry should be better able to estimate the risk involved with the selection of a basic model for testing and thus better manage their financial risk, and 2) the investment required for personnel training should be reduced since the tolerances recommended by Part 431 follow those currently used by industry.

The Sampling Plan has an additional advantage: If a manufacturer is in compliance with the voluntary NEMA guidelines for motor efficiency labeling, the probability of demonstrating compliance by actual testing is high. As discussed, the probability of failure during enforcement testing due to a low mean value is not greater than 2.5%. A motor may also fail during enforcement testing due to high variability. We next estimate the likelihood that a motor labeled in accordance with the MG 1 guidelines would fail during enforcement testing due to insufficient sample size. Step 7 of the NOPR Sampling Plan for Enforcement Testing sets a condition on the sample size. To demonstrate compliance, the initial sample size n_1 must satisfy the following condition:

$$n_1 \geq \left[\frac{tS_1(120 - 0.2RE)}{RE(20 - 0.2RE)}\right]^2, \tag{19}$$

where RE is the rated efficiency and S_1 is the standard deviation of the sample. This equation may be rearranged to yield a condition on the value of t:

$$t \leq \left[\frac{\sqrt{n_1}RE(20 - 0.2RE)}{S_1(120 - 0.2RE)}\right]. \tag{20}$$

Following our earlier discussion, we assume that the difference between the NEMA Nominal and Minimum efficiencies corresponds to three standard deviations, and use the following approximation:

$$S_1 \approx \frac{\sigma}{\sqrt{n_1}} \approx \frac{0.20(100 - RE)}{3\sqrt{n_1}}.$$

Upon substitution into Eq. 20, the following condition on t is obtained:

$$t < 3n_1 \frac{RE(20 - 0.20RE)}{0.20(100 - RE)(120 - 0.2RE)}.$$

For an initial sample of five, t must exceed ten for the sample to fail due to insufficient sample size. The probability that t would exceed 10 by chance is less than 1 in 1000, for a sample of five. We conclude that it is highly unlikely that a product that is labeled in accordance with the MG 1 guidelines would require testing beyond the initial sample of five.

The operating characteristics of the Sampling Plan for Enforcement Testing are shown in Figure 2, which present data for an initial sample of five and testing as many as 20. It may be noted that the 97.5% contour is independent of the standard deviation.

7.2 Enforcement testing example

A specific example of the use of the Sampling Plan for Enforcement Testing follows. We assume that a basic model is tested for a rated efficiency of 89.5.

The manufacturer may select at random no fewer than five units and no more than 20 units from a representative population of motors.

Step 1. The first sample size (n_1) must be five or more units.

In this example, we assume that the manufacturer elects to test an initial sample of five motors, $n_1 = 5$.

Step 2. Compute the mean ($\bar{X}_1$) of the measured energy performance of the n_1 units in the first sample as follows:

$$\bar{X}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i \tag{21}$$

where X_i is the measured full-load efficiency of unit i.

Assume, as in the earlier example of compliance testing, that the results of testing yield efficiencies of 89.9, 89.2, 89.0, 89.3, 89.4. The mean of the sample is thus 89.4.

Step 3. Compute the sample standard deviation (S_1) of the measured full-load efficiency of the n_1 units in the first sample as follows:

$$S_1 = \sqrt{\frac{\sum_{i=1}^{n_1}(X_i - \bar{X}_1)^2}{n_1 - 1}}. \tag{22}$$

The sample standard deviation is 0.3.

Step 4. Compute the standard error ($SE(\bar{X}_1)$) of the mean full-load efficiency of the first sample as follows:

$$SE(\bar{X}_1) = \frac{S_1}{\sqrt{n_1}}. \tag{23}$$

The standard error of the mean is 0.2.

Step 5. Compute the lower control limit (LCL_1) for the mean of the first sample using RE as the desired mean as follows:

$$LCL_1 = RE - tSE(\bar{X}_1) \tag{24}$$

where RE is the applicable EPCA nominal full-load efficiency when the test is to determine compliance to the applicable statutory standard, or is the labeled nominal full-load efficiency when the test is to determine compliance with the labeled efficiency value, and t is the 2.5th percentile of a t-distribution for a sample size of n_1, which yields a 97.5% confidence level for a one-tailed t-test.

Next select the t-coefficient for a sample of five units and a statistical confidence of 97.5%. The value of the t-coefficient may be obtained from standard mathematical tables. A table of t-coefficients is provided here in Table 3. It should be noted that the t-coefficient is often tabulated according to the "degrees of freedom," which is $(n_1 - 1)$.

For a sample size of five the degrees of freedom is 4, thus the t-coefficient is 2.776 and the lower control limit is 89.1.

Step 6. Compare the mean of the first sample ($\bar{X}_1$) with the lower control limit (LCL_1) to determine one of the following:

(i) If the mean of the first sample is below the lower control limit, then the basic model is in noncompliance and testing is at an end.

(ii) If the mean is equal to or greater than the lower control limit, no final determination of compliance or noncompliance can be made; proceed to Step 7.

Since the mean of the initial sample is greater than the lower control limit, proceed to Step 7.

Step 7. Determine the recommended sample size (n) as follows:

$$n = \left[\frac{tS_1(120 - 0.2RE)}{RE(20 - 0.2RE)}\right]^2 \tag{25}$$

where S_1 and t have the values used in Steps 4 and 5, respectively. The factor

$$\frac{120 - 0.2RE}{RE(20 - 0.2RE)}$$

is based on a 20% tolerance in the total power loss at full load and fixed output power.

The recommended sample size is two.

Given the value of n, determine one of the following:

(i) If the value of n is less than or equal to n_1 and if the mean energy efficiency of the first sample ($\bar{X}_1$) is equal to or greater than the lower control limit (LCL_1), the basic model is in compliance and testing is at an end.

(ii) If the value of n is greater than n_1, the basic model is in noncompliance. The size of a second sample n_2 is determined to be the smallest integer equal to or greater than the difference $n - n_1$. If the value of n_2 so calculated is greater than $20 - n_1$, set n_2 equal to $20 - n_1$.

Since the initial sample size is greater than the recommended sample size, the basic model is in compliance and testing is at an end.

Table 3: An abridged t-table. t coefficients based on a statistic confidence of 97.5%, sample size n, and degrees of freedom ν.

n	ν	t
5	4	2.776
6	5	2.571
7	6	2.447
8	7	2.365
9	8	2.306
10	9	2.262
11	10	2.228
12	11	2.201
13	12	2.179
14	13	2.160
15	14	2.145
16	15	2.131
17	16	2.120
18	17	2.110
19	18	2.101
20	19	2.093

8 Further information

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References

[1] Energy efficiency program for certain commercial and industrial equipment: Test procedures, labeling, and certification requirements for electric motors. *Federal Register*, 64(192):54114–54172, Tuesday, October 5, 1999.

[2] Energy conservation program for certain commercial and industrial equipment: Test procedures, labeling, and certification requirements of electric motors; proposed rule. *Federal Register*, 61(230):60439–60475, Wednesday, November 27, 1996.

[3] Energy Policy and Conservation Act of 1975, Pubic Law 94-163; National Energy Conservation Policy Act of 1978, Public Law 95-619; National Appliance Energy Conservation Act of 1987, Public Law 100-12; National Appliance Energy Conservation Amendments of 1988, Public Law 100-357; and Section 122(d), Energy Policy Act of 1992, Public Law 102-486, October 24, 1992.

[4] K. L. Stricklett and M. G. Vangel. Electric motor efficiency testing under the new part 431 of chapter II of title 10, Code of Federal Regulations: Enforcement testing. NIST Technical Note 1422, National Institute of Standards and Technology, Technology Administration, U.S. Department of Commerce, December 1996.

[5] National Electrical Manufacturers Association Standards Publication MG1-1993, Motors and Generators, and Revisions 1, 2, 3 and 4. National Electrical Manufacturers Association, Rosslyn, VA, 22209.

[6] Institute of Electrical and Electronics Engineers, Inc., Standard test procedure for polyphase induction motors and generators, IEEE Std 112-1996, and the correction to the calculation at item (28) in section 10.2 Form B-Test Method B issued by IEEE on January 20, 1998. Institute of Electrical and Elecronics Engineers, Inc., Piscataway, NJ, 08855.

[7] CSA International (or Canadian Standards Association) Standard C390-93, Energy Efficiency Test Methods for Three-Phase Induction Motors. CSA International, Etobicoke (Toronto), Ontario, Canada, M9W 1R3.

[8] M. G. Vangel. The joint distribution of a normal mean and extremum with applications to quality control. In *Proceedings of the Section on Physical and Engineering Sciences*. American Statistical Association, 1998.

[9] Statistical methods for motor efficiency data. Motor and Generator Section Publication, National Electrical Manufacturers Association, Washington, DC 20037, May 1978.

[10] ASTM Standard E 122-89, Standard practice for choice of sample size to estimate a measure of quality for a lot or process, 1989.

[11] C. Stein. A two-sample test for a linear hypothesis whose power is independent of variance. *Annals of Mathematical Statistics*, 16:243–258, 1945.

[12] P. J. Bickel and K. A. Doksum. *Mathematical Statistics*. Holden Day, Oakland, CA, 1977.

[13] Energy conservation program for consumer products. Code of Federal Regulations, Chapter II, Vol. 10, Part 430, pp. 93–263, Office of the Federal Register, National Archives and Records Administration, Washington, DC, 1-1-97 1997.

[14] R. S. Schulman. *Statistics in Plain English with Computer Applications*. Van Norstand Reinhold, New York, 1992.

Appendix A

Exerpts from the Final Rule

This appendix contains exerpts from 10 CFR Part 431, "Energy Efficiency Program for Certain Commercial and Industrial Equipment: Test Procedures, Labeling, and Certification Requirements for Electric Motors," *Federal Register*, Vol. 64, No. 192, Tuesday, October 5, 1999, pp. 54114–54172. The criteria for establishing compliance with the EPCA standard efficiencies and for substantiation and subsequent verification of an AEDM are prescribed in §431.24, pp. 54152–54154. The Sampling Plan for Enforcement Testing is published in the Appendix B to Subpart G, pp. 54171 and 54172.

A.1 Certification testing

431.24(b)(2) *Selection of units for testing.* For each basic model selected for testing, a sample of units shall be selected at random and tested. The sample shall be comprised of production units of the basic model, or units that are representative of such production units. The sample size shall be not fewer than five units, except that when fewer than five units of a basic model would be produced over a reasonable period of time (approximately 180 days), then each unit shall be tested. In a test of compliance with a represented average of nominal efficiency:

(i) The average full-load efficiency of the sample $\bar{X}$ which is defined by

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

where X_i is the measured full-load efficiency of unit i and n is the number of units tested, shall satisfy the condition:

$$\bar{X} \geq \frac{100}{1 + 1.05\left(\frac{100}{RE} - 1\right)}$$

where RE is the represented nominal full-load efficiency, and

(ii) The lowest full-load efficiency in the sample X_{min}, which is defined by

$$X_{min} = \min(X_i)$$

shall satisfy the condition

$$X_{min} \geq \frac{100}{1 + 1.15\left(\frac{100}{RE} - 1\right)}$$

A.2 AEDM substantiation

§431.24(a)(3) *Substantiation of an alternative efficiency determination method.* Before an AEDM is used, its accuracy and reliability must be substantiated as follows:

(i) The AEDM must be applied to at least five basic models that have been tested in accordance with §431.23 of this subpart, and

(ii) The predicted total power loss for each basic model, calculated by applying the AEDM, must be within plus or minus 10% of the mean total power loss determined from testing of that basic model.

(4) *Subsequent verification of an AEDM.*

(i) Each manufacturer shall periodically select basic models representative of those to which it has applied an AEDM, and for each basic model selected shall either:

(A) Subject a sample of units to testing in accordance with §§431.23 and 431.24(b)(2) by an accredited laboratory that meets the requirements of §431.25,

(B) Have a certification body recognized under §431.27 certify its nominal full load efficiency, or

(C) Have an independent state-registered professional engineer, who is qualified to perform an evaluation of electric motor efficiency in a highly competent manner and who is not an employee of the manufacturer, review the manufacturer's representations and certify that the results of the AEDM accurately represent the total power loss and nominal full load efficiency of the basic model.

A.3 The Sampling Plan for Enforcement Testing

Appendix B to Subpart G of Part 431—Sampling Plan for Enforcement Testing

Step 1. The first sample size (n_1) must be five or more units.

Step 2. Compute the mean ($\bar{X_1}$) of the measured energy performance of the n_1 units in the first sample as follows:

$$\bar{X_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} X_i \tag{26}$$

where X_i is the measured full-load efficiency of unit i.

Step 3. Compute the sample standard deviation (S_1) of the measured full-load efficiency of the n_1 units in the first sample as follows:

$$S_1 = \sqrt{\frac{\sum_{i=1}^{n_1}(X_i - \bar{X_1})^2}{n_1 - 1}}. \tag{27}$$

Step 4. Compute the standard error ($SE(\bar{X_1})$) of the mean full-load efficiency of the first sample as follows:

$$SE(\bar{X_1}) = \frac{S_1}{\sqrt{n_1}}. \tag{28}$$

Step 5. Compute the lower control limit (LCL_1) for the mean of the first sample using RE as the desired mean as follows:

$$LCL_1 = RE - tSE(\bar{X_1}) \tag{29}$$

where RE is the applicable EPCA nominal full-load efficiency when the test is to determine compliance to the applicable statutory standard, or is the labeled nominal full-load efficiency when the test is to determine compliance with the labeled efficiency value, and t is the 2.5th percentile of a t-distribution for a sample size of n_1, which yields a 97.5% confidence level for a one-tailed t-test.

Step 6. Compare the mean of the first sample ($\bar{X_1}$) with the lower control limit (LCL_1) to determine one of the following:

(i) If the mean of the first sample is below the lower control limit, then the basic model is in noncompliance and testing is at an end.

(ii) If the mean is equal to or greater than the lower control limit, no final determination of compliance or noncompliance can be made; proceed to Step 7.

Step 7. Determine the recommended sample size (n) as follows:

$$n = \left[\frac{tS_1(120 - 0.2RE)}{RE(20 - 0.2RE)}\right]^2 \tag{30}$$

where S_1 and t have the values used in Steps 4 and 5, respectively. The factor

$$\frac{120 - 0.2RE}{RE(20 - 0.2RE)}$$

is based on a 20% tolerance in the total power loss at full load and fixed output power.

Given the value of n, determine one of the following:

(i) If the value of n is less than or equal to n_1 and if the mean energy efficiency of the first sample ($\bar{X}_1$) is equal to or greater than the lower control limit (LCL_1), the basic model is in compliance and testing is at an end.

(ii) If the value of n is greater than n_1, the basic model is in noncompliance. The size of a second sample n_2 is determined to be the smallest integer equal to or greater than the difference $n - n_1$. If the value of n_2 so calculated is greater than $20 - n_1$, set n_2 equal to $20 - n_1$.

Step 8. Compute the combined mean ($\bar{X}_2$) of the measured energy performance of the n_1 and n_2 units of the combined first and second samples as follows:

$$\bar{X}_2 = \frac{1}{n_1 + n_2} \sum_{i=1}^{n_1+n_2} X_i. \tag{31}$$

Step 9. Compute the standard error ($SE(\bar{X}_2)$) of the mean full-load efficiency of the n_1 and n_2 units in the combined first and second samples as follows:

$$SE(\bar{X}_2) = \frac{S_1}{\sqrt{n_1 + n_2}}. \tag{32}$$

(Note that S_1 is the value obtained above in Step 3.)

Step 10. Set the lower control limit (LCL_2) to

$$LCL_2 = RE - tSE(\bar{X}_2), \tag{33}$$

where t has the value obtained in Step 5, and compare the combined sample mean ($\bar{X}_2$) to the lower control limit (LCL_2) to find one of the following:

(i) If the mean of the combined sample ($\bar{X}_2$) is less than the lower control limit (LCL_2), the basic model is in noncompliance and testing is at an end.

(ii) If the mean of the combined sample ($\bar{X}_2$) is equal to or greater than the lower control limit (LCL_2), the basic model is in compliance and testing is at an end.

MANUFACTURER-OPTION TESTING

If a determination of noncompliance is made in Steps 6, 7, or 11, above, the manufacturer may request that additional testing be conducted, in accordance with the following procedures.

Step A. The manufacturer requests that an additional number, n_3, of units be tested, with n_3 chosen such that $n_1 + n_2 + n_3$ does not exceed 20.

Step B. Compute the mean full-load efficiency, standard error, and lower control limit of the new combined sample in accordance with the procedures prescribed in Steps 8, 9, and 10, above.

Step C. Compare the mean performance of the new combined sample to the lower control limit (LCL_2) to determine one of the following:

(a) If the new combined sample mean is equal to or greater than the lower control limit, the basic model is in compliance and testing is at an end.

(b) If the new combined sample mean is less than the lower control limit and the value of $n_1+n_2+n_3$ is less than 20, the manufacturer may request that additional units be tested. The total of all units tested may not exceed 20. Steps A, B, and C are then repeated.

(c) Otherwise, the basic model is determined to be in noncompliance.