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Distributions of Measurement Errors for Single-Axis Magnetic Field Meters During Measurements Near Appliances

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Comparisons are made between the average magnetic flux density as it would be measured with a single-axis coil probe and the flux density at the center of the probe, assuming that the probe is oriented to measure the maximum field at that point. Probability distributions of the differences between the two quantities are calculated assuming a dipole magnetic field and are found to be asymmetric. The distributions are used to estimate the uncertainty for maximum magnetic field measurements at distances that are large compared with the dimensions of the field source. *Bioelectromagnetics 18:273–276, 1997.* © 1997 Wiley-Liss, Inc.[†]

Key words: coil probe; dipole field; magnetic field; measurement uncertainty; power frequency

INTRODUCTION

Recently, calculations were performed that considered the influence of the size of magnetic field probes or sensors on the accuracy of measurements near electrical appliances [Misakian, 1993; Misakian and Fenimore, 1994, 1996]. These calculations examined the difference between the average magnetic flux density as determined by using magnetic field meters with a single- or three-axis circular coil probe (the three-axis circular coil probes referred to have a common central point) and the calculated magnetic flux density at the center of the probe, B_o. The field was assumed to be produced by a small loop of alternating current, i.e., a magnetic dipole. The magnetic dipole field was chosen as the field of interest, because, to a good approximation, its geometry simulated the field geometry of many electrical appliances and equipment [Mader and Peralta, 1992]. The ''average'' arose as a consequence of the averaging effects of coil probes over their cross-sectional areas when placed in a nonuniform magnetic field. The difference between the average magnetic field and B_o can be thought of as a measurement error, because the center of the probe is normally considered to be the measurement location.

Although the first two of these calculations [Misakian, 1993; Misakian and Fenimore, 1994] consider the worst case or largest errors for three-axis and single-axis field meters, Misakian and Fenimore [1996] report on the statistical distributions of the errors for three-axis field meters. This paper presents the statistical distribution of errors when single-axis field meters are used to measure the maximum magnetic field as a function of r/a, where r is the distance between the dipole and center of the probe, and a is the radius of the probe. The error distribution associated with measuring the maximum magnetic field is of interest, because maximum magnetic field levels are often used to characterize the magnetic field when single-axis field meters are employed [Gauger, 1985; IEEE Magnetic Fields Task Force, 1993]. The calculated errors are approximate in the sense that the calculations assume a purely $1/r^3$ spatial dependence of the magnetic field, i.e., the dimension of the magnetic field source is assumed to be small compared with r. A ''rule of thumb'' for this approximation is that r must be greater than three times the side dimension of the source [Zaffanella et al., in press]. For example, the magnetic field that is responsible for the vertical sweep of many computer monitors will be dipolar in character at the surface of the screen.

AVERAGE MAXIMUM MAGNETIC FLUX DENSITY AND PROBABILITY DISTRIBUTIONS

The average magnetic flux density, B_{av}, for a circular coil probe with cross-sectional area A is given by

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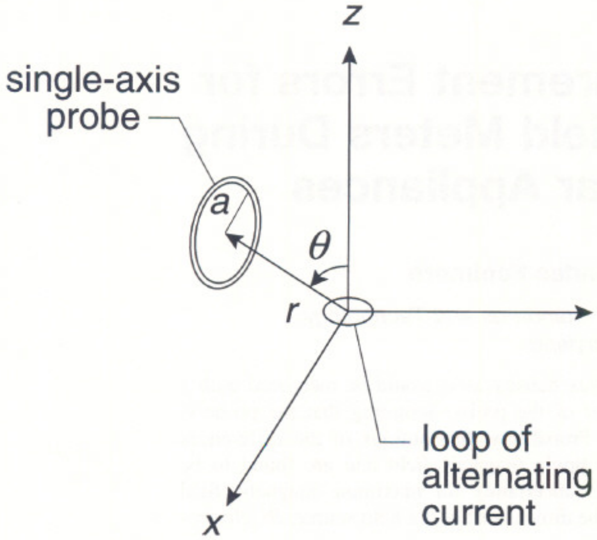

Fig. 1. A single-axis magnetic field probe with radius *a* is rotated until a maximum value of the dipole magnetic field produced by a small circular loop of alternating current is observed. It is assumed that the distance *r* is much greater than the dimensions of the loop.

$$B_{av} = \frac{1}{A}\iint_A \mathbf{B}\cdot\mathbf{n}\, dA, \tag{1}$$

where dA is an element of probe area, **n** is a unit vector perpendicular to A, and **B** is the magnetic flux density. Expressions for B_o and for B_{av} as a function of r, a, and probe orientation with respect to the axis of the magnetic dipole are developed in Misakian [1993] and Misakian and Fenimore [1994]. The reader is also referred to Misakian [1993] for details related to the computation of Equation 1.

The earlier calculation for single-axis field meters reports the worst-case measurement errors as a function of normalized distance r/a; i.e., the maximum value of the average magnetic field, B_{av1}, was determined by rotating the probe while holding r/a and the magnetic dipole axis orientation fixed. Figure 1 shows the geometry of the problem, where θ indicates the orientation of the dipole axis with respect to the position of the probe. The difference between B_{av1} and B_o, expressed in percent, was taken as the measurement error and was designated as ΔB_{av1}. Because the orientation of the dipole axis is not known in typical measurement situations, the earlier calculations evaluated ΔB_{av1} for different values of θ until the largest or worst-case error was determined for a given r/a.

This paper reports the probability distribution of the error, ΔB_{av1}, as a function of r/a. The distribution is found first by evaluating ΔB_{av1} for fixed r/a as θ is varied between 0° and 90°. For the purposes of this study, determination of ΔB_{av1} every 0.1° is adequate, and, because of symmetry considerations, calculations for $\theta > 90°$ are unnecessary. On the interval from 0° to 90°, the error is found to increase monotonically. The unnormalized distribution is found by noting [Misakian and Fenimore, 1996] that the probability of performing a measurement at angle θ with error ΔB_{av1} is sin θ (given the fact that the orientation of the dipole is not known) and plotting sin θ as a function of $\Delta B_{av1}(\theta)$.

Examples of typical ΔB_{av1} distributions are shown in Figure 2 for $r/a = 3$ and $r/a = 8$. For all values of r/a, the distributions have a nonzero mean and are asymmetric. In addition, the extreme negative and positive values of ΔB_{av1} occur for θ equal to 0° and 90°, respectively, and the positive extreme is also the most probable value. The extreme negative values of ΔB_{av1} exceed the extreme positive values in magnitude. Unlike the normal probability distribution, the distributions for ΔB_{av1} are nonzero only on an interval of finite extent. As r/a increases, the magnetic field becomes more uniform, and the distributions become more narrow, as expected.

Table 1 lists the probability or confidence intervals (CIs) of 68% and 95% and the extreme values of the ΔB_{av1} distributions for a range of r/a values. The 68% CI, which, in the case of a normal distribution, corresponds to ± one standard uncertainty (one standard deviation), is found by calculating the 16th and 84th percentile points of the ΔB_{av1} cumulative probability distribution for each r/a. Similarly, the 95% CI that is normally associated with two standard uncertainties is determined by calculating the 2.5 and 97.5 percentile points. The CIs are calculated from the distributions by performing Riemann summations [Thomas and Finney, 1988]. The percentages in Table 1 have uncertainties of ±0.1% in the units of the abscissa. The upper and lower Riemann sums differed by less than 0.1%.

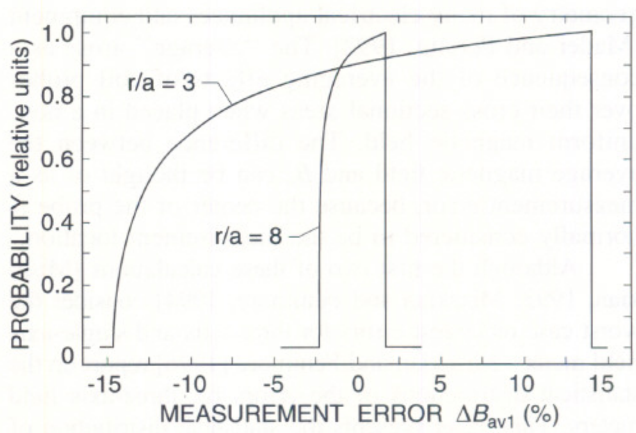

Fig. 2. Examples of unnormalized ΔB_{av1} error distributions for r/a equal to 3 and 8.

TABLE 1. Data for ΔB_{av1} Probability Distributions

r/a	Most Probable Values (%)	Extreme Values (%)	68% CI (%)	95% CI (%)
3	14.1	−14.6/14.1	−7.9/10.2	−12.9/13.5
4	7.5	−8.7/7.5	−5.2/5.2	−7.8/7.1
5	4.7	−5.7/4.7	−3.5/3.2	−5.2/4.4
6	3.2	−4.0/3.2	−2.5/2.2	−3.7/3.0
8	1.8	−2.3/1.8	−1.5/1.2	−2.1/1.7
10	1.1	−1.5/1.1	−0.9/0.7	−1.4/1.1
15	0.5	−0.7/0.5	−0.4/0.3	−0.6/0.5

DISCUSSION

Because the distributions of measurement error are asymmetric, the common measures of central tendency (the mean, median, and mode) do not coincide for our results, and the common measure of spread in the data (the standard deviation) does not have the customary interpretation in determining confidence in a measurement. The calculated distributions of ΔB_{av1} indicate that the mode (the most probable error) is the measure of central tendency most affected by the asymmetry. It corresponds to the largest positive error for each r/a and occurs when $\theta = 90°$.

The results in Table 1 can help to explain discrepancies between measurements of the maximum magnetic field at the same location with probes of different size. The results are also helpful when estimating the total uncertainty. Normally, there will be other sources of error, and estimates can be made of the total uncertainty by using the above results. For example, a rough estimate of the total standard uncertainty (standard deviation or 68.3% CI; CI_{68}) for the error distribution when $r/a = 3$ is given by

$$CI_{68} \approx -\sqrt{(-7.9)^2 + \sigma_t^2}\,,\ + \sqrt{(10.2)^2 + \sigma_t^2}\,, \quad (2)$$

where −7.9 and 10.2 are taken from Table 1, and σ_t^2 is the variance of all other independent sources of uncertainty. An estimate of the expanded uncertainty of "2σ" or 95.4% CI is given by

$$CI_{95} \approx -\sqrt{(-12.9)^2 + (2\sigma_t)^2}\,, \quad + \sqrt{(13.5)^2 + (2\sigma_t)^2}\,. \quad (3)$$

Consideration was given to using the error distributions to determine correction factors for maximum field measurements performed with single-axis field meters. However, as discussed in Misakian and Fenimore [1996], use of correction factors has adverse effects on significant numbers of measurements and should be avoided. Therefore, we recommend that no "corrections" be applied to the field values that are affected by the averaging effects discussed in this paper. Rather, the measurements should be reported with an indication of the total measurement uncertainty determined by combining the CIs provided in Table 1 with other sources of uncertainty according to Equations 2 and 3.

Finally, at measurement locations where the dipole approximation is valid, the results in Table 1 can also be used for guidance in selecting the size of a probe for measurement environments in which the field geometry is expected to be that of a dipole and highly nonuniform. For example, if the maximum magnetic field is to be measured at a distance r from a dipole source with a standard uncertainty of less than about 5%, then magnetic field meter probes with radii a such that $r/a \leq 4$ would be unsuitable. Single-axis probes having radii such that $r/a \geq 5$ would be suitable if the standard uncertainty from other independent sources of uncertainty amounted to 3.6% or less, i.e.,

$$CI_{68} \approx -\sqrt{(-3.5)^2 + (3.6)^2} = -5.0\%, \quad +\sqrt{(3.2)^2 + (3.6)^2} = 4.8\%,$$

where −3.5 and 3.2 are taken from Table 1 for $r/a = 5$.

CONCLUSIONS

Calculations have been performed of the probability distribution of errors (ΔB_{av1}) that can occur when magnetic field meters with single-axis circular coil probes are used to measure the maximum magnetic field produced by a miniature magnetic dipole. Because the magnetic dipole field approximates fields produced by many electrical appliances, the results may be helpful in explaining discrepancies in maximum magnetic field measurements at a given location because of differences in probe size. Knowledge of the 68 and 95% CIs of the asymmetric error distribution allows one to assign estimates of uncertainties associated with the measurements. The results are valid to the extent that the dipole approximation is appropriate and that the distance r between the probe and the magnetic field source can be reasonably estimated.

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