

Dynamic Routing and Congestion Control Through Random Assignment of Routes

Fern Y. HUNT

Information Technology Laboratory
National Institute of Standards and Technology
Gaithersburg, Maryland 20899, USA

Vladimir MARBUKH

Information Technology Laboratory
National Institute of Standards and Technology
Gaithersburg, Maryland 20899, USA

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Abstract

We introduce an algorithm for maximizing utility through congestion control and random allocation of routes to users. The allocation is defined by a probability distribution whose degree of randomness as measured by its entropy, is controlled. We seek to use the entropy to modulate between the two extremes of path specification; minimum cost path selection on one end and equiprobable path allocation on the other. We explore the trade-off between stability and utility in two sample topologies.

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constrained optimization model were proved in [2].

Single path problems are on the other end of the spectrum of path specification. Here multiple paths linking source and destination are available as in the multipath case, but only a single path is selected, for example a path with minimum cost. Wang et al [6] obtained sufficient conditions for the existence of a solution to a single path utility maximization problem. One of their main results states that when there is no duality gap, optimal solutions of the utility maximization problem are equilibrium points of their TCP/AQM-IP algorithm (see [1] for a discussion of this point). Here the congestion prices are used as link costs. However the existence of an equilibrium is not guaranteed. Indeed when a duality gap exists, there is no equilibrium, moreover if it does exist it may be unstable. They propose adding a static component to the link cost in order to stabilize it. Wang et al also establish that there is no duality gap for the multipath problem. Since the utility of the optimal solutions of the latter are an upper bound to the single path utility equilibrium we can ask the following question. Can the requirement for a single path of minimum cost be relaxed so that one can achieve stability and larger utility on the one hand and yet not be so relaxed as the multipath problem where (implicitly) one would have to identify all the paths joining a source and destination with equal and minimum cost? This paper presents a model that attempts to address this question.

In what follows we present an optimization problem for congestion control and route allocation where users are randomly assigned routes according to a probability distribution whose degree of randomness is controlled by a fixed parameter h_s , the entropy of the distribution for a source s . The case $h_s = 0$ corresponds to the single

1 INTRODUCTION

Recent progress in the development of protocols that maximize utility through joint congestion control and routing has uncovered a number of interesting questions regarding the algorithms that represent these protocols. Building on earlier work (see [3] and [2] for references) Kelly and co-workers examined the stability of global optimization algorithms that represent TCP/AQM protocols with routing. In [2] a fluid flow model of a multipath route allocation policy was analyzed. Here a number of routes connecting a source and destination may be available and the goal is to assign traffic and bandwidth rates along routes so that network utility is maximized. If one assumes there is no time delay associated with communication between source and link (resource) layers, global stability of equilibria of the model in [3] was proved and the global stability of equilibria associated with a more

path allocation problem while the maximum value $h_s = \log(\# \text{ of routes for } s)$ corresponds to the problem of allocation among equiprobable routes.

2 MODEL

The optimization problem that our protocol seeks to solve is:

$$\max_{\hat{x} \geq 0} \max_{\hat{\beta} \geq 0} \sum_s U_s(x_s) \quad (1)$$

$$\sum_s \sum_{r \in R_s(l)} \beta_{sr} x_s \leq c_l \quad (2)$$

$$\forall s \quad \beta_{sr} \geq 0, \quad \sum_{r \in R_s} \beta_{sr} = 1 \quad (3)$$

$$- \sum_{r \in R_s} \beta_{sr} \log \beta_{sr} \leq h_s \quad (4)$$

where s is the index of the source-destination pair, U_s is the corresponding utility function, $\hat{x} = \{x_s\}$ is the vector of source rates, and $\hat{\beta} = \{\beta_{sr}\}$ is the matrix whose rows are the route probability distributions for source s . The constraints (2) require that all routes that use link l , i.e. routes r in $R_s(l)$ of source s be assigned bandwidth rates $\beta_{sr} x_s$ so that the total link load does not exceed the capacity c_l . Finally, (3) is the usual requirement for probability distributions and (4) places an upper bound on the degree of randomness for the distribution $\{\beta_{sr}\}$ for source s . Small values of h_s mean that the problem is closer to the single path case and as h_s increases we are closer to the equiprobable case- a version of the multipath problem. Following the approach of [4] we developed an algorithm based on a gradient projection iteration method for the dual optimization problem. We will discuss the convergence of this algorithm and its stability as a function of h_s for two sample topologies. As in [4] and [6], the dual variables are link costs changing in response to congestion. The route allocation distributions follow dynamics that minimize the average route cost and this plays a role in the onset of instability when h_s is small. In [5] where the idea of random route allocation was first proposed, the authors introduced an adaptive algorithm involving non-constant values of h_s . Through simulations they demonstrated the trade-off between stability and utility. They found that the maximum utility occurs near the boundary between stability and instability. We find similar results in our algorithm.

3 Summary of Results

We give a brief summary of some results for the first sample topology, a network consisting of a single source-destination pair of nodes connected by two links. In this situation each link is also a route. Given capacities $c_1 > c_2$ where c_1 and c_2 are the capacities of link 1 and link 2 respectively, suppose for some h_s we have $\frac{c_1}{c_2} = \frac{\beta_1^{(h_s)}}{\beta_2^{(h_s)}}$.

Here $\beta_1^{(h_s)}$ and $\beta_2^{(h_s)}$, the proportion of traffic allocated to links 1 and 2 respectively define a probability distribution with entropy h_s . Note that when $h_s < \log(2)$ is fixed, there are two such distributions. There exists a steady state $(x_s^*, p_1^*, p_2^*, \beta_1^{(h_s)}, \beta_2^{(h_s)})$. Here x_s^* is the source rate, $\{p_i^*\}$ are the link costs for $i = 1, 2$. We give sufficient conditions for this steady state to be an optimal solution of (1)-(4) and sufficient conditions for its stability. For fixed values of c_1, c_2 , we computed the time averaged utility and found the maximum to occur near h_s for which the above mentioned steady state exists. For smaller values of h_s we see instability of our algorithm in the sense of sustained periodic behavior, which could be an analogue of the route flapping phenomenon observed in single path allocation protocols and mentioned in [2],[5] and [6].

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