NIST STRATEGIES FOR REDUCING TESTING REQUIREMENTS

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ABSTRACT

For the past several years, research has been carried out in the Electricity Division at the National Institute of Standards and Technology (NIST) to reduce the testing requirements for analog and mixed-signal devices. The most significant testing technique to result has been a model-based approach to the testing and calibration of such devices. The model is developed from empirical data, physical information, *a priori* information, or a combination of the three. Algebraic operations are performed on these data to create a model. The model approximately spans the vector space within which the device behavior can be described. With this model, the device can be characterized using significantly fewer measurements than is possible with traditional methods. A brief description of the techniques will be presented along with a summary of the results achieved in testing analog and mixed-signal devices.

INTRODUCTION

The research at NIST on the testing of complex electronic systems grew out of the desire to understand analog testing better and to reduce its cost during manufacture or calibration. Initially, test techniques were predominantly applied to data converters. However, it was realized that linear models could be used effectively to model the behavior of other devices (Stenbakken 1985).

The information comprising the model can be any combination of empirical data, physical information, and *a priori* information. Once gathered in vector form, the information

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is placed in a matrix model.

Empirical data, obtained from exhaustive testing of representative devices coming off the production line, are very robust sources of information for device modeling. The usefulness of empirical modeling is based on the assumption that a selected lot of devices will manifest all the variability of the manufacturing process (Souders 1990).

If detailed design information for a device is available, then physical information such as circuit component sensitivity matrices can be included in the model. When underlying device architecture results in well-defined error behavior, then *a priori* vectors may be used. An example of such a case is the predominance of individual bit errors in many types of data converters. An accurate model for this binary behavior can be constructed from Rademacher functions, which are a subset of the Walsh functions (Souders 1985).

Note that each column of the matrix model represents a piece of information that contributes to the description of the device. Therefore, the desired number of columns in the final reduced model should equal the number of underlying parameters governing the device behavior. Additionally, the individual rows represent specific test points at which measurements should be taken. These testing strategies have achieved the development of a model requiring only a limited number of test points. This reduction represents a significant decrease in test time and cost over exhaustive testing.

MODELING TECHNIQUES

A number of data sets (representing exhaustive device testing) and physical and *a priori* vectors must be obtained in linear matrix form, Y. Y is referred to as the modeling set. Next, the question arises as to what procedures to perform in order to derive a model and use this model to reduce testing requirements.

Linear system theory is a mature science offering a large variety of methods to solve systems of equations. A set of vectors is a basis for a vector space if, and only if, every vector in the vector space is a linear combination of the vectors in the basis (Schilling 1988). It is desirable that the model be an exact basis for the vector space in which the device operates. Every possible device output would then be exactly describable by the model.

The QR factorization or decomposition (QRD) and the singular value decomposition (SVD) are two methods suitable for efficiently finding a set of linearly independent vectors (an approximate basis set for a vector space) from a system of vectors (Stenbakken 1993). The basis so determined will be approximate because of noise in the measurement data.

The Mathematics

The modeling of a device is characterized by the linear constant-coefficient matrix equation

$$y = Ax, \tag{1}$$

where y represents a vector of measurements taken on the device to be tested, A is the model derived from Y, and x is the device parameter vector. Generally, the modeling set, Y, is initially created with the number of rows (test points) corresponding to exhaustive testing and the number of columns several times larger than the expected number of underlying parameters. The number of rows must be greater than the number of columns in order for an accurate estimate of the solution to be obtained. A linear system with more rows than columns is called overdetermined and may be solved using least-squares methods.

The first step in arriving at a model is to determine the number of columns in the model by performing an SVD on the columns of Y. This is a matrix factorization yielding

$$Y = USV^T, \tag{2}$$

where U and V are orthogonal matrices and S is a diagonal matrix whose elements are the

singular values associated with the Y matrix (Leon 1980). The vectors in U are called the left singular vectors. Each singular value along the diagonal of S corresponds to a left singular vector. The ^{T} refers to the transposed matrix.

The U matrix is created by orthogonalizing linear combinations of the vectors in Y. Therefore, U has the property that it spans the same vector space as Y. Both singular values and singular vectors are arranged in descending order of importance in describing the information in Y. As the singular values approach zero, the corresponding vectors in U contribute less new information. Although the singular values will never reach zero for a system containing measurement noise, relatively few of the vectors in U are usually needed to explain the structure of any measurement vector (Stenbakken 1993). Once U is obtained, it is used to estimate A as the model in the reduction process. The number of columns of U that are maintained in the model is established using the following bound:

$$\frac{\sum_{i,j=1}^{n} s_{ii}^2}{mn} \leq t^2, \quad t \geq \sigma_m, \tag{3}$$

where s_{ii}^2 is the i^{th} singular value, n is the original column dimension and m is the row dimension of Y, and σ_m is the standard deviation of the measurement noise. The bound t is determined based on specific accuracy needs. The standard deviation of the measurement noise can be estimated by computing the standard deviation of repeated measurement sets on a single device. Use of this bound inequality can sometimes be avoided by looking at a plot of the singular values and visually determining the desired number of vectors from the plot. Computational efficiency is traded for model accuracy as more vectors are added to the model. However, it is possible to optimize the selection of columns for the model. The optimum model size gives the minimum root-mean-squared (rms) difference between the predicted values and the true values, rather than the measured values (Stenbakken 1993). A specific number of columns, j, is selected and a column-reduced version of U is established. The system equation becomes

$$y = U_C x, \tag{4}$$

where the subscript refers to the fact that the model is not the complete U, but only the first j columns. Note that the individual columns of Y are lost in the transformation to U. However, row information remains intact.

Reducing the number of rows is desirable to obtain a minimum set of test points. The minimization cannot be performed using SVD because individual row (test point) identities would be lost. Therefore, the QR decomposition (QRD) is used to select only linearly independent rows of U_C for inclusion in the model. To use the QRD for test point selection, the matrix U_C must first be transposed, since QRD is a column operation. The QRD maintains the identity of the columns on which it operates. The QRD of the rows of U_C is

$$PU_C^T = QR, \tag{5}$$

where P is a permutation matrix that reorders the columns of U_C^T (the rows of U_C) such that the diagonal of R is monotonically decreasing, the Q matrix is a square matrix with orthogonal columns, and the R matrix is an upper triangular invertible matrix the same size as U_C (Leon 1980). The main diagonal of R contains values showing linear independence of one column relative to the previous columns of U_C^T. To select test points, the columns of U_C^T corresponding to the j largest values on the diagonal of R are selected. This is equivalent to selecting the j most linearly independent rows of U_C. Now the reduced model is a $j \times j$ square matrix. These j test points are the minimum required to solve the system of equations.

It is advisable to use more than the minimum number of test points in order to reduce the effects of measurement noise (lower prediction variance) and to provide redundancy to allow the detection of model errors. One

technique to select more test points is an algorithm that uses the maximum pointwise prediction variance to sequentially choose test points. The prediction variance vector, σ_p^2, is calculated using the equation

$$\sigma_p^2 = diag[U_C(\tilde{U}_C^T\tilde{U}_C)^{-1}U_C^T]\cdot\sigma_m^2, \quad (6)$$

where U_C is already defined as the full-row length column-reduced model, $\tilde{U}_C$ is the row-reduced and column-reduced model, σ_m^2 is the measurement variance (constant scalar for all points), and *diag* refers to taking the diagonal of the bracketed matrix. The diagonal expression is calculated using $\tilde{U}_C$ with size j, and the row with the largest value is selected as a test point. The model $\tilde{U}_C$ now has size *j+1*, and the steps are repeated until the desired model size is achieved. Adding rows to the model reduces the prediction variance at the expense of greater test time. A factor of 2 to 4 times the number of columns has been found to be a reasonable trade-off for the number of rows.

The system of equations corresponding to the reduced model is given as

$$\tilde{y} = \tilde{U}_C x, \quad (7)$$

where the reduced set of measurement data is denoted $\tilde{y}$. Once the reduced model is obtained and the reduced set of measurements is taken, the device parameter vector can be estimated using the least-squares solution

$$\hat{x} = (\tilde{U}_C^T\tilde{U}_C)^{-1}\tilde{U}_C^T\tilde{y}. \quad (8)$$

At this point, the full-length measurement vector can be estimated using the full-length model and the estimate of the device parameter vector as

$$\hat{y} = U_C\hat{x}. \quad (9)$$

The goal of estimating device behavior at all test points from measurements taken at only a limited set of test points has been achieved.

Validating the Model

To validate the model, it is desirable to have a set of complete measurement vectors that have not been used in the model building. The residual errors at the complete set of measurement points for this validation set can be calculated using the equation

$$\varepsilon_v = y - \hat{y} = y - U_C\hat{x}. \quad (10)$$

If the residual errors appear to be random and have standard deviation approximately equal to σ_m the model is considered valid. If the residual errors exhibit systematic structure or have a large standard deviation, then the model development will need to be repeated with a larger and presumably richer modeling set, *Y*.

Testing Devices

After validation, the model can be used to test devices coming off the production line, in a calibration lab, etc. Recall that more than the minimum number of test points are measured so that the residual error at these test points can be calculated using the equation

$$\varepsilon_p = \tilde{y} - \tilde{U}_C\hat{x}. \quad (11)$$

The residual error can be checked periodically to monitor model accuracy. If the rms value is significantly larger than the known measurement error, or if particular test points consistently and systematically differ from zero, the model may need to be updated.

APPLICATION EXAMPLES

One automatic test equipment manufacturer currently offers an application software package to implement this modeling approach, and several integrated circuit (IC)

manufacturers are using or evaluating the method on production lines.

In early verification tests at NIST, positive results were achieved modeling a commercial 12-bit digital-to-analog converter (DAC) using a model made up of eleven Rademacher functions and two vectors of superposition errors calculated from empirical data from two devices. The model characterized the ten most significant bits out of the twelve bits. Thirteen measurements were used out of a possible 1024. With use of only the minimum necessary measurements, errors were predicted with maximum uncertainty no greater than 2.5 percent of the peak error (Souders 1985).

The method was also applied successfully to a commercial 13-bit analog-to-digital converter (ADC). A model was developed using a combination of physical and empirical data. The empirical portion of the model was developed from exhaustive data (8192 test points) taken on 50 devices. The decomposition and test point selection methods produced a model with 18 parameters. Sixty-four test points were selected for the reduced model. Predictions using the model were compared with exhaustive testing of 77 devices and the rms value of the differences was 0.024 least-significant bit (LSB) (0.0003 percent of full-scale where one LSB is 2^{-13} or 0.012 percent of full-scale) (Souders 1991). Results are shown in Figures 1 and 2.

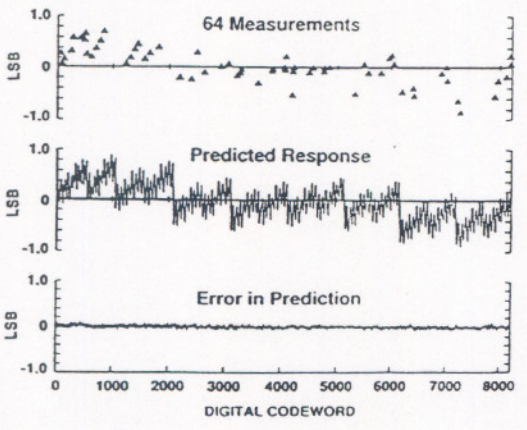

Figure 1 Test results on a 13-bit ADC

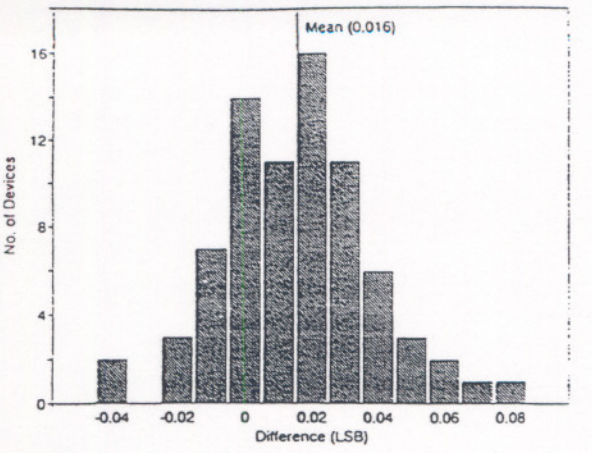

Figure 2 Test result summary for 77 devices

In addition to the mixed-signal IC applications, the NIST strategies have been applied to a multirange thermal voltage converter analog instrument. The manufacturer had specified 255 test points as required for exhaustive testing. This instrument was modeled using the full test data from 100 production units. The decomposition techniques provided 20 parameter coefficients and 50 test points. This 50×20 reduced model produced an rms residual error of 0.036 (or 3.6 percent) of the manufacturer's tolerance specification when tested on 39 additional units. Figure 3 shows a typical measurement vector with its residual error after prediction, offset from zero for clarity. Figure 4 shows the prediction error of the 39 validation vectors. The data have been

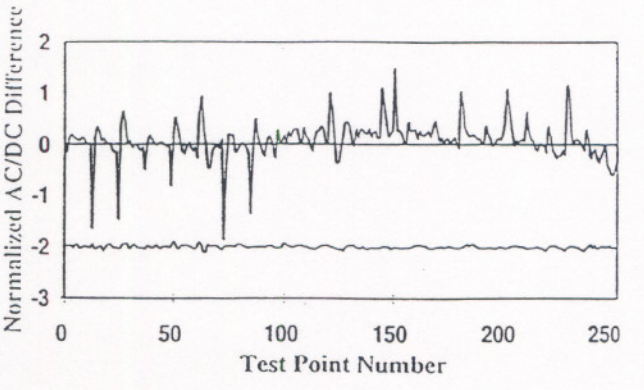

Figure 3 Typical response (top) and residual error using 50 test points

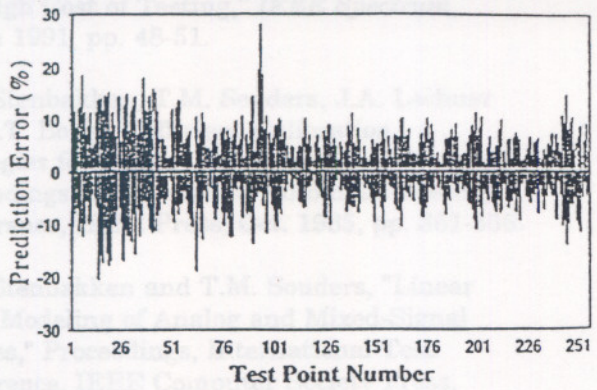

Figure 4 Residual error of the validation set normalized to the manufacturer's tolerance specification so that any measurement error within 100 percent of the tolerance specification meets the manufacturer's requirements (Koffman 1993).

QUALITY ASSURANCE

An individual confidence interval for the true response at any test point can be determined. An interval about the predicted value $\hat{y}$ that contains the true value in $1-\alpha$ percent of infinitely repeated trials is calculated using

$$|y_{true} - \hat{y}| \le diag[\sigma_m^2 U_C(\tilde{U}_C^T \tilde{U}_C)^{-1} U_C^T]^{1/2} t_{1-\alpha/2}, \quad (12)$$

where $t_{1-\alpha/2}$ is a value from a Student's t distribution with $(n-j)(m-j)$ degrees of freedom based on the determination of σ_m (Hwang 1994).

A simultaneous confidence interval containing all m true values in $1-\alpha$ percent of infinitely repeated trials is determined using

$$|y_{true} - \hat{y}|^2 \le diag[j\sigma_m^2 U_C(\tilde{U}_C^T \tilde{U}_C)^{-1} U_C^T] F_{1-\alpha}, \quad (13)$$

where m is the number of test points, j is the number of parameter coefficients, and $F_{1-\alpha}$ is a value from an F distribution using j degrees of freedom for the numerator and $(n-j)(m-j)$ degrees of freedom for the denominator, based on the determination of σ_m^2 (Hwang 1994).

CONCLUSIONS

The NIST strategies for reducing testing requirements have been effectively applied to analog and mixed-signal devices. The methods are currently being used by several private companies with implementation under way at several other organizations. A software package is presently being developed at NIST to expedite the use of these techniques.

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