Effects of Nonmodel Errors on Model-Based Testing

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Abstract—**In previous work, model-based methods have been developed for efficient testing of components and instruments that allow for their full behavior to be predicted from a small set of test measurements. While such methods can significantly reduce the testing cost of such units, these methods are valid only if the model accurately represents the behavior of the units. Previous papers on this subject described many methods for developing accurate models and using them to develop efficient test methods. However, they gave little consideration to the problem of testing units which change their behavior after the model has been developed, for example, as a result of changes in the manufacturing process. Such changed behavior is referred to as nonmodel behavior or nonmodel error. When units with this new behavior are tested with these more efficient methods, their predicted behavior can show significant deviations from their true behavior. This paper describes how to analyze the data taken at the reduced set of measurements to estimate the uncertainty in the model predictions, even when the device has significant nonmodel error. Results of simulation are used to verify the accuracy of the estimates and to show the expected variation in the results for many modeling variables.**

I. Introduction

TESTING of analog and mixed-signal devices affects both the cost and quality of the devices. Engineers must understand both the advantages and pitfalls involved in any proposed techniques to improve the testing process. A comprehensive approach has been developed at the National Institute of Standards and Technology to optimize the tradeoffs in developing an efficient test method for analog and mixed-signal devices [1]–[4]. These methods are applicable to devices ranging in complexity from a/d or d/a converters to complete multirange instruments [5]. The approach is based on the development of a model that allows the prediction of a large number of test results from a much smaller number of tests. This model must include all the significant parameters in the process that can affect the measurement results. The model is validated by comparing the full set of results that it predicts against a full set of actual measurements. This validation process is performed on a number of production devices. A model validated in this manner is adequate so long as the set of validation devices represents a wide range of process parameters and is thus typical of the other devices produced on the same production line.

This test approach also includes an online method, i.e., one that can be used during production testing, to assess the adequacy of the model. This method involves examination of the differences between the reduced set of measurements made and the predicted values for these measurements. These differences or residuals will increase if the production line changes so as to cause changes in the measurements that are not included in the model. Prediction intervals (defined in Section III) for the full set of predicted values can be constructed based on these residuals.

The purpose of this paper is to examine the reliability of this on-line test so as to assess the accuracy of the predictions made by the model. A simulation program is used to generate and analyze data typical of a well-controlled production line and then to show the effects of changes in the underlying model. The results show that prediction intervals based on the residuals from the validation process are valid only if the underlying model does not change; however, prediction intervals that are based on the online test residuals are valid even if the underlying model changes. The on-line test residuals must be used with care, because they can have significant variation if based on a small number of tests.

II. Simulation Program

The simulation program generates data sets that are equivalent to measurements taken on devices or instruments produced by a well-controlled production line. The data simulates a behavior that has been observed for many such devices [1], [3], namely, that the measurements taken on each device are controlled by a relatively small number of underlying parameters. These parameters may in turn be controlled by process parameters of the production line or by variation in the component values of the device. If the number of significant parameters is small compared with the number of measurements taken, then model-based testing can reduce the number of tests that need to be performed without significantly reducing the accuracy of the results. The relation between the measurements and the parameters that control them is generally linear so long as the parameter changes are relatively small.

Consider that the number of performance measurements made on the device is m. Combine the deviations of the m measurements from the ideal values into an $m \times 1$ vector y. Let the number of parameters that control the measurements be given by p and the set of values for these parameters for device i be expressed as the $p \times 1$ vector x^i. The relation between each of these parameters and the measurement deviation at each test can be expressed as an $m \times p$ matrix A. The relation between the parameters controlling the measurements and the measurement deviations can then be written as

$$y^i = Ax^i + \epsilon^i \quad (1)$$

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where ϵ^i is an $m \times 1$ vector that gives the random errors that occurred while measuring device i. In this paper all y vectors will refer to deviations between a measurement and an ideal value. However, the vector will be referred to as a measurement. If all m measurements are similar, then ϵ is often an independent and identically distributed random vector with zero mean and standard deviation of σ.

Often the matrix A is unknown. Parts of it can be derived from circuit models of the device or from physical models of the production process. These parts can be supplemented or the entire model can be developed by empirical methods from full sets of measurements made on a large number of devices. The simulation program used to generate the data for this paper is the same as the one described in [6], with the addition of a nonmodel error capability. The model used for the simulated device has ten parameters, p, and uses random model vectors with amplitudes in arbitrary units that vary from 0.03 to 0.3. The number of measurements, m, is 256; and the average amplitude of the simulated deviation vector, y, is 0.6 in the arbitrary units. The standard deviation σ of the measurement noise vector, ϵ, is 0.1. The derivation of the estimate E of the true model A is described in [6]. That method uses the singular value decomposition (SVD) on a number, n_e, of simulated random device measurement vectors, y^i, and then determines the optimum number of model vectors, k. The reduced set of measurements is derived from E [3]. For the simulation examples considered here, n_e was set to 130, and k was evaluated to be 10. There is not a one-to-one correspondence between the columns of A and E, nor between x and the parameters η used with the estimated model E. Rather, the columns of E span approximately the same space as the columns of A, and the values of η estimate a corresponding linear combination of the parameters x.

III. Random Measurement Error

When the only errors present in the modeling process are the errors due to approximating the true model A with the estimate E and the measurement error ϵ, then an estimate of the accuracy of the predictions can be derived [7], [8]. These estimates are based on the full $m \times k$ model estimate, E, and the reduced $t \times k$ model estimate, $\tilde{E}$, which is the rows of E at the t reduced measurements. The prediction variance, P, of the measurement predictions, due to the measurement noise ϵ, which has a standard deviation of σ, is given by the diagonal of a matrix as

$$P = P_c\hat{\sigma}^2 = \text{diag}\{E(\tilde{E}'\tilde{E})^{-1}E'\}\hat{\sigma}^2 \tag{2}$$

where $\mathbf{P}_c$ is the $m \times 1$ prediction variance coefficients, and $\mathbf{E}'$ is the transpose of $\mathbf{E}$. The noise estimate $\hat{\sigma}$ is obtained by applying the model to a set of n validation devices as

$$\hat{\sigma}^2 = \frac{1}{n(m-k)}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{(y_j^i - \hat{y}_j^i)^2}{1+P_{cj}} \tag{3}$$

where j refers to the jth measurement, and $\hat{y}^i$ is the $m \times 1$ vector of predicted measurements for device i given by

$$\hat{y}^i = E\eta^i = E(\tilde{E}'\tilde{E})^{-1}\tilde{E}'y_s^i. \tag{4}$$

Here η^i is the estimate of the parameters for device i and y_s^i is the vector of t reduced measurements for device i. Define the matrix

$$Z(\tilde{E}) = \mathbf{I} + E(\tilde{E}'\tilde{E})^{-1}E'. \tag{5}$$

Using this matrix, prediction intervals can be constructed for each predicted measurement $\hat{y}$, which have a $1-\alpha$ probability of containing the measured value. The prediction interval for the jth measurement is [7], [8]

$$|y_j - \hat{y}_j| \le \hat{\sigma}w_{jj}^{1/2}\mathrm{t}_{1-\alpha/2} \tag{6}$$

where w_{jj} is the jth diagonal element of $Z(\tilde{E})$, and $\mathrm{t}_{1-\alpha/2}$ is the $1-\alpha/2$ quantile of the t distribution with $n(m-k)$ degrees of freedom. Note that w_{jj} is just one plus the prediction variance coefficient for measurement j, $1+P_{cj}$.

Similar to prediction intervals which define an interval about the predicted value that contains the measured value with some probability, confidence intervals define an interval about the predicted value that contains the true (i.e., expected) value with some probability. For devices that follow (1), the true measurements $\bar{y}$ are

$$\bar{y} = Ax. \tag{7}$$

For simulated devices this true value is known, but for real devices it must be estimated by taking many repeated measurements on a device and averaging the measurements to reduce the random errors. The confidence interval for measurement j,

$$|\bar{y}_j - \hat{y}_j| \le (P_{cj})^{1/2}\hat{\sigma}t_{1-\alpha/2} \tag{8}$$

has a probability of $1-\alpha$ of containing $\bar{y}_j$.

Simulations were run on 32 different models A. For each model, A, an empirical model, E, was developed using 130 simulated devices based on A (130 different random parameter vectors, x, were used) and a measurement error σ of 0.1. Then 32 additional devices generated in the same way, were analyzed as the validation set. Thus, validation data was obtained on 32 times 32 or 1 024 total devices, n. For each model the number of reduced measurements, t, was varied from 11 to 256. The average prediction variance $\bar{P}_c$ for all m measurements was calculated for each number of reduced measurements using the m prediction variances from (2). Equation (6) shows that p_m, the standard deviation of the difference between the measurements and the predicted measurements, is approximately

$$p_m = \sqrt{\frac{\sum_{i=1}^{n}|y^i - \hat{y}^i|^2}{n(m-k)}} \approx \hat{\sigma}\sqrt{1+\bar{P}_c} \tag{9}$$

where $|\cdot|^2$ is the sum of squares of the vector elements. Fig. 1 shows p_m and this estimate for p_m plotted as a function of t. The approximation is close. Equation (8) shows that p_t, the standard deviation of the difference between the true value of the measurement and the predicted measurement, is approximately equal to the square root of the average

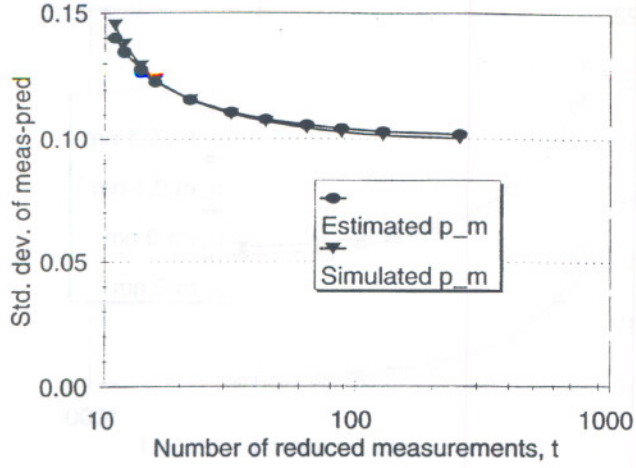

Fig. 1. Standard deviation of difference between the measurements and predicted measurements p_m and the estimate for p_m with no nonmodel error.

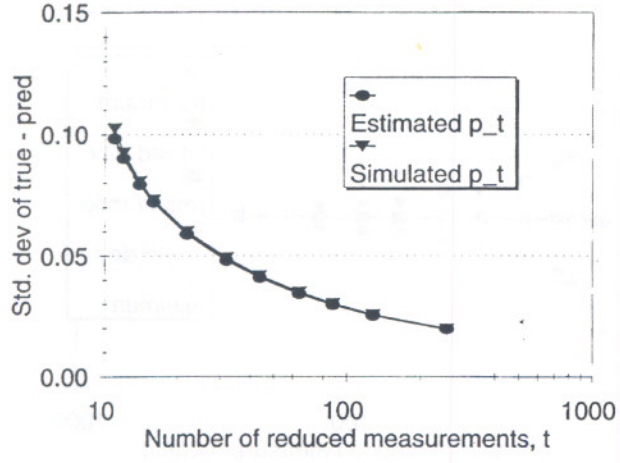

Fig. 2. Standard deviation of difference between the expected value of the measurements and the predicted measurements p_t and the estimate for p_t with no nonmodel error.

prediction variance, as

$$p_t = \sqrt{\frac{\sum_{i=1}^{n} |\bar{y}^i - \hat{y}^i|^2}{n(t-k)}} \approx \hat{\sigma}\sqrt{\bar{P}_c}. \tag{10}$$

Fig. 2 shows that p_t closely follows this estimate for p_t as a function of t. Thus, when there is no nonmodel error the estimate for σ obtained from the validation process, $\hat{\sigma}$, works well for prediction and confidence intervals.

IV. Nonmodel Error

This section will show that when nonmodel error is present, use of the $\hat{\sigma}$ from the validation process with no nonmodel error present gives an inaccurate estimate for p_m, and that use of the online test residuals gives a good estimate for p_m. A device has nonmodel error, if, as a result of changes in the production line, the devices exhibit behavior not captured in the original empirical model. For such devices, the relation between the measurements and the original model is

$$y^i = Ax^i + \mu^i + \epsilon^i \tag{11}$$

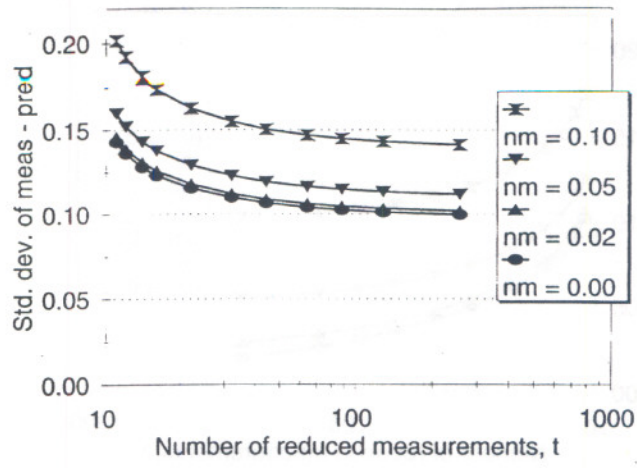

Fig. 3. Standard deviation of difference between the measurements and predicted measurements p_m and the estimate for p_m with 0, 0.02, 0.05 and 0.10 nonmodel error.

where the nonmodel component μ^i is orthogonal to A. The true or expected measurement is then given by

$$\bar{y}^i = Ax^i + \mu^i. \tag{12}$$

The empirical model E will approximate the original model A, and will therefore be approximately orthogonal to μ^i. The μ is fixed for any one device. Over many devices μ is assumed to have a zero mean and a standard deviation of ν independent of the t test points, i.e., the variance of μ at the test points is the same as the variance at the nontest points. Fig. 3 shows the effects on p_m with various amounts of nonmodel error present in addition to the 0.1 level of measurement noise. As the amplitude of the nonmodel error increases, p_m increases proportionally the same for all t. For large t, as

$$t \to m, \qquad p_m \to \sqrt{\sigma^2 + \nu^2}. \tag{13}$$

Fig. 4 shows that as the amplitude of the nonmodel error increases, p_t increases most for the larger number of reduced measurements. At the larger number of measurements the value of p_t is most dramatically reduced so one would expect the effects of nonmodel error to be most significant there.

Figs. 3 and 4 show that the approximations of (9) and (10) are not valid for devices which have nonmodel error. An estimate, which is not sensitive to the presence of nonmodel error, can be expressed in terms of the residuals at the reduced measurements. The standard deviation of the residuals, r_s, is given by

$$r_s = \sqrt{\frac{|y_s - \hat{y}_s|^2}{t-k}} \tag{14}$$

where $\hat{y}_s$ is the vector of predicted measurements at the reduced measurements. This quantity estimates for each device the magnitude of the combination of measurement error, ϵ, and nonmodel error, μ. The accuracy of this estimate is a function of the number of reduced measurements. Fig. 5 shows the rms of the 1 024 r_s's, the rms plus and minus one standard deviation, and the extremes of r_s for each of the reduced number of measurements. Note how large the range of r_s is

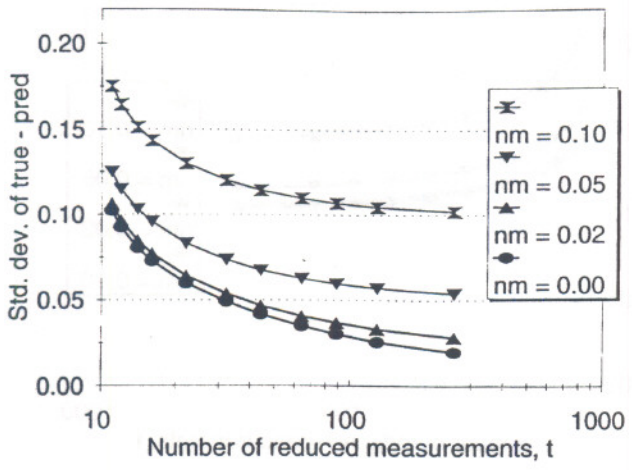

Fig. 4. Standard deviation of difference between the expected value of the measurements and the predicted measurements p_t and the estimate for p_t with 0, 0.02, 0.05, and 0.10 nonmodel error.

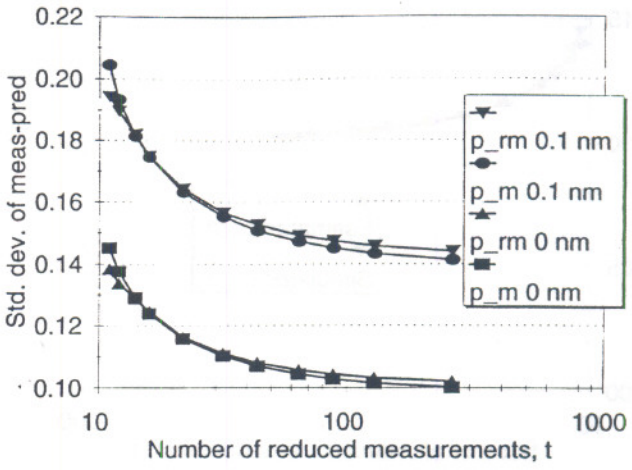

Fig. 6. Standard deviation of difference between the measurements and the predicted measurement p_m and the estimate p_{rm} with 0 and 0.1 nonmodel error.

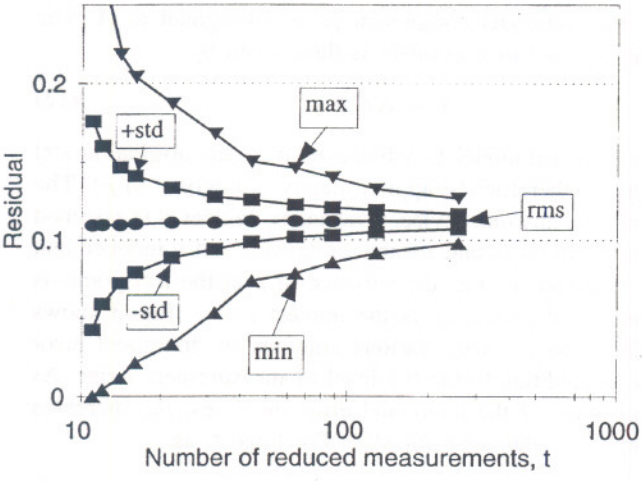

Fig. 5. Rms of residual r_s, rms plus and minus one standard deviation, and extremes for 1 024 simulations with 0.05 nonmodel error.

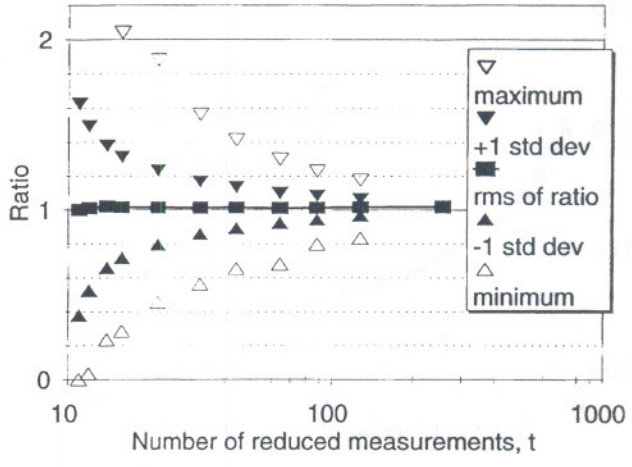

Fig. 7. Rms of ratio of estimate p_{rm} to p_m, rms plus and minus one standard deviation, and extremes for 1 024 simulations with 0.1 nonmodel error.

for small t. This large variability in r_s is due to the small number of degrees-of-freedom, $t-k$, when t is close to k (k is 10 here). The variance of r_s, $v(r_s)$, is proportional to

$$v(r_s) \sim \frac{1}{t-k}. \tag{15}$$

The variance of r_s decreases rapidly as t increases from $k+1$. Thus, a stable estimate for r_s usually requires that t be at least three to five times as large as k. For smaller t's the variance of r_s will be large and so will any prediction intervals based on r_s.

An estimate for p_m that is insensitive to nonmodel error is obtained by using r_s in place of $\hat{\sigma}$ in (9). The new estimate, p_{rm}, is given by

$$p_{\mathrm{rm}} = r_s\sqrt{1+\bar{P}_c}. \tag{16}$$

Fig. 6 shows p_{rm} and p_m for the 1 024 simulations plotted as a function of t for 0 and 0.1 levels of nonmodel error, ν. This new estimate approximates the correct value very well with nonmodel error present, and it appears to be nearly unbiased. However, because of the variation in r_s, this estimate has a wide variation for small t. This can be seen in Fig. 7, which shows the rms of the ratio of p_{rm} to p_m and the rms plus and minus one standard deviation and the ratio extremes of the 1 024 simulations with a nonmodel error of 0.1. When t is greater than 44 (greater than four times k), one standard deviation of p_{rm} is less than 10 percent of the mean value of p_m. This demonstrates the validity of the remark made after (15) that t should be at least three to five times as large as k to get a stable value for r_s.

The estimate for p_t can not be extended to account for nonmodel error as easily as for p_m. However, since p_m is always greater than p_t, p_{rm} can be used as a bound for p_t.

V. Conclusion

With nonmodel error present, the predicted measurements will be inaccurate, and the estimates for the uncertainty in these predictions will be low. However, a nearly unbiased estimate for this uncertainty can be calculated with the use of the residuals at the reduced measurements. This paper has shown that this estimate is accurate if the number of reduced measurements is at least three to five times the number of

parameters being used in the model. Thus, this estimate of the uncertainty in the predictions can be used to assess the reliability of the model-based testing results.

This paper has not described how to monitor the measurements to detect a change in the underlying model or how to update the model estimate. This can be done by performing statistical tests on the ratio of $\hat{\sigma}$ to r_s to detect a change in the model. The appropriate action then depends on the frequency and level of the model changes observed and the desired testing accuracy.

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