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LETTER TO THE EDITOR

Quantised dissipative states at breakdown of the quantum Hall effect

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Abstract. We report the breakdown of the nearly dissipationless quantum Hall effect into a set of distinct, quantised dissipative states in a wide, high-quality GaAs/AlGaAs sample. We found 35 dissipative states on one plateau and 9 on another plateau which have longitudinal voltage drops accurately quantised in units of $\hbar\omega_c/e$ to within our $\pm 0.6\%$ measurement uncertainty. This voltage quantisation implies that the energy dissipation per carrier is quantised in units of the Landau level spacing $\hbar\omega_c$.

The integral quantum Hall resistance [1] $R_H(i) = V_H(i)/I_x = h/(e^2 i)$ is observed when the longitudinal voltage $V_x = R_x I_x$ of the two-dimensional electron gas is very small. Here $V_H(i)$ is the Hall voltage of the ith plateau, i is an integer, and I_x is the current through the sample. There is a critical current above which the voltage V_x rapidly increases by several orders of magnitude [2, 3]. As one approaches the critical current, V_x becomes finite. This is referred to as breakdown of the dissipationless current.

Many mechanisms have been proposed to explain the breakdown phenomena. Some involve classical effects, such as electron heating instabilities [4] and inhomogeneous resistive channels [5]. Others are quantum mechanical, such as intra-Landau level transitions [6], inter-Landau level transitions involving edge currents [7], Zener tunnelling [8], and quasi-elastic inter-Landau level scattering (QUILLS) [9–11].

Bliek *et al* [12] proposed the existence of a new quantum effect to explain the structures in their curves of V_x versus magnetic field at currents near breakdown for samples with narrow constrictions. Their results were interpreted by Eaves and Sheard [9] in terms of the QUILLS model. We report the first observation of dissipative states in which the longitudinal voltage drops V_x are accurately quantised in units of $\hbar\omega_c/e$, and suggest that these phenomena are caused by QUILLS [9–10].

Our sample was a $GaAs/Al_xGa_{1-x}As$ heterostructure grown by molecular beam epitaxy with $x = 0.29$. It is designated as GaAs(7). It has a zero magnetic field mobility of 100 000 $cm^2\,V^{-1}s^{-1}$ at 4.2 K, exhibits excellent integral quantum Hall effect properties, and will be used as the new United States resistance standard from 1 January 1990. The inset of figure 1 shows the geometry of this sample. It is 4.6 mm long and 0.4 mm wide. The two outer Hall potential probe pairs are displaced from the central pair by ± 1 mm. The perpendicular magnetic field direction is such that probes 2, 4 and 6 are near to the potential of the source S, which is grounded. Probes 1, 3 and 5 are nearly at the potential of the drain D.

Figure 1 shows V_x against the magnetic field B for the $i = 2$ (12 906.4 Ω) quantum Hall resistance plateau at $+210\,\mu A$ and 1.5 K, where positive current denotes electrons entering the source and exiting the drain. This current is approaching the $+230\,\mu A$ critical current. V_x was measured between points 2 and 4, hereafter denoted also as $V_x(2,4) \equiv V_x(2) - V_x(4)$. Note the step-like transitions in the magnified region of this x-y recording. They occur at about 5.3 mV spacings of V_x.

The thin solid curve of figure 1 shows a second recording. It has quite different features over the magnetic field range between 10.9 and 11.8 T. Indeed, one obtains many different values of V_x over a long time period if B is held constant within this range. The broken curve shows hysteresis when the magnetic field is decreased.

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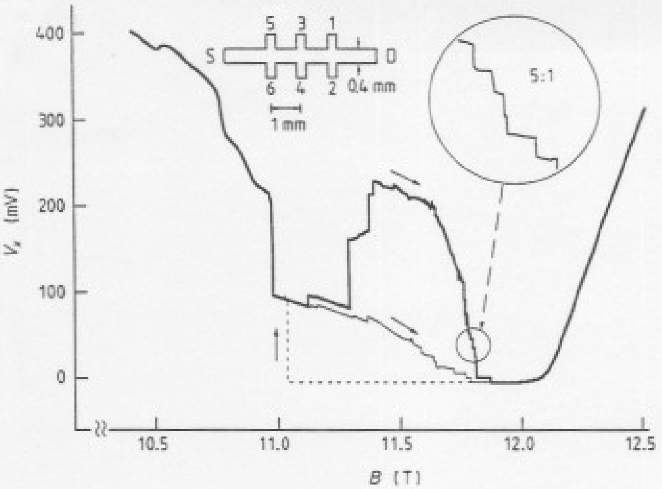

Figure 1. Time-averaged $V_x(2.4)$ signal against B for the $i = 2$ plateau of GaAs(7) at $+210\ \mu$A and 1.5 K. The two full curves are for sweeps in the increasing B direction. The broken curve shows hysteresis for decreasing B. The inset shows the sample geometry.

We investigated the nature of the V_x signals in more detail by using a Hewlett Packard 3458A multimeter† to make voltage histograms. Each histogram consisted of 16 000 measurements in a 2.4 s sampling period. Figure 2(a) displays one of these voltage histograms for the $v_x(2,4)$ signal obtained at 11.71 T and $+210\ \mu$A. The four peaks are spaced about 5.3 mV apart. Figure 2(b) shows the associated time-dependence of this V_x signal. V_x is in only one state at any given time. It remains in that state until electrical noise or other noise processes induces it to switch to another state.

A total of 194 peaks were obtained in the 117 voltage histograms collected at $+210\ \mu$A and 1.5 K over the V_x against B curve region shown in figure 1. We first concentrate on the region near 11.75 T. On ten different occasions ground-state peaks were obtained, with an average value of (0.0374 ± 0.0021) mV. This value included thermally induced voltages, so four additional measurements of V_x were made at zero current. They yielded an average value of (0.0368 ± 0.0006) mV. This agreement confirmed that the ground state is indeed dissipationless. We therefore subtracted this background value of 0.0374 mV from all the excited-state peaks near 11.75 T. We then investigated the 61 excited-state peaks in the vicinty of 11.75 T with voltages less than 54 mV. The states were separated by (5.295 ± 0.003) mV. Although the voltage separations were *different* at other values of magnetic field, the *ratio* V_x/BM was constant, where M represents an integer.

Table 1 shows the result for the 61 excited-state peaks near 11.75 T using the ratio V_x/BM. The value of the ratio is (0.4507 ± 0.0008) mV T^{-1}. The first 10 excited states are all quantised to within, or just outside, the typical $\pm$ 0.6% one-standard-deviation measurement accuracy.

† Brand names are used only for purposes of identification. Such use implies neither endorsement by the National Institute of Standards and Technology nor assurance that the equipment is the best available for the purpose.

Table 1. Experimental determination of the fractional quantum unit V_x/BM and of the quantum number M on the $i = 2$ plateau at $+210\ \mu$A and 1.5 K

M	$V_x/B(M)$ (mV T^{-1})	$V_x/(B)0.4507$ (mV T^{-1})
1	0.4491 ± 0.0020	0.996 ± 0.005
2	0.4542 ± 0.0029	2.016 ± 0.013
3	0.4551 ± 0.0029	3.029 ± 0.013
4	0.4549 ± 0.0026	4.037 ± 0.023
5	0.4525 ± 0.0029	5.020 ± 0.032
6	0.4487 ± 0.0037	5.973 ± 0.049
7	0.4459 ± 0.0029	6.925 ± 0.044
8	0.4464 ± 0.0026	7.924 ± 0.046
9	0.4504 ± 0.0021	8.994 ± 0.042
10	0.4510 ± 0.0024	10.006 ± 0.053

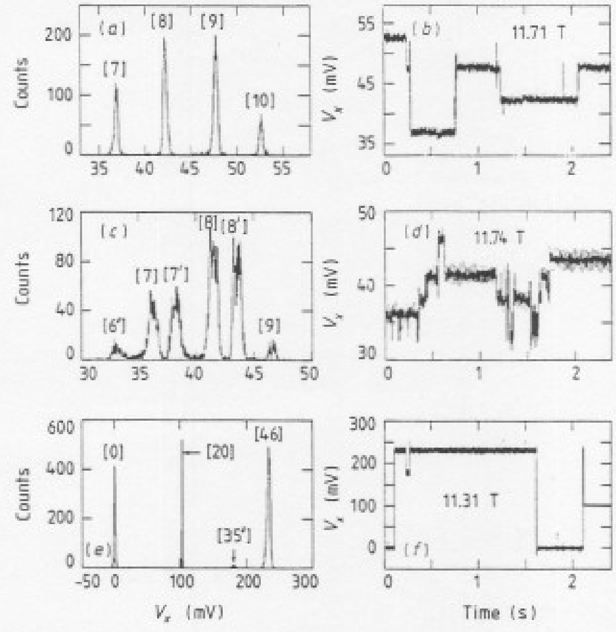

Figure 2. Data obtained on the $i = 2$ plateau at $+210\ \mu$A and 1.5 K. (a) Digital histogram of the $V_x(2, 4)$ signal at 11.71 T. The quantum numbers, M, are indicated in square brackets. (b) The time-dependence of the signal shown in (a). (c), (d) Data for the $V_x(1, 3)$ signal at 11.74 T. The primes denote shifted peaks. The $V_x(1, 3)$ signal is noisier than the $V_x(2, 4)$ signal because it is off-ground. (e), (f) Data for $V_x(2, 4)$ at 11.31 T.

In addition to the 61 peaks discussed above, a total of 26 other peaks also appeared in some of the voltage histograms taken near 11.75 T. Those additional peaks appeared to be shifted by a constant that had two different values. Figures 2(c) and (d) show an example; there are peaks with quantum numbers $M = 7$, 8, and 9, but also three other peaks. The simplest interpretation is that the background or zero is different by the value (1.219 ± 0.024) mV for these three peaks, thus yielding shifted $M = 6$, 7, and 8 peaks denoted by primes in the figure. Ten of these 26 peaks had this shift. The remaining 16 peaks were shifted by the value (2.96 ± 0.16) mV. All the shifted peaks had M larger than 3.

We obtained 87 peaks with voltages greater than 54 mV over the magnetic field interval between 10.44 and 11.75 T. Six of the 87 peaks were shifted by (1.22 ± 0.05) mV and 42 were shifted by (2.79 ± 0.08) mV. These shifts were consistent with those discussed above. In addition, for magnetic fields less than 11.4 T, we obtained six ground state peaks at (0.321 ± 0.024) mV, rather than at (0.037 ± 0.002) mV. Figures 2(*e*) and (*f*) show four peaks at 11.31 T; the ground-state peak and the peaks for $M = 20$ and 46 are sharply defined. Using only the 39 unshifted peaks, and subtracting the appropriate ground-state zeros, we found the additional quantum numbers $M = 11$ to 21, 24, 27, 33, 37, 41 to 47, 57, 76 and 84. All of them occurred within, or slightly outside, the one-standard-deviation uncertainties. For example: $M = 13.98 \pm 0.11$, 19.95 ± 0.16, and 45.91 ± 0.37. If the shifted peaks are included one also obtains quantum numbers $M = 34$, 35, 53, 60, 62 to 65, 73, 74, 79 and 83.

Figure 3 shows one of the V_x against B plots for the $i = 4$ (6 453.2 Ω) plateau at $+300$ μA and 1.5 K. This current was well below the $+340$ μA critical current. Twenty five peaks were obtained from the voltage histograms. Four of them were ground states with an average background value of (0.221 ± 0.014) mV. Seventeen unshifted excited state peaks were found. The V_x/BM ratio was (0.8463 ± 0.0017) mV T^{-1}, and the quantum numbers were $M = 1.0117 \pm 0.0083$, 9.001 ± 0.043, 9.914 ± 0.083, 10.973 ± 0.053, 13.18 ± 0.11, 18.88 ± 0.11, 19.82 ± 0.15, 20.95 ± 0.11 and 25.20 ± 0.15. Again, all of them occurred within, or slightly outside, the one-standard-deviation uncertainties. In addition, there were four peaks shifted by (1.94 ± 0.14) mV.

Figures 2(*b*), (*d*) and (*f*) illustrate why we refer to figures 2(*a*), (*c*) and (*e*) as voltage histograms rather than voltage spectra. Each histogram of 16 000 measurements represents only what occurred during that particular 2.4 s sampling time. For example, peaks 7 and 7′, and 8 and 8′ were occupied for nearly equal time periods in figure 2(*d*), but 6 and 9′ were not occupied at all. The area under a peak is therefore *not* proportional to the occupation probability or lifetime of a state because the switching is noise-induced. Indeed, we could often induce switching to another DC state by resetting the multimeter or by momentarily disconnecting a cable. However, the intrinsic stability of the quantum states does appear to vary significantly over different regions of V_x and B.

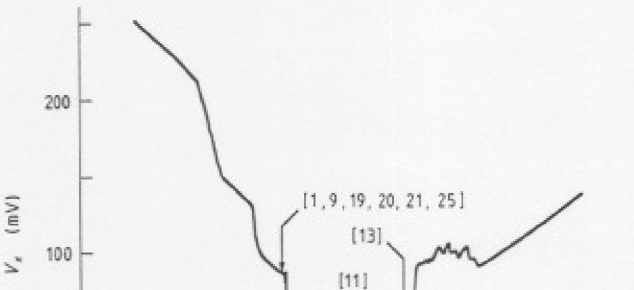

Figure 3. Time-averaged $V_x(2, 4)$ signal against B for the $i = 4$ plateau at $+300$ μA and 1.5 K. The quantum numbers are indicated in brackets.

The quantised dissipation described above can be directly related to Landau level transitions. We propose that originally empty Landau levels are populated by electrons excited from lower, originally full, ground-state Landau levels, and that the quantised dissipation arises from the transition of those electrons back to the original levels. The electrical energy loss per carrier for M Landau level transitions is $M\hbar\omega_c$, and the measured energy loss per carrier is eV_x. If all the electrons of both spin sublevels of a Landau level undergo the transitions then $eV_x = M\hbar\omega_c$. If the ground state involves multiple filled Landau levels, and if we assume that only the upper filled Landau level undergoes transitions, then $eV_x = fM\hbar\omega_c$, where $f = i/2$ for even i plateau numbers. Thus

$$M = \left(\frac{i}{2}\right)\left(\frac{m^*}{\hbar}\right)\left(\frac{V_x}{B}\right) \qquad (1)$$

and the spacing between quantum numbers should be $\Delta M = 1$ if equation (1) is valid.

Applying equation (1) to our V_x/B data for the $i = 4$ plateau, and assuming [13] that $m^* = 0.068m_e$, where m_e is the free-electron mass, we find that $\Delta M = 0.994 \pm 0.002$. This is very close to 1; especially since the uncertainty in the value of m^* must be rather large [14].

However, for our $i = 2$ data, $\Delta M = 0.265 \pm 0.001$ rather than 1. We do not know the reason for this, but we do know [3] that the dissipative region of the $i = 4$ plateau is entirely between the voltage terminals 2 and 4, whereas the dissipation is only partially within that region for the $i = 2$ plateau. Therefore not all the V_x signal occurred within that region.

The same analysis can be applied to the V_x/B data presented in figure 1 of Bliek *et al* [12]. That figure shows structures in V_x against B curves obtained at different currents. We estimate that $\Delta M = 0.96 \pm 0.04$ for their $i = 2$ plateau and that $\Delta M = 1.00 \pm 0.08$ for the $i = 4$ plateau. The presence of a physical constriction probably forced the dissipation to be within the voltage probes for both plateaus.

We have observed dissipative states in the breakdown regime in which the energy per carrier is accurately quantised in units of $\hbar\omega_c$. This suggests that the phenomena are caused by QUILLS [9–11] involving carriers with high-drift velocities and emission of acoustic phonons having energies small compared with $\hbar\omega_c$. This mechanism seems to require that all the carriers undergo the scattering process for breakdown on the $i = 2$ plateau and that half the carriers are scattered for the $i = 4$ plateau.

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