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U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

A Method of Determining the Emission and Susceptibility Levels of Electrically Small Objects Using a TEM Cell

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A Method of Determining the Emission and Susceptibility Levels of Electrically Small Objects Using a TEM Cell

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FOREWORD

This report describes theoretical and experimental analyses developed by staff of the University of Colorado at Boulder in collaboration with the Electromagnetic Fields Division of the National Bureau of Standards (NBS), under a contract sponsored by NBS. Professor David C. Chang heads the University team. Dr. Mark T. Ma of NBS serves as the technical contract monitor. The period covered by this report extends from July 1979 to July 1980.

The work described in this report represents a further aspect of establishing a theoretical basis for the technical analyses of transverse electromagnetic (TEM) transmission lines. The general purpose of pursuing theoretical studies is to evaluate the use of TEM cells for (1) measuring the total rf radiated power by a device inserted into the cell for test, or (2) performing necessary susceptibility tests on a small electronic device.

The particular problem of characterizing emission properties of electrically small radiating sources by tests inside a TEM cell was addressed previously [1]. Basically, an unknown small radiating object was modeled as an equivalent dipole system consisting of three orthogonal electric dipoles and three orthogonal magnetic dipoles, each excited with arbitrary amplitude and phase. Systematic measurement procedures to be taken inside a TEM cell were outlined for determining the individual dipole moments and the corresponding total power that would be radiated by the object in free space. The theory was well supported by the measurement results performed for a self-contained battery-operated spherical dipole. A basic assumption was then made that a constant TEM field distribution existed in the space occupied by the spherical dipole.

The same theory is tested for the susceptibility purpose in this report by representing the test object as an equivalent receiving antenna and by defining the susceptibility level as the per-unit power dissipated in a particular load across some terminals of the object illuminated by a specified incident electromagnetic field. This per-unit dissipated power is further expressed as the mismatch loss factor of the test object in free space, which in turn depends on the equivalent antenna and load impedances. During the course of this study, it is found that that electric quadrupole terms coupled with possible variations of the field distribution around the test area also play an important role in determining accurately the mismatch loss factor. For this reason, the electric quadrupole terms are added to the previous formulation where only electric and magnetic dipoles were considered. Again, necessary measurement procedures for determining the unknown susceptibility characteristics of the test object are described together with some experimental results.

Previous publications under the same effort include:

Tippett, J.C. and Chang, D.C., Radiation characteristics of dipole sources located inside a rectangular coaxial transmission line, NBSIR 75-829 (January, 1976).

Tippett, J.C., Chang, D.C., and Crawford, M.L., An analytical and experimental determination of the cut-off frequencies of higher-order TE modes in a TEM cell, NBSIR 76-841 (June 1976).

Tippett, J.C. and Chang, D.C., Higher-order modes in rectangular coaxial line with infinitely thin inner conductor, NBSIR 78-873 (March 1978).

Sreenivasiah, I. and Chang, D.C., A variational expression for the scattering matrix of a coaxial line step discontinuity and its application to an over moded coaxial TEM cell, NBSIR 79-1606 (May 1979).

Tippett, J.C. and Chang, D.C., Dispersion and attenuation characteristics of modes in a TEM cell with a lossy dielectric slab, NBSIR 79-1615 (August 1979).

Sreenivasiah, I., Chang, D.C., and Ma, M.T., Characterization of electrically small radiating sources by tests inside a transmission line cell, NBS Tech Note 1017 (February 1980).

Wilson, P.F., Chang, D.C., and Ma, M.T., Excitation of a rectangular coaxial transmission line due to a vertical electric Hertzian dipole, NBS Tech Note 1037 (March 1981).

A METHOD OF DETERMINING THE EMISSION AND SUSCEPTIBILITY LEVELS OF ELECTRICALLY SMALL OBJECTS USING A TEM CELL

I. Sreenivasiah, D.C. Chang and M.T. Ma

An electrically small radiating source of arbitrary shape may, to a first order, be modeled by an equivalent dipole system consisting of three orthogonal electric dipoles and three orthogonal magnetic dipoles each excited with arbitrary amplitude and phase. Determination of the individual electric dipole moments and the cross-components of such an equivalent dipole system for the emission case by tests inside a TEM cell, was described in an earlier work by Sreenivasiah, Chang and Ma, assuming a constant TEM modal field distribution in the space occupied by the test object. A method of accounting for the first-order variation in the TEM modal field distribution is presented in this report. This involves the inclusion of electric quadrupole terms in the modeling of the test object. Using the reciprocity principle, the same method is extended to the determination of susceptibility levels of electrically small test objects. Some experimental results for the susceptibility test, demonstrating the importance of the quadrupole terms, are presented.

Key words: Analysis; dipole moments; electrically small objects; emission; quadrupoles; measurements; susceptibility; TEM cell.

1.0 INTRODUCTION

An electrically small radiating source of arbitrary shape may, to a first order, be modeled by an equivalent dipole system consisting of three orthogonal electric dipoles and three orthogonal magnetic dipoles each excited with arbitrary amplitude and phase. In an earlier work [1,2] we described a measurement procedure for determining the individual electric/magnetic dipole moments and the cross-components of such an equivalent dipole radiating system by tests inside a TEM cell (figure 1). This method enables us to determine the free space emission characteristics of an equipment under test (EUT).

A more precise modeling of an arbitrary radiating object requires, in principle, the inclusion of higher order multi-pole moments in its equivalent representation. Even though these multi-pole moments may have only a negligible effect on the total power radiated by the EUT in free space, they can still affect the measurements taken inside a TEM cell if the TEM modal field distribution is not uniform in the test space occupied by the EUT. In order to take into account the variation of the field in the test region, it is necessary to include the electric quadrupoles in the equivalent representation of the EUT.

In the next section we present required mathematical expressions for the total power radiated by a localized system of sinusoidally varying currents, taking the electric quadrupoles into account. In section 3.0 we obtain related expressions for the fields due to a system of dipoles and quadrupoles placed inside a uniform section of waveguide. Based upon these expressions we present a measurement scheme in section 4.0 which enables us to determine the total power radiated by an object from tests taken inside a TEM cell. In section 5.0 we use the reciprocity principle and extend the test procedure to the determination of the susceptibility level of an EUT. Some experimental results for the receiving case demonstrating the importance of the quadrupole terms are presented in section 6.0, followed by concluding remarks in section 7.0.

2.0 FIELDS AND RADIATED POWER DUE TO A LOCALIZED SYSTEM OF CURRENTS

In most practical situations an arbitrary radiating object may be represented by an equivalent localized system of currents. To study the radiation characteristics of such a system of currents, we may restrict ourselves to a sinusoidal time dependence since an arbitrary time variation may be handled by its frequency components by means of Fourier analysis. In this section we start with the vector potential due to a volume distribution of currents and expand it in terms of the dipole and multi-pole moments. The resulting expression will be used to obtain the far fields and the total power radiated by the source under consideration.

The solution for the vector potential in the Lorentz gauge is given by [3]

$$\bar{A}\ (\bar{r}) = \frac{\mu_0}{4\pi} \int_v \bar{J}\ (\bar{r}')\ \frac{e^{-jk|\bar{r} - \bar{r}'|}}{|\bar{r} - \bar{r}'|}\ dv' \tag{2.1}$$

where μ_0 is the free-space permeability, $k = \omega/c$ is the wave number, ω is the radian frequency, c the velocity of light in free space, $\bar{J}$ the current density vector, $\bar{r}$ the field position vector, $\bar{r}'$ the source position vector, and v is the volume occupied by the source. A time dependence in the form of $e^{j\omega t}$ is assumed and suppressed. The electric and magnetic fields are then given, in terms of the vector potential, by the following equations:

$$j\omega\mu_0\varepsilon_0\bar{E} = \nabla \times \bar{B} \tag{2.2}$$

where

$$\bar{B} = \nabla \times \bar{A} \tag{2.3}$$

and ε_0 is the free-space permittivity.

In the far zone we may approximate the exponential in (2.1) by $|\bar{r} - \bar{r}'| \simeq r - \bar{n} \cdot \bar{r}'$ and the inverse distance may be replaced by r, giving

$$\lim_{kr\to\infty} \bar{A}\ (\bar{r}) \approx \frac{\mu_0 e^{-jkr}}{4\pi r} \int_v \bar{J}\ (\bar{r}')\ e^{jk\bar{n} \cdot \bar{r}'}\ dv' \tag{2.4}$$

where $r = |\bar{r}|$ and $\bar{n}$ is a unit vector in the direction of $\bar{r}$. If the source dimensions are small compared to a wave length, we can expand the integral in (2.4) in powers of k and write

$$\bar{A}(\bar{r}) \approx \frac{\mu_o e^{-jkr}}{4\pi r} \sum_{n=0}^{\infty} \frac{(jk)^n}{n!} \int_V \bar{J}(\bar{r}') (\bar{n} \cdot \bar{r}')^n \, dv'. \tag{2.5}$$

Keeping only the first two terms we obtain

$$\bar{A}(\bar{r}) \approx \frac{\mu_o e^{-jkr}}{4\pi r} \int_V [\bar{J} + jk\bar{J}(\bar{n}\cdot\bar{r}')]dv'$$

$$= \frac{\mu_o e^{-jkr}}{4\pi r} \int_V \{\bar{J} + \tfrac{1}{2} jk [(\bar{r}' \times \bar{J}) \times \bar{n} + (\bar{n}\cdot\bar{r}')\bar{J} + (\bar{n}\cdot\bar{J})\bar{r}']\} \, dv' \tag{2.6}$$

where we used the vector identity $\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}$ to write the second term as a sum of an anti-symmetric and a symmetric terms in $\bar{r}'$ and $\bar{J}$.

We see that the first term corresponds to an electric dipole, the anti-symmetric term which is the first inside the square brackets corresponds to a magnetic dipole and, as shown in Appendix I, the symmetric term consisting of the second and third inside the square brackets corresponds to an electric quadrupole. Thus, we write

$$\bar{A}(\bar{r}) = \frac{\mu_o}{4\pi r} \{\bar{P} - jk\,\bar{n} \times \bar{M} + j\frac{k}{2}(\bar{\bar{Q}}\cdot\bar{n})\} e^{-jkr} \tag{2.7}$$

where $\bar{P}$ and $\bar{M}$ are the electric and the magnetic dipole moment vectors given by

$$\bar{P} = \int_V \bar{J}(\bar{r}') \, dv' \tag{2.8a}$$

$$\bar{M} = \tfrac{1}{2} \int_V (\bar{r}' \times \bar{J}) \, dv', \tag{2.8b}$$

$\bar{\bar{Q}}$ is the electric quadrupole tensor whose components are given by

$$Q_{\alpha\beta} = \int_V (\alpha' J_\beta + \beta' J_\alpha) \, dv'; \quad \alpha,\beta = x,y,z \tag{2.8c}$$

and

$$\bar{r}' = \hat{x}\,x' + \hat{y}\,y' + \hat{z}\,z', \quad \hat{x},\hat{y},\hat{z} = \text{unit vectors}. \tag{2.8d}$$

Using the expression (2.7) for the vector potential, we can derive the electric and the magnetic fields in the far zone in accordance with (2.2) and (2.3) and the total power P_o radiated by the source via the Poynting vector formulation. The final expression for P_o as derived in Appendix II is given by

$$P_o = 10\,k^2 \{|\bar{P}|^2 + k^2 |\bar{M}|^2 + \frac{k^2}{20}(3\,Q_1 + Q_2)\} \tag{2.9a}$$

where

$$Q_1 = |Q_{xy}|^2 + |Q_{yz}|^2 + |Q_{zx}|^2 \tag{2.9b}$$

$$Q_2 = |Q_{xx}|^2 + |Q_{yy}|^2 + |Q_{zz}|^2 - R_e\,(Q_{xx}Q_{yy}^* + Q_{yy}Q_{zz}^* + Q_{zz}Q_{xx}^*). \tag{2.9c}$$

The above equations for the total power radiated are equivalent to those obtained by Jackson [4] except for the difference in units and the different manner in which $\bar{P}$ and $\bar{Q}$ are defined.

3.0 MULTIPOLE EXPANSION OF FIELDS GENERATED BY A LOCALIZED DISTRIBUTION OF CURRENTS INSIDE A WAVEGUIDE WITH UNIFORM CROSS-SECTION

The electric and magnetic fields generated by a localized distribution of currents inside a waveguide with uniform cross-section may be expanded in terms of the ortho-normal modes corresponding to the waveguide cross-section. If the walls of the waveguide are perfectly conducting, the coefficients of such an expansion may be obtained by an application of Lorentz's reciprocity principle [5]. When this is done, the amplitudes a_n and b_n for the positive and negative going fields corresponding to n^{th} mode are found to be given by

$$\binom{a}{b}_n = -\tfrac{1}{2} \int_V \bar{J}\cdot\bar{E}_n^{(\mp)}dv' \tag{3.1}$$

where $\bar{J}$ is the volume current density, $\bar{E}_n^{(\mp)}$ is, respectively, the backward and forward progressing ortho-normal electric field distribution function corresponding to the n^{th} mode, and the integration is extended over the volume occupied by the source.

If the size of the radiating source is so small that $\bar{E}_n$ may be assumed to be constant over the volume occupied by the source, the coefficients a_n and b_n are simply given in terms of the electric and/or magnetic dipoles characterizing the source. If $\bar{E}_n$ is not constant over the volume occupied by the source we may expand $\bar{E}_n$ in a Taylor series about a suitably chosen origin and approximate $\bar{E}_n$ by the following

$$\bar{E}_n(\bar{u}) \approx \bar{E}_n(0) + \bar{u}'\cdot\nabla\bar{E}_n(0) \tag{3.2}$$

so that a_n and b_n are now given by

$$\binom{a}{b}_n = -\tfrac{1}{2}\,\{\bar{E}_n^{(\mp)}(0)\cdot\int_V \bar{J}\,dv' + \int_V [\bar{u}'\cdot\nabla\bar{E}_n^{(\mp)}(0)]\cdot\bar{J}\,dv'\} \tag{3.3}$$

where $\bar{u}'$ is the position vector in a localized coordinate system, which may or may not be the same as r'. In Appendix III we reduce (3.3) to the following form, using a method similar to that of Collin [5]:

$$\binom{a}{b}_n = -\tfrac{1}{2}\,\{\bar{E}_n^{(\mp)}(0)\cdot\bar{P} - j\omega\mu_0\,\bar{H}_n^{(\mp)}(0)\cdot\bar{M} + \tfrac{1}{2}\,\nabla\bar{E}_n^{(\mp)}(0) : \bar{\bar{Q}}\} \tag{3.4}$$

where $\bar{H}_n$ is the corresponding ortho-normal magnetic field distribution of the n^{th} mode, $\bar{P}$, $\bar{M}$ and $\bar{\bar{Q}}$ are given by (2.8a-c) and the double dot product between two tensors $\bar{\bar{C}}$ and $\bar{\bar{D}}$ is given by

$$\bar{\bar{C}}:\bar{\bar{D}} = \sum_{\alpha,\beta} C_{\alpha\beta}D_{\alpha\beta}, \ \alpha,\beta = x,y,z. \tag{3.5}$$

If we restrict ourselves to a waveguide or TEM cell with dimensions such that at the operating frequency all the modes, except the fundamental TEM mode, are under cut-off, the output from both ports of the TEM cell consists of only TEM fields with coefficients a_0 and b_0 corresponding to n=0 in (3.4). Further, $\bar{E}_0$ and $\bar{H}_0$ corresponding to the TEM mode are related through

$$\bar{H}_0^{(\pm)} = \pm \hat{z} \times \bar{E}_0^{(\pm)}/\zeta_0 \tag{3.6a}$$

and

$$\bar{E}_0^{\pm} = \bar{e}_0 \, e^{\mp jkz} \tag{3.6b}$$

where $\hat{z}$ is a unit vector in the propagation direction, ζ_0 is the free space characteristic impedance, and $\bar{e}_0$ is the normalized transverse electric field.

Using (3.6a and b) we can write a_0 and b_0 as

$$\binom{a}{b}_0 = -\tfrac{1}{2} \{[\bar{P} \pm jk (\bar{M} \times \hat{z})] \cdot \bar{e}_0 + \tfrac{1}{2} \nabla\bar{E}_0^{(\mp)}(0) : \bar{\bar{Q}}\}. \tag{3.7}$$

Thus the nonuniformity of the field distribution at the cell's center, i.e., $\nabla\bar{E}_0^{(\mp)}(0)$, gives rise to an electric quadrupole contribution to the output fields. If we take more terms in the Taylor series expansion (3.2), we will find that still higher order multi-pole moments contribute to the output fields. However, for many practical purposes it is sufficient to retain terms up to electric quadrupoles. Based upon the relationship (3.7), we describe, in the next section, an experimental procedure for determining the total power radiated by an EUT, in free space, by tests inside a TEM cell.

4.0 EXPERIMENTAL PROCEDURE FOR DETERMINING THE TOTAL POWER RADIATED BY THE EUT

4.1 Principles Underlying the Measurement Procedure

We consider the experimental setup shown in figure 2, where the EUT is placed at a point $(0,y_0,0)$ along the y-axis in the transverse plane midway of the cell. The output yields from the two ports of the cell are added in or out of phase and the resulting power is measured by means of an output indicator. Assuming that the end transitions are perfectly matched, we find from (3.7) that the powers P_s and P_d, corresponding to summing the outputs from the two ports in and out of phase respectively, are given by

$$P_s = |a_0 + b_0|^2 = |\bar{P}\cdot\bar{e}_0 + \tfrac{1}{4}\ (\nabla\bar{E}_0^- + \nabla\bar{E}_0^+)\ :\ \bar{\bar{Q}}|^2 \tag{4.1a}$$

$$P_d = |a_0 - b_0|^2 = |jk(\bar{M} \times \hat{z}) \cdot \bar{e}_0 + \tfrac{1}{4}\ (\nabla\bar{E}_0^- - \nabla\bar{E}_0^+)\ :\ \bar{\bar{Q}}|^2 \tag{4.1b}$$

where $\bar{e}_0$ and $\nabla\bar{E}_0$ are evaluated at the test point $(0,y_0,0)$. For a rectangular co-axial TEM cell, with a symmetrically located center septum, it is known [6] that $\bar{e}_0$ has only y-component and $\nabla\bar{E}_0$ has only three non-vanishing components, namely $(\nabla\bar{E}_0)_{xx}$, $(\nabla\bar{E}_0)_{yy}$, and $(\nabla\bar{E}_0)_{yz}$, at any point $(0,y_0,0)$ along the y-axis of the cell. These are given by

$$-\ (\nabla\bar{E}_0^{\mp})_{xx} = (\nabla\bar{E}_0^{\mp})_{yy} = e'_{oy} \tag{4.2a}$$

$$(\nabla\bar{E}_0^{\pm})_{yz} = \frac{\partial E^{\pm}_{oy}}{\partial z} = \mp\ jke_{oy} \tag{4.2b}$$

where

$$e_{oy} = \bar{e}_0\cdot\hat{y} \tag{4.2c}$$

$$e'_{oy} = \frac{\partial e_{oy}}{\partial y}. \tag{4.2d}$$

Using these relations we can write P_s and P_d as

$$P_s = |P_y e_{oy} - \tfrac{1}{2}\ (Q_{xx} - Q_{yy})\ e'_{oy}|^2 \tag{4.3a}$$

$$P_d = k^2\ |(M_x - \tfrac{1}{2}\ Q_{yz})\ e_{oy}|^2. \tag{4.3b}$$

Note that when the quadrupole terms are not important, (4.3a) and (4.3b) will reduce to the same results obtained previously [1]. The above two equations will form the basis for the experimental procedure that we describe in this section.

Even though (4.3a) and (4.3b) represent a special case, ten unknowns, (P_y, Q_{xx}, Q_{yy}, Q_{yz}, M_x, are all complex numbers in general) are still involved. They may be determined, in principle, by making ten measurements and then solving the resulting simultaneous equations, from which the detailed radiation pattern of the unknown emitter in free space may be computed. This exercise is, however, still not simple because of the presence of non-linear equations with high-order terms. Fortunately, from the practical point of view, we may be only interested to know the total radiated power in free space, P_0, rather than the detailed radiation pattern. If this is the case, the measurement data taken in accordance with the procedure described later will be sufficient to enable us to determine P_0. Before describing the measurement procedure, a remark about the coordinate frame is in order.

We note that P_y, Q_{xx}, Q_{yy} in (4.3a) and M_x and Q_{yz} in (4.3b) refer to the dipole and quadrupole moments of the EUT with respect to $(\bar{x},\bar{y},\bar{z})$ frame of reference where $\bar{x},\bar{y},\bar{z}$-axes are parallel to x,y,z-axes of the TEM cell with origin 0' at the test point $(0,y_0,0)$ as shown in figure 2. Let (x',y',z') be a frame of reference fixed with respect to the EUT and denote the dipole and quadrupole moments of the EUT with respect to this (x',y',z') frame of reference by small letters, i.e., $\bar{p}$, $\bar{m}$, $q_{\alpha\beta}$, etc. When measurements are made with the EUT oriented in a specific direction we must substitute for P_y, Q_{xx}, Q_{yy}, M_x, and Q_{yz} in (4.3a) and (4.3b) in terms of p_α, m_α, $q_{\alpha\beta}$ $(\alpha,\beta = x,y,z)$. Thus different orientations of the EUT result in different equations for the output powers. We make use of this feature to obtain the desired quantities for determining P_0.

In what follows we describe a step-by-step measurement procedure and give an expression for P_0 in terms of the measured powers. The lengthy but straightforward analytical details, which justify the following measurement procedure, are given in Appendix IV.

4.2 Measurement Procedure

a. Remove the phase shifter from the circuit shown in figure 2 so that the output indicator measures the sum powers corresponding to in-phase addition of the fields from the two output ports of the TEM cell. Ensure that the electrical lengths of the cables connecting the output ports to the summing network are equal. All the measurements will be done with the EUT at a fixed test point $(0, y_0, 0)$ but with different orientations, and all the rotations are assumed to be counter-clockwise.

b. Position the EUT at the test point such that x',y',z'-axes of the EUT coincide with $\bar{x},\bar{y},\bar{z}$-axes respectively, and measure the output power and denote it by $P_{z,o}$. Now rotate the EUT about its <u>z'-axis</u> in seven equal steps of 45° and measure the corresponding output powers $P_{z,\pi/4}$, $P_{z,\pi/2}$, $P_{z,3\pi/4}$, $P_{z,\pi}$, $P_{z,5\pi/4}$, $P_{z,3\pi/2}$, $P_{z,7\pi/4}$. Note that the second subscript on P corresponds to the total rotation (θ in figure 3a) of the EUT about its z'-axis, from its starting position.

c. Position the EUT at the test point such that y',z',x'-axes of the EUT coincide with $\bar{x},\bar{y},\bar{z}$-axes respectively, and measure the output power $P_{x,o}$. Rotate the EUT about its <u>x'-axis</u> in seven equal steps of 45° and measure the corresponding output powers $P_{x,\pi/4}$, $P_{x,\pi/2}$, $P_{x,3\pi/4}$, $P_{x,\pi}$, $P_{x,5\pi/4}$, $P_{x,3\pi/2}$, $P_{x,7\pi/2}$. Figure 3b shows a typical orientation of the EUT in this step.

d. Position the EUT at the test point such that z',x',y'-axes of the EUT coincide with $\bar{x},\bar{y},\bar{z}$-axes respectively, and measure the output power $P_{y,o}$. Rotate the EUT about its <u>y'-axis</u> in seven equal steps of 45° and measure the corresponding output powers $P_{y,\pi/4}$, $P_{y,\pi/2}$, $P_{y,3\pi/4}$, $P_{y,\pi}$, $P_{y,5\pi/4}$, $P_{y,3\pi/2}$, $P_{y,7\pi/4}$ (see figure 3c).

e. Insert the 180° phase shifter in the circuit so that the output fields are now added out of phase. In practice this may be accomplished by a cable with one half wave length.

f. Position the EUT at the test point $(0,y_0,0)$ such that its x',y',z'-axes coincide with $\bar{x},\bar{y},\bar{z}$-axes respectively, and measure the output power and denote it by $M_{x,o}$. Now rotate the EUT about its x'-axis in three equal steps of 45° and measure the corresponding output powers $M_{x,\pi/4}$, $M_{x,\pi/2}$, $M_{x,3\pi/4}$ (see figure 3d).

g. Position the EUT at the test point such that its y',z',x'-axes coincide with $\bar{x},\bar{y},\bar{z}$-axes respectively, and measure the output power $M_{y,o}$. Rotate the EUT about its y'-

axis in three equal steps of 45° and measure the corresponding output powers $M_{y,\pi/4}$, $M_{y,\pi/2}$, $M_{y,3\pi/4}$ (see figure 3e).

h. Position the EUT at the test point such that its z',x',y'-axes coincide with $\bar{x},\bar{y},\bar{z}$-axes respectively, and measure the output power $M_{z,o}$. Rotate the EUT about its z'-axis in three equal steps of 45° and measure the corresponding output powers $M_{z,\pi/4}$, $M_{z,\pi/2}$, $M_{z,3\pi/4}$ (see figure 3f).

As shown in Appendix IV, the total power P_0 that would be radiated by the EUT in free space is given by

$$P_0 = \frac{5k^2}{2e_{oy}^2}\{(\sqrt{P_{z,\pi/2}} \pm \sqrt{P_{z,3\pi/2}})^2 + (\sqrt{P_{x,3\pi/2}} \pm \sqrt{P_{x,3\pi/2}})^2 + (\sqrt{P_{y,\pi/2}} \pm \sqrt{P_{y,3\pi/2}})^2\} + k^2\,(6f_1 + 4f_2), \tag{4.4a}$$

with

$$f_1 = \tfrac{1}{2}\sum_{\alpha} (M_{\alpha,o} + M_{\alpha,\pi/2})/e_{oy}^2 \tag{4.4b}$$

$$f_2 = \tfrac{1}{2}\sum_{\alpha} (M_{\alpha,\pi/4} + M_{\alpha,3\pi/4})/e_{oy}^2, \tag{4.4c}$$

where the summation in (4.4b) and (4.4c) is over α = x,y,z and the sign ambiguity in (4.4a) is resolved as explained in Appendix IV.

In the next section we consider the problem of determining the susceptibility characteristics of an object. By making use of the reciprocity principle, we show that a similar test procedure may be used to determine the average power received by the EUT when illuminated by an incident plane wave in free space, where the average is taken over θ and ϕ polarizations and all possible angles of the incident wave.

5. DETERMINATION OF THE SUSCEPTIBILITY LEVEL OF A RECEIVING OBJECT

5.1 Characterization of the EUT as a Receiving Antenna

When studying the susceptibility level of an EUT, we are interested in finding the amount of dissipated power in a load placed across some terminals of the EUT which is illuminated by a specified incident electromagnetic field. In practice, the load under consideration is some component of the EUT that is most likely to fail under the exposure condition. One may wish to know the average power dissipated in the load $\langle P_L \rangle$ where the averaging is taken over all possible orientations of the EUT and over both θ- and ϕ-polarizations of the incident electromagnetic field. To determine such an average susceptibility level, we may conveniently consider the EUT as a receiving antenna system with an antenna impedance Z_a, delivering power to a load Z_L connected across the terminals aa' as shown in the equivalent circuit (figure 4). We know that the average receiving cross-section of a matched electrically small receiving antenna in free-space is equal to

$\lambda^2/4\pi$ where the average is taken over all possible angles of an incident plane wave and the antenna is assumed to be matched to the load as well as to the incident polarization. Then, from the equivalent circuit (figure 4) of the EUT we find that [7]

$$\langle P_L \rangle_0 = \eta \left(\frac{\lambda^2}{8\pi}\right) p_{in} \tag{5.1a}$$

where the subscript 0 refers to free space,

$$\eta = 4 R_a R_L / |(Z_L + Z_a)|^2 \tag{5.1b}$$

is the 'mismatch loss factor' of the EUT in free space, p_{in} is the incident power density, and we have multiplied the average receiving cross-section by a factor of one-half to account for both possible polarizations. Thus, to determine the threshold power density of an incident wave that produces a fixed $\langle P_L \rangle_0$, all we need to know is the 'mismatch loss factor' η. We note that η is dependent upon the transmitting properties of the EUT through $R_a = \mathrm{Re}\,(Z_a)$. The total power transmitted, P_t, by the EUT in free space when driven by a current I at its feed terminals is given by

$$P_t = \tfrac{1}{2}\ |I|^2\ R_a \tag{5.2}$$

This P_t should be the same as P_o in (2.9a) provided that $|I|$ refers to the magnitude of the terminal current responsible for $\overline{P}$, $\overline{M}$ and $\overline{\overline{Q}}$. In devising a procedure for determining the 'mismatch loss factor' in free space of an electrically small EUT, we first determine, as shown in Appendix V, two dimensionless quantities A and B representing the power dissipated in the load of the EUT per unit input power to the TEM cell corresponding to in-phase and out-of-phase excitations, respectively,

$$A = (\eta/8)\ \{|P_y' e_{oy} - \tfrac{1}{2}\ (Q_{xx}' - Q_{yy}')\ e_{oy}'|^2\}/P_t \tag{5.3a}$$

$$B = (\eta/8)\ \{k^2 |M_x' - \tfrac{1}{2}\ Q_{yz}'|^2\ e_{oy}^2\}/P_t \tag{5.3b}$$

where P_α', M_α', $Q_{\alpha\beta}'$; $\alpha = x,y,z$ refer to the equivalent electric dipole, magnetic dipole, electric quadrupole moments of the EUT with respect to the TEM cell axes when it is in the transmission mode with the terminal current I.

These two equations will form the basis for devising an experimental procedure to obtain an expression for determining the 'mismatch loss factor' η needed to find $\langle P_L \rangle_0$ in (5.1a).

5.2 Measurement Procedure for Determining η

We use the experimental setup shown in figure 5 where the EUT is placed at a test point $(0,y_o,0)$ and the TEM cell is excited at both ends by means of a signal generator and a power divider. The isolators prevent the incident power from re-entering the signal generator.

The 180° phase shifter provides a means of exciting the two ports of the cell in- or out-of-phase. The power dissipated P_L in the load is measured by means of a detector and output indicator combination connected across the test terminals of the EUT. By monitoring the input power to the TEM cell, the normalized quantities A and B in (5.3a) and (5.3b) may be obtained for different orientations of the EUT. We establish, as in section 4, two frames of reference $(\bar{x},\bar{y},\bar{z})$ and (x',y',z') fixed with respect to the TEM cell and the EUT, respectively, and go through the same sequence of operations as those described in steps (a) through (h) of section (4.2), except that at each step we now measure the normalized dissipated powers $A_{\alpha,\theta}$, $B_{\alpha,\theta}$ in place of the output powers $P_{\alpha,\theta}$, $M_{\alpha,\theta}$, respectively, where $\alpha = x,y,z$ and θ takes on different values as described in the same measurement procedure.

As mentioned in Appendix V, P_t is given in terms of $\bar{P}'$, $\bar{M}'$, and $\bar{\bar{Q}}'$ in exactly the same manner as P_0 is given in terms of $\bar{P}$, $\bar{M}$ and $\bar{\bar{Q}}$ by (2.9a-c). Now if we compare (5.3a,b) with (4.3a,b) we find that $A_{\alpha,\theta}$ and $B_{\alpha,\theta}$ have the same relation to $\bar{P}'$, $\bar{M}'$ and $\bar{\bar{Q}}'$ as $P_{\alpha,\theta}$ and $M_{\alpha,\theta}$ have with $\bar{P}$, $\bar{M}$ and $\bar{\bar{Q}}$ except for the factor $(\eta/8P_t)$. Thus a weighted sum of $A_{\alpha,\theta}$ and $B_{\alpha,\theta}$ with identical weighting factors as those of $P_{\alpha,\theta}$ and $M_{\alpha,\theta}$ in (4.4a-c) will yield $P_t(\eta/8P_t) = \eta/8$. This implies that η is given by

$$\begin{aligned}\eta = 20\ k^2\ \{&(\sqrt{A_{z,\pi/2}} \pm \sqrt{A_{z,3\pi/2}})^2 + (\sqrt{A_{x,\pi/2}} \pm \sqrt{A_{x,3\pi/2}})^2 \\ &+ (\sqrt{A_{y,\pi/2}} \pm \sqrt{A_{y,3\pi/2}})^2\ \}/e_{oy}^2 \\ &+ 8k^2\ (6f_1' + 4f_2'),\end{aligned} \tag{5.4a}$$

where

$$f_1' = \tfrac{1}{2} \sum_{\alpha} \ (B_{\alpha,o} + B_{\alpha,\pi/2})/e_{oy}^2, \tag{5.4b}$$

$$f_2' = \tfrac{1}{2} \sum_{\alpha} \ (B_{\alpha,\pi/4} + B_{\alpha,3\pi/4})/e_{oy}^2 \tag{5.4c}$$

and again the sign ambiguity in (5.4a) is resolved as explained in Appendix IV.

Using the above equations we calculate η which, when substituted in (5.1a), determines $\langle P_L\rangle_0/P_{in}$.

Under the special condition that P_y' and Q_{yy}' are the only non-vanishing components due to the electric dipole and quadrupole and that no magnetic dipole moment exists, the normalized dissipated power may be expressed as follows, in accordance with the details given in Appendix VI:

$$A_{z,o} = \frac{\eta}{8P_t}|P_y'\ e_{oy} + \tfrac{1}{2}\ Q_{yy}'\ e_{oy}'|^2 \tag{5.5a}$$

$$A_{z,\pi} = \frac{\eta}{8P_t}|P_y'\ e_{oy} - \tfrac{1}{2}\ Q'_{yy}e'_{oy}|^2 \tag{5.5b}$$

and

$$B_{\alpha,\theta} = 0, \tag{5.5c}$$

where

$$P_t = 10\ k^2\ (P_y'^{\,2} + \frac{k^2}{20} Q_{yy}'^{\,2}). \tag{5.5d}$$

From (5.5a) and (5.5b) we obtain

$$\eta = 80\ k^2\ A_{z,o} \frac{1 + \frac{k^2}{5} \Delta^2}{(e_{oy} + \Delta\ e'_{oy})^2} \tag{5.6a}$$

or

$$\eta = 80\ k^2\ A_{z,\pi} \frac{1 + \frac{k^2}{5} \Delta^2}{(e_{oy} - \Delta\ e'_{oy})^2} \tag{5.6b}$$

where

$$\Delta = Q'_{yy}/2\ P'_y = \frac{e_{oy}\ (\sqrt{A_{z,o}} - \sqrt{A_{z,\pi}})}{e'_{oy}\ (\sqrt{A_{z,o}} + \sqrt{A_{z,\pi}})} > 0 \tag{5.6c}$$

can be determined by measuring $A_{z,o}$ and $A_{z,\pi}$ and referring to the field distribution curve given in figure 6.

When, in addition, $Q'_{yy} = o$, we have $\Delta = o$. A simple measurement of either $A_{z,o}$ or $A_{z,\pi}$ will then be sufficient to determine η given by

$$\eta = 80\ k^2\ A_{z,\substack{o\\ \pi}}/e^2_{oy}(y_o). \tag{5.7}$$

6. SOME EXPERIMENTAL RESULTS FOR THE RECEIVING CASE

To verify the theoretical basis presented in previous sections, some measurements were made using a symmetric rectangular co-axial TEM cell of dimensions 1.20 m x 1.20 m x 2.4 m with a center conductor of width 0.992 m. The cell has matched transitions at both ends and is partially loaded with absorbing material to suppress the higher order modes at high frequencies. Earlier measurements indicated that the characteristic impedance of the cell remained unaltered at 50 Ω after placing the absorbing material. However, field measurements using a high impedance calibrated probe showed that the normalized field distribution $\bar{e}_0$ is modified from the ideal theoretical distribution corresponding to an unloaded cell with perfectly conducting walls [1]. A comparative plot of the theoretical and the measured field distribution along the y-axis of the cell is given in figure 6.

Measurement results for the transmitting case when a self-contained battery-operated spherical dipole is placed inside the TEM cell have been used to compare with the results measured in an open space. This comparison indicates that effects of the quadrupole terms in this special case are apparently negligible. Thus, the results reported previously without considering the quadrupole terms remain good [1], and are not repeated here.

In our experiment for the receiving case, monopole antennas of different lengths were used as the test objects and the 'mismatch loss factor' η for each antenna was obtained from measurements taken at different test points inside the TEM cell. These antennas were made simply by soldering a gauge-10 copper wire to the center conductor of an N-type panel connector. A 100 MHz signal source with 1000 Hz square wave modulation was used to excite the two ports of the TEM cell by means of a Tee-connector and cables of equal length. Out-of-phase excitation was accomplished by means of an additional cable with an electrical length of 1.5 m corresponding to one half wave length. Identical 20 dB attenuators were used in place of isolators. An additional 10 dB pad was used as a buffer between the signal source and the power divider. The output from the signal generator was monitored by means of a directional coupler and power meter combination (see figure 5). The test antenna was connected to a crystal detector and the detected output was measured using a VSWR meter. An attempt to use the standard BNC cable to bring out the detected output proved to be impractical. Slight movement of the cable resulted in fluctuations of several decibels in the output indicator, thus rendering the output readings meaningless. This problem was avoided by using a fiber optic transmit-receive link (FOL). The test antenna and crystal detector combination was directly attached to the input end of the transmitter which was battery operated. The output from the transmitter was brought out through a fiber optic cable and fed to the receiver placed outside of the TEM cell and the output from the receiver was fed to a VSWR meter using standard BNC cable. This provided a stable and reliable means of measuring the output from the test antenna. The test antenna, crystal detector, and the fiber optic transmitter combination was enclosed inside a styrofoam box to facilitate the positioning and orientation of the antenna. Initial calibrations were performed to determine (i) the power going into the TEM cell with a known indication on the power meter monitoring the output from the signal generator, and (ii) the input power to the crystal detector, required to produce a known deflection on the VSWR meter. The calibrated results were then the normalized quantities (power dissipated in the load per unit power input to the TEM cell) A and B corresponding to in-phase and out-of-phase excitations, respectively. Thus the crystal detector and the fiber optic link constituted the load to the test antenna. A sketch of the test antenna, crystal detector, and the fiber optic transmitter combination is shown, along with a coordinate frame of reference (x', y', z') in figure 7. The z'-axis is assumed to be pointing outwards from the paper so that x', y', z' constitute a right-handed coordinate system. The geometry of the test object shown in figure 7 suggests that the EUT should physically have only a y'-component of electric dipole (p_y) and no magnetic dipole components. This was verified by noting the absence of any measurable output under the following conditions:

i) out-of-phase excitation of the cell with arbitrary orientation of the test antenna.

ii) in-phase excitation of the TEM cell with the y'-axis of the test antenna in the xz plane of the TEM cell. We may obtain the 'mismatch loss factor' η by measuring the normalized output power $A_{z,o}$ or $A_{z,\pi}$ in accordance with the simplified version (5.7) where $A_{z,o}$ corresponds to orienting the EUT such that x', y', z'-axes coincide with $\bar{x}, \bar{y}, \bar{z}$-axes,

TABLE I

Measured mismatch loss factor η by neglecting the quadrupole term and taking the feed point as the reference point

Monopole Length		5 cm		10 cm		15 cm	
y_o cm	$e_{oy}(y_o)$ V/m	η in units of 10^{-6}		η in units of 10^{-6}		η in units of 10^{-6}	
		$\theta = 0$	$\theta = \pi$	$\theta = 0$	$\theta = \pi$	$\theta = 0$	$\theta = \pi$
20.0	13.80	-	38.82	-	217.2	-	508.9
22.5	13.30	-	38.10	-	217.0	-	504.7
25.0	12.72	-	35.57	-	226.6	-	513.0
27.5	12.28	73.02	36.73	281.3	228.8	576.6	518.0
30.0	11.90	72.26	35.88	277.8	233.5	575.6	518.8
32.5	11.43	72.41	34.33	277.7	240.6	582.0	537.8
35.0	10.90	74.02	-	286.9	-	595.5	-
37.5	10.63	71.61	-	280.2	-	592.2	-
40.0	10.25	72.38	-	287.9	-	611.4	-
Mean		54.76		254.6		552.9	
Maximum		74.02		287.9		611.4	
Minimum		34.33		217.0		504.7	
(Max-Min)/Mean		72.48%		27.85%		19.30%	

$\theta = 0$ means that η is obtained by measuring $A_{z,o}$

$\theta = \pi$ means that η is obtained by measuring $A_{z,\pi}$

respectively, provided that the magnetic dipole and the quadrupole terms are indeed absent. The measured results for monopole antennas of lengths 5 cm, 10 cm, and 15 cm are tabulated in Table I as a function of the test point position $(0, y_0, 0)$.

Since η is characteristic of the EUT, it should be, in principle, invariant to the position of the test point y_0 and also we should obtain the same value for η regardless of whether it is calculated by measuring $A_{z,0}$ or $A_{z,\pi}$. However, the results in Table I show considerable variation in the values of η, particularly so for the case of 5 cm monopole antenna where the minimum value of η is 37.3% below the mean and the maximum value of η is 35.2% above the mean, thus showing a spread of 72.5%. The discrepancy in the measured results may be explained if we recognize that, in obtaining these results, we considered the feed point F in figure 7 to be the reference point where the crystal detector is connected to the antenna and the dipole moment is defined. However, the current distribution on the EUT, consisting of the monopole antenna, crystal detector, and the fiber optic transmitter combination, is highly asymmetric about this point, particularly for the case where the monopole length is 5 cm. Hence, the quadrupole terms will have significant effect on the measured results. To minimize the discrepancy in the measured results, we should either choose the reference point such that the quadrupole moments are negligible with respect to the reference point, or include quadrupole terms in the calculation. In Table II we tabulated results obtained by choosing the geometric center G in figure 7 to be the test point. From the geometry of the EUT it was felt that the current distribution should be essentially symmetric about the geometric center so that the quadrupole moments with respect to this point will be very small. From the results we note a dramatic improvement where the values of η for the 5 cm antenna show a spread of only 19.4% below the mean value and 11.8% above the mean. But the results are still not quite satisfactory. Furthermore, in practice the nature of the current distribution on the test object is generally unknown and the selection of a suitable reference point would be difficult. To overcome this difficulty, we need to account for the quadrupole terms in the measurement and calculation procedure. Let us now consider the results obtained by accounting for the dominant quadrupole term q_{yy}. If we keep only this term in addition to the contribution form p_y, we will need both measurements $A_{z,0}$ and $A_{z,\pi}$ to obtain η (see (5.6a,b) or Appendix VI). The measured results including the quadrupole term are presented in Tables III and IV corresponding to taking the points F and G (figure 7) as the reference points, respectively. The improvement in the results is immediately evident. Comparing the η values for the monopole of 5 cm in length in Table II and III, we see that the 'spread factor', which we defined as the difference between the maximum and the minimum expressed as a percentage of mean value, is now reduced from 31.2% to 10.9% by taking the quadrupole term into account. Comparing the results in Tables III and IV, we do not see appreciable difference between the two. This shows that the results obtained after including the quadrupole term are quite insensitive to the choice of the reference point on the EUT, whereas a comparison of Tables I and II shows that the results obtained by neglecting the quadrupole term are highly sensitive to the choice of the reference point on the EUT. Also, as noted before, the results obtained by neglecting the quadrupole term are sensitive to the location of the EUT inside the cell unless the

TABLE II

Measured mismatch loss factor η by neglecting the quadrupole term and taking the geometric center as the reference point

Monopole Length		5 cm		10 cm		15 cm	
y_0 cm	$e_{oy}(y_0)$ V/m	η in units of 10^{-6}		η in units of 10^{-6}		η in units of 10^{-6}	
		$\theta = 0$	$\theta = \pi$	$\theta = 0$	$\theta = \pi$	$\theta = 0$	$\theta = \pi$
20.0	13.80	57.82	51.34	-	248.2	-	576.2
22.5	13.30	57.85	50.83	239.8	246.5	-	548.2
25.0	12.72	58.46	50.37	243.1	255.6	537.0	551.8
27.5	12.28	58.46	49.02	240.6	254.5	540.6	550.3
30.0	11.90	57.14	47.58	240.6	258.8	536.7	551.6
32.5	11.43	58.21	46.53	242.4	264.1	541.5	562.7
35.0	10.90	59.20	46.62	254.6	287.4	563.5	591.6
37.5	10.63	58.04	44.96	248.8	278.2	568.6	-
40.0	10.25	57.86	42.69	254.8	-	608.2	-
Mean		52.94		253.1		552.0	
Maximum		59.20		278.4		608.2	
Minimum		42.69		239.8		536.7	
(Max-Min)/Mean		31.19%		15.25%		12.95%	

$\theta = 0$ means that η is obtained by measuring $A_{z,o}$

$\theta = \pi$ means that η is obtained by measuring $A_{z,\pi}$

TABLE III

Measured mismatch loss factor η by considering the quadrupole term and taking the feed point as the reference point

Monopole Length			5 cm		10 cm		15 cm	
			η in units of 10^{-6}		η in units of 10^{-6}		η in units of 10^{-6}	
y_o cm	$e_{oy}(y_o)$ V/m	$\lvert e'_{oy}(y_o)\rvert$ V/m^2	$\theta = 0$	$\theta = \pi$	$\theta = 0$	$\theta = \pi$	$\theta = 0$	$\theta = \pi$
20.0	13.80	17.5	-	52.74	-	233.1	-	530.9
22.5	13.30	19.0	-	53.90	-	234.9	-	529.1
25.0	12.72	19.0	-	54.06	-	246.2	-	538.8
27.5	12.28	19.0	54.10	53.60	259.3	249.4	548.4	545.1
30.0	11.90	19.0	53.03	53.03	255.3	255.3	547.1	547.2
32.5	11.43	17.6	53.70	50.00	256.0	262.2	553.9	566.1
35.0	10.90	15.8	55.83	-	265.7	-	568.5	-
37.5	10.63	15.4	54.03	-	259.6	-	565.7	-
40.0	10.25	14.5	54.96	-	267.2	-	684.6	-
Mean			53.58		253.7		552.1	
Maximum			55.83		267.2		584.6	
Minimum			50.00		233.1		529.1	
(Max-Min)/Mean			10.88%		13.44%		10.05%	

$\theta = 0$ means that η is obtained using $A_{z,o}$

$\theta = \pi$ means that η is obtained using $A_{z,\pi}$

The quadrupole correction term (q_{yy}/p_y) is obtained by using (5.6c) at y_o = 30 cm.

TABLE IV*

Measured mismatch loss factor η by considering the quadrupole term and taking the geometric center as the reference point

Monopole Length			5 cm		10 cm		15 cm	
y_0 cm	$e_{oy}(y_0)$ V/m	$\lvert e'_{oy}(y_0)\rvert$ V/m^2	η in units of 10^{-6}		η in units of 10^{-6}		η in units of 10^{-6}	
			θ = 0	θ = π	θ = 0	θ = π	θ = 0	θ = π
20.0	13.80	17.5	53.84	55.28	-	241.2	-	570.1
22.5	13.30	19.0	53.39	55.26	247.8	238.7	-	541.6
25.0	12.72	19.0	53.76	54.97	251.6	247.1	537.0	544.9
27.5	12.28	19.0	53.48	53.67	249.3	245.8	547.7	543.2
30.0	11.90	19.0	52.25	52.25	249.6	249.6	544.0	544.2
32.5	11.43	17.6	53.40	50.92	251.1	255.1	548.6	555.4
35.0	10.90	15.8	54.57	50.75	263.2	269.4	570.5	584.4
37.5	10.63	15.4	53.51	48.94	257.2	269.2	575.6	-
40.0	10.25	14.5	53.44	46.37	263.2	-	615.5	-
Mean			52.78		253.1		558.8	
Maximum			55.28		269.4		615.5	
Minimum			46.37		238.7		537.0	
(Max-Min)/Mean			16.88%		12.13%		14.05%	

*See footnotes under Table III

reference point on the EUT is so chosen that the quadrupole moments are very small with respect to the reference point. Even after including the quadrupole term, our results still show 10 to 15% spread. This may be attributed to a number of factors, the most important one being that we are looking at the results for a very wide range of y_0 (the position of the test object). In this range (20 cm to 40 cm) even the derivative of e_0 is not constant (see figure 6). If we restrict y_0 to a narrower range, the variation in measured results would be considerably smaller. The other factors that might result in systematic errors in the measurement are:

1. The loading of the TEM cell with absorbing materials implies that the field distribution may not be that of a perfect TEM mode. There may be a small axial component of the electric and/or the magnetic field.

2. In our computation we included only the dominant component, q_{yy}, of the electric quadrupole. Inclusion of all the components of the quadrupole tensor should improve the accuracy of the results.

7. CONCLUSIONS

The theoretical analysis and the measurement procedures described in this report should enable one to obtain the free-space radiated power of an arbitrary but electrically small radiating object by measurements taken inside a TEM cell. The method presented is more accurate compared to our previous method [1,2] since the electric quadrupole terms are also included in modeling the test object. We have also extended the analysis and the test procedure to the determination of the susceptibility levels of electrically small objects. Our experimental results for the receiving case clearly demonstrate the following:

i) The nonuniformity of the TEM modal field distribution inside a TEM cell could result in considerable error in the interpretation of the measurement results if the electric quadrupole terms are not included in modeling the EUT.

ii) If the quadrupole terms are not included, the measured results are highly sensitive to the location of the EUT inside the TEM cell and also to the choice of the reference point on the EUT. A poor choice of the reference point may result in errors of the order of 70%.

iii) If the quadrupole terms are included in modeling the EUT, the measured results are not only more accurate, but also much less sensitive to the location of the EUT inside the TEM cell and to the choice of the reference point on the EUT.

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APPENDIX I

REDUCTION OF SYMMETRIC TERM IN (2.6) DUE TO AN ELECTRIC QUADRUPOLE

We examine the symmetric term in (2.6) in its component form

$$\int_V \{(\bar{n}\cdot\bar{r}')\,\bar{J} + (\bar{n}\cdot\bar{J})\bar{r}'\}\,dv'$$

$$= \int \{(n_x x' + n_y y' + n_z z')\,(\hat{x}\,J_x + \hat{y}\,J_y + \hat{z}\,J_z)$$

$$+ (n_x J_x + n_y J_y + n_z J_z)\,(\hat{x}\,x' + \hat{y}\,y' + \hat{z}\,z')\}\,dv'$$

$$= \hat{x}\int \{2n_x x'\,J_x + n_y(x'\,J_y + y'\,J_x) + n_z(z'\,J_x + x'\,J_z)\}\,dv'$$

$$+ \hat{y}\int \{2n_y\,y'\,J_y + n_z(y'\,J_z + z'\,J_y) + n_x(x'\,J_y + y'\,J_x)\}\,dv'$$

$$+ \hat{z}\int \{2n_z z'\,J_z + n_x(z'\,J_x + x'\,J_z) + n_y(y'\,J_z + z'\,J_y)\}\,dv'$$

$$= \bar{\bar{Q}}\cdot\bar{n} \equiv \bar{Q}(n) \qquad \text{(A1.1)}$$

where $\bar{\bar{Q}}$ is an electric quadrupole tensor with the components

$$Q_{\alpha\beta} = \int_V (\alpha'\;J_\beta + \beta'\;J_\alpha)\,dv',\quad \alpha,\beta = x,y,z \qquad \text{(A1.2)}$$

and the vector $\bar{Q}(n)$ has the components given by

$$Q_\alpha = \sum_\beta Q_{\alpha\beta} n_\beta . \qquad \text{(A1.3)}$$

APPENDIX II

POWER RADIATED BY A COMBINATION OF AN ELECTRIC DIPOLE, A MAGNETIC DIPOLE AND AN ELECTRIC QUADRUPOLE

The expressions (2.2) and (2.3) for the electric and the magnetic fields take the following simpler form in the far zone:

$$\bar{E} \approx j\omega\, \bar{n}x(\bar{n} \times \bar{A}), \tag{A2.1a}$$

$$\bar{B} \approx -j\, k\, \bar{n} \times \bar{A} \tag{A2.1b}$$

and the time-averaged power radiated per unit solid angle is given by

$$P\ (\theta,\phi) = \tfrac{1}{2}\ R_e\ \{r^2\, \bar{n}\cdot(\bar{E} \times \bar{H}^*)\} \tag{A2.2}$$

which, upon using (A2.1a,b) may be written as

$$P\ (\rho,\phi) = \frac{\omega^2 r^2}{2\zeta_0}\, |\bar{n} \times \bar{A}|^2 \tag{A2.3}$$

where $\zeta_0 = 120\pi$ is the free space characteristic impedance. Substituting for $\bar{A}$ from (2.7) we can write the total power radiated in free space P_0 as

$$\begin{aligned} P_0 &= \frac{15\ k^2}{4\pi}\int_{4\pi} |\bar{n} \times \{\bar{P} - j\, k\, \bar{n}x\bar{M} + j\,\frac{k}{2}\,\bar{\bar{Q}}\cdot\bar{n}\}|^2\, d\Omega \\ &= \frac{15\ k^2}{4\pi}\int_{4\pi} \{|\bar{n}x\bar{P}|^2 + k^2\, |\bar{n}x(\bar{n} \times \bar{M})|^2 + \frac{k^2}{4}|\bar{n}x(\bar{\bar{Q}}\cdot\bar{n})|^2\}\, d\Omega \end{aligned} \tag{A2.4}$$

since we can show that the following integrals vanish identically.

$$I_1 = \int (\bar{n}x\bar{a}) \cdot [\bar{n}x(\bar{n}x\bar{b})]\, d\Omega \tag{A2.5a}$$

$$I_2 = \int (\bar{n}x\bar{a}) \cdot [\bar{n}x(\bar{\bar{c}}\cdot\bar{n})]\, d\Omega \tag{A2.5b}$$

$$I_3 = \int \{\bar{n}x(\bar{n}x\bar{b})\} \cdot \{\bar{n}x(\bar{\bar{c}}\cdot\bar{n})\}\, d\Omega \tag{A2.5c}$$

where $\bar{a}$, $\bar{b}$ are arbitrary constant vectors and $\bar{\bar{c}}$ is an arbitrary symmetric tensor.

Using the vector identity

$$\bar{a}x(\bar{b}x\bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c} \tag{A.2.6}$$

and noting that $\bar{n}\cdot\bar{n} = 1$ we write the first integral in (A2.5a) as

$$I_1 = \int\ (\bar{n}x\bar{a}) \cdot \{\bar{n}(\bar{n}\cdot\bar{b}) - \bar{b}\}\, d\Omega$$

$$= \int \bar{n} \cdot (\bar{b} \times \bar{a})\, d\Omega = 0 \tag{A2.7}$$

since $\bar{n} \times \bar{a} \cdot \bar{n} = 0$ and

$$\int n_\alpha \, d\Omega = 0; \quad \alpha = x, y, z. \tag{A2.8}$$

Using the vector identity

$$(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = (\bar{a} \cdot \bar{c})(\bar{b} \cdot \bar{d}) - (\bar{a} \cdot \bar{d})(\bar{b} \cdot \bar{c}) \tag{A2.9}$$

we write the second integral in (A2.5b) as

$$I_2 = \int \{\bar{a} \cdot (\bar{\bar{c}} \cdot \bar{n}) - \bar{n} \cdot (\bar{\bar{c}} \cdot \bar{n})(\bar{n} \cdot \bar{a})\}\, d\Omega = 0 \tag{A2.10}$$

which follows from (A2.8) and the vanishing of the following integrals,

$$\int n_\alpha n_\beta n_\gamma \, d\Omega = 0 \quad \alpha, \beta, \gamma = x, y, z. \tag{A2.11}$$

Using (A2.9) we reduce I_3 to

$$I_3 = \int (\bar{n} \times \bar{b}) \cdot (\bar{\bar{c}} \cdot \bar{n})\, d\Omega$$

$$= - \int \bar{b} \cdot (\bar{n} \times (\bar{\bar{c}} \cdot \bar{n}))\, d\Omega$$

$$= - \int \bar{b} \cdot \{\hat{x}\, (n_y c_z - n_z c_y) + \hat{y}\, (n_z c_x - n_x c_z) + \hat{z}\, (n_x c_y - n_y c_x)\}\, d\Omega \tag{A2.12}$$

where

$$c_\alpha = \sum_\beta c_{\alpha\beta} n_\beta; \quad \alpha, \beta = x, y, z. \tag{A2.13}$$

Using the relation

$$\int n_\alpha n_\beta \, d\Omega = \frac{4\pi}{3} \delta_{\alpha\beta}; \quad \alpha, \beta = x, y, z \tag{A2.14}$$

and noting that $\bar{\bar{c}}$ is symmetric, we find that I_3 is identically zero.

Now we consider the integral in (A2.4) term by term. The first term becomes

$$\int |\bar{n} \times \bar{P}|^2 \, d\Omega = \int (\bar{n} \times \bar{P}) \cdot (\bar{n} \times \bar{P}^*)\, d\Omega$$

$$= \int (|\bar{P}|^2 - |\bar{n} \cdot \bar{P}|^2)\, d\Omega = \frac{8\pi}{3} |\bar{P}|^2 \tag{A2.15}$$

where the property (A2.14) is again used. Hence the power radiated by the electric dipole is given by

$$P_e = 10\ k^2\ |\bar{P}|^2. \tag{A2.16}$$

With the aid of (A2.9), the second term in (A2.4) may be written as

$$\int \{\bar{n}x(\bar{n}x\bar{m})\} \cdot \{\bar{n}x(\bar{n}x\bar{m}^*)\}\ d\Omega$$

$$= \int (\bar{n}x\bar{m}) \cdot (\bar{n}x\bar{m}^*)\ d\Omega = \frac{8\pi}{3}\ |\bar{m}|^2 \tag{A2.17}$$

and the power radiated by the magnetic dipole is given by

$$P_m = 10\ k^4\ |\bar{M}|^2. \tag{A2.18}$$

Finally, the third term in (A2.4), the quadrupole radiation, involves the integral,

$$\int |\bar{n}\ x\ \bar{Q}\ (n)|^2\ d\Omega$$

$$= \int \{|\bar{Q}\ (n)|^2 - |\bar{n}\cdot\bar{Q}\ (n)|^2\}\ d\Omega$$

$$= \int \sum_{\alpha,\beta,\gamma} Q_{\alpha\beta}Q^*_{\alpha\beta}n_\beta n_\gamma d\Omega - \int \sum_{\alpha,\beta,\gamma,\delta} Q_{\alpha\beta}Q^*_{\gamma\delta}n_\alpha n_\beta n_\gamma n_\delta\ d\Omega \tag{A2.19}$$

where the summation is over x,y,z.

Using (A2.14) and the result

$$\int n_\alpha n_\beta n_\gamma n_\delta\ d\Omega = \frac{4\pi}{15}\ (\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) \tag{A2.20}$$

we write

$$\int |\bar{n}x\bar{Q}(n)|^2\ d\Omega = \frac{4\pi}{3} \sum_{\alpha,\beta} |Q_{\alpha\beta}|^2 - \frac{4\pi}{15}\ \{ \sum_{\alpha} Q_{\alpha\alpha} \sum_{\gamma} Q^*_{\gamma\gamma} + 2 \sum_{\alpha,\beta} |Q_{\alpha\beta}|^2\} \tag{A2.21}$$

so that the power radiated by the quadrupole term is given by

$$P_q = \frac{k^4}{4}\ \{ 3 \sum_{\alpha,\beta} |Q_{\alpha,\beta}|^2 - | \sum_{\alpha} Q_{\alpha\alpha}|^2\}. \tag{A2.22}$$

Adding (A2.16), (A2.18), and (A2.22) we obtain

$$P_o = P_e + P_m + P_q = 10\ k^2\ \{|\bar{P}|^2 + k^2\ |\bar{M}|^2 + \frac{k^2}{40}\ [3 \sum_{\alpha,\beta} |Q_{\alpha\beta}|^2 - | \sum_{\alpha} Q_{\alpha\alpha}|^2]\}. \tag{A2.23}$$

Using the symmetry property of $\bar{\bar{Q}}$ we write P_o as

$$P_o = 10\ k^2\ \{|\bar{P}|^2 + k^2\ |\bar{M}|^2 + \frac{k^2}{20}\ (3\ Q_1 + Q_2) \qquad ((A2.24a)$$

where

$$Q_1 = |Q_{xy}|^2 + |Q_{yz}|^2 + |Q_{zx}|^2 \tag{A2.24b}$$

$$Q_2 = |Q_{xx}|^2 + |Q_{yy}|^2 + |Q_{zz}|^2 - R_e\,(Q_{xx}Q^*_{yy} + Q_{yy}Q^*_{zz} + Q_{zz}Q^*_{xx}). \tag{A2.24c}$$

APPENDIX III

EXPRESSIONS FOR a_n and b_n IN TERMS OF DIPOLE AND QUADRUPOLE MOMENTS

We consider the second integral on the right hand side of (3.3) and write the integrand in its component form as

$$
\begin{aligned}
(\bar{u}'\cdot\nabla\bar{E})\cdot\bar{J} &= (\hat{x}\, u_x' + \hat{y}\, u_y' + \hat{z}\, u_z') \cdot (\hat{x}\hat{x}\,\frac{\partial E_x}{\partial x} + \hat{x}\hat{y}\,\frac{\partial E_y}{\partial x} + \hat{x}\hat{z}\,\frac{\partial E_z}{\partial x} \\
&+ \hat{y}\hat{x}\,\frac{\partial E_x}{\partial y} + \hat{y}\hat{y}\,\frac{\partial E_y}{\partial y} + \hat{y}\hat{z}\,\frac{\partial E_z}{\partial y} \\
&+ \hat{z}\hat{x}\,\frac{\partial E_x}{\partial z} + \hat{z}\hat{y}\,\frac{\partial E_y}{\partial z} + \hat{z}\hat{z}\,\frac{\partial E_z}{\partial z}) \cdot (\hat{x}\, J_x + \hat{y}\, J_y + \hat{z}\, J_z) \\
&= u_x' \,(J_x \frac{\partial E_x}{\partial x} + J_y \frac{\partial E_y}{\partial x} + J_z \frac{\partial E_z}{\partial x}) \\
&+ u_y' \,(J_x \frac{\partial E_x}{\partial y} + J_y \frac{\partial E_y}{\partial y} + J_z \frac{\partial E_z}{\partial y}) \\
&+ u_z' \,(J_x \frac{\partial E_x}{\partial z} + J_y \frac{\partial E_y}{\partial z} + J_z \frac{\partial E_z}{\partial z})
\end{aligned}
\qquad \text{(A3.1)}
$$

where we dropped the subscripts, superscripts, and argument on $\bar{E}$ and used u_x', u_z', and u_z' to denote the x,y,z-components of $\bar{u}'$. Note that $\bar{u}'$ is identical to $\bar{r}'$ when the coordinate origins of the EUT and TEM cell frames coincide with each other.

Next we use the relation

$$\nabla \times \bar{E} = -\, j\omega\mu_o \bar{H} \qquad \text{(A3.2)}$$

to write the expression $1/2\ j\omega\mu_o\ (\bar{u}'\times\bar{J})\cdot\bar{H}$ in its component form as

$$
\begin{aligned}
-\ 1/2\ j\omega\mu_o\ (\bar{u}'\times\bar{J})\cdot\bar{H} &= 1/2\ (u_y'\ J_z - u_z'\ J_y)(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}) \\
&+ 1/2\ (u_z'\ J_x - u_x'\ J_z)(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) \\
&+ 1/2\ (u_x'\ J_y - u_y'\ J_x)(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}).
\end{aligned}
\qquad \text{(A3.3)}
$$

Adding (A3.1) and (A3.3) and making some rearrangement we can write

$$
\begin{aligned}
(\bar{u}'\cdot\nabla\bar{E})\cdot\bar{J} &= -\ 1/2\ j\omega\mu_o\ (\bar{u}'\times\bar{J})\cdot\bar{H} \\
&+ 1/2\ (u_x'\ J_y + u_y'\ J_x)(\frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial x}) \\
&+ 1/2\ (u_y'\ J_z + u_z'\ J_y)(\frac{\partial E_y}{\partial z} + \frac{\partial E_z}{\partial y})
\end{aligned}
$$

$$+ \frac{1}{2} \left(u'_z J_x + u'_x J_z\right)\left(\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z}\right)$$

$$+ u'_x J_x \frac{\partial E_x}{\partial x} + u'_y J_y \frac{\partial E_y}{\partial y} + u'_z J_z \frac{\partial E_z}{\partial z}. \tag{A3.4}$$

From the definition of the double dot product between two tensors we note that

$$\nabla\bar{E}:\bar{\bar{Q}} = \sum_{\alpha,\beta} \frac{\partial E_\alpha}{\partial \beta} Q_{\alpha\beta} \tag{A3.5}$$

and using the definitions (2.8b and c) we can write

$$\int_V (\bar{u}' \cdot \nabla\bar{E}) \cdot \bar{J}\, dv' = - j\omega\mu_o \bar{H}\cdot\bar{M} + \frac{1}{2} \nabla\bar{E}:\bar{\bar{Q}} \tag{A3.6}$$

which leads to

$$\binom{a}{b}_n = - \frac{1}{2} \left\{\bar{E}_n^{(\mp)} \cdot \bar{P} - j\omega\mu_o \bar{H}_n^{(\mp)} \cdot \bar{M} + \frac{1}{2} \nabla\bar{E}_n^{(\mp)} : \bar{\bar{Q}}\right\}. \tag{A3.7}$$

APPENDIX IV

DERIVATION OF AN EXPRESSION FOR P_o IN TERMS OF THE MEASURED POWERS USING A TEM CELL

1. Equations for Powers Corresponding to in-Phase Measurement.

In order to interpret the measured results using a TEM cell, we write (4.3a) in terms of the dipole and quadrupole moments, p_α, $q_{\alpha\beta}$, (α,β = x,y,z), of the EUT for different orientations of the EUT. A general expression for the output power for arbitrary orientation of the EUT will be too involved. However, in all the measurement steps that we described in section IV, the orientation of the EUT is so specified that one of the coordinate axes of the EUT is made coincident with one of the axes of the TEM cell and EUT is rotated in a plane transverse to this fixed axis. Thus, the rotation is always performed in x'y'-, y'z'- or z'x'-plane of the EUT only. Let us examine step (b) in the measurement procedure where a typical orientation of the EUT is shown in figure 3a. For this choice of the orientation of the EUT, we note that

$$x = x'\cos\theta - y'\sin\theta \qquad \text{(A4.1a)}$$

$$y = x'\sin\theta + y'\cos\theta \qquad \text{(A4.1b)}$$

$$J_x = J_x' \cos\theta - J_y' \sin\theta \qquad \text{(A4.1c)}$$

$$J_y = J_x' \sin\theta + J_y' \cos\theta. \qquad \text{(A4.1d)}$$

So that

$$P_y = p_x\sin\theta + p_y\cos\theta \qquad \text{(A4.2a)}$$

and

$$\begin{aligned}
\tfrac{1}{2}\,(Q_{xx} - Q_{yy}) &= \int (x\, J_x - y\, J_y)\, dxdydz \\
&= \int \{(x'\cos\theta - y'\sin\theta)(J_x'\cos\theta - J_y'\sin\theta) \\
&\quad - (x'\sin\theta + y'\cos\theta)(J_x'\sin\theta + J_y'\cos\theta)\}\, dx'dy'dz' \\
&= \int [(x'\, J_x' - y'\, J_y')(\cos^2\theta - \sin^2\theta) \\
&\quad - 2\,(x'\, J_y' + y'\, J_x'\,)\, \sin\theta\, \cos\theta]\, dx'dy'dz' \\
&= \tfrac{1}{2}\,(q_{xx} - q_{yy})\, \cos 2\theta - q_{xy}\, \sin 2\theta \qquad \text{(A4.2b)}
\end{aligned}$$

where we have made use of the fact that dxdydz = dx'dy'dz' [8]. As we noted in section IV, p_α and $q_{\alpha\beta}$ (α,β = x,y,z) are the components of the electric dipole and quadrupole of the EUT, referred to its own frame of reference (x',y',z'). Equation (4.3a) may be rewritten as

$$|(p_x\sin\theta + p_y\cos\theta)\ e_{oy} - \{\tfrac{1}{2}\ (q_{xx} - q_{yy})\ \cos2\theta - q_{xy}\ \sin2\theta\}\ e'_{oy}|^2 = P_{z,\theta}. \qquad \text{(A4.3a)}$$

A cyclic change of the coordinate axes will yield the following equations corresponding to steps (c) and (d) of the measurement procedure.

$$|(p_y\sin\theta + p_z\cos\theta)\ e_{oy} - \{\tfrac{1}{2}\ (q_{yy} - q_{zz})\cos2\theta - q_{yz}\ \sin2\theta\}\ e'_{oy}|^2 = P_{x,\theta} \qquad \text{(A4.3b)}$$

$$|(p_z\sin\theta + p_x\cos\theta)\ e_{oy} - \{\tfrac{1}{2}\ (q_{zz} - q_{xx})\ \cos2\theta - q_{zx}\sin2\theta\}\ e'_{oy}|^2 = P_{y,\theta}. \qquad \text{(A4.3c)}$$

Note that in (A4.3a) to (A4.3c) we have used the notations $P_{z,\theta}$, $P_{x,\theta}$, and $P_{y\theta}$ to denote the measured sum powers for different EUT orientations in accordance with steps (b), (c) and (d) described in section IV. What we now need is a manipulation of these equations in a manner that $|\bar{p}|^2 = p_x^2 + p_y^2 + p_z^2$ is given in terms of the measured powers, preferably as a weighted sum of the measured powers. Before doing so we derive equations for powers corresponding to out of phase measurement.

2. <u>Equations for Powers Corresponding to out-of-Phase Measurement</u>.

Consider step (f) of the measurement procedure involving out-of-phase measurements. We now need to express (4.3b) in terms of the dipole and quadrupole moments of the EUT for different orientations. A typical orientation of the EUT in this step is illustrated in figure 3d. The coordinate transformation relations are given by

$$y = y'\cos\theta - z'\sin\theta \qquad \text{(A4.4a)}$$

$$z = y'\sin\theta + z'\cos\theta \qquad \text{(A.4.b)}$$

$$J_y = J'_y\ \cos\theta - J'_z\ \sin\theta \qquad \text{(A4.4c)}$$

$$J_z = J'_y\ \sin\theta + J'_z\ \cos\theta \qquad \text{(A4.4d)}$$

which result in the following expression for Q_{yz} in terms of q_{yy}, q_{zz}, and q_{yz}:

$$Q_{yz} = \int\ (y\ J_z + z\ J_y)\ dxdydz = \tfrac{1}{2}\ (q_{yy} - q_{zz})\ \sin2\theta + q_{yz}\ \cos2\theta. \qquad \text{(A4.5a)}$$

It is an easy matter to verify that M_x is unaltered under a rotation in the yz-plane. That is

$$M_x = m_x. \qquad \text{(A4.5b)}$$

We may now write (4.3b) as

$$k^2 \, e_{oy}^2 \, |m_x - \tfrac{1}{4} \, (q_{yy} - q_{zz}) \, \sin 2\theta - \tfrac{1}{2} \, q_{yz} \, \cos 2\theta|^2 = M_{x,\theta}. \qquad \text{(A4.6a)}$$

A cyclic change of coordinate axes results in the following two equations:

$$k^2 \, e_{oy}^2 \, |m_y - \tfrac{1}{4} \, (q_{zz} - q_{xx}) \, \sin 2\theta - \tfrac{1}{2} \, q_{zx} \, \cos 2\theta|^2 = M_{y,\theta} \qquad \text{(A4.6b)}$$

$$k^2 \, e_{oy}^2 \, |m_z - \tfrac{1}{4} \, (q_{xx} - q_{yy}) \, \sin 2\theta - \tfrac{1}{2} \, q_{xy} \, \cos 2\theta|^2 = M_{z,\theta}. \qquad \text{(A4.6c)}$$

We note that when θ is a multiple of $\pi/4$, each of the equations (A4.6a)-(A4.6c) involves either a diagonal or an off-diagonal element of q. Because of this, the expression for the power radiated due to a combination of magnetic dipole and electric quadrupole of the EUT takes on a simpler form which we derive next. Also note that we have used the notations $M_{x,\theta}$, $M_{y,\theta}$, and $M_{z,\theta}$ in (A4.6a) to (A4.6c) to denote the measured difference powers for different EUT orientations in accordance with steps (f), (g) and (h) described in section IV.

3. Power Radiated, $P_m + P_q$, due to the Magnetic Dipole and Electric Quadrupole Moments of the EUT

By a simple substitution we can derive the following equations from (A4.6a)-(A4.6c)

$$k^2 \, e_{oy}^2 \, \{|m_x|^2 + \tfrac{1}{4} |q_{yz}|^2\} = \tfrac{1}{2} \, (M_{x,o} + M_{x,\pi/2}) \qquad \text{(A4.7a)}$$

$$k^2 \, e_{oy}^2 \, \{|m_y|^2 + \tfrac{1}{4} |q_{zx}|^2\} = \tfrac{1}{2} \, (M_{y,o} + M_{y,\pi/2}) \qquad \text{(A4.7b)}$$

$$k^2 \, e_{oy}^2 \, \{|m_z|^2 + \tfrac{1}{4} |q_{xy}|^2\} = \tfrac{1}{2} \, (M_{z,o} + M_{z,\pi/2}) \qquad \text{(A4.7c)}$$

$$k^2 \, e_{oy}^2 \, \{|m_x|^2 + \frac{1}{16} \, |q_{yy} - q_{zz}|^2\} = \tfrac{1}{2} \, (M_{x,\pi/4} + M_{x,3\pi/4}) \qquad \text{(A4.7d)}$$

$$k^2 \, e_{oy}^2 \, \{|m_y|^2 + \frac{1}{16} \, |q_{zz} - q_{xx}|^2\} = \tfrac{1}{2} \, (M_{y,\pi/4} + M_{y,3\pi/4}) \qquad \text{(A4.7e)}$$

$$k^2 \, e_{oy}^2 \, \{|m_z|^2 + \frac{1}{16} \, |q_{xx} - q_{yy}|^2\} = \tfrac{1}{2} \, (M_{z,\pi/4} + M_{z,3\pi/4}) \qquad \text{(A4.7f)}$$

where m_α, $q_{\alpha\beta}$ $(\alpha,\beta = x,y,z)$ are complex in general.

Taking the sum of (A4.7a)-(A4.7c) we obtain

$$k^2 \, (|\bar{m}|^2 + \tfrac{1}{4} q_1) = f_1 \qquad \text{(A4.8a)}$$

where

$$|\bar{m}|^2 = |m_x|^2 + |m_y|^2 + |m_z|^2 \qquad \text{(A4.8b)}$$

$$q_1 = |q_{xy}|^2 + |q_{yz}|^2 + |q_{zx}|^2 \quad \text{(A4.8c)}$$

$$f_1 = 1/2 \sum_{\alpha} (M_{\alpha,o} + M_{\alpha,\pi/2})/e_{oy}^2. \quad \text{(A4.8d)}$$

Similarly, taking the sum of (A4.7d)-(A4.7f) we obtain

$$k^2 \left(|\bar{m}|^2 + \frac{1}{8} q_2\right) = f_2 \quad \text{(A4.9a)}$$

where

$$f_2 = 1/2 \sum_{\alpha} (M_{\alpha,\pi/4} + M_{\alpha,3\pi/4})/e_{oy}^2, \quad \text{(A4.9b)}$$

$$q_2 = |q_{xx}|^2 + |q_{yy}|^2 + |q_{zz}|^2 - \text{Re}\,(q_{xx}q_{yy}^* + q_{yy}q_{zz}^* + q_{zz}q_{xx}^*) \quad \text{(A4.9c)}$$

and Re stands for 'real part of'. From (2.9a) we note that power radiated by the magnetic dipole $\bar{m}$ and electric quadruple $\bar{\bar{q}}$ is given by the sum $P_m + P_q$,

$$P_m + P_q = 10\, k^4 \left(|\bar{m}|^2 + \frac{3}{20} q_1 + \frac{1}{20} q_2\right). \quad \text{(A4.10)}$$

From (A4.8a) and (A4.9a) we obtain

$$\frac{3}{5} f_1 + \frac{2}{5} f_2 = k^2 \left(|\bar{m}|^2 + \frac{3}{20} q_1 + \frac{1}{20} q_2\right) \quad \text{(A4.11)}$$

so that

$$P_m + P_q = 2\, k^2\, (3\, f_1 + 2\, f_2), \quad \text{(A4.12)}$$

where f_1 and f_2 are given in (A4.8d) and (A4.9b) in terms of the measured difference powers, $M_{\alpha,o}$, $M_{\alpha,\pi/4}$, $M_{\alpha,\pi/2}$, and $M_{\alpha,3\pi/4}$, $\alpha = x,y,z$. This is the precise reason why steps (f), (g) and (h) in section IV are proposed.

4. Power Radiated, P_e, by the Electric Dipole Moment of the EUT

To derive an expression for P_e we first write, using (A4.3a), the following eight equations corresponding to $\theta = 0,\pi,\ \pi/2,\ 3\pi/2,\ \pi/4,\ 3\pi/4,\ 5\pi/4$, and $7\pi/4$, respectively.

$$|(p_y - \tilde{q}_{zz})|^2 = P_{z,0}/e_{oy}^2 \quad \text{(A4.13a)}$$

$$|(p_y + \tilde{q}_{zz})|^2 = P_{z,\pi}/e_{oy}^2 \quad \text{(A4.13b)}$$

$$|(p_x + \tilde{q}_{zz})|^2 = P_{z,\pi/2}/e_{oy}^2 \quad \text{(A4.13c)}$$

$$|(p_x - \tilde{q}_{zz})|^2 = P_{z,3\pi/2}/e_{oy}^2 \quad \text{(A4.13d)}$$

$$|(p_x/\sqrt{2} + p_y/\sqrt{2} + \tilde{q}_{xy})|^2 = P_{z,\pi/4}/e_{oy}^2 \quad \text{(A4.13e)}$$

$$|(p_x/\sqrt{2} - p_y/\sqrt{2} - \tilde{q}_{xy})|^2 = P_{z,3\pi/4}/e_{oy}^2 \quad \text{(A4.13f)}$$

$$|(p_x/\sqrt{2} + p_y/\sqrt{2} - \tilde{q}_{xy})|^2 = P_{z,5\pi/4}/e_{oy}^2 \quad \text{(A4.13g)}$$

$$|(p_x/\sqrt{2} - p_y/\sqrt{2} + \tilde{q}_{xy})|^2 = P_{z,7\pi/4}/e_{oy}^2 \quad \text{(A4.13h)}$$

where $\tilde{q}_{xy}$ and $\tilde{q}_{zz}$ are defined as

$$\tilde{q}_{xy} = q_{xy}\ e'_{oy}/e_{oy} \quad \text{(A4.14a)}$$

$$\tilde{q}_{zz} = 1/2\ (q_{xx} - q_{yy})\ e'_{oy}/e_{oy}. \quad \text{(A4.14b)}$$

The radiation from a typical EUT arises from the surface current distribution over the body of the EUT. Hence, for an electrically small EUT, it is fair to assume that the equivalent electric dipoles and quadrupoles are in phase. Then we can verify the following by a simple substitution:

$$p_x^2 + \tilde{q}_{zz}^2 = 1/2\ (P_{z,\pi/2} + P_{z,3\pi/2})/e_{oy}^2 \quad \text{(A4.15a)}$$

$$p_x\tilde{q}_{zz} = 1/4\ (P_{z,\pi/2} - P_{z,3\pi/2})/e_{oy}^2 \quad \text{(A4.15b)}$$

$$p_y\tilde{q}_{zz} = 1/4\ (P_{z,\pi} - P_{z,0})/e_{oy}^2 \quad \text{(A4.15c)}$$

$$p_xp_y = 1/4\ [P_{z,\pi/4} + P_{z,5\pi/4} - P_{z,3\pi/4} - P_{z,7\pi/4}]/e_{oy}^2. \quad \text{(A4.15d)}$$

Using the last three equations we write $\tilde{q}_{zz}^2$ as

$$\tilde{q}_{zz}^2 = \frac{(P_{z,\pi/2} - P_{z,3\pi/2})(P_{z,\pi} - P_{z,0})}{4\ e_{oy}^2\ (P_{z,\pi/4} + P_{z,5\pi/4} - P_{z,3\pi/4} - P_{z,7\pi/4})}. \quad \text{(A4.16)}$$

There are a number of ways of obtaining p_x^2. A direction substitution of (A4.16) into (A4.15a) yields one value for p_x^2. Substitution of (A4.16) into the square of (A4.15b) yields another. A third way is to take the square root of the product of (A4.13c) and (A4.13d), giving

$$p_x^2 - \tilde{q}_{zz}^2 = \pm\sqrt{P_{z,\pi/2}\ P_{z,3\pi/2}}\ /e_{oy}^2, \quad \text{(A4.17)}$$

which can be combined with (A4.15a) to obtain

$$
\begin{aligned}
p_x^2 &= \frac{1}{4e_{oy}^2} \left(P_{z,\pi/2} + P_{z,3\pi/2} \pm 2\sqrt{P_{z,\pi/2}\,P_{z,3\pi/2}}\right) \\
&= \frac{1}{4e_{oy}^2} \left(\sqrt{P_{z,\pi/2}} \pm \sqrt{P_{z,3\pi/2}}\right)^2 \\
&= \frac{1}{4e_{oy}^2} \left(\sqrt{P_{z,\pi/2}} + \sqrt{P_{z,3\pi/2}}\right)^2 \quad \text{if } p_x^2 > \tilde{q}_{zz}^2 \\
&= \frac{1}{4e_{oy}^2} \left(\sqrt{P_{z,\pi/2}} - \sqrt{P_{z,3\pi/2}}\right)^2 \quad \text{if } p_x^2 < \tilde{q}_{zz}^2 .
\end{aligned}
\tag{A4.18}
$$

In (A4.18), the relative values of p_x^2 and $\tilde{q}_{zz}^2$ can be confirmed by comparing (A4.15a) and (A4.16). The values for p_x^2 obtained by different substitutions may be used as a check to see (i) whether or not the measured data ($P_{z,\theta}$) are accurate, and (ii) whether or not the assumption of no phase difference between p_x and $\tilde{q}_{zz}$ is reasonable.

Similarly, the following formulas for p_y^2 and p_z^2 can also be derived:

$$
p_y^2 = \frac{1}{4e_{oy}^2} \left(\sqrt{P_{x,\pi/2}} \pm \sqrt{P_{x,3\pi/2}}\right)^2 , \tag{A4.19}
$$

$$
p_z^2 = \frac{1}{4e_{oy}^2} \left(\sqrt{P_{y,\pi/2}} \pm \sqrt{P_{y,3\pi/2}}\right)^2 \tag{A4.20}
$$

where the sign in (A4.19) is determined by the relative values of p_y^2 and $\tilde{q}_{xx}^2$, and that in (A4.20) is determined by the relative values of p_z^2 and $\tilde{q}_{yy}^2$. The expressions for $\tilde{q}_{xx}^2$ and $\tilde{q}_{yy}^2$ can be obtained by replacing the subscript z in (A4.16) by x and y, respectively.

Even though p_x^2, p_y^2 or p_z^2 may appear to involve only two measured powers as evidenced by (A4.18) to (A4.20). The choice of "+" or "-" sign, however, is determined by $\tilde{q}_{xx}^2$, $\tilde{q}_{yy}^2$ or $\tilde{q}_{zz}^2$, and all of these three latter quantities involve eight different measurements. This explains the reason why steps (b), (c) and (d) in section IV are proposed.

The radiated power, P_e, due to the electric dipole moment, is given by

$$
P_e = 10\, k^2 \,(p_x^2 + p_y^2 + p_z^2). \tag{A4.21}
$$

The total equivalent power radiated by the EUT in free space is then

$$
P_o = P_e + P_m + P_q \tag{A4.22}
$$

which after substituting for P_e, P_m, and P_q, yields (4.4).

APPENDIX V

POWER RECEIVED BY A RECEIVING ANTENNA INSIDE A TEM CELL

To derive an expression for the induced open circuit voltage at the terminals of a receiving antenna inside a TEM cell we consider two sets of fields $(\bar{E}_1, \bar{H}_1)$ and $(\bar{E}_2, \bar{H}_2)$ corresponding to the following cases:

1. $\bar{E}_1, \bar{H}_1$ are produced by exciting the antenna at its feed terminals by a voltage source. The feed terminals are assumed to be centered at a point $(x_o, y_o, 0)$ inside the cell as shown in figure 8. Let $\bar{J}_1$ be the corresponding volume current density distribution on the antenna.

2. $\bar{E}_2, \bar{H}_2$ are maintained by feeding the TEM cell with external sources.

The fields $(\bar{E}_1, \bar{H}_1)$ and $(\bar{E}_2, \bar{H}_2)$ satisfy the following identity in accordance with the Lorentz reciprocity principle [5],

$$\int_S (\bar{E}_1 x \bar{H}_2 - \bar{E}_2 x \bar{H}_1) \cdot \bar{n} ds = \int_v \bar{E}_2 \cdot \bar{J}_1 \, dv \tag{A5.1}$$

where S is any surface enclosing the antenna, v is the volume enclosed by S, and $\bar{n}$ is the outward directed normal to S.

When the EUT is in the receiving mode, the right-hand side of (A5.1) represents $V_{oc} I$, where V_{oc} is the induced open circuit voltage across the antenna terminals of the equivalent circuit shown in figure 4, and I is the input current. When the EUT is in the transmitting mode and the TEM cell and frequency are such that only the dominant TEM mode exists, $a_1 = 0$, $b_1 = 0$ (see figure 8). Under this condition, the left-hand side reduces to $-2\,(a_o a_2^i + b_o b_2^i)$, which can certainly be expressed in terms of the measured powers given in (4.3a) or (4.3b). In particular, when $x_o = 0$ (EUT is positioned at the cell center), and the cell is measured with two output ports connected in phase ($a_2^i = b_2^i$), we have

$$|V_{oc}^+| \; |I| = 2|a_2^i| \, |a_o + b_o|,$$

or

$$|V_{oc}^+|^2 \; |I|^2 = 4|a_2^i|^2 \; |P_y' \, e_{oy} - 1/2 \; (Q_{xx}' - Q_{yy}') e_{oy}'|^2 . \tag{A5.2}$$

When the cell is measured with two output ports connected out of phase ($a_2^i = -\, b_2^i$), we have

$$|V_{oc}^-| \; |I| = 2|a_2^i| \, |a_o - b_o|$$

or

$$|V_{oc}^-|^2 \; |I|^2 = 4|a_2^i|^2 k^2 |M_x' - 1/2 \; Q_{yz}'|^2 \; e_{oy}^2 . \tag{A5.3}$$

The total power, P_t, that would be radiated in free space by a transmitting EUT corresponding to an excitation terminal current I is given by

$$P_t = \tfrac{1}{2} \, |I|^2 \, R_a, \tag{A5.4}$$

where R_a is the radiation resistance or the real part of the antenna impedance. Note that P_t can be expressed in terms of $\bar{P}'$, $\bar{M}'$ and $\bar{\bar{Q}}'$ in exactly the same manner as P_o is given in terms of $\bar{P}$, $\bar{M}$ and $\bar{\bar{Q}}$ by (2.9a).

The power, P_L, dissipated in the load of a receiving EUT (see figure 4) is given by

$$P_L = \tfrac{1}{2} \, \frac{|V_{oc}^{\pm}|^2 R_L}{|Z_L + Z'_a|^2} \tag{A5.5}$$

where Z_L is the load impedance with R_L as its real part, and Z'_a is the equivalent antenna impedance of the EUT inside a TEM cell.

In general, Z'_a is different from the corresponding impedance Z_a in free space. However, most of the equipment to be tested for susceptibility purposes are inefficient when viewed as receiving antennas. Thus, Z_a and Z'_a both have very large reactive parts and small resistive parts. Since Z_a and Z'_a differ mainly in their resistive parts, we may assume that

$$|Z_L + Z'_a| \approx |Z_L + Z_a|. \tag{A5.6}$$

Substituting (A5.2) or (A5.3) into (A5.5) and using (A5.4), we obtain, for the in-phase case,

$$P_L^+ = \frac{R_a R_L |a_2^i|^2}{P_t |Z_L + Z_a|^2} \, |P'_y \, e_{oy} - \tfrac{1}{2} \, (Q'_{xx} - Q'_{yy}) e'_{oy}|^2 \tag{A5.7a}$$

and for the out-of-phase case,

$$P_L^- = \frac{R_a R_L |a_2^i|^2}{P_t |Z_L + Z_a|^2} \, k^2 |M'_x - \tfrac{1}{2} \, Q'_{yz}|^2 \, e_{oy}^2. \tag{A5.7b}$$

Defining the dimensionless quantities A and B as the power dissipated per unit input power to the TEM cell, we have

$$A \equiv P_L^+/2|a_2^i|^2 = \frac{\eta}{8P_t} \, |P'_y \, e_{oy} - \tfrac{1}{2} \, (Q'_{xx} - Q'_{yy}) \, e'_{oy}|^2 \tag{A5.8a}$$

and

$$B = P_L^-/2|a_2^i|^2 = \frac{\eta}{8P_t} \, k^2 |M'_x - \tfrac{1}{2} \, Q'_{yz}|^2 \, e_{oy}^2, \tag{A5.8b}$$

where η is the mismatch loss factor defined in (5.1b).

APPENDIX VI

EXPRESSION FOR η IN TERMS OF $A_{z,o}$ and $A_{z,\pi}$

When p_y and q_{yy} are the only non-vanishing components of the electric dipole moment and electric quadrupole moment, the radiated power P_t is given by

$$P_t = 10\, k^2 \left(p_y^2 + \frac{k^2}{20}\, q_{yy}^2\right). \qquad \text{(A6.1)}$$

Using (5.3a) and (A4.3a) with the assumption that p_y and q_{yy} are of the same phase, we write

$$A_{z,o} = (\eta/8)(p_y\, e_{oy} + \tfrac{1}{2}\, q_{yy}\, e'_{oy})^2/P_t \qquad \text{(A6.2a)}$$

$$A_{z,\pi} = (\eta/8)(p_y\, e_{oy} - \tfrac{1}{2}\, q_{yy}\, e'_{oy})^2/P_t. \qquad \text{(A6.2b)}$$

Since q_{yy} is in general smaller than p_y, and e'_{oy} is negative according to figure 6, we have $A_{z,\pi} > A_{z,o}$ and $\tfrac{1}{2} \left|\frac{q_{yy}\, e'_{oy}}{p_y\, e_{oy}}\right| < 1$. We then have

$$\Delta = q_{yy}/2p_y = \frac{e_{oy}}{e'_{oy}} \left(\frac{\sqrt{A_{z,o}} - \sqrt{A_{z,\pi}}}{\sqrt{A_{z,o}} + \sqrt{A_{z,\pi}}}\right) > 0, \qquad \text{(A6.3)}$$

which can be determined by measuring $A_{z,o}$ and $A_{z,\pi}$ and the field distribution curve given in figure 6. Once Δ is known, η can be obtained by substituting for P_t from (A6.1) in (A6.2a,b) and the result is

$$\eta = 80k^2\, A_{z,\substack{o\\ \pi}} \cdot \frac{1 + \frac{k^2}{5}\Delta^2}{(e_{oy} \pm \Delta\, e'_{oy})^2}\,. \qquad \text{(A6.4)}$$

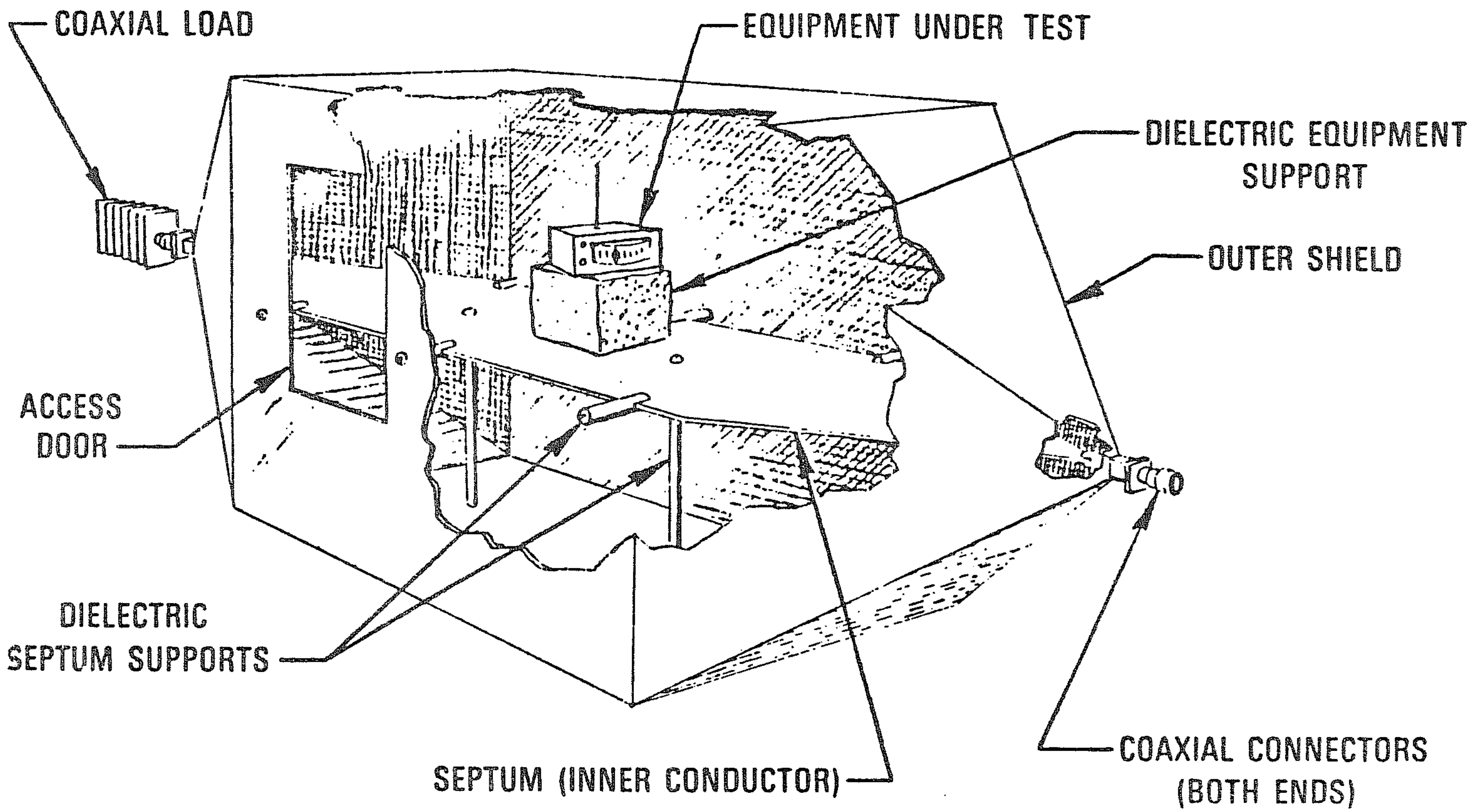

Figure 1. A conceptual NBS TEM cell

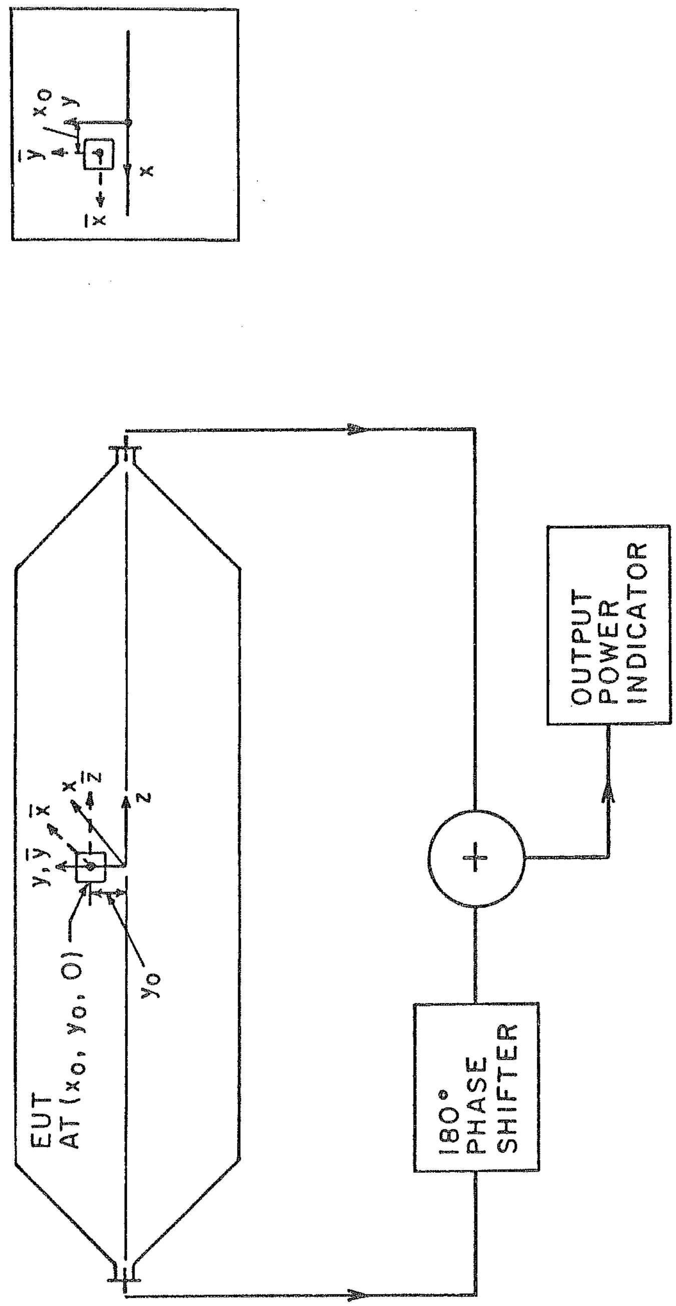

Figure 2. Test set-up for determining total power radiated, in free space, by an electrically small EUT

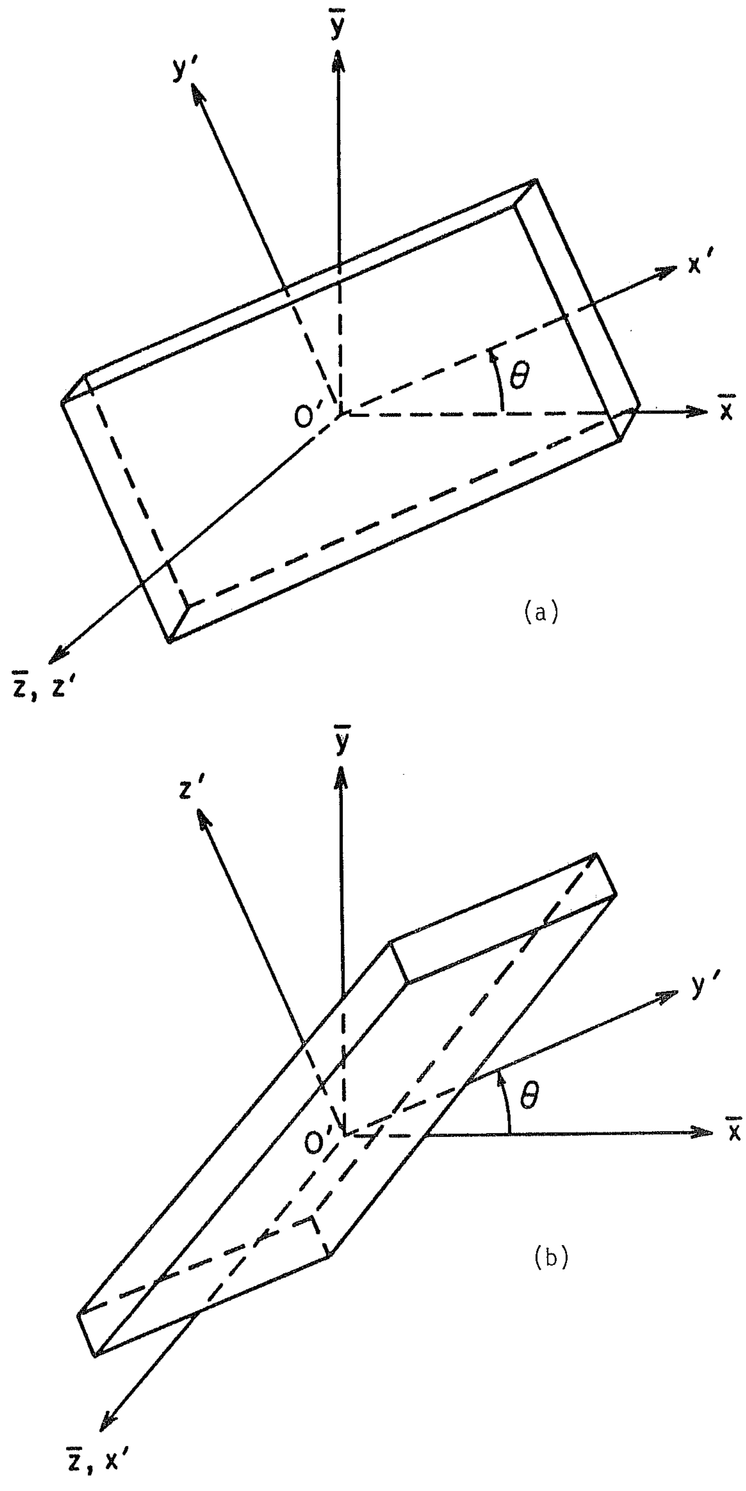

Figure 3. (a) Orientation of the EUT in step (b), section 4.2
(b) Orientation of the EUT in step (c), section 4.2

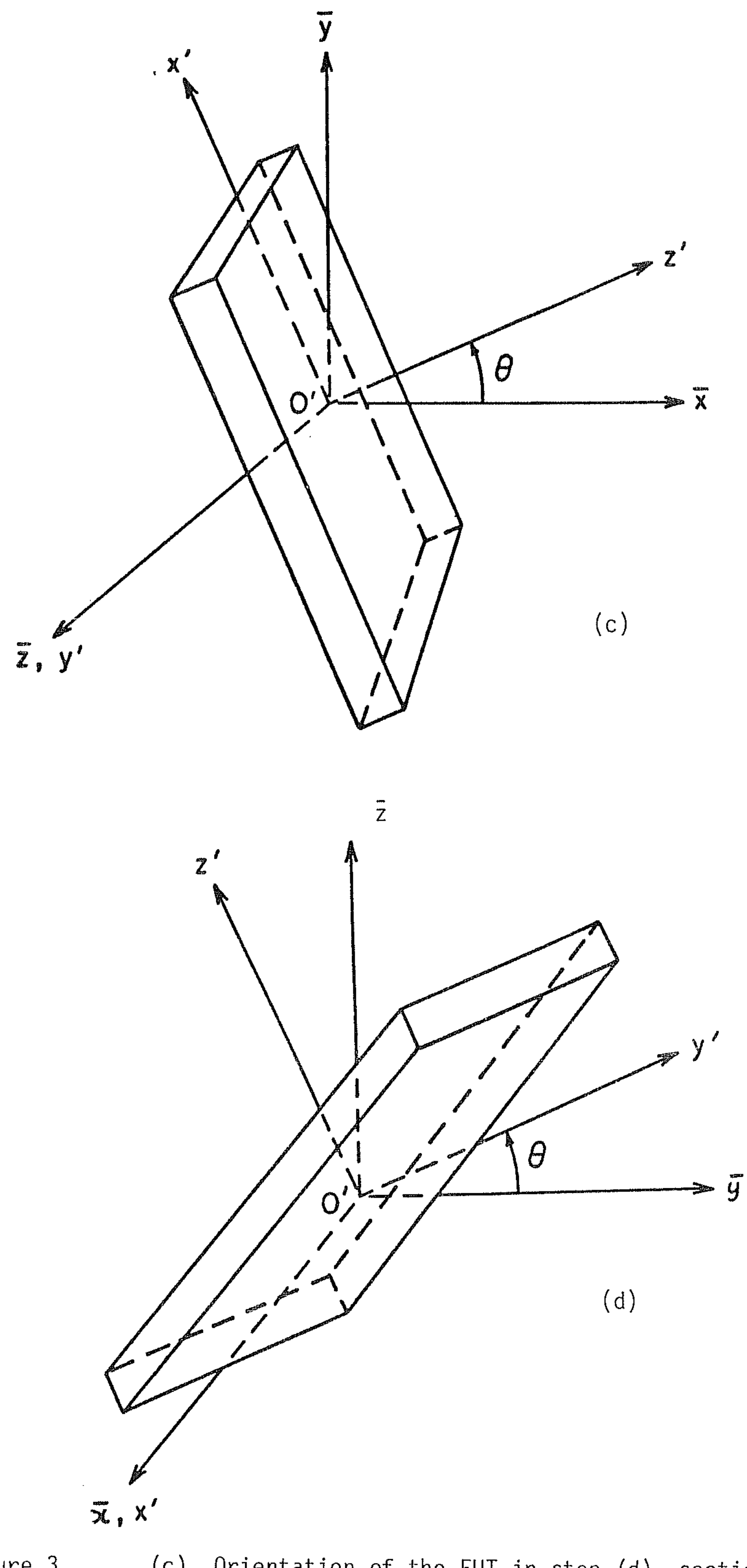

Figure 3. (c) Orientation of the EUT in step (d), section 4.2
(d) Orientation of the EUT in step (f), section 4.2

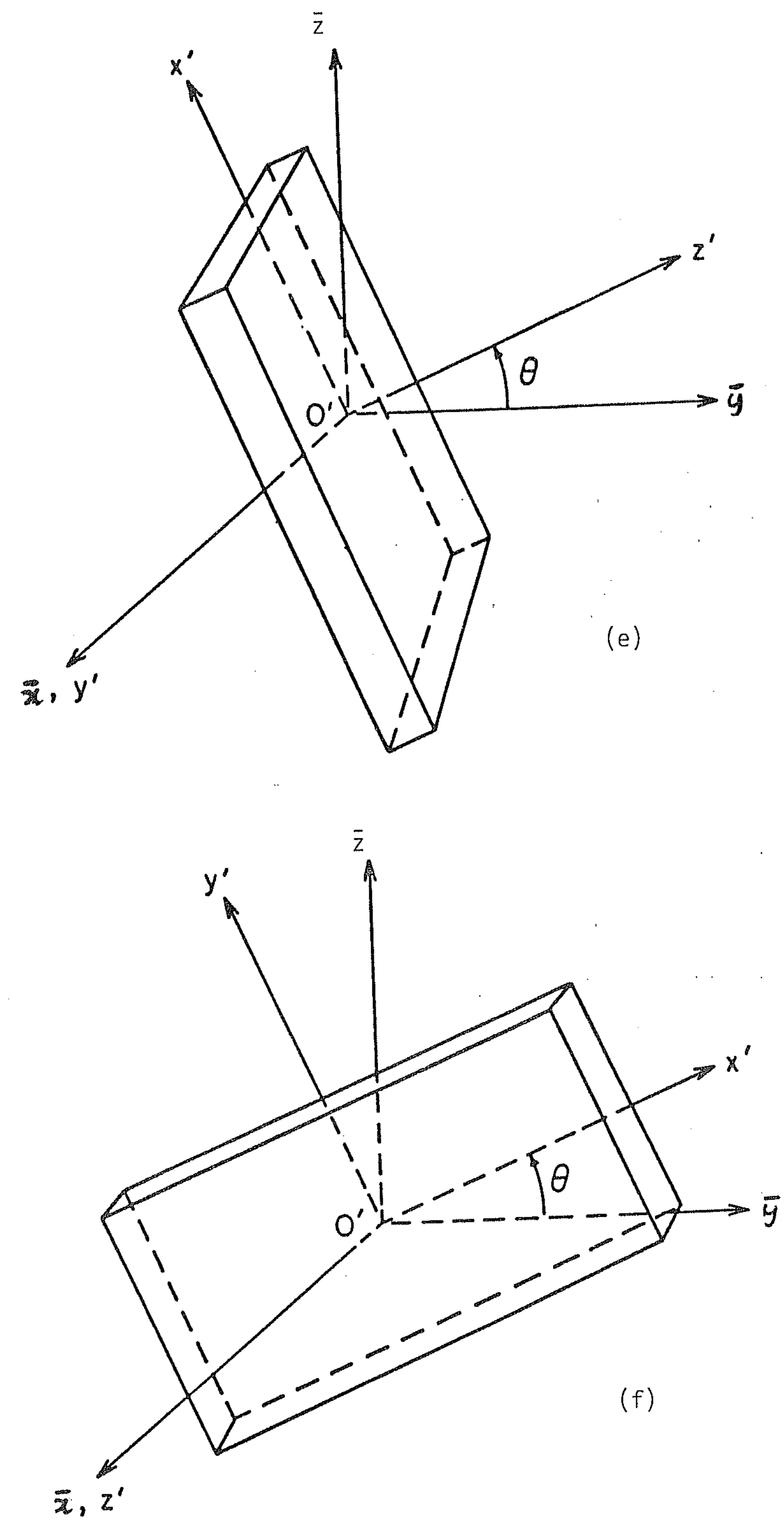

Figure 3. (e) Orientation of the EUT in step (g), section 4.2
(f) Orientation of the EUT in step (h), section 4.2

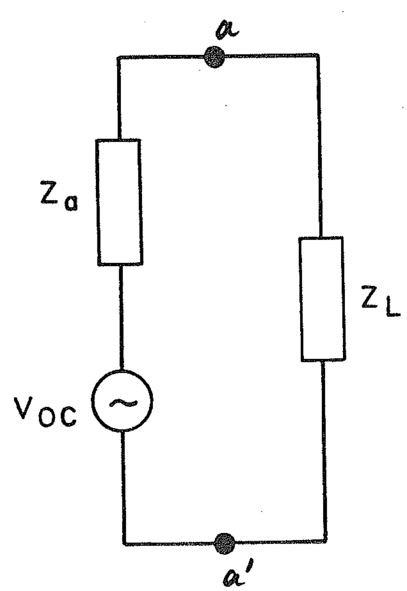

Figure 4. Equivalent circuit representation of the EUT as a receiving antenna system

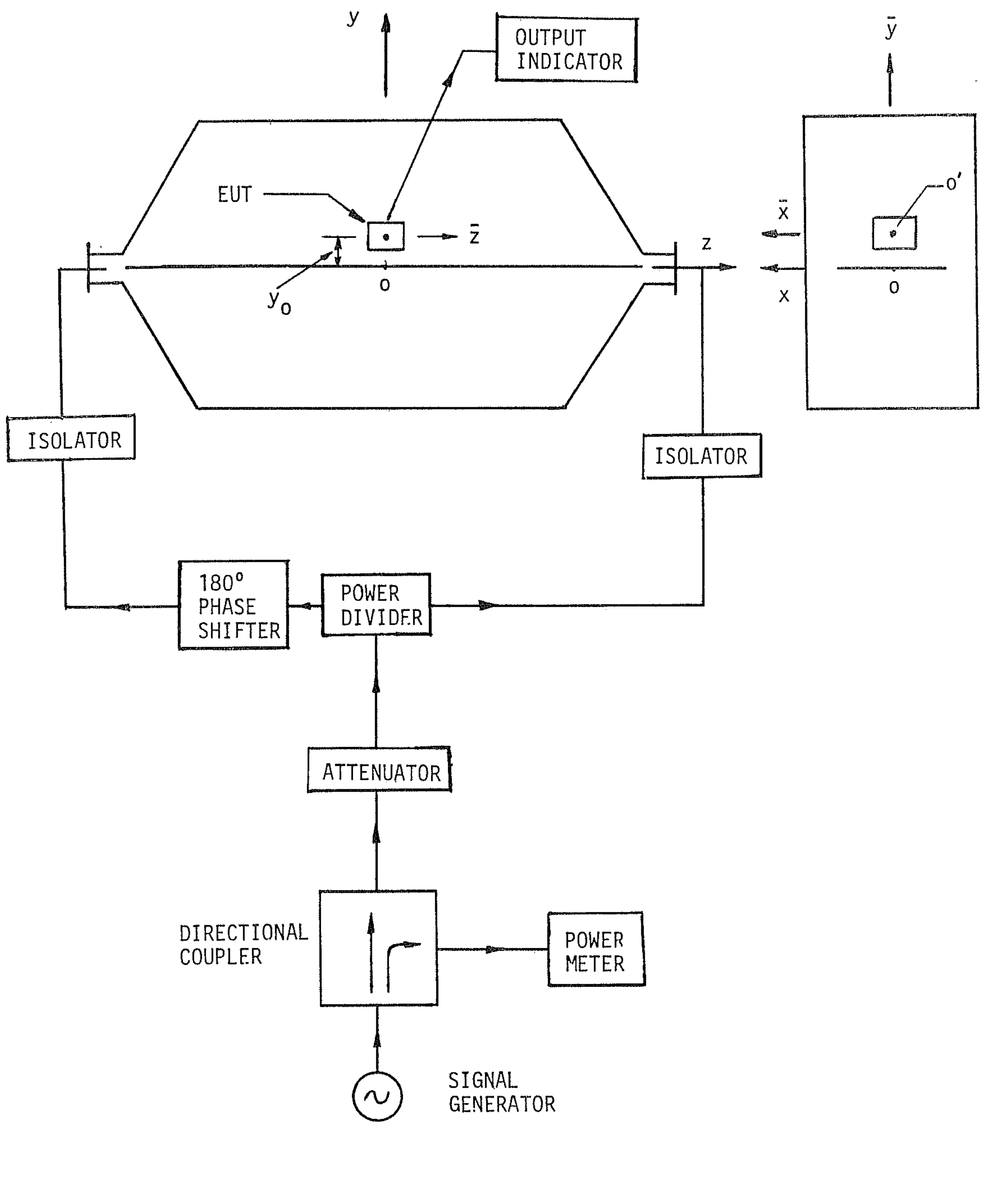

Figure 5. Test set-up for determining the average susceptibility level of an electrically small EUT

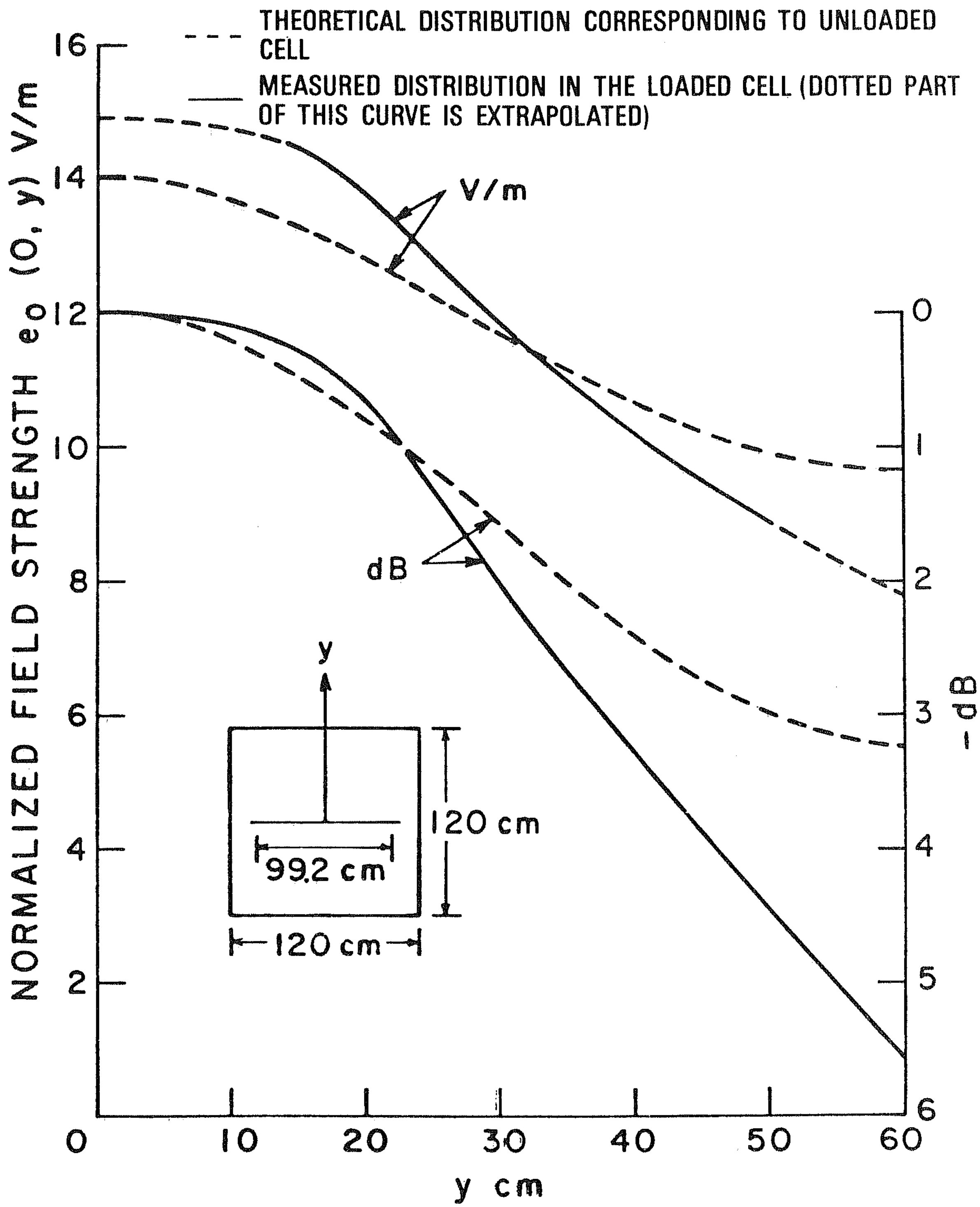

Figure 6. Normalized transverse electric field distribution inside a TEM cell

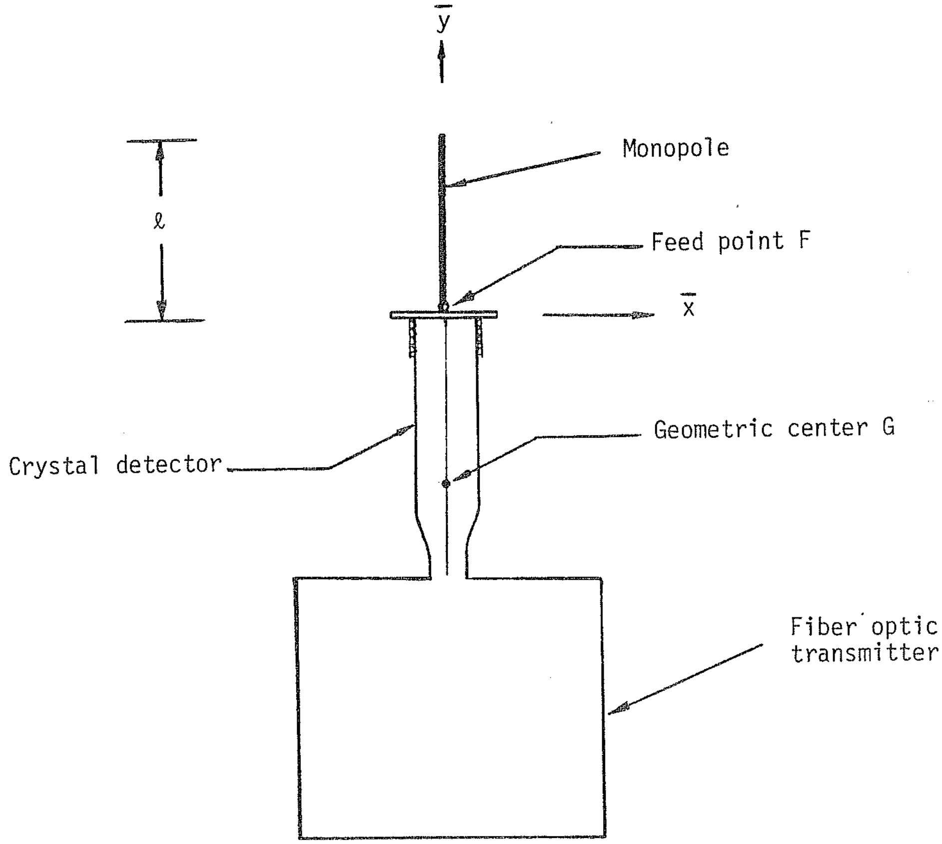

Figure 7. Monopole antenna, crystal detector and fiber optic transmitter combination

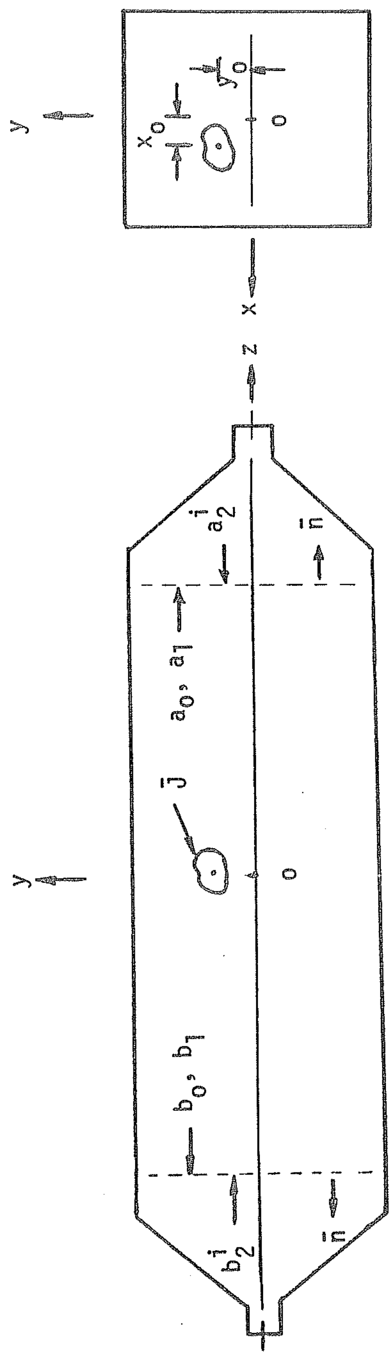

Figure 8. Receiving object inside a TEM cell

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5. AUTHOR(S)

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11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

An electrically small radiating source of arbitrary shape may, to a first order, be modeled by an equivlenet dipole system consisting of three orthogonal electric dipoles and three orthogonal magnetic dipoles each excited with arbitrary amplitude and phase. Determination of the individual electric dipole moments and the cross-components of such an equivalent dipole system for the emission case by tests inside a TEM cell, was described in an earlier work by Sreenivasiah, Chang and Ma, assuming a constant TEM modal field distribution in the space occupied by the test object. A method of accounting for the first-order variation in the TEM modal field distribution is presented in this report. This involves the inclusion of electric quadrupole terms in the modeling of the test object. Using the reciprocity principle, the same method is extended to the determination of susceptibility levels of electrically small test objects. Some experimental results for the susceptibility test, demonstrating the importance of the quadrupole terms, are presented.

12. KEY WORDS (Six to twelve entries; alphabetical order; capitalize only proper names; and separate key words by semicolons)

Analysis; dipole moments; electrically small objects; emission; quadrupoles; measurements; susceptibility; TEM cell.

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