

# Simulations of Noise-Parameter Verification Using Cascade with Isolator or Mismatched Transmission Line\*

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**Abstract** — Results are presented for simulations of a verification process for noise-parameter measurements. The verification process consists of first measuring separately both a passive device and the amplifier or transistor of interest (the device under test, or DUT) and then measuring the tandem configuration of passive device plus DUT. The results of the measurements of the tandem configuration are then compared to the predictions obtained by cascading the noise parameters and S-parameters of the two individual components. In order that the comparisons be meaningful, uncertainties are computed for both predictions and simulated measurements.

**Index Terms** — Amplifier noise, measurement uncertainties, noise measurements, noise parameters, transistor noise, verification methods.

## I. INTRODUCTION

It is usually helpful, and often essential, to have a method to confirm that measurements are correct, or at least not obviously incorrect. This is all the more important for complicated measurements, in which failure modes of the measurement process may not be at all evident. Measurements of noise parameters and noise figure of amplifiers or transistors certainly fall in that category. Several verification methods have been suggested [1 – 4]. One of the most attractive methods employs an auxiliary passive two-port device; in this method, one measures the S-parameters of the auxiliary passive device (APD) and the DUT, measures the noise parameters of the DUT, and then attaches the auxiliary device to the DUT input and measures the S-parameters and noise parameters of the tandem configuration [3, 4]. With the noise and S-parameters of the DUT measured, and with the noise parameters of the APD calculable from its measured S-parameters, one can also compute what the noise and S-parameters of the tandem configuration should be and compare those predictions to the values measured for the tandem configuration. There are several attractive features of this verification method. One is that the verification measurements are performed on an active device (the tandem configuration), whose gain and noise figure can be comparable to those of the DUT. If an isolator is used as the auxiliary two-port, some of the noise parameters are

approximately independent of the amplifier properties, and thus constitute a sort of independent standard [3], as opposed to just a consistency check. If a section of mismatched transmission line is chosen as the auxiliary two-port, the tandem configuration can be a highly reflective two-port, posing a more difficult measurement challenge [4]. Furthermore, with an attenuator or a mismatched transmission line, the method can also be implemented in an on-wafer environment.

To test and compare the use of this method with different auxiliary passive devices, we have performed simulations and have compared the measurement simulations to the simulated predictions for the noise parameters. We have evaluated the uncertainties in both the simulated measurements and in the predictions, so that meaningful comparisons can be performed. The following section describes the verification method, the simulations, and the uncertainty analysis. Section III contains the results, and Section IV is devoted to a discussion and conclusions.

## II. THEORY AND SIMULATOR

### A. Verification Method

The measurements required for the verification method are shown in Fig. 1. The S-parameters are measured for all three configurations shown: the DUT, the auxiliary passive device, and the tandem configuration. The noise parameters of the DUT and the tandem configuration are also measured, and the noise parameters of the auxiliary two-port are obtained from

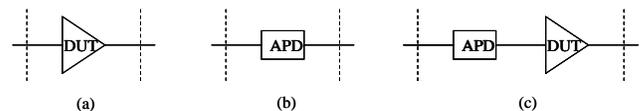


Fig. 1 Three configurations to be measured for verification method: (a) the DUT alone, (b) the auxiliary passive device alone, and (c) the tandem configuration.

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Bosma's theorem [5]. (A more recent and available derivation of Bosma's theorem can be found in [6].) From the measured characteristics of the DUT and the APD, one can compute the noise parameters of the tandem configuration and compare this prediction to the actual measurement results.

### B. Simulator

The NIST uncertainty analysis for noise-parameter measurements [7] relies on a Monte Carlo (MC) program for evaluation of the type-B uncertainties. The MC program performs repeated simulations of noise-parameter measurements and computes the type-B uncertainties from the standard deviations of the distributions of the different variables. The core of the MC program has now been extracted for use in its own right, as a simulator to investigate different strategies for noise-parameter measurements on amplifiers and transistors. The MC program and the simulator will be described in detail elsewhere, but some features that merit mentioning are that the measurements of all S-parameters, reflection coefficients, and noise temperatures (input, output, and ambient) are simulated separately, and correlations among the various measurements are included. The set of input terminations is chosen by the user, and the set of simulated output noise-temperature measurements is analyzed in the same way as are real measurements, to yield a set of noise parameters. The uncertainties in the underlying measurements (the reflection coefficients, S-parameters, noise temperatures), as well as the correlations in the errors, are read into the program.

For purposes of the simulations discussed in this paper, we assumed a set of terminations consisting of one hot noise source, with noise temperature around 700 K, and seven near-ambient terminations, with noise temperatures in the range 300 K – 350 K. The near-ambient terminations were obtained from an electronic calibration unit used for VNA calibrations. The values of noise temperatures and reflection coefficients used in the simulations all correspond to values actually measured for the different terminations. Thus, the noise temperatures and reflection coefficients of the set of input terminations vary with frequency. It is highly unlikely that any of the qualitative conclusions of the present study depend on the particular set of input terminations chosen. The values of uncertainties and correlations that were used in the simulations correspond approximately to those used in amplifier noise measurements at NIST. The number of simulated measurement sets for each data point shown in the results was 40,000. This is enough to ensure that the deviation of any uncertainty from its asymptotic value is less than 10 %, and in most cases it is 3 % or less.

### C. Uncertainties

To determine whether predictions agree with measured results, we must know the uncertainties for the predictions, as

well as for the measurements. For the uncertainties in the measurements, we use the uncertainty analysis for NIST noise-parameter measurements [7]. For the simulated measurements discussed here, we evaluated the type-B uncertainties in the same manner as we would for real measurements, from the standard deviation of the distribution of simulated results for each noise parameter. Each simulated measurement set has its own type-A uncertainty, obtained from the fit to the measured output noise temperatures; we use the RMS average of these type-A uncertainties and then add the type-A and type-B uncertainties in quadrature to obtain the combined uncertainties for our results. Thus, the error bars for the simulated measurements correspond to the (RMS) average standard uncertainties that would be expected in these sorts of measurements.

The uncertainties in the predictions require additional work. When an isolator is used as the auxiliary passive device, the uncertainty analysis for the predictions simplifies greatly [3], but for the case of the mismatched transmission line, the simplifications do not occur, and the full calculation must be done. For the uncertainties in the predictions, we use the usual formulas for propagation of errors [8]. The predictions are obtained by cascading the S-parameters and noise parameters of the two individual components, yielding equations of the form

$$X_i' = f_i(\mathbf{X}, \mathbf{S}, \mathbf{S}_{APD}), \quad (1)$$

where  $X_i'$  denotes any of the noise parameters of the tandem configuration,  $\mathbf{X}$  is the set of noise parameters of the amplifier alone,  $\mathbf{S}$  is the amplifier scattering matrix, and  $\mathbf{S}_{APD}$  is the scattering matrix of the auxiliary passive device. If the uncertainties in  $\mathbf{X}$ ,  $\mathbf{S}$ , and  $\mathbf{S}_{APD}$  are known, the uncertainties in the  $X_i'$  are given by

$$u_c^2(X_i') = \sum_{j,k} \frac{\partial f_i}{\partial x_j} \frac{\partial f_i}{\partial x_k} u(x_j, x_k), \quad (2)$$

where the sums over  $j$  and  $k$  run over all the elements of  $\mathbf{X}$ ,  $\mathbf{S}$ , and  $\mathbf{S}_{APD}$ .  $x_j$  and  $x_k$  denote elements of  $\mathbf{X}$ ,  $\mathbf{S}$ , and  $\mathbf{S}_{APD}$ , and  $u(x_j, x_k)$  is the  $j, k$  element of the covariance matrix for the  $x$ 's. A further complication arises due to the fact that our analysis is all done in terms of a set of noise parameters [3,7] based on the noise correlation matrix, but we also want the results in terms of IEEE noise parameters. This requires a second invocation of the process embodied in (1) and (2) to get the uncertainties in the IEEE noise parameters of the tandem configuration from the  $X'$  and  $S'$  parameters. The number of variables involved and the complexity of the equations in (1) and in the transformations from  $X'$  to IEEE parameters combine to render the full calculation rather painful. We do not subject the reader to details of that calculation here.

### III. RESULTS

The verification method was simulated for two different amplifiers with significantly different noise parameters. Since one of the amplifiers was owned by NIST and the other by Agilent, we refer to them as the NIST and Agilent amplifiers. The NIST amplifier is a low-noise amplifier (LNA) with a minimum noise figure between 1.4 dB and 1.6 dB between 8 GHz and 12 GHz. The Agilent amplifier has a higher noise figure (2.5 dB to 2.9 dB). Both amplifiers have values of  $|\Gamma_{opt}|$  below 0.2 in the frequency range of interest (8 GHz – 12 GHz). For each amplifier, two different auxiliary passive devices were considered, an isolator and a Beatty standard. Simulations were performed at four frequencies: 8 GHz, 9 GHz, 11 GHz, and 12 GHz. All input quantities for the simulations corresponded to real measured values. The Beatty standard was highly reflective near the odd integral frequencies (9 GHz and 11 GHz), and matched near the even integral frequencies (10 GHz and 12 GHz).

Results of the simulated verifications are shown in Figs. 2 – 5. The quantities for which we show results are the 50  $\Omega$  noise figure  $F_0$ (dB), the minimum effective input noise temperature  $T_{min}$ , the noise resistance  $R_n$ , and the magnitude of the optimum source reflection coefficient  $|\Gamma_{opt}|$ . Except for  $F_0$ , these quantities appear in the common IEEE parameterization of the effective input noise temperature of an amplifier,

$$T_e = T_{min} + \frac{4R_n T_0}{Z_0} \frac{|\Gamma_{opt} - \Gamma_{source}|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_{source}|^2)}, \quad (3)$$

where  $T_0 = 290$  K, and  $Z_0$  is the reference impedance. The 50  $\Omega$  noise figure is defined by

$$F_0(dB) = 10 \log_{10} \left[ 1 + \frac{T_e(\Gamma_{source} = 0)}{T_0} \right]. \quad (4)$$

We have not shown the results for the gain or the phase of  $\Gamma_{opt}$ , because those are similar and do not add much to the discussion. In each figure, we show the prediction and the simulated measurement for the DUT alone (called the “amp” in the figures), for the isolator-DUT tandem configuration, and for the Beatty standard-DUT tandem configuration. The predicted values for the amplifier alone are taken to be the “true” values, which were results of actual measurements performed earlier. The particular mismatched transmission line that we used was reflective at odd integer frequencies (9 GHz and 11 GHz), and matched (nonreflective) at even integer frequencies (8 GHz, 10 GHz, and 12 GHz). Results are shown for both the NIST LNA and the Agilent amplifier.

The salient feature of the figures is that predictions and simulated measurements agree very well in most cases. Not surprisingly, the differences between using an isolator versus using a mismatched transmission line are most evident at the frequencies for which the transmission line is highly reflective, 9 GHz and 11 GHz. At these frequencies, the transmission line-amplifier tandem presents a more serious measurement challenge. At the frequencies for which the mismatched transmission line is reflective (9 GHz and 11 GHz), there is some difference between prediction and simulated measurements in the case of the NIST LNA, but not for the Agilent amplifier. This is a reflection of the greater measurement challenge posed by low-noise amplifiers that are poorly matched, as is the case for the mismatched line-LNA tandem configuration. Nonetheless, the predictions and simulated measurements agree within the uncertainties even in this case. Any differences in the case of the Agilent amplifier, which has a significantly larger noise figure, are imperceptible.

The point was made in [3] that the isolator-amplifier tandem configuration provides an absolute verification standard, in that three of its four noise parameters (in a particular representation) are *calculable* from the scattering parameters of the isolator; they do not depend on the amplifier. The best noise parameter for this purpose is the quantity  $X_{12}$ , which is defined as the 1,2 element of the noise correlation matrix in the wave representation, divided by  $S_{12}^*$ ,

$$X_{12} = \left\langle c_1 c_2^* / S_{21}^* \right\rangle, \quad (5)$$

where  $c_1$  and  $c_2$  are the amplitudes of the intrinsic noise waves emanating from ports one and two of the amplifier, or in this case the tandem configuration. The fact that  $X_{12}$  is very nearly independent of the amplifier in the isolator-amplifier tandem configuration can be seen by comparing the results for  $X_{12}$  for the amplifier-isolator tandem configuration for the two different amplifiers. This comparison is shown in Figs. 6 and 7, where we see that the real part of  $X_{12}$  for the isolator-amplifier tandem configuration is approximately equal for the two different amplifiers, as is the imaginary part of  $X_{12}$ .

### IV. CONCLUSIONS

We have performed simulations demonstrating the use of an auxiliary passive device in a tandem configuration with an amplifier to verify measurements of the amplifier’s noise parameters. If the uncertainties are understood, then the predictions and the measured noise parameters for the tandem configuration should agree within the uncertainties, and this

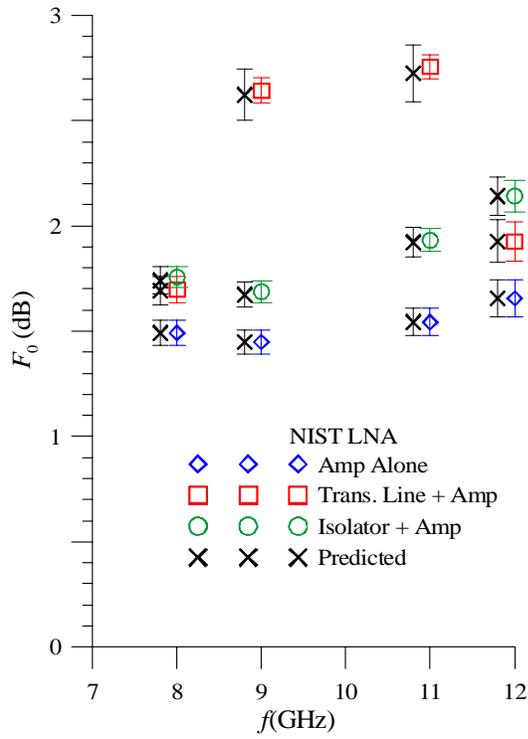


Fig. 2(a) Simulated predictions and measurement results for 50  $\Omega$  noise figure, NIST LNA.

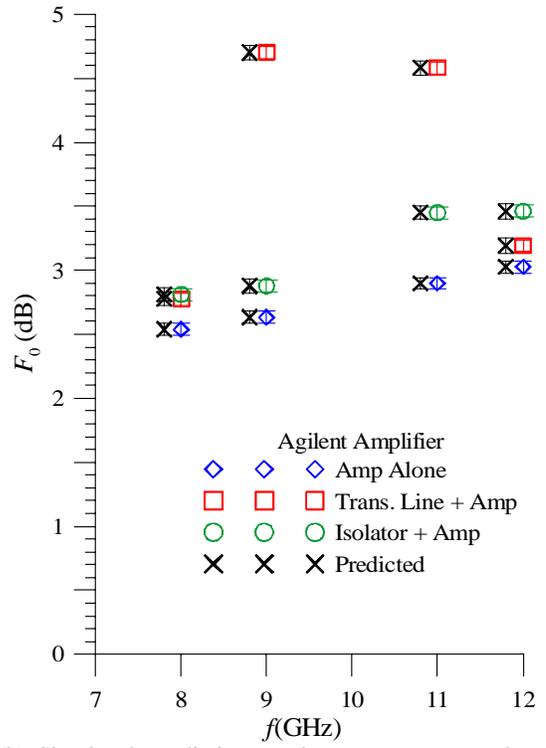


Fig. 2(b) Simulated predictions and measurement results for 50  $\Omega$  noise figure, Agilent Amplifier.

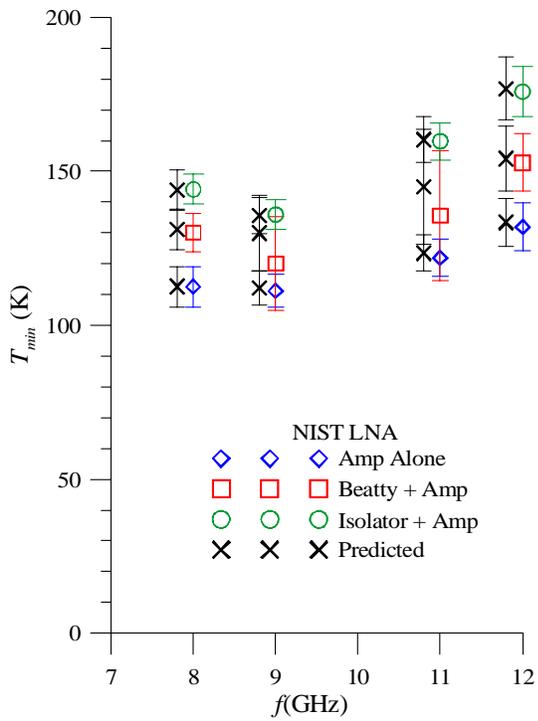


Fig. 3(a) Simulated predictions and measurement results for  $T_{min}$ , NIST LNA.

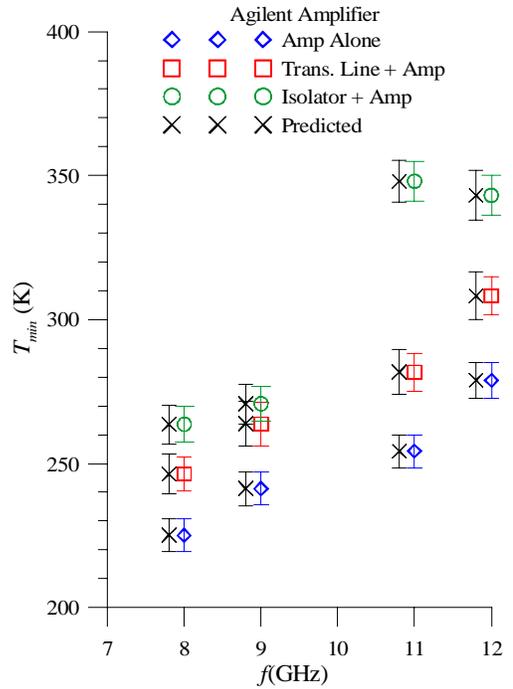


Fig. 3(b) Simulated predictions and measurement results for  $T_{min}$ , Agilent amplifier.

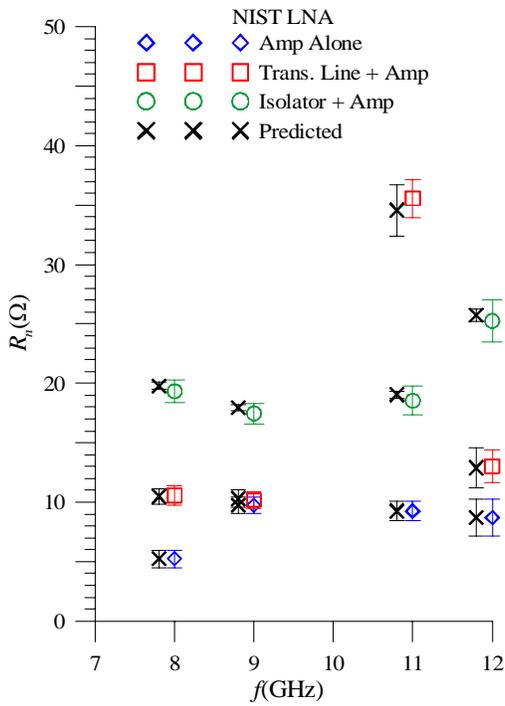


Fig. 4(a) Simulated predictions and measurement results for  $R_n$ , NIST LNA.

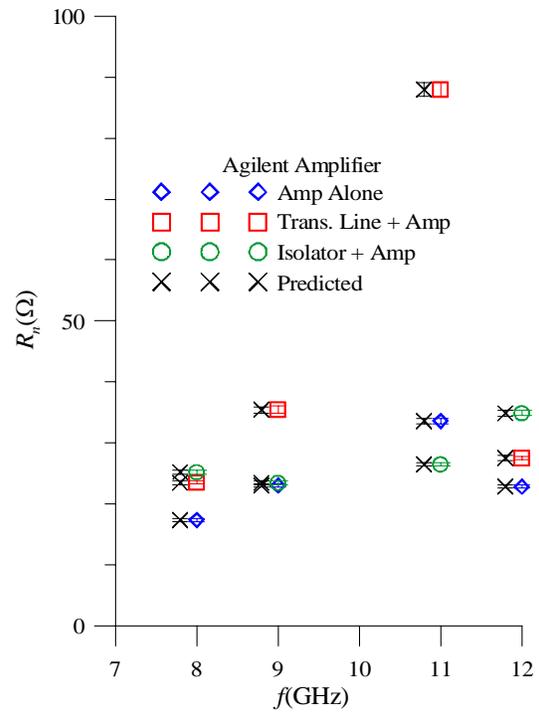


Fig. 4(b) Simulated predictions and measurement results for  $R_n$ , Agilent amplifier.

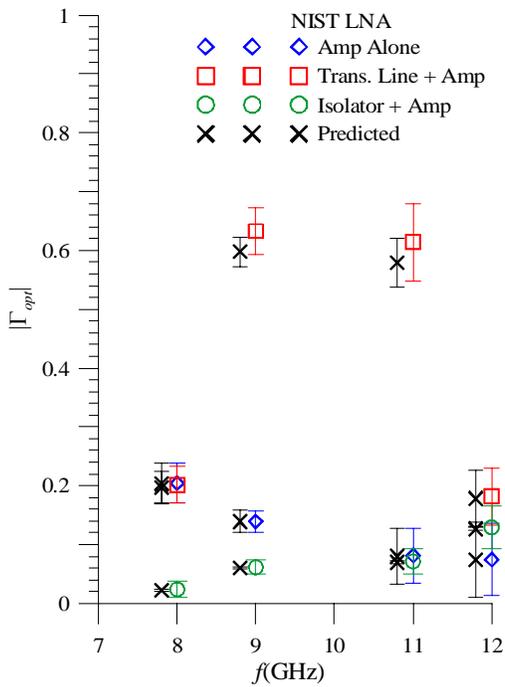


Fig. 5(a) Simulated predictions and measurement results for the magnitude of  $\Gamma_{opt}$ , NIST LNA.

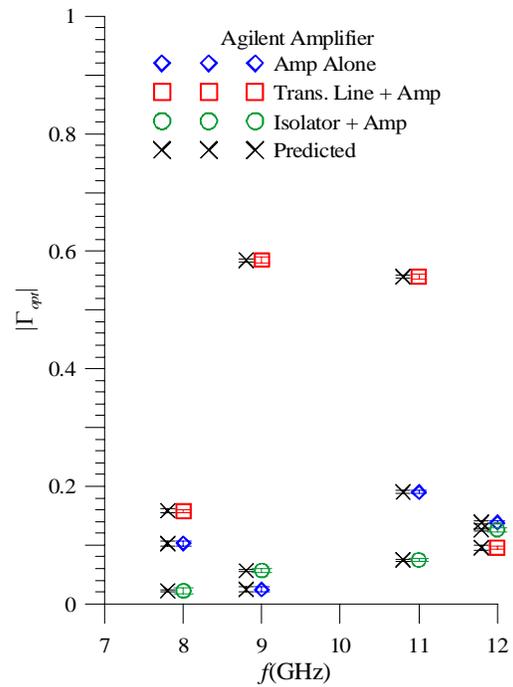


Fig. 5(b) Simulated predictions and measurement results for the magnitude of  $\Gamma_{opt}$ , Agilent amplifier.

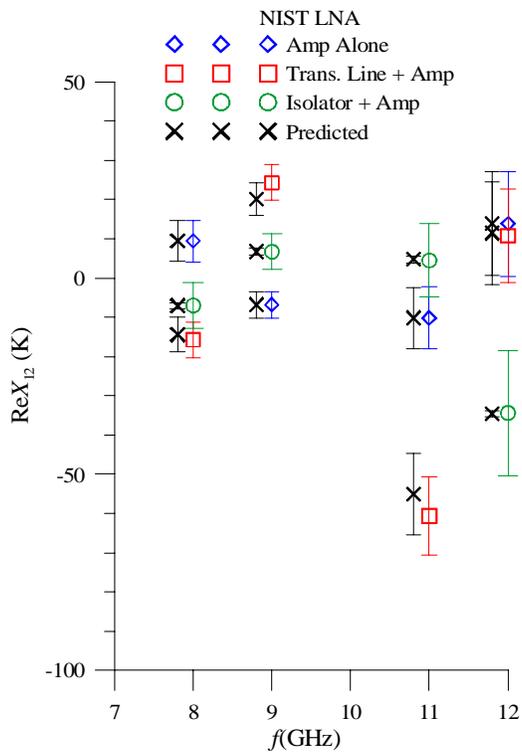


Fig. 6(a) Simulated predictions and measurement results for  $ReX_{12}$ , NIST LNA.

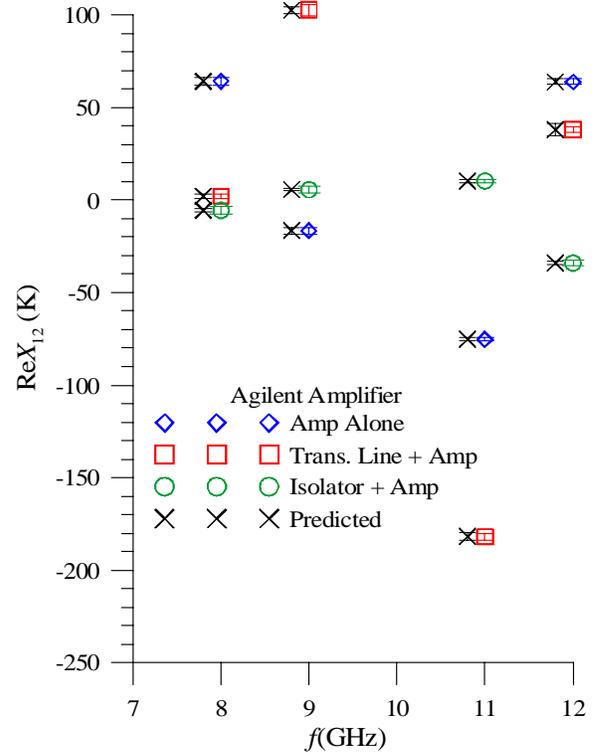


Fig. 6(b) Simulated predictions and measurement results for  $ReX_{12}$ , Agilent amplifier.

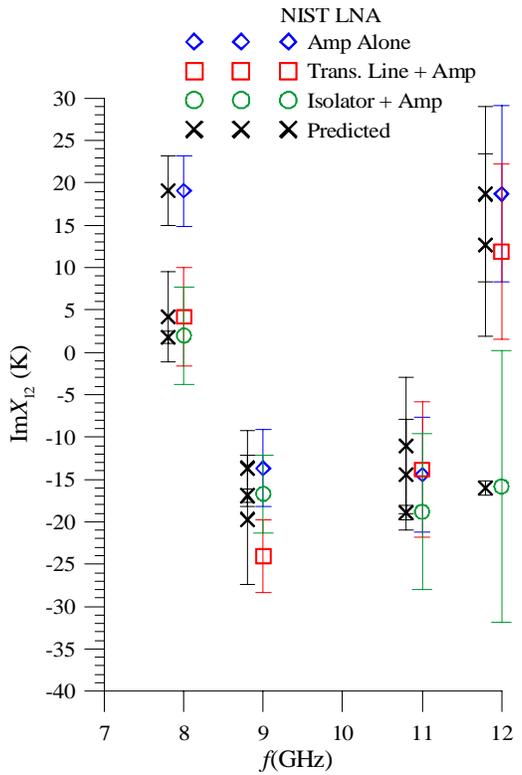


Fig. 7(a) Simulated predictions and measurement results for  $ImX_{12}$ , NIST LNA.

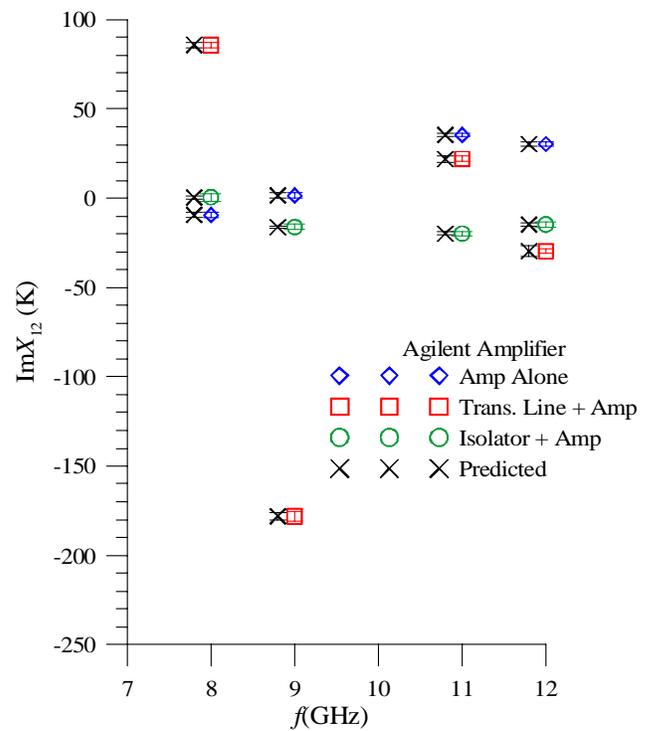


Fig. 7(b) Simulated predictions and measurement results for  $ImX_{12}$ , Agilent amplifier.

agreement was confirmed by the simulations. Some previous, related work has been done [3,4]; the major advances in the present work are the inclusion of full uncertainties for both the predictions and the (simulated) measurements, the use of two amplifiers with different properties, and the comparison of isolator and transmission-line results. Future work will implement and compare the methods with actual measurements and full uncertainties.

Some simulation work also remains to be done. The present work indicates that predictions agree with simulated measurement results if the measurements are correct and the uncertainties are estimated correctly. For completeness, we should also demonstrate that the comparison will fail if there are errors present that have not been included in the uncertainties, or if the uncertainties have been underestimated.

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