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# METHODS, APPARATUS, AND PROCEDURES FOR THE COMPARISON OF PRECISION STANDARD RESISTORS

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**ABSTRACT**

Reference is made to some of the more important contributions that have been made to the subject of precise measurements of electrical resistance. The sensitivity of bridges when used with the modern high-sensitivity moving-coil galvanometer is discussed rather fully. Special consideration is given to the methods and apparatus used and to the procedures followed in the National Bureau of Standards in those comparisons in which the precision desired is of the order of 1 part in a million.

The more important factors limiting the precision of the comparisons, such as load coefficients, terminals, and contacts, thermoelectromotive forces, insulation, and the optical system of the galvanometer, are discussed rather fully. A method of analysis of networks containing both linear and nonlinear four-terminal conductors is given, and the theoretical basis for the experimental procedure used in determining the effect of slight defects in the insulation is pointed out. This is followed by a brief discussion of Ohm's law from the standpoint of precise resistance measurements and by a brief discussion of units of resistance. Finally, reference is made to more than 100 publications having a more or less direct bearing on the subject of resistance comparisons.

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## I. INTRODUCTION

Over a period of many years information pertinent to the comparisons of the resistances of precision standard resistors has been accumulating in this Bureau. This paper presents in a connected form the more important parts of this information not adequately explained in previous publications.

The paper is intended to present:

1. For those having a general interest in electrical measurements, a somewhat comprehensive discussion of the Wheatstone bridge method and of the Thomson bridge method, and a brief description of the apparatus used and procedures followed in this Bureau in making resistance comparisons to a precision of 1 part in 1 million.

2. For those making precise resistance comparisons, a discussion of various factors having a bearing on the precision of such comparisons.

To serve this twofold purpose the work of others previous to 1910 is reviewed briefly, and in this review it is pointed out that the conclusions reached by Heaviside, Maxwell, and others were based on assumptions which do not conform to present conditions. Fundamentals are considered in the earlier parts of the paper, and these are explained rather fully, while highly technical discussions are, for the most part, placed in appendices. The manner of presentation makes it possible for those whose interest is only general to familiarize themselves with some of the fundamentals of precision resistance comparisons without reading more than approximately the first third of the paper, and for those who may be interested only in some particular phase of the problem, such as thermoelectromotive forces or the analyses of networks, to find readily (by referring to the index at the end of the paper) what is given on the subject.

## II. REVIEW OF EARLIER WORK

The first measurements, by a null method, of what later was recognized as a definite property of a conductor, called the resistance, were made by Becquerel [3].[1] Using a differential galvanometer, he obtained the first definite proof of the relation between the resistance, length, and cross section of wires. His results were published in 1826. It was in the same year that Ohm [66] published his most important paper, though it has hitherto been generally considered that Ohm's law is of a somewhat later origin. In 1833 Christie [12], who was not familiar with the work of Ohm, described an arrangement of wires which later became known as the Wheatstone bridge. With this connection he verified the results obtained by Becquerel and also determined the relative conductivities of a number of metals. As the significance of Ohm's work was not understood until later, Christie was in the position of having devised one of the best methods of measuring resistance before the concept of resistance had become definitely established.

In 1843 Wheatstone [117, 118] presented a paper before the Royal Society of London, in which he defined resistance and referred to "standard of resistance" and "resistance coils."

In 1862 Thomson [94] published a paper in which he described what he called a "New Electrodynamic Balance for the resistances of short bars and wires." This later became known as the Thomson bridge, the Thomson double bridge, the Kelvin bridge, and the Kelvin double bridge. Thomson seems to have been the first to attempt measurements of the highest precision attainable with the apparatus then available and the first to have even an approximate understanding of the factors limiting the precision of measurement.

Since then the sensitivity of bridges has been a subject of much discussion. Of the more important of the earlier contributions, mention should be made of a paper by Schwendler [79] published in 1866, of a paper by Heaviside [32] published in 1873, a paper by T. Gray [26] published in 1881, and the second and third editions [58] of Maxwell's Electricity and Magnetism, 1892. The conclusions given in these publications were based on an assumption that the battery used was not capable of supplying all of the power desired. Obviously, this assumption would seldom be valid at the present time. Furthermore, no account was taken of the electromotive force developed by the relative motion of the winding and magnet of the galvanometer. This constitutes another reason why the conclusions given are not applicable in case a modern high-sensitivity moving-coil galvanometer is used. However, not all of the earlier writers on this subject were of the opinion that the then available sources of electric power constituted a limiting factor in the attainable sensitivity, since as early as 1862 Thomson, in the paper to which reference has been made, said "I shall conclude by remarking that the sensibility of the method which has been explained, as well as of Wheatstone's balance, is limited solely by the heating effect of the current used for testing." In 1889 Paalzow and Rubens [68], in connection with a study of bolometers, made a rather thorough investigation of the effects of heating by the current in a Wheatstone bridge, but their conclusions are not

[1] Figures in brackets indicate literature references at the end of this paper.

directly applicable to resistance measurements. In 1892 Guye [28], in a further study of bolometers, pointed out that the effect of heating by the test current is proportional to the temperature coefficient of the material from which a bridge arm is constructed, proportional to the square of the current in it, and inversely proportional to its facilities for dissipating heat. He also pointed out that by making all, or pairs, of the bridge arms alike in all respects, the effects of heating by the test current could be almost completely compensated, unless the power dissipation in the bridge arms were unusually large.

In 1893 Glazebrook [23] gave the results of a series of measurements of standard resistors, using different test currents. He found the resistance of 10-ohm standard resistors increased perceptibly as the test current was increased from 0.05 to 0.15 ampere, but he concluded that since the increase in resistance was proportional to the square of the test current, the effect of the smaller test current was extremely small.

In 1895 this was again pointed out by Schuster [78], who evidently was not familiar with the work of Thomson, though he was familiar with that of Guye. Schuster's conclusion was that "The highest percentage accuracy with which a given resistance can be measured is directly proportional to the square root of the maximum electric work which can be done on it without overheating." In 1906 Jaeger [40] and Smith [82, 83, 84], independently, and more recently Von Steinwehr [86], discussed the subject from the same point of view. Nevertheless, the conclusions of Heaviside, Gray, and Maxwell still persist.

The effect of the electromotive force developed by the relative motion of the winding and magnet of the galvanometer is less obvious. However, it has been taken into consideration by Jaeger [40] and by Von Steinwehr [86], both of whom give a different formula for the sensitivity of the Wheatstone bridge (also the Thomson bridge), according to the use of a moving-magnet galvanometer or a suitably damped moving-coil galvanometer.

## III. SENSITIVITY OF BRIDGES

### 1. DAMPING OF GALVANOMETERS

In the modern high-sensitivity moving-coil galvanometer the electromotive force generated in the galvanometer during the time the deflection is changing at its maximum rate may be of the same order of magnitude as the impressed electromotive force. Therefore, during this time it has a marked effect on the current and consequently on the motion of the coil. What is observed is a damping of the motion of the coil, and, among other factors, this depends upon the resistance of the complete galvanometer circuit, that is, the resistance to an electromotive force in the galvanometer branch of the bridge. If this resistance is much less than that which results in critical damping, the movement of the coil toward any new equilibrium position is very sluggish. If, on the other hand, this resistance is considerably more than that which results in critical damping, the coil continues for some time to oscillate about any new equilibrium position. Neither condition is conducive to rapid nor accurate measurements. To obtain a satisfactory performance, either a magnetic shunt or an auxiliary resistance is used for adjusting the damping. As the latter is more convenient and is more generally used, it only will be considered in deriving expressions for the sensitivity. However, it should be pointed out that in general somewhat higher sensitivities may be obtained by the use of an adjustable shunt on the magnet of the galvanometer.

It is convenient to think of the resistance to an electromotive force in the galvanometer branch as consisting of two parts, namely, the resistance of the galvanometer and the resistance external to the galvanometer. In case the resistance of the bridge between its galvanometer terminals, with the galvanometer branch open, is less than the external resistance which gives a desired damping the auxiliary resistance, $U$, is connected in series with the galvanometer, as shown in figure 1, and adjusted so as to give the desired damping. In case the resistance of the bridge between the galvanometer terminals is so high as to give insufficient damping, the auxiliary resistance is placed in parallel with the bridge and galvanometer, and so adjusted as to give the desired damping. Some prefer to have the damping critical in all cases. However, measurements can be made somewhat more quickly when the damping is approximately two-thirds critical, which results in an "overshoot" of 6 percent. Also in some cases the sensitivity is higher with the damping less than critical, while in others it is higher with critical damping. Consequently, it is not desirable to use the same damping in all cases. However, to avoid undue complications, it will be assumed, for the present, that the auxiliary resistance will be so adjusted as to give the same damping in all cases, and this will be referred to as the specified damping.

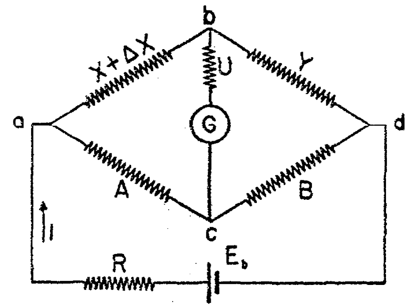


FIGURE 1.—*Circuit diagram of unbalanced Wheatstone bridge.*

Here $X$, $Y$, $A$, and $B$ represent values of the resistances of the four arms for which the bridge would be balanced.

### 2. WHEATSTONE BRIDGE

In what follows, the arms of the Wheatstone bridge will be designated $X$, $Y$, $A$, and $B$. When the bridge is balanced, the resistances of the arms will also be designated $X$, $Y$, $A$, and $B$. To indicate that a reduction of the resistance of the $X$ arm by an amount $\Delta X$ would establish a balance of the bridge, the resistances of all arms of the bridge will be designated as shown in figure 1.

Therefore,

$$X/Y=A/B. \tag{1}$$

With the galvanometer branch open, let

$E$ = the potential drop from $a$ to $d$,
$E_x$ = the potential drop from $a$ to $b$,
$E_a$ = the potential drop from $a$ to $c$, and
$E_g$ = the difference between the potential of $b$ and $c$.

Obviously then

$$E_x = \frac{E(X+\Delta X)}{X+\Delta X+Y}, \tag{2}$$

$$E_a = \frac{EA}{A+B}, \text{ and} \tag{3}$$

$$E_g = \mp \frac{E(X+\Delta X)}{X+\Delta X+Y} \pm \frac{EA}{A+B}. \tag{4}$$

Since the direction of $E_g$ taken as positive is immaterial, to avoid complications in what follows the $\mp$ sign will be taken as positive, and the $\pm$ sign will be taken as negative. Then, since from eq 1 it follows that

$$\frac{A}{A+B} = \frac{X}{X+Y}, \tag{5}$$

eq 4 takes the form

$$E_g = \frac{EX}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)} - 1\right] \text{ or} \tag{6}$$

$$E_g = \frac{EX}{X+Y}\left[(1+\Delta X/X)\left(1-\frac{\Delta X}{X+Y}+\left(\frac{\Delta X}{X+Y}\right)^2-\text{etc.}\right)-1\right]. \tag{7}$$

If the bridge is nearly balanced, second- and higher-order terms may be neglected, in which case eq 7 reduces to

$$E_g = EY\Delta X/(X+Y)^2. \tag{8}$$

Letting $dX$ represent the proportional decrease in the resistance of the $X$ arm, which would establish a balance of the bridge,

$$dX = \Delta X/X. \tag{9}$$

Consequently,

$$E_g = EXYdX/(X+Y)^2. \tag{10}$$

Since the potential difference which would appear across a break were a branch of a network opened, may be considered as an electromotive force acting in that branch, $E_g$ may be considered as an electromotive force in the galvanometer circuit.

Now let $D$ be the change (in scale divisions) of the deflection of the galvanometer resulting from unit change of the electromotive force in the galvanometer circuit when the resistance $V$ connected in series with the galvanometer is that which gives the specified damping. Hereafter $D$ will be referred to as the sensitivity of the galvanometer. Also let $W$ be the resistance of the bridge between its galvanometer terminals, $b$ and $c$ of figure 1, with the galvanometer branch open. The case in which $W$ is less than $V$ will be considered first. Then $U$ is placed in series with the galvanometer, as shown in figure 1, and so adjusted that

$$U+W=V.$$

Consider that the zero of the galvanometer scale is at an end of the scale, that $Q$ is the scale reading with the battery branch of the bridge open, that $Q_1$ is the scale reading with the battery connected as shown in figure 1, that $Q_2$ is the scale reading with the leads to the battery interchanged, and that the galvanometer is so connected that, with $dX$ positive, $Q_1>Q>Q_2$. Then from eq 10 it follows that

$$Q_1-Q=DEXYdX/(X+Y)^2, \tag{11}$$

and

$$Q_2-Q=-DEXYdX/(X+Y)^2. \tag{12}$$

Subtracting eq 12 from eq 11 eliminates $Q$ and gives

$$dQ=2DEXYdX/(X+Y)^2. \tag{13}$$

where $dQ(=Q_1-Q_2)$ is the change in the deflection of the galvanometer following a reversal of the connections to the battery.

Now let $S$ represent the combined sensitivity of the bridge and galvanometer, that is, define $S$ by the equation

$$S=dQ/dX. \tag{14}$$

Then it follows from eq 13 and 14 that

$$S=2DEXY/(X+Y)^2. \tag{15}$$

Since from eq 1 it follows that

$$XY/(X+Y)^2=AB/(A+B)^2, \tag{16}$$

another expression for the sensitivity is

$$S=2\ DEAB/(A+B)^2. \tag{17}$$

However, if the resistance $W$ of the bridge between its galvanometer terminals is greater than the resistance $V$, which gives the specified damping of the galvanometer, the resistance $U$ is placed in parallel with the galvanometer and adjusted so that

$$UW/(U+W)=V. \tag{18}$$

From eq 10 it follows that with breaks in both the $U$ and galvanometer branches the potential drop across each of the breaks

$$E_{gu}=EXYdX/(X+Y)^2. \tag{19}$$

With the galvanometer branch only open, the current in the $U$ branch is $E_{gu}/(U+W)$, while the potential drop across the break in the galvanometer branch, $E_g$, is $U$ times this current, or $E_{gu}U/(U+W)$ Therefore, since $U/(U+W)=V/W$.

$$E_g=EXYdXV/W(X+Y)^2. \tag{19a}$$

That is, the effect of the resistance in parallel with the galvanometer (in parallel with the bridge, if considered from the standpoint of the electromotive force developed in the galvanometer coil as a result of its motion), when of such value as to give the specified damping of the galvanometer, is a reduction of the electromotive force in the galvanometer circuit, and consequently of the combined sensitivity of the bridge and galvanometer by the ratio of $V$ to $W$ [101]. Therefore, instead of the relations given by eq 15 and 17,

$$S=2DE\ \frac{XYV}{(X+Y)^2W}, \tag{20a}$$

and

$$S=2DE\ \frac{ABV}{(A+B)^2W}. \tag{20b}$$

With reference to these equations it should be noted: (1) That although they are first-order approximations, for the purpose at hand they may be considered as exact, and it is immaterial whether the potential drop across the bridge is measured with the galvanometer branch (and its parallel branch) open or closed. (2) That if they are to apply with the resistance used in adjusting the damping of the galvanometer either in series or in parallel with the galvanometer, in all cases in which the ratio of $V$ to $W$ is greater than one it is to be taken as one.

In what follows, the relative positions of the battery and galvanometer shown in figure 1 (and fig. 2) will be considered as their normal positions. With the position of the galvanometer and battery and their respective rheostats interchanged,

$$S=2D\epsilon\frac{AXV}{(A+X)^2W}, \tag{21a}$$

and

$$S=2D\epsilon\frac{BYV}{(B+Y)^2W}. \tag{21b}$$

Here $\epsilon$ is the potential difference between branch points $b$ and $c$, which now are the battery terminals of the bridge.

Except for limitations imposed by the heat developed in the bridge arms by the test current, eq 20 and 21 are in convenient form for use. However, if $E$ (or $\epsilon$) exceeds a certain magnitude, the heating in the bridge will result in a change of the resistance of one or more of the bridge arms by an amount in excess of that permissible or in excess of that corresponding to the precision sought in the measurement. There is, therefore, a fairly definite upper limit to the sensitivity which may be used in any particular case. This will be referred to as the permissible sensitivity. The permissible sensitivity depends on the sensitivity of the galvanometer to an electromotive force in a circuit giving the specified damping, the resistance external to the galvanometer which gives the specified damping, the relative magnitudes of the bridge arms, the resistance of the bridge between its galvanometer terminals (factors which have been considered above), the precision sought in the measurement, and the load coefficients of the bridge arms.

The load coefficient of a conductor will be defined as the ratio of the proportional change in its resistance to the power dissipated in it. It would be logical therefore to develop formulas for the sensitivity of bridges based explicitly on the power dissipated in each bridge arm. However, there is some advantage in using either the current [40] or the potential drop in each bridge arm, and here the potential drop will be used. How these factors, especially the precision desired in the measurement and the load coefficients, limit the permissible sensitivity may be seen by considering the following example.

Assume that

$D=10$ millimeters per microvolt,
$V=15$ ohms,
$X=10$ ohms,
$Y=5$ ohms,
$A=50$ ohms, and
$B=25$ ohms.

Assume that the bridge is to be balanced to 1 part in a million and that investigation of the load coefficients has shown that a change of 1 part in a million occurs

in $X$ when $E_x$, the potential drop in $X$, is 0.75 volt,
in $Y$ when $E_y$, the potential drop in $Y$, is 0.75 volt,
in $A$ when $E_a$, the potential drop in $A$, is 2 volts, and
in $B$ when $E_b$, the potential drop in $B$, is 1.5 volts.

Obviously, if the resistance of $X$ is to be calculated from values assigned to $A$, $B$, and $Y$, the potential drop in none of the four resistances can be permitted to exceed the value just stated, and preferably it should not exceed two-thirds this value. In the absence of known compensating effects, it will be assumed that the maximum permissible sensitivity is obtained when the potential drop in the bridge is as high as possible, without that in any arm of the bridge being higher than two-thirds that which results in a proportional change in the resistance equal to the precision sought in the measurement.

With the battery and galvanometer in their normal positions, it is readily seen that $W=20$ ohms, also that as $E$ is increased $E_x$ is the first to reach the maximum permissible value and that $E_x=EX/(X+Y)$. Therefore, eq 20*a* may be written

$$S=2DE_x\frac{YV}{(X+Y)W}, \tag{22}$$

and taking $E_x=0.5$ volt gives

$$S=\frac{2\times10\times10^6\times.5\times5\times15}{15\times20}=2.5\times10^6. \tag{22a}$$

That is, the change in deflection of the galvanometer following a reversal of the connections to the battery is 2.5 mm per part per million lack of balance of the bridge.

With the positions of the battery and galvanometer interchanged, it is readily seen that $W$ is less than $V$; that, as $\epsilon$ is increased, $E_b$ is the first to reach the maximum permissible value; and that $E_b=\epsilon B/(B+Y)$. Therefore, eq 21*b* may be written

$$S=2DE_b\frac{YV}{(B+Y)W}. \tag{23}$$

Taking $E_b=1$ volt and $V/W=1$ gives

$$S=\frac{2\times10\times10^6\times1\times5\times1}{30}=3.3\times10^6, \tag{23a}$$

or a deflection of 3.3 mm per part per million lack of balance of the bridge.

It will thus be seen that either arrangement of battery and galvanometer gives a permissible sensitivity more than ample for establishing balances of the bridge to 1 part per million, and that there is not much choice between the two.

If the bridge were to be balanced to 1 part in 4 million, the permissible potential drop in each bridge arm, and consequently the permissible sensitivity, would be only half as large, while the precision sought is higher by a factor of 4. Consequently, the permissible residual deflection of the galvanometer could be only one-eighth of what it might equally well be in making a balance to 1 part in a million. On the other hand, if the load coefficients were smaller by a factor of 4, the permissible potential drops would be higher by a factor of 2, and consequently the permissible sensitivity would be higher by a factor of 2.

Returning now to a general consideration of the sensitivity of the Wheatstone bridge, it should be noted that with the battery and galvanometer in their normal positions,

$$E=E_x(X+Y)/X=E_y(X+Y)/Y=E_a(A+B)/A=E_b(A+B)/B, \qquad (24)$$

and with the positions of the battery and galvanometer interchanged,

$$\epsilon=E_x(A+X)/X=E_y(B+Y)/Y=E_a(A+X)/A=E_b(B+Y)/B, \qquad (25)$$

Therefore, it follows from eq 20 and 24 that

$$S=2DE_x\frac{YV}{(X+Y)W}, \qquad (26x)$$

$$S=2DE_y\frac{XV}{(X+Y)W}, \qquad (26y)$$

$$S=2DE_a\frac{BV}{(A+B)W}, \qquad (26a)$$

and

$$S=2DE_b\frac{AV}{(A+B)W}, \qquad (26b)$$

and from eq 21 and 25 that

$$S=2DE_x\frac{AV}{(A+X)W}, \qquad (27x)$$

$$S=2DE_y\frac{BV}{(B+Y)W}, \qquad (27y)$$

$$S=2DE_a\frac{XV}{(A+X)W}, \qquad (27a)$$

and

$$S=2DE_b\frac{YV}{(B+Y)W}. \qquad (27b)$$

Now, if it is understood that $E_x$, $E_y$, $E_a$, and $E_b$ each represents the maximum permissible potential drop in $X$, in $Y$, in $A$, and in $B$, each of eq 26 and each of eq 27, in general, gives a different sensitivity. However, that one of eq 26 which gives the lowest sensitivity gives the maximum permissible sensitivity with the battery and galvanometer in their normal positions; while that one of eq 27 which gives the lowest sensitivity gives the maximum permissible sensitivity with the positions of the battery and galvanometer interchanged. This statement concerning the maximum permissible sensitivity is based on the assumption that the resistance of one arm of the bridge is to be calculated from known values of the resistances of the other arms. Later it will be shown that measurements may be made in such a way as to largely eliminate the effect of heating by the test current in one or more of the bridge arms, and that the effect of heating in all bridge arms, if not excessive, may be determined experimentally and an appropriate correction applied. In either case, a somewhat higher sensitivity may be permissible.

It will be noted that by keeping the galvanometer branch closed and reversing the connections to the battery, the permissible sensitivity is twice that which would be obtained by first closing the battery branch and then the galvanometer branch. In addition this eliminates the effects of thermoelectromotive forces and leakage from power circuits, insofar as these remain constant over a time corresponding to a few periods of the galvanometer. Furthermore, a few reversals of connections to the battery, at intervals corresponding approximately to the period of the galvanometer, with the galvanometer branch closed eliminate the effect of gradual drifts and hysteresis in the galvanometer deflections. This is of great importance, since usually in measurements of the highest precision, adjustments must be carried to a point at which the changes in the deflection of the galvanometer resulting from a lack of perfect balance of the bridge are much less than the changes in the deflection resulting from disturbing influences.

## 3. THOMSON BRIDGE

Referring to figure 2, a material simplification in the analysis may be brought about by using Kennelly's [48] Δ to $Y$ transformation, that

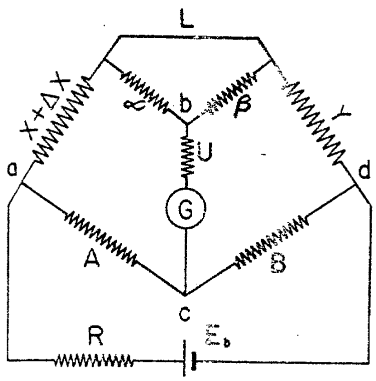


FIGURE 2.—*Circuit diagram of unbalanced Thomson bridge.*

Here $X$, $Y$, $A$, $B$, $\alpha$, and $\beta$ represent resistances of the six arms for which the bridge would be balanced.

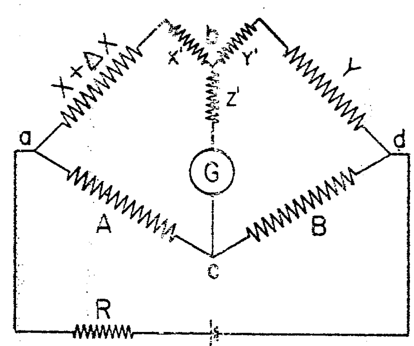


FIGURE 3.—*Circuit diagram shown in figure 2, with the delta connected resistances, $\alpha$, $\beta$, and $L$, replaced by the equivalent star-connected resistances, $X'$, $Y'$, and $Z'$.*

is, by considering the three Δ connected conductors, $L$, $\alpha$, and $\beta$, as replaced by three $Y$ connected conductors, $X'$, $Y'$, and $Z'$, as is shown in figure 3.

This is the same as the Wheatstone bridge, except that there is a resistance $X'$ in series with $X$, $Y'$ in series with $Y$, and $Z'$ in series with the galvanometer.

The procedure followed in deriving eq 26 and 27 then leads to

$$S=2DE_x\frac{(Y+Y')V}{(X+X'+Y+Y')W}, \tag{28x}$$

$$S=2DE_y\frac{(X+X')V}{(X+X'+Y+Y')W}, \tag{28y}$$

$$S=2DE_a\frac{BV}{(A+B)W}, \tag{28a}$$

and

$$S=2DE_b\frac{AV}{(A+B)W}, \tag{28b}$$

if the positions of the battery and galvanometers are as shown in figure 2, and

$$S=2DE_x\frac{AV}{(A+X+X')W}, \tag{29x}$$

$$S=2DE_y\frac{BV}{(B+Y+Y')W}, \tag{29y}$$

$$S=2DE_a\frac{(X+X')V}{(A+X+X')W}, \tag{29a}$$

and

$$S=2DE_b\frac{(Y+Y')V}{(B+Y+Y')W}, \tag{29b}$$

if the relative positions of the battery and galvanometer are interchanged.

With reference to the differences between eq 26 and 28 and eq 27 and 29, it should be pointed out that for the $Y$-connected conductors to be equivalent to the $\Delta$-connected conductors it is necessary that

$$X'=L\alpha/(L+\alpha+\beta),$$

$$Y'=L\beta/(L+\alpha+\beta), \tag{30}$$

and

$$Z'=\alpha\beta/(L+\alpha+\beta).$$

In all cases, $X'$ and $Y'$ are each less than $L$, and usually $L$ is very small in comparison with $A+X$ and $B+Y$. Consequently, there are relatively few cases in which $X'$ and $Y'$ may not be omitted from eq 29$x$ and 29$y$, which then become identical with eq 27$x$ and 27$y$. In few if any cases is the permissible sensitivity determined by the power dissipated in $A$ or $B$. Therefore there is no need for drawing conclusions from either eq 29$a$ or eq 29$b$. The Thomson bridge method, with the positions of the battery and galvanometer interchanged, is not used in the comparison of precision standard resistors. However, the Thomson bridge method may be used in resistance thermometry, and then there are advantages in interchanging the positions of the galvanometer and battery.

With reference to eq 28*x* and 28*y*, which apply when the battery and galvanometer are in their normal positions (see figs. 2 and 9), as a balance is approached by the procedure to be described later, $(Y+Y')/(X+X'+Y+Y')$ approaches $Y/(X+Y)$ and $(X+X')/(X+X'+Y+Y')$ approaches $X/(X+Y)$. Equations 28 may, therefore, be considered to be the same as eq 26. However, with the battery and galvanometer in their normal positions and with $A$, $B$, X, and $Y$ the same in both the Thomson bridge and the Wheatstone bridge, $W$ is higher in the Thomson bridge than in the Wheatstone bridge. In the comparisons of precision standard resistors made in this Bureau by the Thomson bridge method, this is of no consequence, since $W$ is less than $V$. Under these conditions, the sensitivity of the Thomson bridge is the same as that of the Wheatstone bridge.

It is improbable that a case might arise in which the power dissipated in the $\alpha$ arm or the $\beta$ arm of the bridge would limit the permissible sensitivity, whether the positions of the battery and galvanometer are normal or interchanged. Therefore, no equation containing the potential drop in $\alpha$ or in $\beta$ is given.

## IV. LOAD COEFFICIENTS

Reference has already been made to load coefficients defined as the ratio of the proportional increase in the resistance to the power dissipated. This definition requires some amplification, since, in all cases, time and the medium surrounding the resistor are involved. In the following discussion it will be assumed that the resistors are immersed in oil of low viscosity and that this oil is kept in fairly rapid circulation. Fortunately, most precision resistors come to an approximate temperature equilibrium with the oil in a fairly short time, usually less than 1 minute. With resistors of the type developed in this Bureau about 30 years ago [76], which are sealed in cases containing oil, the change in resistance is rapid at the start of the measuring current and later is very gradual. These resistors apparently have two thermal-time constants, one of about 30 seconds and one of about 30 minutes. Stated in another way, the difference in temperature between the resistance element and the oil in the sealed container becomes nearly constant in a minute, while the difference in temperature between the oil in the container and the oil of the bath becomes nearly constant in an hour. The load coefficients stated for standards of this type apply when the current has been passing from 1 to 2 minutes, the time usually required for making a measurement.

From what has just been said it might be assumed that load coefficients are proportional to temperature coefficients and inversely proportional to the facilities provided for dissipating heat. Experience shows that, in general, this is so only if the design is such that the heating by the test current results in no marked mechanical strain. Consequently, in measuring load coefficients it may be better, when possible, to use a procedure such that this assumption is reduced from a first to a second or third order of importance.

As an illustration of this point consider that it is desired to determine the load coefficient of a 1-ohm standard. In that case the procedure might be as follows:

(1.) From among the available 0.1- and 0.01-ohm standards select from each denomination one of the better from the standpoint of facilities for dissipating heat and low temperature coefficient.

(2.) Inspect these two standards and make an estimate of their relative facilities for dissipating heat.

(3.) From the estimate of their relative facilities for dissipating heat and their known temperature coefficients make an estimate of the ratio of the load coefficient of the 0.01-ohm standard to the load coefficient of the 0.1-ohm standard, and designate this ratio $k$.

(4) Place the 0.1-ohm standard in the $X$ arm and the 0.01-ohm standard in the $Y$ arm of a bridge, and balance the bridge by adjustments of the $A$ arm, first with 0.1-watt and then with 0.5-watt power dissipation in the 0.1 ohm-standard. Assuming that the power dissipated in the $A$ and $B$ arms of the bridge has no appreciable effect on their resistances, the load coefficient of the 0.1-ohm standard (that is, the change in the resistance of the 0.1-ohm standard resulting from the dissipation of 1 watt in it) is taken as 2½ times the proportional increase in the resistance of the $A$ arm divided by $(1-0.1\ k)$.

(5) Place this 0.1-ohm standard in the $Y$ arm of the bridge and a 1-ohm standard in the $X$ arm, and balance the bridge first with one and then another potential drop across the bridge. The load coefficient of the 1-ohm standard is taken as equal to the proportional increase in the resistance of the $A$ arm of the bridge divided by the increase in power dissipation in the 1-ohm standard plus 0.1, the load coefficient of the 0.1-ohm standard as determined in (4). It will be noted that the result obtained involves the initial estimate of the load coefficients of the 0.01-ohm standard relative to that of the 0.1-ohm standard to the extent of only 1 percent. The value thus obtained for the load coefficient of the 1-ohm standards may, therefore, be presumed to be somewhat more precise than the value obtained for the load coefficient of the 0.1-ohm standard.

Load coefficients of standards of higher nominal values may be determined as outlined above, but for standards of the lowest nominal value used a different procedure is required. If among these there are two standards of such construction that it may be assumed that the temperature rises for equal power dissipations are equal and that the temperature inequalities cause little or no mechanical strain, and if these two standards have markedly different resistance-temperature coefficients, their load coefficients may be determined from their resistance-temperature coefficients and the difference of their load coefficients obtained by direct comparison.

Having determined the load coefficients of one or more standard resistors, the load coefficients of others of the same nominal value are readily determined by direct comparisons, whether or not their performances are normal.

Investigation of wire standard resistors of the Physikalisch-Technische Reichsanstalt design having winding areas of approximately 40 cm$^2$ has shown that the proportional change in their resistance resulting from the dissipation of 1 watt in their windings is about the same as the proportional change in their resistance resulting from increasing the temperature of the oil bath 1° C. It may be concluded therefore that the temperature rise of the resistance material above the temperature of the oil is about 1° C per watt power dissipation.

On the same basis it has been concluded that for the sealed standard resistors developed in this Bureau about 1907 and the double-walled type constructed prior to 1930 [89] the temperature rise is also about 1° C per watt power dissipation. For standard resistors of lower denominations in which the resistance material is in the form of sheets, the temperature rise per watt dissipation is less. For those of the Physikalisch-Technische Reichsanstalt design, of the smaller size, the temperature rise is about 0.2° C for the 0.01 ohm, 0.4° C for the 0.001 ohm, and 0.3° C for the 0.0001 ohm per watt dissipation.

Since most of the temperature coefficients are less than 20 parts per million, errors resulting from heating will, in general, be less than 1 part in 2 million if the potential drops are limited to the following values:

0.15 volt for 1-ohm,
.5 volt for 10-ohm,
1.5 volts for 100-ohm,
5. volts for 1,000-ohm,
15. volts for 10,000-ohm,
0.05 volt for 0.1-ohm,
.035 volt for 0.01-ohm,
.0075 volt for 0.001-ohm, and
.0025 volt for 0.0001-ohm standard resistors.

The temperature rises, and consequently the load coefficients, depend on the viscosity and other factors affecting the circulation of the oil, the design of the standards, and their temperature coefficients, so there must of necessity be large variations. However, the potential drops stated above are about the maxima permissible in measurements to 1 part per million, unless the load coefficients are known to be abnormally low, or a procedure is followed for eliminating the error which otherwise would result from the heating by the test current.

In routine testing the schedule of potential drops given above is followed approximately in the comparisons of resistors having nominal values of 1 ohm and less. In the comparisons of resistors having nominal values of 10 ohms and more the potential drops used are much less than those given in the schedule. In special cases the criterion used for the maximum permissible potential drops, without an application of a correction for the heating, is a very small but definitely noticeable change in the balance of the bridge on increasing the potential drop across the bridge by a factor of 2.

If there is occasion to use larger potential drops, if there is reason to suspect one or more abnormally large load coefficients, or if the load coefficient of one or more of the bridge arms is not known to the accuracy necessary for obtaining the desired precision, a valid correction for the effect of the heating may be obtained by balancing the bridge first with what is presumed to be a suitable potential drop across the bridge and second with a somewhat larger potential drop across the bridge. If the second potential drop is twice the first, the effects of the temperature rise in each arm of the bridge will be 4 times as large as with the first potential drop. Consequently, if the balances are established by adjustments of the $A$ arm (or $A$ and $\alpha$ arms) of the bridge and the value taken for $A$ is four-thirds the first minus one-third the second, a correction is applied for the effects of heating by the test current in all arms of the bridge.

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## V. GALVANOMETER USED WITH NBS PRECISION BRIDGE

The galvanometer which has been and still is being used in most of the precision resistance measurements made in the National Bureau of Standards was designed and constructed especially for the purpose about 1914 [116]. It has an adjustable shunt on the magnet, all-copper circuit, and taut suspensions with the center of gravity of the moving system slightly off the axis of rotation. This latter feature makes it possible by tilting to adjust the period, $T$, over a range from about 5 to about 15 seconds. In cases in which really high sensitivity is desired the performance seems to be most satisfactory with a period of about 10 seconds. The shunt of the magnet is so adjusted that with a period of 10 seconds the external resistance giving critical damping is 35 ohms. This adjustment once made is seldom changed as the shunt is not readily accessible. The galvanometer is used with a scale distance of 1.5 m. With critical damping the operating constants then are

$$T=10 \text{ seconds},$$
$$D=30 \text{ mm}/\mu\text{v},$$
$$V=35 \text{ ohms},$$

or with two-thirds critical damping, which results in a 6 percent overshoot and gives about the maximum speed of operation with the 10 second period, the operating constants are

$$T=10 \text{ seconds},$$
$$D=20 \text{ mm}/\mu\text{v},$$
$$V=60 \text{ ohms}.$$

It may be of interest to see what the permissible sensitivity is in the comparison of 10-ohm standards when the $A$ and $B$ arms of the bridge are each 25 ohms, the battery is in the normal position, and the galvanometer is used critically damped. In this case the resistance, $W$, of the bridge between galvanometer terminals is 17½ ohms. With $E_x$ or $E_y=0.5$ volt, it follows from eq $26_x$ or $26_y$ that $S=2\times30\times.5\times10/20=15$ mm per part per million lack of balance of the bridge. This is not only much higher than is needed but is higher than it is desirable to use. In some cases, however, the permissible sensitivity is none too high, and in a very few cases it is not quite sufficient for the establishment of balances to 1 part per million. In these the potential drop may be increased to the point at which errors resulting from heating (calculated from a knowledge of the load coefficients of the particular standards used) and from lack of sensitivity are approximately equal, or even beyond this point, and then a correction for the heating is determined and applied. In a very few cases there would be an advantage in using a galvanometer of higher sensitivity. However, for a considerable part of the measurements the galvanometer is adjusted so as to have a period of from 6 to 8 seconds. Then both the sensitivity and the external resistance giving a specified damping are less than with the 10-second period.

## VI. NBS PRECISION BRIDGE

The bridge now in use in the comparisons of precision standard resistors was designed and constructed in this Bureau in 1918. All parts of the bridge arms and the adjustable resistor used in regulating the damping of the galvanometer are immersed in oil. The ammeter, voltmeter, and rheostats used in regulating the test current and other auxiliary equipment are conveniently located outside the oil bath. The entire bridge circuit is shielded against leakage from direct-current power circuits. The oil bath is thermostatically controlled at a temperature of 25° C, and during the time the apparatus is in use the oil is kept in vigorous circulation. During the summer the normal dew point is occasionally very nearly 25° C. To prevent the condensation of moisture in the oil bath and to maintain good insulation of the battery, galvanometer, and other parts of the circuit outside the oil bath, the air of the laboratory is dried by refrigeration.

The more important resistance sections of the bridge are of the double-walled sealed type developed jointly with others [98] of the Bureau's staff. The cases contain no oil, since the use of oil in permanently sealed resistors has long been considered inadvisable. To obtain low load coefficients the cases were made considerably larger than those first described, and the resistance wire was selected on the basis of low temperature coefficient. The primary of a well-insulated variable mutual inductor is connected in series with one of the battery leads, and its secondary is connected in series with one of the galvanometer leads. This inductor (not shown in fig. 5) serves to balance the electromotive force induced in the galvanometer circuit on reversing the test current.[2] The *A*-arm of the bridge is of the adjustable direct-reading type, such as is discussed in a recent publication [63] of this Bureau. It consists of five resistance sections, three of which are adjustable by means of dial switches. These three sections are shown in figure 4. With each of the dial switches set at its mid-position, the current in each shunt is one-tenth of that in the section shunted, and the sum of the resistances of the three

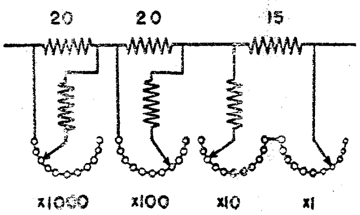


FIGURE 4.—*The part of the .. arm of the NBS precision bridge which is adjustable in steps of 0.0001 ohm over the range from approximately 49.4445 ohms to approximately 50.5555 ohms.*

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[2] The lack of an inductive balance manifests itself as ballistic deflection of the galvonometer, following a reversal of the test current. If this ballistic deflection is large, it limits, somewhat, the precision of the resistive balance. However, there is a more important reason why the ballistic deflections should be kept small. Most sensitive galvanometers when deflected alternately in one and then in the other direction, by equal amounts, have their rest points shifted slightly in the direction of the last deflection. For the galvanometer used with the NBS precision bridge, if the rest point is observed after a deflection in one direction and again after an equal deflection in the opposite direction, the difference between the two observed rest points is from 1 to 2 percent of the amplitude of the deflections. If, therefore, systematic errors from this source are to be insignificant, inductive balances must be such that the ballistic deflections are less than 50 times the change in deflection resulting from lack of resistive balances corresponding to the precision sought in the comparisons.

In the comparisons of standard resistors of the usual construction, having resistances in the range from 0.1 to 100 ohms, usually this condition is realized without a special device for making inductive balances and without special precautions on the part of the observer. However, if the resistors in the *X* and *Y* arms of the bridge have low resistances of different nominal values (such, for example, as 0.001 and 0.0001 ohm), and if a high precision is desired, a means for compensating the effect of the difference between their time constants and of mutual inductances between different parts of the bridge circuit is necessary. Likewise, if the resistors in the *X* and *Y* arms of the bridge are of the usual bifilar construction and have high resistances of different nominal values (such for example, as 10,000 and 1,000 ohms), and if a high precision is desired, provision should be made for compensating the effect of the distributed capacitances. The effects of self and mutual inductances and of distributed capacitances may be compensated by a mutual inductor having one of its windings connected in a galvanometer lead and the other winding connected in a battery lead between the current-reversing switch and a current terminal of the bridge, provided the inductance is adjustable over uitable range of positive and negative values.

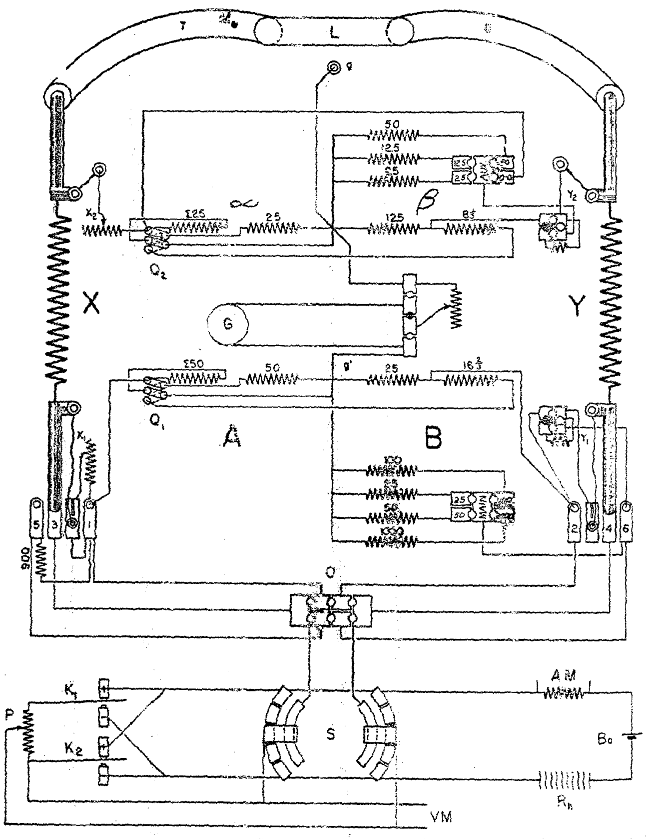


FIGURE 5.—*Diagram of essential features of the NBS precision bridge—connected as a Thomson bridge.*

In general, the notation here is the same as in figures 2, 9, and 16. Here 1 and 2 represent main terminal blocks (shown in detail in fig. 11), 3 and 4, current-terminal blocks; 5 and 6, terminal blocks for the less frequently used ratios of *A* to *B*; 7 and 8, hinged terminal blocks to accommodate standard resistors with different spacing of terminals; Σ 50, the three adjustable resistance sections shown in figure 4; $R_h$, compression carbon-plate rheostat for adjusting large test currents; *P*, potential divider for adjusting small test currents; *AM*, ammeter shunt; *VM*, voltmeter leads; $k_1$, $k_2$, and *S*, keys and switch for closing battery branch and reversing connections to battery leads, $k_1$ and $k_2$ for small test currents, *S* for large test currents; $Q_1$ and $Q_2$, copper links and terminal blocks with amalgamated contacts for connecting two sections of the *A* arm and of the α arm, either in series or in parallel. For the Wheatstone bridge method usually terminals 1 and 2 are used instead of 3 and 4, $x_1$ and $y_1$ are left disconnected, and the galvanometer circuit is closed by a connection between *M* and *g*.

sections with their shunts is 50 ohms. The resistance sections of the shunts are so chosen that the steps of the dial switches change the combined resistances of sections and shunts respectively by 0.0001, 0.001, 0.01, or by 0.1 ohm. The positions of each dial switch are numbered from 0 to 10. When the three shunted sections designated Σ50 in figure 5, are connected in series, or in parallel, with the 50-ohm section of the *A* arm, and any section of the *B* arm of the bridge used, differences in the readings of the dial switches correspond to differences in the ratio of *A* to *B* in parts per million of the nominal ratio of *A* to *B*. When the three shunted sections are connected in series with the 50-ohm section and the 900-ohm section, differences in the readings of the dial switches correspond to differences in the ratio of *A* to *B* in parts in 10 million of the nominal ratio of *A* to *B*. In all cases the *A* arm has its nominal resistance when the reading of the dial switches is approximately 5555.

The dial switches may be rotated indefinitely in either direction, while the complete range of adjustment is covered by a rotation of slightly less than 120°. The brushes, three in number for each switch, are of the multiple-leaf type, such as are used by Otto Wolff. They are mounted on what amounts to the feet of a rigid tripod, and the contact pressure is supplied by three coiled steel springs. This arrangement largely eliminates rocking of the brushes as the switch is rotated and gives a nearly constant pressure of the switches against the contact blocks. With the switch in any position, one of the brushes rests on an insulated segment and the circuit is through the other two brushes in series.

The dial switches are of good quality and operate under oil. Presumably, therefore, variation of the resistance of the contacts of each dial switch never exceeds 0.001 ohm. Since the resistance of each shunt is 10 times the resistance of the section shunted, a variation of 0.001 ohm in each of the four dial switches in the same direction results in a variation of the combined resistance of sections and shunts of 0.004/121 ohm. This is 1 part in 3 million in case *A* is nominally either 25 or 100 ohms, and 1 part in 30 million in case *A* is nominally 1,000 ohms. However, the probable effects of the variations of the resistances of the dial switches are approximately one order smaller than this.

The *B* arm (see fig. 5) has resistance sections of 16⅔, 25, 50, 100, and 1,000 ohms, most of which may be used singly or in combinations. Therefore, a number of values for the ratios of *A* to *B* may be had, such as 1 to 1, 2 to 1, 4 to 1, 10 to 1, 1 to 2, 1 to 4, and others seldom used. Each of these ratios may be varied by changing the readings of the dial switches of the *A* arm, and for the most part the range of variation is from 0.5 percent below to 0.5 percent above the nominal ratio.

Adjustments are such that differences in the readings of the dial switches, not in excess of 500, correspond to differences in the ratio of *A* to *B* well within 1 part in a million of the nominal ratio providing no one of the readings differs by more than 500 from that for which *A* has its nominal value. In exceptional cases corrections must be applied to the readings to give differences accurate to 1 part in a million of the ratio.

The $\alpha$ and $\beta$ arms (see fig. 5) are similar to the *A* and *B* arms, except that the resistance of corresponding sections is half as large, the $\alpha$ arm does not have a 450-ohm section corresponding to the 900-ohm section of the $A$ arm, and the $\beta$ arm does not have a 500-ohm section corresponding to the 1,000-ohm section of the $B$ arm. The dial switches of $A$ and $\alpha$ arms are mechanically connected so as to always have the same reading.

A general view of the bridge removed from the oil is shown in figure 6. A general view of the bridge and oil bath is shown in figure 7. Two features of the oil bath which should be mentioned are the means employed for securing a reasonably uniform temperature throughout the bath and of maintaining the desired temperature. The oil bath has a false bottom supported about 3 cm above the true bottom of the bath and extending to within about 1 cm of the side walls, which are of nickel-plated copper. Near the center of the false bottom there is an opening about 8 cm in diameter, and the circulating propeller is located beneath this opening and concentric with it. The circulation of the oil in the central part of the bath is downward, through this opening, outward between the false and true bottoms, upward next to the side walls to near the surface of the oil, and inward on and under the oil surface. The rate of the circulation is approximately 3 liters per second, so that the volume of oil passing through the central opening in 1 minute is approximately the same as the volume of oil contained in the bath. The heat for maintaining the temperature is supplied by small carbon-filament lamps located slightly above and near the opening in the false bottom. By means of radiation, a part of the heat developed in the lamp filaments is distributed through the oil almost instantly. That part of the radiation from the lamp filaments absorbed in and near the lamp bulbs is carried quickly to the side walls of the bath and surface of the oil, the parts from which heat is being lost by radiation and air convection.

## VII. BRIDGE EQUATIONS

In the usual discussion of the Wheatstone bridge it is assumed that if $X$ represents the unknown resistance, its magnitude is to be calculated from the equation

$$X = YA/B \tag{32}$$

That is, it is assumed that magnitudes of $Y$, $A$, and $B$ are each known to an accuracy at least as high as that expected for $X$, or that either the magnitude of $Y$ or $A$ and the ratio of $A$ to $B$ or $Y$ to $B$ are known to this accuracy. In measurements of the type under consideration neither of these assumptions can, in general, be made. It is therefore necessary to use some procedure which avoids the use of these assumptions.

### 1. SUBSTITUTION METHOD

If a number of nominally equal resistances $X_1$, $X_2$, $X_3$, $X_4$, etc. are to be measured and the resistances of two or more of these are known, for example $X_2$ and $X_4$, usually a standard of the same nominal value as those being measured is placed in the $Y$ arm, and the $B$ arm is made nominally equal to the $A$ arm. Then all of the $X$'s may be substituted one after the other in the $X$ arm, and the bridge balanced in each case by an adjustment of the $A$ arm. If $a_1$, $a_2$, $a_3$, $a_4$, etc., are the read-

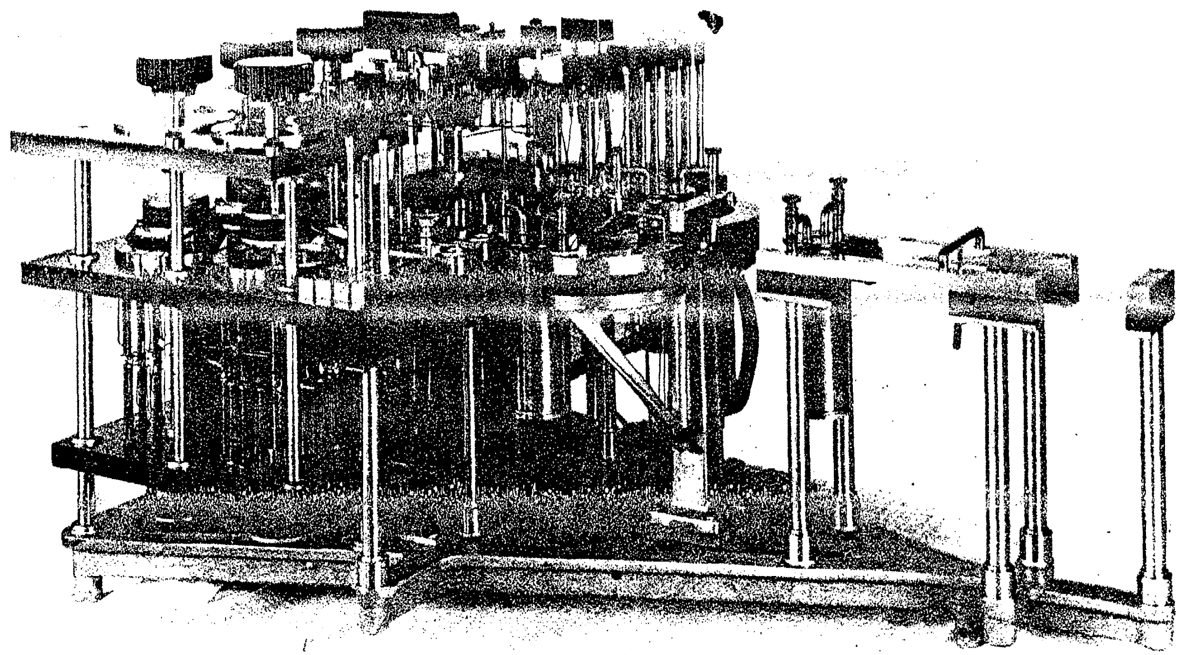

FIGURE 6. *General view of the NBS precision bridge (removed from oil bath).*

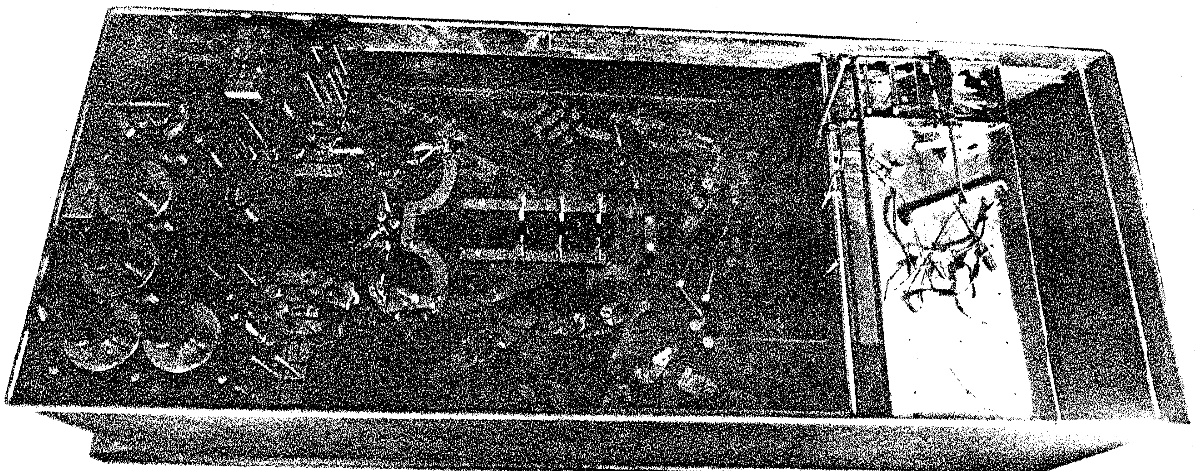

Figure 7.—*General view of the NBS precision bridge and oil bath.*

ings of the dial-switch settings, $A_n$ is the nominal value of $A$, and $a_n$ is the reading of the dial-switch settings for which $A=A_n$,

$$\left.\begin{aligned} X_1&=\frac{Y}{B}A_n(1+a_1-a_n)\\ X_2&=\frac{Y}{B}A_n(1+a_2-a_n)\\ X_3&=\frac{Y}{B}A_n(1+a_3-a_n)\\ X_4&=\frac{Y}{B}A_n(1+a_4-a_n)\end{aligned}\right\} \quad (33)$$

etc.

Letting $N$ represent the nominal value of $X$ and of $YA_n/B$, and $x$, $y$, and $b$ represent the amounts in proportional parts by which $X$, $Y$, and $B$ exceed their nominal values, eq 33 may be written in the form

$$N(1+x)=N(1+y)(1+a-a_n)/(1+b). \quad (34)$$

Expanding eq 34 and neglecting 3d and higher-order terms gives

$$x=y+(a-a_n)-b+y(a-a_n)-b(a-a_n)+b^2-by. \quad (35)$$

Letting $z$ represent that part of eq 35 which, for the series of measurements, is a constant, and, for the present, leaving out of consideration the second-order term $(y-b)(a-a_n)$ gives

$$\left.\begin{aligned} x_1&=a_1+z\\ x_2&=a_2+z\\ x_3&=a_3+z\\ x_4&=a_4+z\end{aligned}\right\} \quad (36)$$

etc.

Since $X_2$ and $X_4$ are known, $x_2$ and $x_4$ are known. Furthermore, all $a$'s will be read from the bridge. Therefore, $z$ may be determined from the second or fourth of eq 36, or what is better, from both of these equations and the mean value used. Substituting the mean value of $z$ in each of eq 36 gives a value for each of the $x$'s and consequently for $X_1$, $X_2$, $X_3$, $X_4$, etc. in terms of the unit in which values of $X_2$ and $X_4$ are expressed.

To see to what extent the values thus found are in error as a result of the approximations involved in eq 36, it is necessary to consider the second-order term $(y-b)(a-a_n)$ of eq 35, the magnitudes of $x_1$, $x_2$, $x_3$, $x_4$, etc. and what constitutes nominal values of each of the bridge arms. It was intended that resistances of the sections of the $A$ arm be so proportioned that differences of readings of the dials equal differences in resistance in parts per million of the resistance the $A$ arm has when the reading of the dials is 5555. This resistance (instead of 25, 100, or 1,000 ohms, depending on what sections are used and the way they are connected) should be taken as the nominal value of $A$. The appropriate multiple of the unit used is taken as the nominal value of the $X$'s, and nominal values of $B$ and $Y$ are so chosen that

$$B_n/Y_n=A_n/X_n.$$

As a result of the way in which the eq 36 are used $z$ includes all of $(y-b)(a-a_n)$ except the part equal to $(y-b)[a-(a_2+a_4)/2]$, while the nominal value chosen for $A$ makes

$$y-b=5555-a+x, \text{ approximately.} \tag{37}$$

Therefore $(5555-a+x)$ $[a-(a_2+a_4)/2]$ may be taken as the error in or a correction term for any one of eq 36. From the data obtained in a series of measurements it is therefore possible to determine, almost at a glance, the correction which, if significant, should be applied to one or more of eq 36. The first factor in this correction term is a constant for the series of measurements and depends upon the relative departures of $Y$ and $B$ from their nominal values. By a selection of the standard resistor to be used in the $Y$ arm, usually it is possible to make this factor less than 0.02 percent. If the first factor is less than 0.02 percent and the resistance of no one of the standard resistors substituted one after the other in the $X$ arm differs from the mean of $X_2$ and $X_4$ by more than 0.05 percent, in no case will the error or correction term exceed 1 part in 10 million.

Returning now to a further consideration of the sensitivity of bridges, the substitution method not only obviates the requirement of an accurate knowledge of the resistances of the $A$, $B$, and $Y$ arms of the bridge, but, for the most part, eliminates the effects of heating by the test current in these arms. Therefore, if the battery and galvanometer are in their normal positions, it is permissible to use in the $Y$ arm a standard resistor having a higher nominal value than those being substituted alternately in the $X$ arm and thus realize a somewhat higher permissible sensitivity, as may be seen by reference to eq 26$x$. If, on the other hand, the positions of the battery and galvanometer are interchanged, the use in the $Y$ arm of a standard of lower nominal value than those being substituted in the $X$ arm gives a higher permissible sensitivity only in case $W$ is larger than $V$, as may be seen by reference to eq 27$x$. Furthermore, it is not the load coefficients of the resistors being compared, but their differences or spread which limits the permissible potential drop in these resistors.

Equation 36, and others which follow under the heading "Bridge Equations," are applicable if the Wheatstone bridge method is used. They are also applicable if the Thomson bridge method is used, provided the adjustment is made in the manner which will be described later (or its equivalent), and provided further that the resistance of the potential lead which then constitutes a part of the $A$ arm of the bridge is not so large but that differences in readings of the dials may be taken as differences in the ratio of $A$ to $B$ in parts per million of its nominal value. If $x_1$ is the resistance of this lead, including the adjustable rheostat (see fig. 9), the effect of this resistance is taken into account by the addition to the right-hand members of eq 36 the second-order term $(x_1/A_n)[a-(a_2+a_4)/2]$. Usually $x_1/A_n$ does not exceed 0.05 percent, and $a-(a_2+a_4)/2$ does not exceed 0.05 percent, so usually this second-order term amounts to less than 1 part in 1 million.

A point which should be brought out here is that if a change in the unit of resistance is made, no change will be required either in the bridge or the procedure followed in the reduction of the observational data. A change in the unit will change $x$ in the correction term

$$[5555-a+x][a-(a_2+a_4)/2],$$

but an approximately equal change in $a$, $a_2$, and $a_4$ may be made by the selection of a different standard resistor for use in the $Y$ arm of the bridge.

## 2. INTERCHANGE METHOD

If two nominally equal resistances, $X_1$ and $X_2$, are to be compared, the bridge may be balanced by an adjustment of the $A$ arm, first with $X_1$ in the $X$ arm and $X_2$ in the $Y$ arm and then with $X_2$ in the $X$ arm and $X_1$ in the $Y$ arm. This gives two relations, which may be written as follows:

$$X_1=X_2\frac{A_n}{B}(1+a_1-a_n). \tag{38}$$

$$X_2=X_1\frac{A_n}{B}(1+a_2-a_n). \tag{39}$$

From eq 38 and 39 it follows that

$$X_2=X_1\sqrt{1+a_2-a_1+(a_2-a_n)(a_n-a_1)+(a_1-a_n)^2} \tag{40}$$

If, therefore, conditions are such that the second-order terms under the radical and terms of the order of $(a_2-a_1)^2$ may be neglected,

$$X_2=X_1\ [1+(a_2-a_1)/2]. \tag{41}$$

If, in addition, $X_1$ has so nearly its nominal value that $x_1(a_2-a_1)/2$ may be neglected,

$$x_2=x_1+(a_2-a_1)/2, \tag{42}$$

where $x_2$ and $x_1$ are the departures of $X_1$ and $X_2$ from their nominal values.

## 3. KNOWN RATIO METHOD

In the comparison of two resistances, $X_1$ and $X_2$, whose nominal values differ by a factor such as 2, 3, 5, or 10, different procedures may be used, but most of these only in special cases. The following is almost universally applicable and most frequently used in this Bureau.

The ratio of the bridge is set nominally equal to the ratio of the resistances of the two standards, which are placed one in the $X$ arm and the other in the $Y$ arm. Then, if $X_2$ is known and it is placed in the $Y$ arm of the bridge,

$$X_1=RX_2(1+a_1-a_r), \tag{43}$$

where $R$ is the nominal ratio of $X_1$ to $X_2$, $a_1$ is the reading of the dials of the $A$ arm of the bridge, and $a_r$ is the reading (as yet undetermined) of the dials of the $A$ arm, for which the ratio of $A$ to $B$ has the nominal ratio of $X_1$ to $X_2$. From eq 43 it follows that

$$x_1=x_2+a_1-a_r, \text{ approximately,} \tag{44}$$

where $x_1$ and $x_2$ are the departures of $X_1$ and $X_2$ from their nominal values.

If, however, $X_2$ is known and it is placed in the $X$ arm of the bridge

$$x_1=x_2+a_r-a_1. \tag{45}$$

If $A_n/B$ differs from the nominal ratio of $X_1$ to $X_2$ (or $X_2$ to $X_1$) by less than 3 parts in 10,000, and $X_1$ and $X_2$ differ from their nominal values by less than 3 parts in 10,000, the approximations are not likely to exceed 1 part in 10 million. The procedures followed in determining $a_r$ will be considered in the next section.

## VIII. ESTABLISHMENT OF KNOWN RATIOS

The principle underlying one of the procedures followed in finding the reading of the $A$ arm (that is, $a_r$ of eq 43) of the bridge for which the ratio of $A$ to $B$ is accurately an integer may be illustrated as follows. If there are at hand $n$ standards or coils having nearly equal values, $M_1$, $M_2$, $M_3$, etc., and $S$ is their resistance when connected in series,

$$S=M_1+M_2+M_3+\text{etc.} \tag{46}$$

Also, if $P$ is their resistance when connected in parallel,

$$\frac{1}{P}=\frac{1}{M_1}+\frac{1}{M_2}+\frac{1}{M_3}+\text{etc.} \tag{47}$$

Now, let $M$ be their mean resistance and $m_1$, $m_2$, $m_3$, etc. be the departures of each in proportional parts from the mean of all. Then

$$S=M(1+m_1+1+m_2+1+m_3+\text{etc.}) \tag{48}$$

and

$$\frac{1}{P}=\frac{1}{M(1+m_1)}+\frac{1}{M(1+m_2)}+\frac{1}{M(1+m_3)}+\text{etc.} \tag{49}$$

Assuming that the resistances of the standards or coils are so nearly equal that the third and higher powers of $m$ may be neglected, expansion by the binomial theorem of the terms $1/(1+m_1)$, $1/(1+m_2)$, $1/(1+m_3)$, etc. in eq 49 gives

$$\frac{1}{P}=\frac{1}{M}(1-m_1+m_1^2+1-m_2+m_2^2+1-m_3+m_3^2+\text{etc.}). \tag{50}$$

Since by definition $m_1+m_2+m_3+$etc. equals zero,

$$S=nM, \tag{51}$$

and

$$\frac{1}{P}=\frac{n}{M}\left(1+\frac{1}{n}\sum m^2\right), \tag{52}$$

where $\Sigma m^2 = m_1^2 + m_2^2 + m_3^2 +$ etc. It follows, therefore from eq 51 and 52 that

$$\frac{S}{P}=n^2\left(1+\frac{1}{n}\sum m^2\right). \tag{53}$$

There is no difficulty in adjusting a group of standard resistors or coils so that the resistance of no one differs from the mean of all by more than a few, or even 1 part in 10,000, in which case $\Sigma m^2/n$ would be not more than 1 part in 10 million or 1 part in 100 million, so usually $\Sigma m^2/n$ may be neglected. However, the conductors used in making the series and parallel connections will in general have resistances which cannot be neglected.

Presumably Lord Rayleigh was the first to use the same coils connected alternately in parallel and in series [72] in building up from unit standards. Steps of 4, 9, 16, 25, etc. are readily obtained simply by changing the connections of the appropriate numbers of coils of approximately equal resistances from parallel to series. Unfortunately, the square root of 10 is not an integer, so a step of 10, which is most frequently needed, cannot be obtained directly in this way.

In this Bureau an auxiliary apparatus constructed in 1912 is used for finding the reading of the dial switches of the $A$ arm of the bridge for which the ratio of $A$ to $B$ is accurately 10 to 1. The circuit of this apparatus is shown in figure 8. It will be observed that there are seven resistance sections, six of 150 ohms each, and one of 50 ohms, all connected in series. Two amalgamated copper links not shown in the figure serve to connect either group of three of the 150-ohm sections in parallel. Also other amalgamated copper links are provided for use in determining the relative resistances of the sections. The resistances of the links are relatively low and definite and the arrangement is such that the resistance of each link, including two amalgamated contacts and the appropriate portion of each of two terminal blocks may readily be measured. In the appendix on terminals and contacts, it will be shown that the effect of the resistances of parts of terminal blocks not included as parts of links or as parts of resistance sections is negligibly small. All other connectors are so arranged that the effect of their resistance is eliminated in the procedure followed in the use of the apparatus.

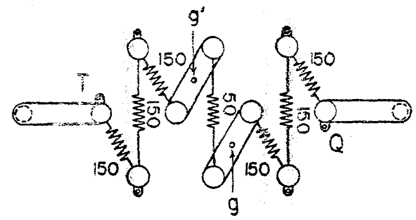

FIGURE 8.—*Circuit diagram of the auxiliary apparatus used for determining the reading of the dial switches of the NBS precision bridge, for which the ratio of the resistance of the A arm to the resistance of the B arm is 10 to 1.*

When this apparatus is placed in the bridge with a galvanometer connection made at $g$, it constitutes the $X$ and $Y$ arms of the bridge. With the three sections on the right connected in parallel and all other sections in series, the ratio of the resistance of the series sections to the resistance of the parallel sections, that is, the ratio of $X$ to $Y$, is very nearly 10 to 1. With $A$ set at a nominal value of 100 ohms, $B$ set at a nominal value of 10 ohms, and with no part of the auxiliary apparatus in the $A$ or $B$ arms, the bridge is balanced by an adjustment of the $A$ arm and the reading of the dial switches recorded. Then with the battery leads transferred to the points $T$ and $Q$, the bridge is again balanced by an adjustment of the $A$ arm and the reading of the dial switches recorded.

In the first balance, the resistance of the connection between the normal junction of the $X$ and $A$ arms and the point $T$ is in the $X$ arm, while in the second balance this resistance is in the $A$ arm. Likewise, in the first balance the resistance between the normal junction of $Y$ and $B$ and the point $Q$ is in the $Y$ arm, while in the second it is in the $B$ arm. Since the ratio of $A$ to $X$ and $B$ to $Y$ is 1 to 5, one-sixth of the difference between the two readings [111] of the dial switches applied to the first in the direction which moves it towards the second corrects for the error which otherwise would result from the resistances of the end connectors of the auxiliary apparatus. Now the auxiliary apparatus is turned through 180°, the parallel connectors are transferred to what is now the right side, and the galvanometer connection is made at $g'$, so as to again place the 50-ohm section in the $X$ arm of the bridge. Then two additional balances of the bridge are established and weighted as just described, giving a second reading of the dial switches, corrected for the effect of the resistances of the end connectors. Taking the mean of these two readings corrects for the difference in the mean resistance of the three sections on the right and the three sections on the left, as shown in figure 8. It is therefore the reading of the dial switches for which the ratio of $A$ to $B$ is accurately 10 to 1, excepting a correction to account for the lack of strictly proper adjustment of the 50-ohm section and a further correction to account for the resistances of the paralleling connectors. The first of these is $-0.1$, the amount expressed in proportional parts by which the resistance of the 50-ohm section exceeds the mean resistance of the other six sections taken three at a time in parallel, and corrected for the resistances of the paralleling connections, while the second is $+8/9$ the mean resistance of the paralleling connections expressed in proportional parts of 50 ohms. These corrections (which amount to only a few parts in 1 million and are easily determined to 1 part in 10 million) applied to the mean of the two readings, referred to above, give the reading $a$, of eq 43, 44, and 45, corresponding to a ratio of $A$ to $B$ equal to 10 to 1, with a probable error not in excess of 1 part in 3 million. However, to obtain this precision requires interpolation from galvanometer deflections, since the apparatus reads directly only to 1 part in 1 million. The reading of the dial switches of the $A$ arm of the bridge for which the ratio of $A$ to $B$ is 10 to 1 must be known to a high accuracy, since (starting with 1-ohm standards, which are used in maintaining the unit) any error in this reading enters once in the evaluation of the 10-ohm standards, twice in the evaluation of the 100-ohm standards, three times in the evaluation of the 1,000-ohm standards, and four times in the evaluation of the 10,000-ohm standards. It also enters once in the evaluation of the 0.1-ohm standards, twice in the evaluation of the 0.01-ohm standards, three times in the evaluation of the 0.001-ohm standards, and four times in the evaluation of the 0.0001-ohm standards.

When it is desired to find the reading of the $A$ arm for which the ratio of $A$ to $B$ is 2 to 1, the procedure is as follows:

1. The $A$ arm is connected so that its nominal resistance is 100 ohms, the $B$ arm is connected so that its nominal resistance is 50 ohms, the $X$ arm is arranged to receive two standard resistors connected in series, and the $Y$ arm is arranged to receive one standard resistor.

2. Three standard resistors having the same nominal value (preferably 100 ohms) are used.

3. The bridge is balanced by an adjustment of the $A$ arm with each of the three standard resistors placed one after the other in the $Y$ arm, the other two being in the $X$ arm.

4. The mean of the three readings of the $A$ arm, after a correction is applied to account for the resistance of connectors, is taken as the reading of the $A$ arm for which the ratio of the resistance of the $A$ arm to the resistance of the $B$ arm is 2 to 1.

A similar procedure is followed in determining the reading of the $A$ arm for which the ratio of $A$ to $B$ is 3 to 2, 3 to 1, 2 to 3, 1 to 3, 1 to 2, or involves other small integers.

## IX. ADJUSTMENTS OF THOMSON BRIDGE

A simplified diagram of the bridge circuit when the Thomson bridge method is used, is shown in figure 9. Here, $x_1$ represents one of the

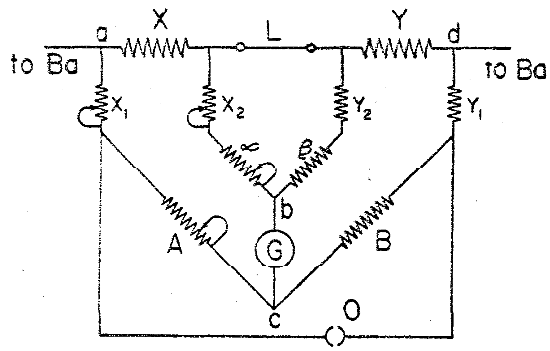


FIGURE 9.—*Circuit diagram of the Thomson bridge arranged for explanation of the procedure followed in establishing balances of the bridge in such a way as to make $A/B=X/Y$.*

potential terminals of $X$, the potential lead, and a rheostat continuously adjustable through a range of about 0.01 ohm; $y_1$ represents one of the potential terminals of $Y$, the potential lead, and a resistance which for the purpose at hand may be considered as fixed, though actually it is adjustable in steps; $x_2$ is similar to $x_1$; and $y_2$ is similar to $y_1$. $L$ and $O$ represent connectors of rather low resistance, which may be opened or closed at will.

The adjustments are carried out as follows:

(1) With the connector $L$ closed and the connector $O$ open, the direct-reading arms $A$ and $\alpha$ are so adjusted as to give a balance of the bridge. This makes

$$\frac{A+x_1}{B+y_1}=\frac{X}{Y} \text{ approximately.} \qquad (54)$$

(2) With the connectors $L$ and $O$ both closed, the bridge is balanced by an adjustment of $x_1$. These two adjustments make

$$\frac{x_1}{y_1}=\frac{X}{Y} \text{ approximately.} \tag{55}$$

(3) With the connectors $L$ and $O$ both open, the bridge is balanced by an adjustment of $x_2$ [74]. This makes

$$\frac{X+x_2+\alpha}{Y+y_2+\beta}=\frac{A+x_1}{B+y_1}. \tag{55a}$$

(4) With $L$ closed and $O$ open, the bridge is again balanced by an adjustment of the direct-reading arms which, as stated above, are mechanically connected so always read the same.

These four adjustments give

$$\frac{X}{Y}=\frac{A}{B}, \tag{56}$$

usually to the precision desired or attainable. The process is really one of successive approximations, but the observer soon learns to judge from the magnitude of the change in the reading of the dials in making the fourth adjustment and the known precision required in the various adjustments, if the adjustments need be repeated. Equation 56 is the same as the Wheatstone bridge equation. Therefore, in making a series of measurements, the data recorded and all calculations are the same as though the Wheatstone bridge method had been used. That is, only the readings of the dials for each of the final balances of the bridge are recorded, and these are substituted in the appropriate equation 36, 42, 44, or 45 considered above.

Since $x_1/A$ and $y_1/B$ seldom exceed 0.001, while in the final adjustment $x_1$ is in series with $A$ and $y_1$ is in series with $B$, the precision required in the second adjustment may be two or three orders lower than that sought in the measurement. Furthermore, the precision required in the third adjustment need not be high, and in many cases this adjustment may be omitted. An important feature of the procedure is that the adjustments result in making $X/Y=A/B$, whether or not the four-terminal conductors, $X$ and $Y$, are linear.

The simplicity of the adjustments results more from having suitably designed and well-constructed apparatus than from the procedure followed, which differs only in a minor detail from that described by Jaeger and Diesselhorst [45]. Procedures for determining the correction terms [P] of the Thomson bridge equation or for making these terms negligibly small have been discussed by several of those who have had occasion to use the Thomson bridge method.

## X. PRECISION AND ACCURACY

The expected precision of the measurements may not be fully realized on account of slightly faulty insulation in the bridge or in one of the resistances being measured, static electrical effects, rapidly varying thermoelectromotive forces, etc. Whether or not it is actually obtained depends to a large extent on the temperament and skill of the observer. To be properly qualified he should have a desire to do the job well, be neither easily fatigued nor perturbed, recognize slight disturbances promptly and be able to locate their source and correct the difficulty, and, above all else, be able to differentiate between that which is essential and that which is not essential in each of eight orders of magnitudes.

At present the resistance of no standard resistor can be presumed to be known (either in international ohms [3] or in absolute ohms) to an accuracy within one order of the precision to which two or more nominally equal resistances can be compared (if within the range considered here) or quite to the precision to which an 0.0001-ohm standard may be compared with a 10,000-ohm standard (provided a sufficient number of intermediate steps is used). It might seem, therefore, that the precision considered here is unnecessarily high. However, the precision of the resistance measurements is a factor contributing somewhat to the accuracy to which units of resistance may be realized and is an important factor in selecting the particular standard resistors used in maintaining a unit of resistance. For judging the relative quality of standard resistors, a precision of 1 part in 1 million is really needed, and it is a great convenience that for the most part this is readily obtained. As an indication of a limited need for a somewhat higher precision, it may be pointed out that the resistance of each of a fairly large group of 1-ohm standard resistors (of the double-walled type constructed by Thomas in 1933) is remaining so constant relative to their mean resistance that only by making the comparisons to a precision of about 1 part in 10 million can the changes occurring during a few months be detected.

In the above discussions a stated precision represents twice the probable error of a single measurement. The precision is estimated from the extent of the agreement of results on repetitions of the measurement under different conditions, or the consistency of results obtained in case more than the minimum required number of measurements is made.

For example, if five 1-ohm standards are to be intercompared, they may be substituted one after another in the same arm of the bridge. The five balances of the bridge thus obtained give data from which the resistance of each standard may be calculated in terms of the resistance of any one or the mean resistance of any two or more. Repetitions of these measurements under conditions giving different distributions of the systematic errors furnish data for determining the probable error of a single measurement by the substitution method, that is, the probable error of each of the five results obtained from five balances of the bridge.

The same five standards may be compared by interchange between the $X$ and $Y$ arms of the bridge in all possible combinations. The data obtained from the 20 balances of the bridge then serve in calculating the resistance of each standard in terms of the resistance of any one or the mean resistance of any two or more, the probable error in the results and the probable error in a single measurement by the interchange method. If all balances of the bridge were equally reliable, the probable error in the results obtained from the 20 balances of the bridge would be smaller than that obtained from five balances

[3] Here by "international ohm" is meant a unit of resistance realized by the use of a column of mercury under specified conditions, as distinguished from the unit of resistance of this country or the mean of the units of resistance of several countries.

of the bridge; also the probable error of a single measurement by the interchange method would be smaller than by the substitution method.

In the course of time many sets of measurements are made and repeated, so fairly definite estimates may be made of the precision obtained in individual measurements under various conditions. In repetitions of measurements after one or more days, the agreement of results occasionally is not as good as would be expected from the estimated precision of the measurements. In such cases, extending the measurements over longer periods of time usually leads to the conclusion that changes in the standard resistors are mainly responsible for the lack of agreement of results. Consequently, there is little increase in the accuracy of the measurements other than the detection of a possible error in recording a bridge reading, by making more than the minimum required number of balances of the bridge. Therefore, in comparing a number of standard resistors, usually this is done by the substitution method, making the minimum required number of balances of the bridge, and then repeating this series of measurements one or more days later, under conditions giving different readings of the *A* arm of the bridge. In case the results obtained from the two series of measurements are as consistent as should have been expected, means of the values thus obtained are taken as the results of the measurements. In case of a lack of a reasonable agreement in the results, further measurements are made to determine the cause and to remedy the difficulty should it be found to be a defect in the measuring apparatus instead of changes in the resistance of one or more of the standard resistors used in the series of measurements. In certificates for standard resistors, usually the stated accuracy of the values given is determined not by the precision of the measurements but by the uncertainty of the values in international ohms of the standard resistors used in maintaining units of resistance and the estimated changes with time of the resistances of the standard resistors being certified. More frequently than otherwise, the stated accuracy is 0.005 percent.

## XI. APPENDICES

In the above discussion a number of important points have been passed over with little or no consideration. Some of these will now be considered somewhat in detail under the headings: Terminals and Contacts; Thermoelectromotive Forces; Insulation; Optical System; Methods of Analyses; Ohm's Law; and Units of Resistance; while others will be found discussed in publications to which reference is made.

### APPENDIX 1. TERMINALS AND CONTACTS

#### (a) STANDARD RESISTORS

For the resistance of a conductor to be definite, one of several requirements is that the current always enter and leave the conductor in such a way as to give always the same or an equivalent distribution of the current density in that part of the conductor between the particular equipotential surfaces which serve in limiting or defining the resistance. In addition, the potential drop must always be taken between the same two or equivalent equipotential surfaces. Each time a standard resistor is removed from and replaced in a circuit in which there is an electromotive force some change occurs in the current distribution over the surfaces through which the current enters and leaves the standard, and the potential drop cannot always be taken between exactly the same two or equivalent equipotential surfaces. It will therefore be of interest to see what conditions are necessary in order that the resistance be definite to 1 part in 1 million, considering only the effects of contacts and current and potential distributions in terminals. But first it will be necessary to state more precisely what is to be understood by the resistance, or rather the resistances, of a conductor.

Since the more important resistances of a four-terminal conductor are more definite than is the resistance of a two-terminal conductor, four-terminal conductors will be considered first. Referring to figure 10, the heavy lines designated 1, 2, 3, and 4 will be considered as representing surfaces on which connections to other conductors may be made, that is, they will represent terminals. In case 1 and 4 are normally used as current terminals and 2 and 3 are normally used as potential terminals, the resistance may be defined as the ratio of the drop in potential from 2 to 3 to the current entering on 1 and leaving on 4. This will be designated the resistance (1234). In case 2 and 4 are used as current terminals and 1 and 3 as potential terminals, the resistance may be defined as the ratio of a drop in potential from 1 to 3 to the current entering on 2 and leaving on 4. This will be designated the resistance (2134).

It will thus be seen that a four-terminal conductor has as many four-terminal resistances as there are permutations of four numbers. However, only two of these resistances are independent of each other, and the relations between all are rather simple. Since there are so many resistances, it is a convenience to divide them into groups and to attach distinctive names to these groups. The four equal resistances (1234), (2143), (4321), and (3412) will be called the direct resistance. The four equal resistances (1432), (4123), (2341), and (3214) will be called the cross resistance, and the four equal resistances (1243), (2134), (3421), and (4312) will be called the diagonal resistance. When defined in this way, the direct resistance minus the cross resistance equals the diagonal resistance, as was pointed out in a previous paper [101]. Furthermore, any one of the 12 remaining four-terminal resistances is equal in magnitude but opposite in sign to either the direct, the cross, or the diagonal resistance.

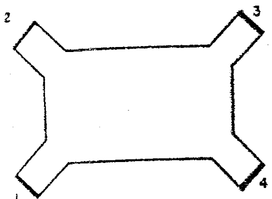

FIGURE 10.—*Generalized four-terminal conductor.*

It is also a convenience to divide four-terminal conductors into groups in accordance with the relative magnitudes of the resistances and attach names to these groups. Conductors for which the cross resistance is less than a millionth part of the direct resistance will be called linear,[4] and all others will be called nonlinear.

If variations in the current and potential distribution on terminals are to affect the direct resistance by certainly less than 1 part in a million, the terminals must be so designed that the sum of the following potential ratios will be less than 0.000 001. With 1 and 4 used as current terminals, (1) the potential difference between any two points on 2 to the potential difference between 2 and 3; (2) the potential difference between any two points on 3 to the potential difference between 2 and 3, and with 2 and 3, used as current terminals; (3) the potential difference between any two points on 1 to the potential difference between 1 and 4; and (4) the potential difference between any two points on 4 to the potential difference between 1 and 4. This condition is generally realized in standard resistors designed for precise measurements, but is seldom realized in standard resistors designed for use with large currents. In many cases the resistance depends upon the manner of connecting the current leads to the current terminals to the extent of 0.01 percent, and in exceptional cases to the extent of 1 percent.

The resistance of a two-terminal resistor may be defined as the ratio of the potential drop to the current, taken between two equipotential surfaces lying partially in the terminals of the resistor and partially in the conductors through which the current enters and leaves the resistor. From an experimental standpoint, it is better to define the resistance of a two-terminal resistor as the difference in resistance of two four-terminal conductors which are the same in all respects, except that one does and the other does not include the two-terminal resistor.

[4] It should be noted that the distinction made here between a linear and nonlinear conductor does not involve a proportionality and lack of proportionality of the potential drop to the current.

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To illustrate what is meant by this statement and to show that the resistance so defined is not strictly a constant, let it be assumed that there is at hand a short copper rod 1 cm in diameter and having plane end surfaces which are amalgamated and that the resistance of this rod is to be measured. Also let it be assumed that there are at hand two sets of cylindrical copper terminal blocks having plane amalgamated end surfaces, the diameter of one set of blocks being 1 cm and the diameter of the other set of blocks being 2 cm. Each terminal block has a flexible current lead and a flexible potential lead. If (1) the copper rod is placed between the amalgamated surfaces of the terminal blocks having the smaller diameter and the resistance of the four-terminal conductor thus constituted is measured; if (2) the copper rod is removed, the amalgamated surfaces of terminal blocks are placed in contact, and the resistance of the four-terminal conductor thus constituted is measured; and if (3) the latter measured value is subtracted from the former measured value, a value is obtained for the resistance, as defined, of the copper rod. This resistance will in general be slightly higher than that which may be calculated from the known resistivity and dimensions of the copper rod. Now, if this procedure is repeated, using the terminal blocks having the larger diameter, a still higher value will be obtained for the resistance of the copper rod, provided the contacts between the rod and terminals are equally good in the two cases, and provided also that when terminal blocks are placed together, the contact is such as to give a substantially uniform current density over the contact area. In the first case, one amalgamated contact is included in the measured resistance. In the second case, there is in addition a nonuniformity of current density in the vicinity of the contacts, which in effect increases the measured resistance.

Obviously, therefore, the resistance of a two-terminal standard resistor depends to a greater or less extent on the manner in which it is connected into a circuit. For example, when a two-terminal standard resistor is placed in an arm of a Wheatstone bridge, the bridge arm, which is a four-terminal conductor, consists of the standard and the end connectors. If the terminals of the standard and of the end connectors are amalgamated, the end connectors consist of two amalgamated surfaces or mercury cups and two terminal blocks of the bridge. With end connectors of the usual type, that is, having deep mercury cups, the resistance of the bridge arm may depend on the amount of the mercury in the cup and the position of the terminals of the standard in the cup to the extent of a few microhms. This difficulty is obviated to a considerable degree by the use of plane amalgamated surfaces, instead of mercury cups.

What is of more importance than the form of the contact surfaces is the gradual accumulation of copper amalgam in the solid phase on the terminals of standard resistors and on end connectors or contact blocks. Cases have been observed in which the removal of the amalgam in the solid phase has resulted in a lowering of the resistance by more than 10 microhms (though a definite reason for so large a change has not been found). Even when the amalgamated surfaces of the terminals of the standard resistor and of the terminal blocks with which it may be used are apparently in good condition, the resistance may be expected to depend on the resistances of the amalgamated contacts and the distribution of the current density in their vicinity to the extent of a few microhms.

### (b) TERMINAL BLOCKS OF BRIDGE

The more important terminal blocks of the NBS precision bridge and all terminal blocks of the auxiliary apparatus used in establishing known resistance ratios have four or more terminals.

A diagram of a main terminal block of the bridge which is used when the *A* arm is 25 or 100 ohms is shown in figure 11 (a). The block is constructed of copper, and terminals 1, 2, 4, and 6 are copper wires soldered in holes drilled in the underside of the block. Terminal 3 is a plane amalgamated surface (not a mercury cup), and terminal 5 is a small mercury cup. Terminals 1 and 5 are so located with respect to the other terminals that they may be considered as equivalent, unless both are used at the same time. Terminal 6 is connected to the 900-ohm section of the *A* arm. The main terminal block normally used with the *B* arm of the bridge, when this arm has a resistance of 25 ohms or 10 ohms, is similar to that shown in figure 11 (a) except that it has no terminal 6. When the Wheatstone bridge method is used, terminals 1 (or 5), 3 and 4 are used as shown in figure 11 (b). When the Thomson bridge method is used, terminals 1, 2, and 4 are used as shown in figure 11 (c). Consequently, the current distributions in these blocks are not the same in the two cases, and as a result the contributions of one terminal [illegible] to the resistance of the $A$ arm and of the other terminal block to the resistance of the $B$ arm are not the same. When the Thomson bridge method is used the resistance contributed to the $A$ arm or to the $B$ arm by its terminal block [illegible] than when the Wheatstone bridge method is used, by the four-terminal resistance (1234).

[illegible] the reading of the $A$ arm of the bridge for which the ratio of $A$ to $B$ is [illegible] is determined by the Wheatstone bridge method and this reading is used with both the Wheatstone and the Thomson bridge methods, the effect of these resistances should be known. Measurements of these resistances give values ranging from $-0.3$ to $+0.3$ microhm, depending upon the point on the amalgamated

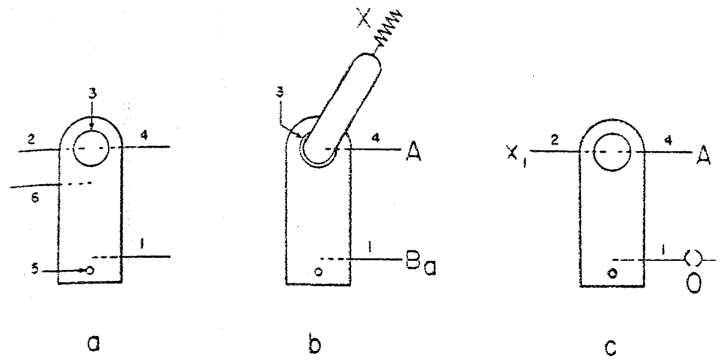


FIGURE 11.—*A main terminal block of the NBS precision bridge.*

*a* shows the six terminals of the terminal block, *b* shows the three terminals used with the Wheatstone bridge method, and *c* shows the three terminals used with the Thomson bridge method.

surface serving as terminal 3. However, as used, the connection is distributed somewhat uniformly over the central portion of the surface, so probable values lie in the range from $-0.1$ to $+0.1$ microhm. Therefore, since the lowest resistance used in the $A$ arm is 25 ohms and the lowest resistance used in the $B$ arm is 10 ohms, changes in the current distribution resulting from a change from one bridge method to another and from variations in contacts on the amalgamated surfaces affect the ratio of $A$ to $B$ only a few parts in 100 million.

The contribution of these terminal blocks to the resistances $x_1$ and $y_1$ when the Thomson bridge method is used is of no consequence, since adjustments make $(A+x_1)/(B+y_1)=(A/B)=(X/Y)$. In addition to the resistance of an amalgamated surface, these terminal blocks contribute approximately 1 microhm to the resistance of the $X$ arm and $Y$ arm of the bridge when the Wheatstone bridge method is used. However, if a precision of 1 part in 1 million is expected, the resistances of the $X$ and $Y$ arms must be 10 ohms or more, or the conductors must be of the four-terminal type. When the conductors are of the four-terminal type, all connecting conductors can be changed from one to another of two adjacent arms of the bridge [111], and this makes it possible to apply a correction to account for the resistances of connectors, including amalgamated contacts.

### (c) TERMINAL BLOCKS OF AUXILIARY APPARATUS

In the previous discussion of the auxiliary apparatus, used in determining the reading of the dial switches of the $A$ arm of the bridge for which the ratio of $A$ to $B$ is 10 to 1, very little was said concerning corrections to account for the effect of the resistances of the terminal blocks. This matter will now be considered in detail.

Each of these terminal blocks is a four-terminal conductor and consequently has two independent four-terminal resistances. One of the end terminal blocks is shown in elevation in figure 12 and in plan in figure 8. Here terminal 3 is an amalgamated surface on which a copper link rests when the three resistance sections adjacent to the terminal block are connected in parallel, terminal 4 is a part of one of the 150-ohm resistance sections, terminal 2 is a small mercury cup which is used as a potential (or current) terminal in measuring the resistance of the paralleling connection, that is, of the copper link, the two amalgamated contacts and a portion of each of two terminal blocks. Terminal 2 is also used a current or potential terminal, both when the three 150-ohm resistance sectio adjacent to it are connected in series and when they are connected in paral Terminal 1 serves in connecting the apparatus into the bridge. With the para connection two-thirds of the current is through terminal 3, one-third of the cu rent is through terminal 4, and all of the current is through terminal 2 or termina In measuring the resistance of the paralleling connection, all of the current through terminal 3 and all through terminal 1 or terminal 2. With the ser connection, all the current is through terminal 1 or terminal 2, and all throu terminal 4.

One of the terminal blocks to which two 150-ohm sections are connected shown in elevation in figure 13 and in plan in figure 8. Here terminal 3 is a amalgamated surface on which a copper link rests when the resistance sectio terminating in the block are connected in parallel, and 2 is a small mercury cu used as a terminal in measuring the resistance of the paralleling connectio

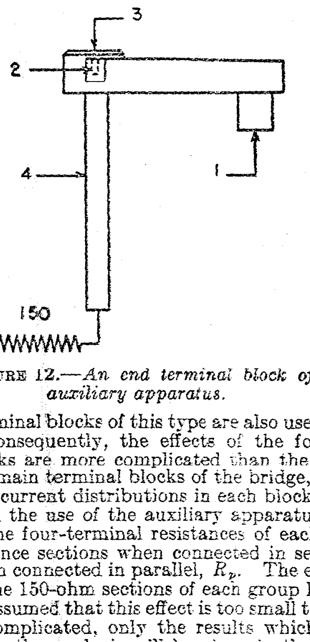


FIGURE 12.—*An end terminal block of auxiliary apparatus.*

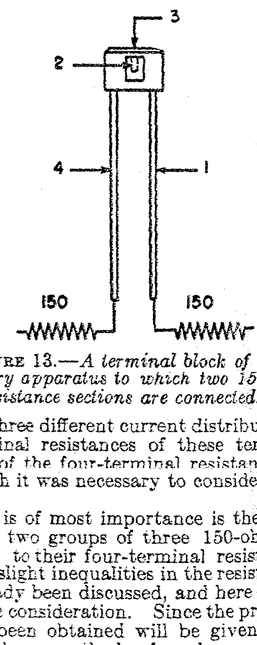


FIGURE 13.—*A terminal block of auxiliary apparatus to which two 150-ohm resistance sections are connected.*

Terminal blocks of this type are also used with three different current distributions. Consequently, the effects of the four-terminal resistances of these terminal blocks are more complicated than the effects of the four-terminal resistances of the main terminal blocks of the bridge, in which it was necessary to consider only two current distributions in each block.

In the use of the auxiliary apparatus, what is of most importance is the ratio of the four-terminal resistances of each of the two groups of three 150-ohm resistance sections when connected in series, $R_s$, to their four-terminal resistances when connected in parallel, $R_p$. The effect of slight inequalities in the resistances of the 150-ohm sections of each group has already been discussed, and here it will be assumed that this effect is too small to require consideration. Since the problem is complicated, only the results which have been obtained will be given here, while the analysis will be given in the appendix on methods of analyses. With the notation shown in figure 14, the ratio

$$R_s/R_p=9\left\{1-4[L_1+L_2+(1234)_a-(1432)_a/2-(1234)_b+ (1432)_b-(1234)_c+(1432)_c+(1234)_d-(1432)_d/2]/450\right\} \quad (57)$$

Also the resistance of terminal 1 of terminal block $a$, that is, the potential drop from 1 to 2 of $a$ to the current entering on 1 of $a$ and leaving on 1 of $d$, and which will be designated the resistance $(1_a1_a2_a1_d)$, is higher by two-thirds the resistance $(1432)_a$, and the resistance of terminal 1 of terminal block $d$, $(1_d1_d2_d1_a)$, is higher by two-thirds the resistance $(1432)_d$ with the parallel connection than with the series connection. The former is taken into account by the procedure followed in the use of the apparatus. However, the latter represents the increase in resist-

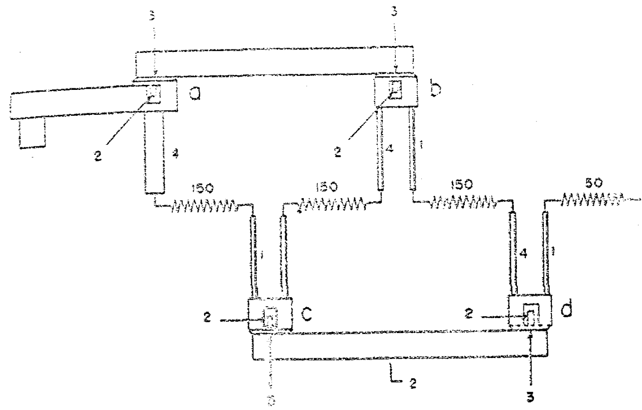


FIGURE 14.—*Parallel connection of three 150-ohm resistance sections of the auxiliary apparatus.*

This connection involves the four terminal blocks, disignated *a*, *b*, *c*, and *d*, and two paralleling connectors, designated $L_1$ and $L_2$. The series connection is made by removing the paralleling connectors.

ance of the 50-ohm section resulting from the change of the connection of three 150-ohm sections from series to parallel. In eq 57, $L_1$ represents the four-terminal resistance of the paralleling connection between terminal blocks *a* and *b*, using 1 and 2 of terminal block *a*, and 1 and 2 of terminal block *b*, as the terminals (the measurement being made with the other paralleling connection open). A similar statement applies to $L_2$, and $(1234)_a$ represents the direct resistance of terminal block *a*, $(1432)_a$ the cross resistance of terminal block *a*, etc. A similar solution applies to the other group of three 150-ohm sections. To this point, three different current distributions in each of the eight terminal blocks of the auxiliary apparatus have been considered.

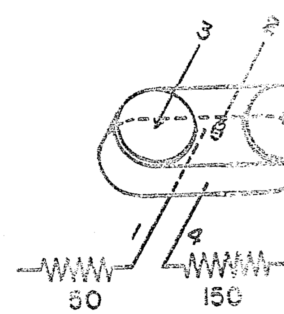


FIGURE 15.—*A central terminal block of the auxiliary apparatus.*

However, the relative values of resistance sections must be determined, and measurements of these require additional current distributions in at least some of the terminal blocks. Of these additional current distributions, the two in each of the central terminal blocks, *d* and *d′*, which are of the type shown in figure 15, are of most importance. The additional current distributions in the central terminal blocks are involved in a determination of the ratio of the resistance of the 50-ohm section to the resistance of the six 150-ohm sections connected in series. The measurements used in this determination consist in comparisons of the resistance of the 50-ohm section with the resistance of one and then of the other of the groups of three 150-ohm sections connected in parallel.

In the measurement of the resistance of one group of three 150-ohm sections connected in parallel, 1 and 2 of terminal block *a* and 5 and 2 of terminal block *d* are used as the four terminals. In the measurement of the resistance of the other group of three 150-ohm sections connected in parallel, 1 and 2 of terminal block *a′* and 5 and 2 of terminal block *d′* are used as the four terminals. These resistances will be referred to as the resistances $(1_a2_a2_d5_d)_p$ and $(1_{a'}2_{a'}2_{d'}5_{d'})_p$. The resistance of the 50-ohm section is measured, using 5 and 2 of terminal block *d* and 5 and 2 of terminal block *d′* as the four terminals. This will be referred to as the resistance $(5_d2_d2_{d'}5_{d'})$. The addition of the resistance $(2_a5_d1_d2_d)_p$ to the resistance $(1_a2_a2_d5_d)_p$ gives the resistance $(1_a2_a2_d1_d)_p$, which, when multiplied by the ratio given by eq 57, gives the resistance $(1_a2_a2_d1_d)_s$, that is, the four-terminal resistance of the three 150-ohm sections in series and with the normal distribution of current in each of the four-terminal blocks involved. The resistances of the other group of three 150-ohm sections in series is obtained in the same way. The addition of the resistances $(2_d4_d5_d2_{d'})$ and $(2_{d'}4_{d'}5_{d'}2_d)$ to the resistance $(5_d2_d2_{d'}5_{d'})$, the measured resistance of the 50-ohm section, gives the resistance $(4_d2_d2_{d'}4_{d'})$, that is, the resistance of the 50-ohm section with what may be considered as the normal distribution of current in the its terminal blocks, $d$ and $d'$. The result of these measurements may be expressed in the form

$$\frac{18R_{50}}{R_{150}+R_{150}+R_{150}+R_{150}+R_{150}+R_{150}}=1+a. \qquad (5$$

Here $a$ is the amount, expressed in proportional parts, by which the resistance of the 50-ohm section exceeds the resistance of the six 150-ohm sections in series divided by 18, with what may be considered as the normal current distribution in all terminal blocks. As has previously been pointed out, $-a/10$ is a correction which must be applied in the use of the apparatus.

In the design of these terminal blocks and in the selections of values for the resistances of sections, an effort was made to make the effects of the different current distributions in the terminal blocks negligibly small. The extent to which this aim was realized may be seen from the relations stated above and the fact that measurements of the four-terminal resistances of the terminal blocks give values for each ranging from about $-0.5$ to $+0.5$ microhm, depending mainly on the part of the amalgamated surface serving as terminal 3. However, in use, each connection is distributed somewhat uniformly over the entire amalgamated surface, and under these conditions it is hardly probable that any of the four-terminal resistances lie outside the range from $-0.3$ to $+0.3$ microhm. Presumably, therefore, changes in current distribution in terminal blocks and systematic errors in the determination of the resistances of paralleling connections of the auxiliary apparatus contribute not more than 1 part in 50 million to the uncertainty of the reading of the $A$ arm of the bridge, for which the ratio of $A$ to $B$ is 10 to 1.

Consequently, measurements of the relative values of the resistance sections and of the parelleling connections of the auxiliary apparatus may be made with any connection found convenient, provided only that each resistance section and each paralleling connection is considered as a four-terminal conductor.

What is of more importance than the resistances of the terminal blocks is the resistance of the amalgamated contacts included in the paralleling connections. Each of these contacts has a surface on the terminal block and another surface on the connecting link. The surfaces of the terminal blocks lie substantially in the same plane, and each has an area of approximately 1.25 cm$^2$. The under side of each connecting link is a plane, and the dimensions are such that when two terminal blocks are connected by a link, the entire contact areas of each of the two terminal blocks are effective. A few weeks, at most, previous to the use of the apparatus, the surfaces of terminal blocks and connecting links are polished with fine emery paper backed by a piece of plate glass or other plane surface having an area of not less than 300 cm$^2$ until the copper may be seen distinctly, indicating the removal of practically all of the accumulated copper amalgam in the solid phase. The surfaces are then reamalgamated, using the sodium-amalgam process. In placing links on the terminal blocks, they are moved back and forth parallel to the amalgamated surfaces until they slide freely indicating the removal from between the surfaces of any loose solid copper-amalgam or other solid material. With these precautions, a link may be removed and replaced by another of the same dimensions without changing the resistance of the paralleling connection by more than a few tenths of a microhm. In addition, the measured resistance of each parallel connector is in agreement with the value estimated from the dimensions and the resistivity of copper. The liberal design of these contacts, the possibility of removing the copper-amalgam in the solid phase without impairing the mechanical fit, and the exceptional precautions taken in their use result in a very much better performance than is realized with the contacts between standard resistors and terminal blocks of the NBS precision bridge.

Furthermore, if the average resistance of all contacts used in the paralleling connections were 1 microhm higher, or lower, at the time of use than at the time of measurement, this would result in an error in the determination of the reading of the $A$ arm of the bridge, for which the ratio of $A$ to $B$ is 10 to 1, of less than 1 in 25 million. Consequently, it is not necessary that the resistances of the paralleling connectors be measured when and as used.

If, for example, terminal 1 of $X$ were connected to the $A$ arm of the bridge, terminal 2 were used as a connection to the battery, and other connections were as shown in figure 16, the adjustment would establish the relation

$$\frac{A}{B}=\frac{\text{the diagonal resistance of } X}{\text{the direct resistance of } Y}. \tag{6}$$

The conductors, $X$ and $Y$, might be connected into the bridge in such a way that the adjustment would make the ratio of $A$ to $B$ equal to any one of several ratios of a four-terminal resistance of one, to a four-terminal resistance of the other. Finally, it should be noted that whether the Thomson bridge method, the potentiometer method, the Matthiessen and Hockin bridge method, or the Kohlrausch differential galvanometer method is used the same value will be found for the ratio of a four-terminal resistance of $X$ to a four-terminal resistance of $Y$, provided proper adjustments are made in each case and equal precisions are attained in these adjustments.

## APPENDIX 2. THERMOELECTROMOTIVE FORCES

In measurements to a precision of 1 part in 1 million, the potential drops in the bridge arms resulting from the test current must be balanced in many cases to 0.01 microvolt and in exceptional cases to 0.001 microvolt. However, conditions under which the measurements must be made are such that unless careful consideration were given to the thermoelectromotive forces they would often amount to a few or even several microvolts and change rather rapidly. As pointed out above, keeping the galvanometer circuit closed and judging the balance of the bridge from the changes in the deflection of the galvanometer following reversals of the test current eliminate the effect of the thermoelectromotive forces insofar as these remain constant during time intervals corresponding to a few periods of the galvanometer. While it is very desirable that the thermoelectromotive forces be kept constant, experience shows that a thermoelectromotive force which is changing slowly and at a fairly uniform rate does not preclude accurate balances of the bridge by a skillful observer. The principal means used for steadying the thermoelectromotive forces, most of which also reduce their magnitudes, are mentioned in the following numbered paragraphs:

1. Bridge arms, terminal blocks, the rheostat used in regulating the damping of the galvanometer, and a part of the leads between the bridge and galvanometer are placed under oil which is kept at very nearly a uniform temperature throughout by vigorous circulation.

2. The galvanometer is mounted on an inside wall, where it is fairly free from erratic changes in temperature.

3. A tight metal case completely surrounds the galvanometer, except that there is a glass window of sufficient size to permit of a view of the coil and a reflection of a light beam from the mirror.

4. The coil, coil terminals, suspensions, and terminals of the galvanometer, and the leads between galvanometer and connections in the oil bath are of copper, but the necessary connections between them are soldered.

5. That part of the galvanometer circuit outside the oil bath is thermoelectrically balanced insofar as this is practicable.

6. Most of the soldered connections outside the oil bath are thermally shunted, at least to some extent.

7. Thermal resistances are made high in parts of the circuit in which this is an advantage.

8. All soldered connections are protected from direct exposure to the general circulation of the air of the laboratory.

9. Soldered connections outside both the galvanometer case and the oil bath are kept to a minimum number.

10. Soldered connections outside the oil bath are made with the copper parts as nearly in direct contact as is practicable over areas equal to or larger than the cross section of the conductors and with only a sufficient amount of solder to produce a good thermal contact.

11. Abrupt changes in the thermal capacity per unit length of conductor in the vicinity of possible sources of changing thermoelectromotive force are avoided to as great an extent as is practicable.

12. Masses of material having relatively large thermal capacities are placed in the vicinity of more probable sources of changing thermoelectromotive force for the purpose of reducing the rate of change of the temperature.

13. The dial switches serving to adjust balances of the bridge operate in the oil bath, so are well lubricated, and this reduces the heat developed in the contacts by the operation of the switches.

14. The resistances in series with the dial switches are high relative to the resistances of the sections shunted. Consequently, variations of the thermoelectromotive force, introduced into the A arm of the bridge by the dial switches, are fully one order smaller than the variations of the thermoelectromotive forces in the dial switches.

Since thermoelectric balances and thermal shunts are not generally used elsewhere, an explanation of each may be in order.

### (a) THERMOELECTRIC BALANCES

If a complete electric circuit is made up in part from one kind of metal and in part from another kind of metal, there must be at least two and in any case an even number of junctions between dissimilar metals. That is, the junctions occur in pairs. When these junctions are at different temperatures usually there is a thermoelectromotive force in the circuit. However, it is not necessary that all junctions be at the same temperature to avoid a thermoelectromotive force. The thermoelectromotive force would be zero if the two junctions constituting one pair were at one temperature, the two junctions constituting another pair were at another temperature, etc. This is the end sought in making thermoelectric balances. In the simplest case a thermoelectric balance consists in making the two junctions of a pair of similar construction and of placing these two junctions in such relative positions that both will be exposed to very nearly the same ambient temperature. More generally, a thermoelectric balance consists of constructions and arrangements of parts such that some type of symmetry both from the thermal and from the thermoelectric points of view is realized for two or more of several junctions in an electric circuit. For a circuit to be thermoelectrically balanced, each junction between dissimilar metals must be taken into consideration.

For the purpose of illustrating a method of realizing a thermoelectric balance, consider a soldered connection between two copper wires of the same size which are exposed to air currents of varying temperature. The connection is made similar in the two directions parallel to the circuit. The conductor is supported from opposite walls of the laboratory, and the connection is placed at some distance from either support. Consequently, air currents heat and cool the wires on opposite sides of the connection at very nearly the same rate, and keep them at very nearly the same temperature. Therefore, the two junctions, solder to copper and copper to solder, always assume very nearly the same temperature, so both the magnitude and the rate of change of the thermoelectromotive force are very small. In this case, one of the junctions is balanced against the other.

For the purpose of illustrating another method of realizing a thermoelectric balance, consider two sections of a circuit in each of which there is a soldered connection between a copper wire 1 mm in diameter, and a copper wire 0.5 mm in diameter. The two soldered connections are made similar and so located with respect to each other that in case of a changing temperature, the rate of heat transfer from a larger to a smaller wire is the same through one as through the other of the soldered connections. As a consequence, the difference in temperature of two junctions, copper to solder and solder to copper, of one connection, is very nearly the same as that of the other connection. Therefore, the two soldered connections, each constituting a thermocouple, develop very nearly the same electromotive force. Further more the arrangement is made such that in case of a current in the circuit, this current is from the larger to the smaller wire through one of the soldered connections, and from the smaller to the larger wire through the other connection. Therefore, the electromotive force developed in one of the soldered connections is in opposition to that developed in the other. In this case, one of the thermocouples is balanced against the other.

### (b) THERMAL SHUNTS

A thermal shunt consists of an electrically insulated or insulating heat conductor which serves to equalize the temperature of two sections of a circuit between which there are possible sources of thermoelectromotive force. As an illustration, consider a soldered connection between a relatively large and a relatively small insulated copper wire that is exposed to air currents of varying temperature. Only sufficient insulation is removed from the ends of the wires to permit of soldering, after which the smaller wire is wrapped a few times around the larger wire, and this portion thoroughly impregnated with shellac. Then, although air currents may heat and cool the smaller wire more rapidly than the larger, the heat transfer between the two is mainly through the electrical insulation. The result is a material reduction in the temperature difference and rate of change of the temperature difference between the two junctions, copper to solder and solder to copper, and consequently, a material reduction in the thermoelectromotive force and its rate of change.

Thermoelectric balances and thermal shunts constitute two of the simpler and more generally applicable of the effective means of reducing troubles from thermoelectromotive forces. Furthermore, each may be made to supplement the other and sundry means employed for this purpose, such as a thermal shield the circulation of air in an enclosed space, the blocking of air currents, etc.

With the arrangement used and procedure followed in establishing balances of the bridge, thermoelectromotive forces seldom constitute a limiting factor on the precision of measurement, even when there is forced circulation of the air from the general heating system at a temperature 10°C above the temperature of the bridge and 15°C above the general temperature of the laboratory or from the refrigerating system at a temperature 15°C below the temperature of the bridge and laboratory. Only in exceptional cases is it necessary to place temporarily an obstruction to the normal circulation of the air in the vicinity of the galvanometer, the terminals of standard resistors, or the connectors extending above the surface of the oil.

## APPENDIX 3. INSULATION

The maintenance of sufficiently good insulation constitutes a troublesome factor, especially in the measurement of higher resistances. Referring to figure 1, it will be seen that the battery and galvanometer leads are at different potentials. Consequently, at least a small part of the current supplied by the battery passes through the insulation between them, and as a result each arm of the bridge is shunted by a resistance of the order of the resistance of the insulation between a battery lead and a galvanometer lead. In case the resistance of one arm of the bridge is 10,000 ohms, the resistance of the insulation between the battery lead and the galvanometer lead connected to it must be in excess of the $10^{10}$ ohms, or be compensated in some way, such for example, as by the use of a substitution method, if the error from this source is to be less than 1 part in a million. Resistances of the insulation between the battery and galvanometer leads in excess of $10^{10}$ ohms are not readily realized, and having been realized, it is not safe to assume that they are being maintained unless certain precautions are taken, and then only in case occasional checks justify the assumption. Usually it is assumed that a Wagner branch [96, 97] if properly designed and properly used, constitutes an effective means of avoiding errors which otherwise might result from slightly defective insulation. Originally, the bridge was equipped with a Wagner branch, but it was removed for reasons, the more important of which will be discussed later under the heading "Wagner branches." Since the removal of the Wagner branch, efforts have been made to maintain the resistance of the insulation between the battery branch and the galvanometer branch sufficiently high that the leakages between these branches do not constitute a limiting factor in the precision of measurement. However, if the insulation were to be sufficiently good to obviate errors which might arise as a result of passage of leakage current from direct current power circuits through the bridge, its resistances would have to be two or possibly three orders higher. While errors from this source are eliminated for the most part by the reversal of the connections to the battery used with the bridge, and this is the regular practice followed in balancing the bridge, a grounded metal guard is placed under all mechanical supports of the bridge, the battery and its leads, and the galvanometer and its leads. This is a very effective means of isolating the bridge from power circuits. This guard also serves in testing the insulation of battery and galvanometer leads. Furthermore, the power used in regulating the temperature and driving the stirring motor is supplied from an alternating current circuit, one side of which is grounded.

The means employed for securing good insulation are as follows: (1) The air of the laboratory is dried by refrigeration when this is necessary to keep the dew point well below the temperature of the oil bath and laboratory, not only while the measurements are being made but during a sufficient time in advance to thoroughly dry all insulating material exposed to the air. Incidentally, drying the air of the laboratory reduces the moisture content of the oil, and this helps in the maintenance of its insulating property. (2) Care is taken never to reduce the temperature of the oil below the dew point of the air of the laboratory, as this would result in the condensation of water in the oil. (3) The oil is selected on the basis of original freedom from acids and tendency to develop acids, is protected from strong light, is kept covered when the apparatus is not in use, and is renewed at intervals of about 2 years. (4) Most of the insulating material is protected from strong light, especially when not in use. (5) Care is taken not to permit the accumulation of excessive amounts of dust or hygroscopic sulfur compounds on the surfaces of the insulators of the hard-rubber type. (6) The bridge proper and the rheostat used in regulating the damping of the galvanometer are kept under oil. (7) The galvanometer leads have but few supports and these are of amber. (8) The galvanometer case is supported on amber posts, and the galvanometer circuit is insulated from the case. (9) The battery is supported on blocks of paraffin, and the battery leads are well insulated.

However, the resistance of the insulation of the battery and its leads, the rheostats for regulating the test current, the ammeter-voltmeter, keys, etc., generally is from two to four orders lower than the resistance of the insulation of the galvanometer and galvanometer leads. It should be noted that it is not necessary that the insulation of both the battery branch and the galvanometer branch be exceptionally good. Nevertheless, when the humidity is very low, the insulation of the entire bridge circuit is so good that there may be trouble from electrostatic actions. To avoid electrostatic effects arising from the stirring of the oil, the circulating propeller is electrically connected to the guard, and the belt driving the circulating propeller is located beneath the metal tank containing the oil bath. The more serious of the remaining sources of electrostatic effects arise from movements of the observer. In case these movements cause trouble, the guard is temporarily extended under the observer so he becomes a part of it, the observer is thoroughly insulated from the floor, or a galvanometer lead is connected to the galvanometer case. Should slight defects in the insulation be suspected, in any case, their presence or absence may be confirmed easily. If present, but not very serious, usually it is a simple matter to carry out the measurement in such manner as to avoid an error from this cause.

### (a) THEORY IN CASE OF DEFECTIVE INSULATION

The correction for the effect of slightly defective insulation will be designated as $f$. Then, following the notation used in the paper, the Wheatstone bridge equation may be written

$$X=\frac{Y}{B}A_n[1+a-a_n+f]. \qquad (62)$$

In cases in which the resistances of the battery leads and galvanometer leads are negligibly small in comparison with the resistances of the bridge arms, as is usually assumed in discussions of this subject, figure 17 represents the bridge circuit with sufficient exactness for the purpose at hand. Here $a$, $b$, $c$, and $d$ represent branch points of the bridge; $e$ represents the guard extending under all mechanical supports of the apparatus; and $m$, $n$, $o$, and $p$ represent the resistances of the insulation between leads, or branch points of the bridge and the guard. For this somewhat ideal arrangement, if the bridge is balanced, the relation between $A$, $B$, $X$, $Y$, $m$, $n$, $o$, and $p$ is

$$\frac{X(1+Ymp/F)}{Y(1+Xop/F)}=\frac{A(1+Bmn/F)}{B(1+Ano/F)}, \qquad (63)$$

where

$$F=mno+nop+opm+pmn. \qquad (64)$$

Equation 63 is readily obtained by the use of Rosens [77] star-polygon transformation. If put in the form of eq 62, and if $F$ is sufficiently large relative to $Ypm$, $Xop$, $Bnm$, and $Ano$, that terms divided by the second and higher power of $F$ may be neglected,

$$X=\frac{Y}{B}A_n[1+a-a_n+(Xop+Bmn-Ano-Ymp)/F]. \qquad (65)$$

Therefore, to a first order approximation,

$$f = \frac{Xop + Bmn - Ano - Ymp}{mno + nop + mop + mnp}. \tag{66}$$

Equation 66 shows the effect of slight defects in the insulation and the relation between these defects necessary for them to compensate each other. However, it is of little use in the solution of the particular problem at hand, since, in general, $m$, $n$, $o$, and $p$ will not be known. Furthermore, it is not sufficiently general to be applicable in all cases.

Frequently a resistance is placed in one or both battery leads to reduce the potential drop across the bridge to a value less than the electromotive force of the battery and in one or both galvanometer leads or in parallel with the galvanometer

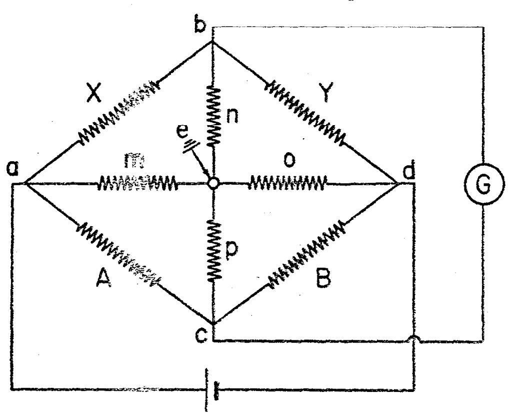


FIGURE 17.—*Wheatstone bridge circuit arranged to show the effect of slightly defective insulation of the battery leads and of the galvanometer leads.*

to adjust the damping of the galvanometer. Also major parts of the leakage through or over insulating material may be distributed near the battery and galvanometer. When this is the case, resistances in the leads make slight defects in the insulation more serious.

### (b). PROCEDURE IN CASE OF DEFECTIVE INSULATION

For such a bridge circuit as shown in fig. 18 or the simpler one (shown in fig. 17), to a first-order approximation, the correction term, $f$, of eq 62 may be considered to be the product of two factors.[5] One of these, which will be designated $J$, comprises two or more resistance ratios involving only the resistances of bridge arms, galvanometer leads, and leakage paths between galvanometer leads and the guard. The other, which will be designated $K$, is the potential drop from the guard to the branch point $b$ (or branch point $c$) divided by the potential drop from $a$ to $d$.

Following the notation used above and replacing $f$ by the product of $J$ and $K$,

$$X = Y\frac{A_n}{B}\,[1 + a - a_n + JK]. \tag{67}$$

With branch point $a$ connected to the guard

$$K_a = X/(X+Y), \tag{68}$$

so

$$X = Y\frac{A_n}{B}\,[1 + a_a - a_n + JX/(X+Y)]. \tag{69}$$

With branch point $d$ connected to the guard,

$$K_d = -Y/(X+Y), \tag{70}$$

so

$$X = Y\frac{A_n}{B}\,[1 + a_d - a_n - JY/(X+Y)]. \tag{71}$$

[5] The basis of the discussion given here will be explained in the appendix on methods of analysis first expedient.

From eq 69 and 71 it follows that

$$J = a_d - a_a. \tag{72}$$

From eq 69 and 72, or 71 and 72, it follows that

$$X = Y\frac{A_n}{B}\,[1 + (a_a Y + a_d X)/(X+Y) - a_n]. \tag{73}$$

In like manner it may be shown that if balances of the bridge are made with branch points $b$ and $c$ alternately connected to the guard

$$X = Y\frac{A_n}{B}\,[1 + (a_b A + a_c X)/(A+X) - a_n]. \tag{74}$$

Referring to eq 73 or eq 74, it will be seen that by making two balances of the bridge instead of the usual one, the effect of slight defects in the insulation of battery and galvanometer leads is taken into account. For eq 73 or 74 to give results accurate to 1 part in 1 million, $a_a - a_d$ or $a_b - a_c$ must lie in the range 0.001 to −0.001. However, these two equations are independent of each other, so usually one or the other will be applicable.

It should be pointed out that the situation is somewhat more complicated than would appear from the above discussion or an inspection of figure 18. For reasons which are stated on page 239, the balance of the bridge is judged from changes in the deflection of the galvanometer following reversals of connections to the leads of the battery. These reversals are made at points at some distances from the battery terminals. As a consequence, the direction of the potential gradient in the insulation may be reversed in some and not in others of the leakage paths, so the correction term may be different in the two cases. Consequently, when eq 73 or 74 is used an appropriate procedure should be followed in establishing the extra balances of the bridge.

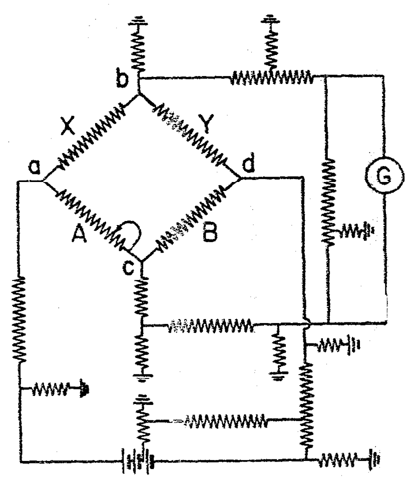


FIGURE 18.—*Wheatstone bridge circuit with leakage paths between battery leads and the guard, and between the galvanometer leads and the guard.*

### (c) TEST OF INSULATION

However, what is desired is not an evaluation of a correction term but an assurance that it is so small that it may be neglected without reducing the precision of the measurements. Keeping in mind the relations given above, it is a simple matter to more or less definitely locate defects in the insulation if these defects are really serious, although it may or may not be a simple matter to remedy the difficulty. Since the insulation of the galvanometer, galvanometer leads and rheostats used in regulating the damping of the galvanometers is generally much better than the insulation of the battery, battery leads, switches, and rheostats used in adjusting the current through the bridge, the insulation may be tested by making $K$ of eq 67 have first a positive and then a negative value somewhat larger than the value it can possibly have as the bridge is used, and noting the resulting changes, if any, in the balances of the bridge.

The value $K$ naturally assumes presumably will never lie outside the range from plus to minus the electromotive force of the battery, supplying the test current, divided by the potential drop from $a$ to $d$. Usually, therefore, an appropriate change in $K$ may be made by connecting an auxiliary battery (having an electromotive force of about one order higher than the electromotive force of the battery supplying the test current) between the guard and branch point $a$ (or $d$) and then reversing connections to this auxiliary battery. This should be done with the reversing switch in the battery branch of the bridge in one position and then repeated with the reversing switch in the other position, the battery branch being closed in both cases.

Of the locations not so far considered, at which slightly defective insulation might introduce errors, the more probable is in the auxiliary apparatus used in finding the reading of the dial switches of the $A$ arm of the bridge for which the ratio of $A$ to $B$ is 10. At the time this apparatus was constructed the importance of providing a means of testing the insulation was not appreciated. For example, should the resistance of the insulation between one of the end terminal blocks and one of the central terminal blocks be as low as $10^8$ ohms, the reading obtained would correspond to a ratio differing from 10 by about 2 parts in 1 million. However, checks by a number of standard resistors singly and then in series have not indicated any significant error from this source.

It should be pointed out that when nominally equal resistances are being compared, either by substitution one after another in the same arm of the bridge or by interchange between arms, errors which otherwise would result from slightly defective insulation are automatically eliminated to the extent that the shunting effects remain constant during a series of measurements. Also, that as the apparatus is used, leakages within the $A$ and $B$ arms of the bridge are automatically taken into account to the extent that they remain constant during a series of measurements.

In general, leakages within resistance standards have the effect of lowering their resistances, rather than limiting the precision of the measurements. In exceptional cases in which the conduction through the insulation is sufficient to result in noticeable electrolytic polarization the usual precision of measurement cannot be obtained.

It has been assumed that direct leakage from any part of the bridge circuit to points within $X$ or $Y$ is negligible. Also that direct leakage between battery and galvanometer leads is negligible. The bridge is kept under oil of good quality, and the battery and galvanometer leads are brought out, so that there is little possibility of a current from one to the other except through bridge arms or by way of the guard. So presumably these assumptions are justifiable.

### (d) WAGNER BRANCHES

It was mentioned that originally the bridge was equipped with a Wagner branch. Usually when a Wagner branch is used with a Wheatstone or other four-arm bridge, it is attached as shown in figure 19. Here $W_a$ and $W_b$ represent the arms of the Wagner branch. The bridge and the Wagner branch are adjusted by successive approximations. First, the bridge is adjusted with the key $k_w$ open, so that there is no current in the galvanometer. Second, with the galvanometer connected between branch point $b$ (or $c$) and the guard or with the galvanometer in its normal position, and the key, $k_w$, closed, the Wagner branch is so adjusted that there is no current through the galvanometer. This second adjustment, if it involves any marked changes in the ratio of $W_a$ to $W_b$, disturbs the first adjustment to some extent. Therefore, the first adjustment is repeated and this disturbs the second adjustment slightly, etc. However, successive adjustments require smaller and smaller changes, and after the changes become insignificant, the galvanometer branch is at the potential of the guard. Then there is no leakage current through the leakage path, $q$, or any arm of the bridge. Therefore, since there is no current through the galvanometer, the Wheatstone bridge equation gives the relation between $X$, $Y$, $A$, and $B$.

If this adjustment were valid with the key $k_1$ against the + battery terminal block and the key, $k_2$, against the − battery terminal block, the relation between the resistances would be as follows:

$$\frac{W_a(1+W_b/q)}{W_b}=\frac{X}{Y}=\frac{A}{B}. \tag{75}$$

However, with this adjustment the galvanometer branch will not be at the potential of the guard with both keys $k_1$ and $k_2$ against + battery terminal blocks, as shown in figure 18, or with $k_1$ against a −, and $k_2$ against a + battery terminal block. More specifically, when a switch, such as is shown in figure 19, is used in reversing the current through the bridge and this switch is located between the bridge and a leakage path between a battery lead and the guard, no adjustment can be made such that the galvanometer branch will be at the potential of the guard for all permutations of the settings of the keys $k_1$, $k_2$, and $k_w$. If then there is a leakage path from a galvanometer lead to the guard, no adjustment can be made such that there is no current through the galvanometer with the current through the bridge alternately in one and then in the other direction, when the key $k_w$ is open and also when the key $k_w$ is closed.

Notwithstanding this difficulty it is possible, in some cases only, to make an adjustment such that the relation between $X$, $Y$, $A$, and $B$ is that given by the

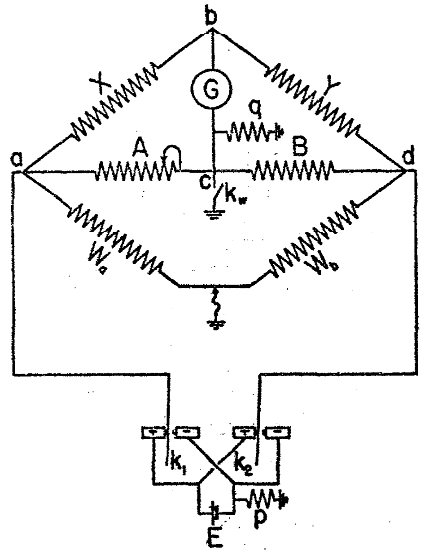


FIGURE 19.—*A Wagner branch connected in parallel with the battery branch of a Wheatstone bridge.*

$W_a$ and $W_b$ represent the arms of the Wagner branch, and $p$ and $q$ represent leakage paths between the bridge circuit and the guard.

Wheatstone bridge equation. Adjustments of the bridge and of the Wagner branch are made by successive approximations, judging balances by changes in the deflection of the galvanometer following reversals of the test current. In adjusting the bridge, the key $k_w$ is left open, while in adjusting the Wagner branch, it is kept closed. When proper adjustments have been realized there may be a fairly large current through the galvanometer with the key $k_w$ open and a different fairly large current through the galvanometer with the key $k_w$ closed. However, the magnitudes and directions of these currents are independent of the direction of the current through the bridge.

The situation is not nearly so complicated if the Wagner branch is attached to branch points $b$ and $c$ of the bridge, as is shown in figure 20. With this arrangement adjustment by successive approximations leads to a condition such that there is no current through the galvanometer with any possible combination of the settings of the keys $k_1$, $k_2$, $k_w$, and $k_w$. While this adjustment does not bring the galvanometer to the potential of the guard or prevent a leakage between a galvanometer lead and the guard, it nevertheless leads to the relation between $X$, $Y$, $A$, and $B$ given by the Wheatstone bridge equation.

This discussion of the Wagner branch is by no means comprehensive. It should however serve to show that an attachment of a Wagner branch to the NBS precision bridge would not constitute a simple and convenient means of obviating the need of good insulation.

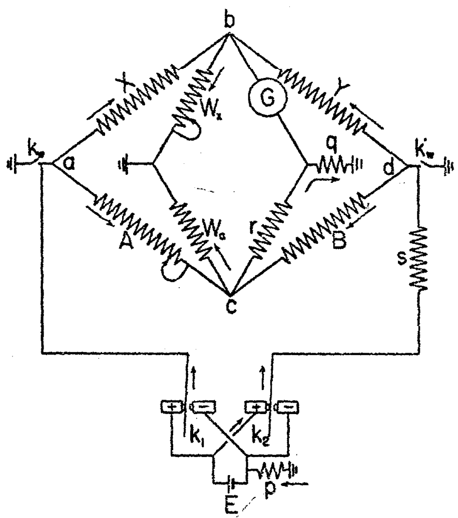


FIGURE 20.—*A Wagner branch connected in parallel with the galvanometer branch of a Wheatstone bridge*

Here the arrows indicate the direction of the leakage current in each branch of the network when the potential of all bridge arms is higher than the potential of the guard.

## APPENDIX 4. OPTICAL SYSTEMS OF GALVANOMETERS

While balances of the bridge are being established, the deflection of the galvanometer is ever changing, so it requires the concentrated attention of the observer. Watching the deflection and separating those components of the deflection resulting from a lack of balance of the bridge from those resulting from other causes constitute the most fatiguing part of a series of measurements. As fatigue is a factor affecting the precision of a series of measurements the optical system should be as suitable for the purpose as can be obtained without excessive refinements. What constitutes the best obtainable optical system depends on the skill of the available optician, personal preference, and physiological factors.

### (a) DESCRIPTION OF OPTICAL SYSTEM

In the system used in the precision measurements of resistance made in this Bureau an image of a line source of light is focused on a ground-glass scale. The scale is placed at a distance of about 1.5 m in front of the galvanometer mirror and is adjustable in height so as to accommodate it to different observers while standing. The graduations and ground surface are on the observer's side of the scale, which is a matter of very considerable importance, since it serves to almost completely eliminate direct reflections from the scale.

The light source is a 35-w straight filament lamp with a cylindrical bulb. However, it is operated far below normal intensity for reasons which will be explained presently. The lamp is enclosed in a cylindrical metal case with openings at the top and bottom for ventilation and an opening on the side which can be so adjusted as to exclude from the light beam reflected from the galvanometer mirror, images resulting from reflections within the lamp bulb. The galvanometer mirror is 1 cm in diameter and about 0.5 mm in thickness. The quality of the mirror and galvanometer window is such that practically theoretical resolving power is realized.

All direct illumination back of the ground-glass scale is avoided. Consequently, the only marked contrasts arise from the graduations and the image of the light source. The grinding is rather fine but not so fine but that both the graduations and the image are seen distinctly when viewed at an angle of as much as 20° with respect to the normal to the scale. This permits the use of both eyes and does not require that the observer keep his head in a very definitely fixed position. The graduations are fine, distinct, black lines with 1-mm spacing.

The lamp and scale are supported from a side wall and the ceiling, both of which are masonry. Also the galvanometer is supported from a side wall. Consequently, mechanical vibrations of the scale and image are seldom troublesome. With the lamp operated at normal intensity, the image appears rather wide and excessively bright. Therefore, the power supplied to the lamp is reduced somewhat below the point at which, on dull days and with the window shades open, the diffraction fringes do not contribute to the apparent width of the image and a suitable contrast is obtained. As the outside illumination changes the intensity of the light source is not changed but occasionally one or more of the window shades are closed or the lights of the laboratory turned on. The general illumination is so adjusted that the contrast between the graduations and ground glass is approximately the same as the contrast between the ground glass and image.

In cases in which the combined sensitivity of the bridge and galvanometer are barely sufficient for obtaining the desired precision, repetitions of series of measurements show that an exceptionally skillful observer can so nearly balance the bridge that that component of the electromotive force resulting from the lack of balance of the bridge does not exceed 0.001 microvolt. Since the sensitivity of the galvanometer then would be 30 mm/microvolt, it follows that 0.001 microvolt corresponds to changes in the deflection of 0.06 mm, following reversals of the direction of the test current. To associate a change in the deflection that is only slightly larger than this with a change in the direction of the test current (while the deflection is changing gradually as a result of one or more of several causes, abruptly on reversals of the test current because of a lack of a perfect inductive balance, and erratically as a result of one or more of several possible causes); requires that the deflection be observed continuously while the test current is reversed several times, at intervals corresponding approximately to the period of the galvanometer. However, a balance of the bridge to 0.001 microvolt can be made only when the thermoelectromotive forces are remaining reasonably constant, the inductances of the bridge are reasonably well balanced, local and microseismic vibrations are not troublesome, electromotive forces induced by changes of the magnetic field in the laboratory are not troublesome, and conditions otherwise are favorable.

### (b) QUALITY OF OPTICAL SYSTEMS AND SENSITIVITY OF BRIDGES

Since the optical system of the galvanometer is the last of the factors affecting the sensitivity of bridges to be discussed at this time, it is in order to give an illustration of how a consideration of most of these factors leads to a fairly definite conclusion. For this purpose let the reader assume that he has the problem of comparing a number of 10-ohm standard resistors, that he has at hand a bridge suitable for the purpose, and that he has prepared a specification for a galvanometer, but before purchasing a galvanometer wishes to know if the specification is adequate and satisfactory for the realization of a precision of 1 part in 1 million in the comparisons.

Let it be assumed that:

1. The period of the galvanometer is to be 7 seconds.
2. The galvanometer is to be critically damped with an external resistance of 60 ohms.
3. The sensitivity of the galvanometer is to be such that with 1 microvolt, in the circuit giving critical damping, the deflection is five scale divisions, with the distance between the mirror and scale that which is to be used.
4. The optical system of the galvanometer is to be of such quality that a change of the deflection of 0.1 scale division is readily detectable.
5. It is estimated that local disturbances will cause random change in the apparent equilibrium position of the galvanometer of not more than 0.1 scale division during time intervals of 7 seconds.
6. The resistance between the galvanometer terminals of the bridge is 60 ohms.
7. The comparisons are to be made by substituting the 10-ohm standard resistors successively in the $X$ arm of the bridge.
8. A 10-ohm standard resistor is to be placed in the $Y$ arm of the bridge and is not to be disturbed during the series of measurements.
9. Balances of the bridge are to be checked by a single observation of the change in the deflection of the galvanometer following a reversal of the test current, with no provision for establishing inductive balances.

10. The divergence of the load coefficients of the series of 10-ohm standard resistors, to be compared, is 10 parts in 1 million per watt dissipation.

As previously pointed out (eq $26x$), the change in the deflection of the galvanometer following a reversal of the test current is

$$2\,D\,E_x\,Y/(X+Y),$$

provided, as according to 2 and 6, the specified damping is realized without the use of a shunt. Here $E_x$, the potential drop in the $X$ arm, is not assumed to be known but is to be determined from the data given above. Since the sensitivity of the galvanometer, $D$, is five scale divisions per microvolt and $X$ and $Y$ are equal, the change in the deflection of the galvanometer resulting from 1 part in 1 million lack of balance of the bridge is five $E_x$ scale divisions, provided $E_x$ is expressed in volts.

From 1, 2, 4, 5, and 9 it follows that the bridge may be out of balance nearly by an amount corresponding to a change of the deflection of the galvanometer of 0.2 scale division without being detected. Consequently, the minimum discernible

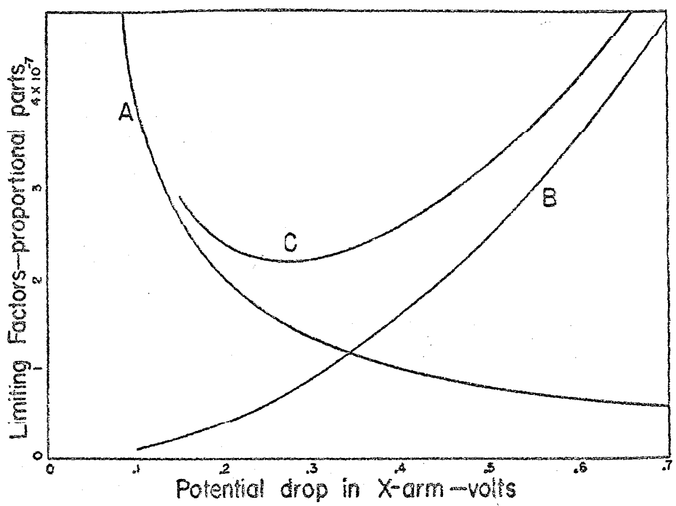


FIGURE 21.—*Sensitivity of bridge and galvanometer (an illustrative problem).*

Curve $A$ shows minimum discernible lack of balance of the bridge; curve $B$, effect of heating; and curve $C$, sum of factors limiting the sensitivity.

lack of balance of the bridge may be taken as 1 in 25 million $E_x$. Plotting the minimum discernible lack of balance of the bridge as ordinates and $E_x$ as abscissas gives curve $A$ of figure 21.

From 7 and 10, since the resistance in the $X$ arms is 10 ohms, it follows that for the series of measurements the divergence of the effects of heating is $E_x^2 \times 10^{-6}$. Plotting $E_x^2$ as ordinates and $E_x$ as abscissas gives curve $B$. Adding the ordinates of curves $A$ and $B$ gives curve $C$, the sum of the various factors limiting the sensitivity. It will be observed that the maximum of what may be called the effective sensitivity is 2.2 in 10 million, or about 1 in 4½ million, and that this sensitivity is obtained with a potential drop of approximately 0.27 volt in the $X$ arm of the bridge. Obviously this sensitivity is somewhat more than that required for a precision of 1 part in 1 million.

In this analysis it has been assumed that the combined effect of the various factors limiting the effective sensitivity is obtained by the addition of their individual effects, that is, the analysis is based on the worst, not the probable, of the assumed conditions. It may be concluded, therefore, that a galvanometer having the operating characteristics specified would be suitable for obtaining the desired precision in the series of measurements, and inasmuch as there would be no difficulty in obtaining a galvanometer having these operating constants, it may be concluded that the specification is satisfactory.

## APPENDIX 5. METHODS OF ANALYSES (THREE EXPEDIENTS)

Instead of Kirchhoff's laws or Maxwell's "mesh currents" simpler expedients have been used in this paper in the solution of network problems. For the most part these are so obvious or so well known that no explanation need be given, or reference is made to publications in which they are discussed. However, three of the expedients used deserve special mention. The first of these consists in the substitution of a battery in place of a resistance in which there is a current, the second consists in the substitution of a very high resistance and battery of very high electromotive force in place of a low (or zero) resistance and battery of low electromotive force, while the third consists in a separation and synthesis of the effects of the independent resistances of a nonlinear four-terminal conductor. This is accomplished by the substitution of first four, then five, and finally another five linear conductors in place of the nonlinear four-terminal conductor. These substitutions present no experimental difficulties, since they are made not in an actual circuit but in a diagram representing the circuit or merely in a mental picture.

### (a) CLASSICAL FORM OF SOLUTION OF UNBALANCED WHEATSTONE BRIDGE

Before considering any one of these three expedients a few of the many possible expressions for the electromotive force or current in the galvanometer circuit of an unbalanced Wheatstone bridge will be stated, for the purpose of comparison with corresponding expressions, one of which will be derived by the use of the first and another by the use of the second of these expedients.

The expression given by Maxwell and others, and which will be referred to as the "classical solution," is

$$I_g=\frac{E_bB\Delta X}{RG(A+B+X+\Delta X+Y)+R(A+X+\Delta X)(B+Y)+G(A+B)(X+\Delta X+Y)+XY(A+B)+AB(X+\Delta X+Y)+\Delta XY(A+B)}. \tag{76}$$

Here $I_g$ is the current in the galvanometer branch and $G$ is the resistance of the galvanometer branch. The significance of the other characters are as explained in connection with figure 1.

It has been shown in this paper (eq 6) that

$$E_g=\frac{EX}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right], \tag{77}$$

from which it follows [30, 71, 88, 108] that

$$I_g=\frac{EX}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right]/R_g, \tag{78}$$

where $R_g$ is the resistance to an electromotive force in the galvanometer branch, that is, the resistance of the galvanometer branch plus the resistance of the bridge between its galvanometer terminals with the galvanometer branch open and the battery branch closed.

### (b) FIRST EXPEDIENT (HYPOTHETICAL BATTERY OF ZERO RESISTANCE)

For a battery having no resistance to be equivalent to a resistance in which there is a current, the electromotive force of the battery must be equal to minus the product of the current and resistance. As simple illustrations of the application of this expedient, consider the circuit shown in figure 1. Here the current distribution in the bridge arms and galvanometer branch may be considered to depend on the potential drop from $a$ to $d$, regardless of how this potential drop is produced. We are therefore at liberty to consider that the potential drop is produced in any way which suits our convenience. One of the ways in which the potential drop might be produced would be by replacing the resistance, $R$, in the battery branch in which there is a current, $I$, by a second battery having no resistance and an electromotive force equal to $-RI$, or by replacing the entire battery branch by a battery having no resistance and an electromotive force $E_1=E_b-RI$. If this were done, the potential drop from $a$ to $d$ would not change on opening the galvanometer branch. The potential drop across the break in the galvanometer branch is given by eq 6, if $E_1$ is substituted for $E$. Another simplification accomplished by this expedient is that from the standpoint of the resistance to an electromotive force in the galvanometer branch, branch points $a$ and $d$ are connected by a conductor of zero resistance. Consequently, an expression for the resistance to an electromotive force in the galvanometer branch can be written from an inspection of figure 1, and is

$$G+\frac{AB}{A+B}+\frac{(X+\Delta X)Y}{X+\Delta X+Y}.$$

Consequently,

$$I_g=\frac{E_1X}{X+Y}\left[\frac{1+\Delta X/X}{1+\Delta X/(X+Y)}-1\right]\Big/\left(G+\frac{AB}{A+B}+\frac{(X+\Delta X)Y}{X+\Delta X+Y}\right). \tag{79}$$

This equation or its equivalent was communicated to me orally by Leo Behr without an analytical proof. However, his original analysis was the same as that given here.

This expedient, in some cases, is helpful in the solution of complicated network problems as well as simpler problems, such as that just considered. As an illustration, consider the bridge circuit shown in figure 18 and consider that the bridge is balanced. Hypothetically, the battery, battery leads, and leakage paths between the guard and battery, and guard and battery leads may be replaced by two or more batteries (each having the appropriate electromotive force) connected as shown in figure 22, or otherwise, without changing the current or potential relative to the guard at any point in the bridge arms, galvanometer leads, and leakage paths between the guard and galvanometer leads.

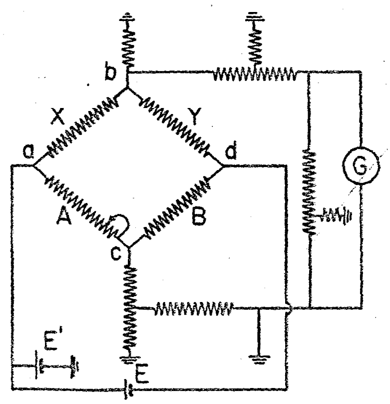


FIGURE 22.—*The battery, battery leads, and leakage paths (between battery leads and the guard only) shown in figure 18 replaced by two batteries.*

Obviously, if $E'=EX/(X+Y)$, no current is drawn from the battery whose electromotive force is $E'$ and the usual Wheatstone bridge equation gives the relation between $X$, $Y$, $A$, and $B$. However, in general the relation between $X$, $Y$, $A$, and B will depart from the Wheatstone bridge relation proportionally to $(E'/E)-(X/(X+Y))$, while the proportionality factor depends upon the relative resistances of the bridge arms, leakage paths from the galvanometer leads to the guard, and sections of the galvanometer leads. By deliberately changing $(E'/E)-(X/(X+Y))$ first to one and then to another known value and noting the resulting changes in the balance of the bridge, both the initial value of $(E'/E)-(X/(X+Y))$ and the proportionality factor may be found. This is the basis of the discussion given in the appendix on insulation.

### (c) SECOND EXPEDIENT (HYPOTHETICAL BATTERY OF HIGH ELECTROMOTIVE FORCE AND HIGH RESISTANCE)

For a high resistance and battery of high electromotive force to be equivalent to a low (or zero) resistance and battery of low electromotive force, it is necessary that the high electromotive force minus the product of the high resistance and current be equal to the low electromotive force minus the product of the low resistance and current, and that the current be the same in the two cases. As a simple illustration of the application of this expedient, consider again the circuit shown in figure 1. Here the current distribution in the bridge arms and galvanometer branch may be considered as dependent on the current in the battery branch regardless of how this current is produced. We are therefore at liberty to consider that the current is produced in any way which suits our convenience.

One of the ways in which the current $I$ in the battery branch might be produced would be by increasing the resistance $R$ indefinitely and increasing the electromotive force $E_b$ sufficiently to give this current. If this were done the galvanometer branch could be opened without changing the current $I$ in the battery branch. Furthermore, from the standpoint of an electromotive force in the galvanometer branch, the resistance of the battery branch would be infinite. Consequently, an expression for the resistance to an electromotive force in the galvanometer branch could be written from an inspection of figure 1. With the galvanometer branch open it is readily seen that the current in the $X$ arm is $I(A+B)/(X+\Delta X+Y+A+B)$ and in the $A$ arm is $I(X+\Delta X+Y)/(X+\Delta X+Y+A+B)$. These currents multiplied by the resistances $(X+\Delta X)$ and $A$ give the potential drop from $a$ to $b$ and from $a$ to $c$, while the difference between these two potential drops is the electromotive force appearing in the galvanometer circuit when the galvanometer branch is closed. With the galvanometer branch closed it is readily seen that the resistance to an electromotive force in the galvanometer branch is $G+(X+\Delta X+A)(Y+B)/(X+\Delta X+Y+A+B)$. Consequently, the current in the galvanometer branch

$$I_g=\frac{I[(X+\Delta X)(A+B)-A(X+\Delta X+Y)]/(X+\Delta X+Y+A+B)}{G+(X+\Delta X+A)(Y+B)/(X+\Delta X+Y+A+B)}, \tag{80}$$

which reduces to

$$I_g=\frac{IB\Delta X}{G(A+B+X+\Delta X+Y)+(A+X+\Delta X)(B+Y)}. \tag{81}$$

This is an equation given without proof by Jaeger [39].

Equations 76, 77, 78, 79, and 81, are exact, and two or more of them might be applied to the same bridge at the same time. That is, each of these equations is a different form of the solution of the same problem. To reduce them to a common form, eq 76 for example, would require a determination of the relation between $E$, $E_t$, $I$, and $E_b$, and of an expression for $R_g$ in terms of $G$, $X+\Delta X$, $Y$, $A$, $B$, and $R$. A comparison of the derivation of eq 76 either by the method used by Heaviside or by the method used by Maxwell with that used in deriving eq 79 and that used in deriving eq 81 would serve to show the relative simplicity of the methods used here.

### (d) THIRD EXPEDIENT (SEPARATION AND SYNTHESIS OF THE EFFECTS OF THE SIX INDEPENDENT RESISTANCES OF A FOUR-TERMINAL CONDUCTOR)

Before considering the separation and synthesis of the effects of the independent resistances of a nonlinear four-terminal conductor, it is important to know the number and nature of these resistances and the purpose of the separation and synthesis. In the appendix on terminals and contacts it was pointed out that a four-terminal conductor has but two independent four-terminal resistances. These are the direct resistance and the cross resistance, which here will be designated $D$ and $C$. The remaining independent resistances are the terminal resistances, which are of the three-terminal type. These are

$$\left.\begin{array}{l} R_1=(1124)=(1142)=(4211)=(2411)\\ R_2=(2231)=(2213)=(1322)=(3122)\\ R_3=(3342)=(3324)=(2433)=(4233) \text{ and}\\ R_4=(4431)=(4413)=(1344)=(3144).\end{array}\right\} \tag{82}$$

(See appendix on terminals and contacts for notation)

The purpose of the separation and synthesis of the effects of these six resistances is to obtain analytical solutions of complicated networks composed of linear conductors and nonlinear four-terminal conductors. Normally, little or nothing would be gained by separating the effect either individually or collectively of the terminal resistances of the four-terminal conductors from the effect of the resistances of the linear conductors connected to the terminals. Therefore the only separations which will be considered are (a) of the effect of the direct resistances from the combined effect of the cross resistances, the terminal resistances, and the resistances of the linear conductors; and (b) of the effect of the cross resistances from the combined effect of the direct resistances, the terminal resistances, and the resistances of the linear conductors. Since the solution must contain the combined effect of the terminal resistances and the resistances of linear conductors, in general it is necessary to determine three effects. These three effects may be determined by considering (1) that each nonlinear four-terminal conductor, constituting a part of a network, has four linear resistances arranged as shown in figure 23 (a), (2) that each nonlinear four-terminal conductor has five linear resistances arranged as shown in figure 23 (b), and (3) that each nonlinear four-terminal conductor has five linear resistances arranged as shown in figure 23 (c).

When in the analysis of a network problem any of the well-known procedures is followed in the treatment of the linear conductors, the first consideration gives a partial solution which is complete, except that it contains neither the effect of the direct resistances nor the effect of the cross resistances of the nonlinear four-terminal conductors. The second consideration gives a partial solution which is complete, except that it does not contain the effect of the cross resistances of the nonlinear four-terminal conductors. The third consideration gives a partial solution which is complete, except that it does not contain the effect of the direct resistances of the nonlinear four-terminal conductors.

The second partial solution minus the first partial solution gives the effect of the direct resistances, that is, the separation (a); while the third partial solution minus the first partial solution gives the effect of the cross resistances, that is, the separation (b). The addition of the effect of the direct resistances and the effect of the cross resistances to the combined effect of the terminal resistances and resistances of linear conductors constitutes a synthesis of these three effects, and consequently gives a complete solution of the network problem. Obviously, the complete solution may be obtained by the addition of the second and third, and

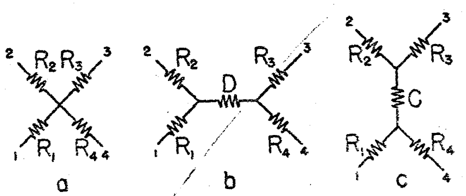


FIGURE 23.—*Hypothetical arrangements of linear resistances for separating the effect of the direct resistance and the effect of the cross resistance from the effect of the terminal resistances of a nonlinear four-terminal conductor.*

*a*, four linear resistances equivalent to the four terminal resistances of a nonlinear four-terminal conductor; *b*, five linear resistances equivalent to the direct resistance and the four terminal resistances of a nonlinear four-terminal conductor; *c*, five linear resistances equivalent to the cross resistance and the four terminal resistances of a nonlinear four-terminal conductor.

subtraction of the first of the partial solutions. If the second or third of these partial solutions contains the effect of the direct resistances or of the cross resistances as additive terms only, that is, if the separation (a) or the separation (b) occurs more or less automatically, the transfer of the additive terms occurring in one of these partial solutions to the other of these partial solutions gives the complete solution, so the first partial solution is not required. In cases in which the combined resistances of terminals of the nonlinear four-terminal conductors and linear conductors connected to the terminals are sufficiently high relative to the four-terminal resistances of the nonlinear four-terminal conductors that first order approximations of the effects of the four-terminal resistances are sufficient, the effect of the direct resistances can always be obtained as additive terms in the second partial solution, and the effect of the cross resistances can always be obtained as additive terms in the third partial solution.

To show how this method of analysis works out in practice, consider that three linear conductors having resistances of 0.43, 1, and 1.37 ohms respectively, are connected to a nonlinear four-terminal conductor, $Q$, as shown in figure 24 (a), that the six independent resistances of $Q$ are

$$R_1 = .07 \text{ ohm},$$
$$R_2 = .13 \text{ ohm},$$
$$R_3 = .05 \text{ ohm},$$
$$R_4 = .03 \text{ ohm},$$
$$D = .08 \text{ ohm, and}$$
$$C = .03 \text{ ohm},$$

and that the problem is to find first the potential drop from $M$ to 3 with 1 ampere in the circuit and second the resistance of that part of the circuit included between $M$ and $N$. To more definitely visualize these problems, it may be assumed that $Q$ consists of a circular piece of sheet metal to which terminal posts are soldered.

The first partial solution is obtained from the arrangement of linear conductors in figure 24 (b). For this arrangement the resistance between M and the junction $R_1$, $R_2$, $R_3$, and $R_4$ (see fig. 23, a) is $(.5\times1.5)/2$, or .3750 ohm. Therefore, since the current is 1 ampere, the potential drop from *M* to 3, $E_1=.3750$ volt.

The second partial solution is obtained from the arrangement of linear conductors shown in figure 24 (c). If $E_2$ represents the potential drop from *M* to 3 for

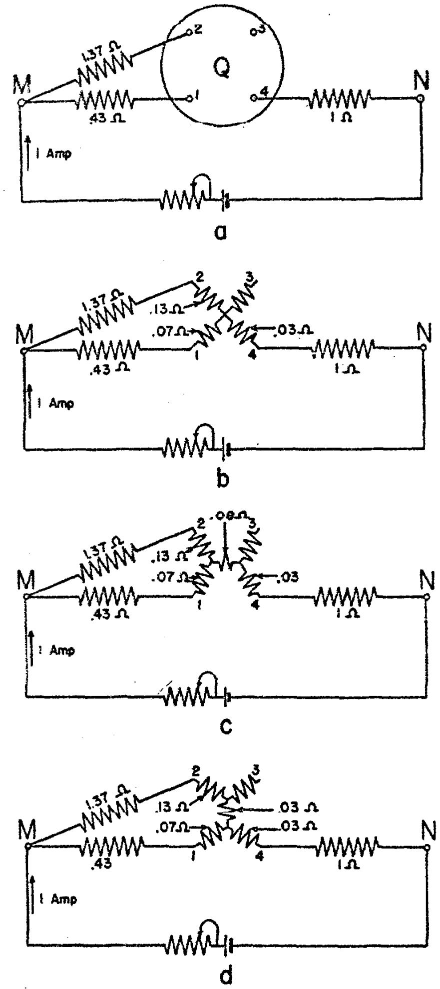


FIGURE 24.—*A simple network containing one nonlinear four-terminal conductor.*

*a*, a complete circuit in which *Q* represents the nonlinear four-terminal conductor; *b*, a circuit showing the terminal resistances of *Q* as linear resistances; *c*, a circuit showing the terminal resistances and the direct resistance of *Q* as linear resistances; and *d*, a circuit showing the terminal resistances and the cross resistance of *Q* as linear resistances.

this arrangement, obviously, $E_2$ is then larger than $E_1$ by 0.08 volt, so $E_2=.4550$ volt.

The third partial solution is obtained from the arrangement shown in figure 24 (d). If $E_3$ represents the potential drop from *M* to 3 for this arrangement, it is readily seen that the current in the upper branch is (.43+.07)/(.43+.07+.03+.13+1.37)=.2463 ampere. This current multiplied by 1.5, the resistance of the upper branch between $M$, and junction of $R_2$, $C$, and $R_3$ (see fig. 23, c) gives $E_3$=.3695 volt. Therefore, the potential drop from $M$ to 3 of the actual circuit, $E=E_2+E_3-E_1$=.4495 volt.

Since in the second partial solution the effect of the direct resistance of $Q$ appeared as an additive term, namely 0.08 volt, the first partial solution might have been omitted and the complete solution obtained by transferring this additive term to the third partial solution. This would have given $E=E_3$ +.08=.4495 volt.

Considering now the second problem and letting $r_1$, $r_2$ and $r_3$ be the resistances of that part of the circuit between $M$ and $N$ for the arrangements shown in figures 23 *b*, *c*, and *d*, it will be seen that the effect of the direct resistance, which is 0.08 ohm, will appear in the solution for $r_2$ as an additive term with a coefficient of unity. Therefore, it is not necessary to solve for $r_1$ or $r_2$.

Referring to figure 23 (d), it is readily seen that

$$r_3=\frac{(1.37+.13+.03)\ (.43+.07)}{1.37+.13+.03+.43+.07}+.03+1=1.4069 \text{ ohms.}$$

Adding to this the resistance contributed by the direct resistance of $Q$, namely .08 ohm, gives for the resistance of that part of the actual circuit between $M$ and $N$ of figure 23 (a)

$$r=1.4069+.08=1.4869 \text{ ohms.}$$

The values stated for the six independent resistances of the nonlinear four-terminal conductor, $Q$, may be realized by the use of eight linear conductors having the resistances and arrangement shown in figure 25. If therefore these eight linear conductors are substituted for the nonlinear four-terminal conductor, the problems just considered become problems in linear networks, and the solutions given may be checked by well-known methods.

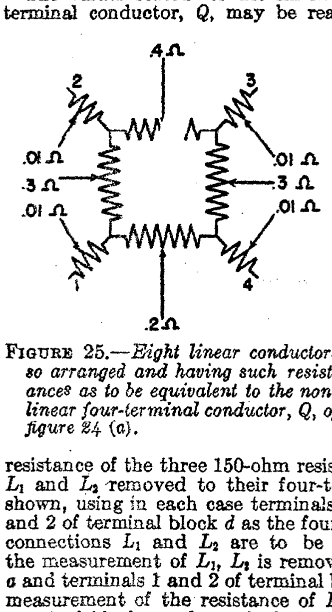


FIGURE 25.—*Eight linear conductors so arranged and having such resistances as to be equivalent to the nonlinear four-terminal conductor, Q, of figure 24 (a).*

A somewhat more complicated problem is a determination of the effects of the three different current distributions in each of the four terminal blocks involved in changing three resistance coils from a series to a parallel connection and in measuring the resistances of the paralleling connections. A solution of this problem would serve to give a better idea as to how the method of analysis under consideration works out in practice, and at the same time give an answer to a question that frequently arises in connection with precise resistance comparisons.

Referring to figure 14, the problem is to find the ratio of the four-terminal resistance of the three 150-ohm resistance sections with the paralleling connectors $L_1$ and $L_2$ removed to their four-terminal resistance with these connectors as shown, using in each case terminals 1 and 2 of terminal block *a* and terminals 1 and 2 of terminal block *d* as the four terminals. The resistances of the paralleling connections $L_1$ and $L_2$ are to be measured as four-terminal conductors. In the measurement of $L_1$, $L_2$ is removed and terminals 1 and 2 of terminal block *a* and terminals 1 and 2 of terminal block *b* are used as the four terminals. In the measurement of the resistance of $L_2$, $L_1$ is removed and terminals 1 and 2 of terminal block *c* and terminals 1 and 2 of terminal block *d* are used as the four terminals.

It will be assumed that all conductors are linear except the terminal blocks *a*, *b*, *c*, and *d*; and that each of these is a nonlinear four-terminal conductor. Aside from what is obvious, the problem resolves itself into a determination of the effects of the direct and cross resistances of the terminal blocks *a*, *b*, *c*, and *d*.

With reference to the terminal resistances of the terminal blocks, it will be seen by reference to figure 14 that $R_2$ of each terminal block is included in the measurement of $L_1$ or $L_2$, $R_1$ of terminal blocks *a* and *d* are outside the resistances under consideration, $R_1$ of terminal blocks $c$ and $d$ may be considered as a part of a 150-ohm resistance section, and in each case $R_4$ may be considered as a part of a 150-ohm resistance section. Consequently, none of the terminal resistances of any of the terminal blocks need appear explicitly in the analyses of the problem. Furthermore, the effects of the direct resistances and of the cross resistances of the terminal blocks must be small relative to the effects of the 150-ohm resistance sections, so there is no need of making the first partial solution.

Considering that the terminal blocks are replaced by linear conductors representing the direct resistances of the terminal blocks gives the arrangement shown in figure 26 (a), and considering that the terminal blocks are replaced by linear conductors representing the cross resistances gives the arrangement shown in figure 26 (b). Therefore, the second partial solution may be obtained from a

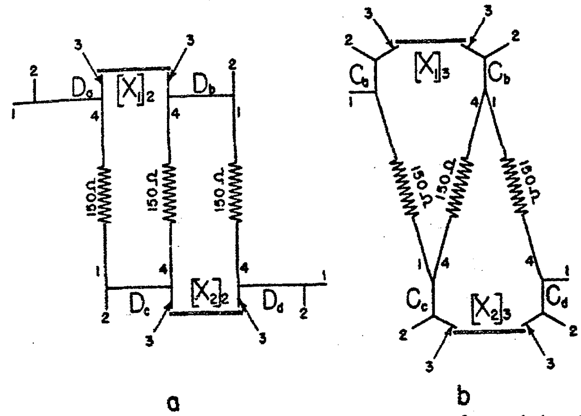


FIGURE 26.—*Two arrangements of linear resistances for determining the the effect of three different current distributions in each of the four nonlinear four-terminal blocks shown in figure 13.*

consideration of the arrangement shown in figure 26(a), and the third partial solution may be obtained from a consideration of the arrangement shown in figure 26 (b). Here $D_a$, $D_b$, $D_c$, and $D_d$ represent the direct resistances and $C_a$, $C_b$, $C_c$, and $C_d$ represent the cross resistances of the terminal blocks, $a$, $b$, $c$, and $d$.

Let

$[R_s]_2$ = the four-terminal resistance of the entire combination with the series connection, excepting that part contributed by the cross resistances $C_a$, $C_b$, $C_c$, and $C_d$ of the terminal blocks.

$[R_p]_2$ = the four-terminal resistance of the entire combination with the parallel connection, excepting that part contributed by the cross resistances.

$[X_1]_2$ and $[X_2]_2$ = the resistances of the paralleling connections, excepting that part contributed by the cross resistances.

$[R_s]_3$ = the four-terminal resistance of the entire combination with the series connection, excepting that part contributed by the direct resistances, $D_a$, $D_b$, $D_c$, and $D_d$.

$[R_p]_3$ = the four-terminal resistance of the entire combination with the parallel connection, excepting that part contributed by the direct resistances.

$[X_1]_3$ and $[X_2]_3$ = the resistances of the paralleling connections, excepting that part contributed by the direct resistances.

$L_1$ and $L_2$ = the four-terminal resistances of the paralleling connections.

$R_s$ = the four-terminal resistance of the entire combination with the series connection, and

$R_p$ = the four-terminal resistance of the entire combination with the parallel connection.

Referring to figure 26(a), it is readily seen that

$$[R_s]_2 = 450 + D_a + D_b + D_c + D_d, \tag{83}$$

$$[R_p]_2 = 50 + (9D_a + 4\,[X_1]_2 + D_b + D_c + 4\,[X_2]_2 + 9D_d)/9, \tag{84}$$

$$[X_1]_2 = L_1 - D_a - D_b, \text{ and} \tag{85}$$

$$[X_2]_2 = L_2 - D_c - D_d. \tag{86}$$

From eq 83 it follows that

$$[R_s]_2 = 450\,[1 + (D_a + D_b + D_c + D_d)/450], \tag{87}$$

and from eq 84, 85, and 86 it follows that

$$[R_p]_2 = 50\,[1 + (4L_1 + 4L_2 + 5D_a - 3D_b - 3D_c + 5D_d)/450]. \tag{88}$$

Referring to figure 26 (b), it is easily seen that

$$[R_s]_3 = 450, \tag{89}$$

$$[R_p]_3 = 50 + (4\,[X_1]_3 + 4\,[X_2]_3 - 6C_a - 6C_d)/9 \tag{90}$$

$$[X_1]_3 = L_1 + C_a + C_b, \text{ and} \tag{91}$$

$$[X_2]_3 = L_2 + C_c + C_d. \tag{92}$$

From eq 89, 90, 91, and 92 it follows that

$$[R_s]_3 = 450, \text{ and} \tag{93}$$

$$[R_p]_3 = 50 + (4L_1 + 4L_2 - 2C_a + 4C_b + 4C_c - 2C_d)/9. \tag{94}$$

Transferring the additive terms of eq 88 which contain the direct resistances to eq 94 gives

$$R_p = 50\,[1 + (4L_1 + 4L_2 + 5D_a - 2C_a - 3D_b + 4C_b - 3D_c + 4C_c + 5\,D_d - 2C_d)/450]. \tag{96}$$

Transferring the additive terms of eq 87 which contain the direct resistances to eq 93 gives

$$R_s = 450\,[1 + (D_a + D_b + D_c + D_d)/450]. \tag{95}$$

From eq 95 and 96 it follows

$$R_s/R_p = 9[1 - 4(L_1 + L_2 + D_a - C_a/2 - D_b + C_b - D_c + C_c + D_d - C_d/2)/450], \tag{97}$$

which is eq 57 in the appendix on terminals and contacts.

It is readily seen that the resistance $R_1$, of terminal block $a$ is higher by $2C_a/3$ and the resistance $R_1$, of terminal block $d$ is higher by $2C_d/3$ with the parallel connection than with the series connection. In fact, this is the reason why $C_a$ and $C_d$ appear in eq 90.

Obviously the effects of the different current distributions in each of the four-terminal terminal blocks depend on their four-terminal resistances. Furthermore, by design or adjustment, these four-terminal resistances may be made exceedingly small. As an illustration of a design having exceedingly small four-terminal resistances consider an equilateral tetrahedron, using the apices as the terminals. Also, by design, it is possible to make the four-terminal resistances definite to almost any extent desired. In cases in which the four-terminal resistances are sufficiently small to be neglected, and are equally definite, a nonlinear four-terminal conductor may be considered as equivalent to four linear conductors connected in star.

In the solution of network problems, it is frequently possible to select for the independent variable any one of two or more quantities which are dependent on each other. Making different selections leads to different forms. Then, by making approximations, still other forms are obtained of the solution. As an illustration of exact expressions for the current in the galvanometer branch of an unbalanced bridge consider eq 76, 78, 79, and 81. The second of these involves the drop in potential across the bridge with the galvanometer branch open, the third the drop in potential across the bridge with the galvanometer branch closed, the fourth the current through the bridge, while the first, that is the usual classical solution, involves the electromotive force in the battery branch. The second of these involves the resistance of the battery branch in a rather complicated way, as does also the classical solution. Therefore, if only these four were available it is conceivable that cases might arise in which it would be more convenient to use either the third or the fourth rather than the second or the classical form of solution. In many cases approximate solutions may be obtained in forms much more convenient to use than any of the forms of exact solutions, yet giving all the accuracy required.

## APPENDIX 6. OHM'S LAW

The work of Ohm contributed materially to our concepts of current, electromotive force, and potential difference; originated our concepts of resistance and resistivity; gave the generally used relation between current, electromotive force, and resistance; and led to the formulas used in expressing the resistances of combinations of conductors in terms of their individual resistances and to various laws and theorems pertaining to the distribution of current in systems of conductors. However, more than 50 years before Ohm published his more important papers, Cavendish very probably had concepts of the quantities now called potential difference, current, resistance, and of the property now called resistivity. Furthermore, from experiments with electrolytic conductors he reached the conclusion that the current is proportional to the first power of the potential difference.

During 1825, 1826, and 1827, Ohm published a number of papers in which he described measurements of the electric conductance of metal wires, drew some general conclusions from his experimental data, and made an analysis of the electric circuit. These papers led to those general ideas and relations which taken collectively may be considered to be Ohm's law. From the standpoint of resistance comparisons, the most important of these general ideas is that the resistance of a conductor is independent of the current in it, while the more important of these relations are the equations used in expressing the resistances of combinations of conductors in terms of the resistances of the individual conductors.

The various conclusions reached by Ohm were not accepted unless or until verified by others. However, by 1843, when Wheatstone published an important paper, they were more or less generally accepted. Yet the feeling persisted that more accurate measurements would show Ohm's law to be merely an approximation. In the early 1870's it was thought that definite departures from the law had been found. Accordingly, the British Association appointed a committee to investigate and report on the matter. Maxwell was the chairman of this committee, and presumably he devised the methods of test, but the measurements were made by Chrystal. A conclusion reached from an analysis of the experimental data obtained in the tests and reported by Maxwell in 1876 was that "If a conductor of iron, platinum or German silver one square centimeter in cross section has a resistance of one ohm for infinitely small currents its resistance when acted upon by an electromotive force of one volt (provided the temperature is kept the same) is not altered by so much as $1/10^{12}$ part." These tests have usually been considered as proof that Ohm's law is exact, at least to the extent likely to be significant in electrical measurements. Nevertheless, the subject is deserving of some discussion.

An expression for the potential drop between the terminals of a standard resistor (in which there is a current) that presumably is more exact than that generally used is

$$e=RI+x+y+z, \tag{98}$$

where

$e$ = the potential drop between the potential leads of the standard resistor,
$R$ = the direct resistance of the standard resistor,
$I$ = the component of the current under the control of the operator and which enters and leaves the standard resistor through the current terminals,
$x=Ri$, where $i$ is the current not under the control of the operator,
$y$ = the component of the potential difference having its origin in the standard resistor and potential leads and which is not caused by the current $I$, and
$z$ = the component of the potential drop in the standard resistor and potential leads caused by changes in the magnetic field.

Letting $E$=that component of the potential drop caused by the current, $I$, it follows from eq 98 that

$$E=RI. \tag{99}$$

Pertinent questions concerning the application of these and other equations based on Ohm's law, to standard resistors and resistance coils under normal conditions of use are:

1. To what extent is $R$ independent of $I$?
2. To what extent is eq 99 reliable?
3. To what extent are the formulas used in expressing the resistance of combinations of conductors in terms of the resistances of the individual conductors reliable?
4. To what extent can $E$ be separated from $e$?

*Question 1.*—A precision method of determining the extent to which $R$ is independent of $I$ consists in the use of a bridge (either Wheatstone or Thomson) and observing the changes in balance with changes in the magnitude of the test current and the changes in balance with the time following the establishment or reversal of the test current. The procedure followed in establishing balances must be such as to give a separation of $E$ and $e$. Questions 2 and 3 are not involved.

Obviously only relative effects of the test current upon the resistances of the conductors constituting the arms of the bridge can be determined in this way. Therefore, a quantitative conclusion as to the effect upon some particular standard resistor can be reached only after a number of observations have been made, and this conclusion must involve some assumption such as that the effect for a selected group of standard resistors is, on the average, zero, except to the extent that causes are found and the effects of these causes are taken into consideration.

In the normal use of standard resistors the test current causes the resistance material to assume a temperature above that of the surrounding medium. This results in what may be considered as the normal change in the resistance of the conductor corresponding to the difference in temperature. In addition, the difference in temperature between the resistance material and its mechanical support may cause strains in the conductor, and if so, these strains result in a further change in the resistance. These changes in resistance are at least approximately proportional to the square of the current and reach their full magnitude only after the current has been maintained a sufficient time for the difference in temperature to become substantially constant.[6] These will be referred to as the primary effects of the current upon the resistance. A procedure for determining and applying a correction for the primary effects of the test current has been considered under the heading "Load Coefficients."

In addition, the current affects the resistance, in some cases at any rate, in other ways. When a number of standard resistors are substituted one after another in the $X$ arm of the bridge and balances are established by the procedure outlined above, it not infrequently happens that the precision attainable for these balances is less with some of the standards than with others, although all may have low temperature coefficients and equal facilities for dissipating the heat developed by the test current. In cases in which the precision attainable in establishing a balance of the bridge is well below the average, it not infrequently happens that the resistance of the resistor under investigation is low immediately following a reversal of the test current and then increases more or less gradually attaining a practically constant value in a time ranging from a few seconds to a minute or more. It happens less frequently that the resistance changes in an irregular way and does not come to a practically constant value.

A comparison of the performance characteristics with the details of construction, properties of the materials used in the construction, insulation resistance, etc., frequently give a definite clue as to the cause of the changes in resistance. In some cases it appears that following a change in the current the Peltier effect, especially at junctions between copper and the resistance material, causes a local increase and a local decrease in the temperature, which results in a thermoelectromotive force. This electromotive force, after it reaches a constant value, is proportional to the current and reverses sign following a reversal of the direction of the current. In other cases, it appears that an appreciable part of the current passes through the insulation, and that this part decreases for a time (relative to

[6] It is conceivable (in cases in which the conductor is not rigidly supported) that the forces resulting from electromagnetic action might produce strains of sufficient magnitude to cause perceptible changes in the resistance. If so, these changes would appear almost instantly following a change in the magnitude of the current.

the total current) following the establishment or reversal of the test current. In still other cases it appears that there is a gradual mechanical yielding of the insulating material under the stresses caused by the temperature difference between the resistance material and its mechanical support. In such cases the effect of the test current upon the resistance may be larger at the end of 15 seconds than at the end of a longer time.

Changes in resistance resulting from the test current, whether or not the way in which they are brought about is known, and provided they are not proportional to the square of the current, will be referred to as the secondary effects of the current upon the resistance. There are many possible causes of secondary effects, so it is not possible in all cases to obtain as complete information concerning them as might be desired.

However, correlation of the secondary effects with details of construction of many resistors makes it possible to predict with some degree of certainty their magnitudes in others in which they are too small to be observed. For relatively few resistance coils and standard resistors under normal conditions of use is it probable that the secondary effects of the current upon the resistance amount to less than 1 in 100 million. On the other hand, for relatively few standard resistors having resistances in the range from 0.0001 to 10,000 ohms, if of the precision type and of good quality, are the secondary effects of the normally used test currents in excess of 1 in 100,000.

*Question 2.*—It appears that the first precise measurements having a direct bearing on the relation between a change in current from one steady value to another steady value and the resulting change in potential drop were made by F. A. Wolff of this Bureau. At a meeting of the American Association for the Advancement of Science in the summer of 1899 he presented a paper entitled "Experimental Test of the Accuracy of Ohm's Law." Unfortunately, no further record of this investigation has been found either in the form of a published paper or unpublished notes or data. The circuit used was essentially that shown in figure 27. Here $x_1$, $x_2$, $x_3$, and $x_4$ represent the four arms of a balanced Wheatstone bridge. This bridge constitutes the $X$ arm of a second Wheatstone bridge, the other arms of which are designated $Y$, $A$, and $B$. This bridge is also balanced. Dr. Wolff refers to the bridge whose arms are designated $x_1$, $x_2$, $x_3$, and $x_4$ as the auxiliary bridge, the other bridge as the main bridge, and the circuit as a bridge within a bridge.

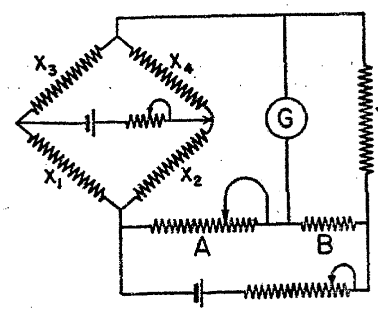


FIGURE 27.—*Wolff's circuit for testing the accuracy of Ohm's law.*

This method was tried about 1920 [105], when it gave promise of reasonably satisfactory results, though some difficulties were encountered. These difficulties were, for the most part, obviated by replacing the battery of the auxiliary bridge by a source of alternating current and omitting a provision for balancing the auxiliary bridge. Standard resistors having nominal values of 10 ohms were used as the $x_1$, $x_2$, $x_3$, and $x_4$ arms of the auxiliary bridge and the $Y$ arm of the main bridge. The direct current in the $Y$ arm was adjusted to a value somewhat larger than that regularly used in testing 10-ohm standard resistors and the alternating current in the supply branch of the auxiliary bridge was adjusted to a root mean square value equal to approximately twice the value of the direct current in the $Y$ arm of the main bridge.

Since $x_1$, $x_2$, $x_3$, and $x_4$ were approximately equal in resistance very little of the alternating current passes through the $A$, $B$, and $Y$ arms, or through the battery or galvanometer branch of the main bridge. Presumably, therefore, the alternating current had no appreciable effect except in $x_1$, $x_2$, $x_3$, and $x_4$, the arms of the auxiliary bridge. Following the usual procedure in balancing the main bridge, the precision of the balances was about 1 part in 10 million. On opening the connection to the source of the alternating current, the balance was changed by an amount not perceptibly different from that corresponding to the change in resistance of the series parallel combination of $x_1$, $x_2$, $x_3$, and $x_4$ resulting from the reduced power dissipation.

A conclusion to be drawn from this experiment, for standard resistors such as were used and currents having values over a limited range, including values normally used in testing such standards, is that part of the potential drop caused by the current is equal to the product of the resistance and current to an extent at least somewhat beyond that to which the resistance may be considered as independent of the current. It appears that if apparatus were designed especially for the purpose and the most suitable resistance wire obtainable were used, the method might be expected to yield a precision of a few parts in 100 million.

*Question 3.*—The test made by Chrystal, using "the first method which occurred to the Committee," might have given directly an answer as to the reliability of one of the formulas used in expressing the resistance of a combination of conductors in terms of the resistances of the individual conductors. In this method five resistance coils of equal resistance are compared with each other, four in a 2-and-2 series-parallel combination. According to Ohm's law, the resistance of each coil is the same as the resistance of the other four in the series-parallel combination, and on the average this would be true, even if there were slight differences in the resistances of the five coils. However, the quality of the resistors used by Chrystal was not sufficiently good to permit of measurements to what now would be considered a high accuracy.

An insignificant modification of the method consists in the comparison of the resistances of four standard resistors (of nominally equal resistance) with each other and with the four in the series-parallel combination, by alternate substitution in the same arm of a Wheatstone bridge. In a test made about 1920 [105], using 10-ohm standard resistors of good quality, this method gave reasonably satisfactory results, when corrections were applied to take into account the effects of the resistances of terminals and contacts, but not sufficiently satisfactory to be considered as a definite answer as to the reliability of one of the formulas used in expressing the resistance of a combination of conductors in terms of their individual resistances. However, there seems to be no direct or indirect experimental evidence (such as that given in the answer to question 2) not in conformity with the supposition that the resistances of combinations of conductors, as expressed in terms of their individual resistances, are reliable to the extent that the resistances of the individual conductors are definite. But to realize this accuracy it may be necessary to take into account not only the resistances of connectors but also the effects of the different current distributions in terminal blocks or the different current distributions through or over the surface of the insulation.

*Question 4.*—The extent to which the potential drop $E$ resulting from the current $I$ and the actual potential drop $e$ can be separated may be judged by deliberately introducing into a bridge circuit a disturbing electromotive force and noting the resulting change in the balance and precision of the balance. In cases in which the disturbing electromotive force amounts to only a few microvolts and is reasonably constant or is only two or three orders higher and alternating at frequencies of power distribution systems (or higher) neither the balance nor the precision of the balance is affected by amounts corresponding to as much as 0.01 microvolt. This question has been discussed more fully in appendix 2, Thermoelectromotive Forces.

This brief discussion of Ohm's law has been limited mainly to questions arising in precision resistance comparisons. It represents an attempt to coordinate one of several classes of information obtained more or less incidentally in the design, construction, and testing of standard resistors, bridges, potentiometers, etc.

## APPENDIX 7. UNITS OF RESISTANCE

A comprehensive discussion of units of resistance is not pertinent to the subject of this paper. However, some reference should be made here to the unit now known as the NBS International Ohm and the relations between this unit and some of the other units now being used or whose use is contemplated.

The International Conference at London in 1908 established an International Committee on Electrical Units and Standards to formulate a plan for and direct such work as may be necessary in connection with maintenance of standards, fixing of values—intercomparison of standards and to complete the work of the conference. This International Committee met in Washington in the spring of 1910, and one of its decisions was "to accept (for the present) as the International ohm the mean of the units of the Physikalisch-Technische Reichsanstalt and the National Physical Laboratory" which had been realized in these laboratories from the so-called mercury ohm.

Standard resistors from Germany, France, and England were compared with standard resistors of the United States from time to time during a period of about 6 weeks. Values were then assigned to all the standard resistors used in the intercomparisons in terms of the new unit. In assigning the new values, it was assumed that the mean value of one of the standard resistors from the National Physical Laboratory during the time of the intercomparisons was the same as the value previously assigned in the National Physical Laboratory, and that the mean of the values of the resistances of the two standard resistors from the Physikalisch-Technische Reichsanstalt during the time of the intercomparion was the same as the mean of the values previously assigned in the Physikalisch-Technische Reichsanstalt. Equal weights were given to the previously assigned value in the National Physical Laboratory and the mean of the previously assigned values in the Physikalisch-Technische Reichsanstalt.

Following the assignment of new values to the standard resistors of this Bureau used in the intercomparison, they were intercompared either directly or indirectly with many other standard resistors of this Bureau. From time to time since then, selected groups (usually of 20 or more) 1-ohm standard resistors have been intercompared, and on the basis of relative changes and previous good performances a group of 10 standards has been selected from the larger group. In assigning new values to all the standard resistors of the larger group and to others, it has been assumed that the mean resistance of the group of 10 had not changed since the previous similar intercomparison. This in general is the procedure by which the unit established in 1910 has been maintained in this Bureau.

From 1910 to 1929 only standard resistors of the sealed type developed by Rosa [76] were included in the groups of 10. A total of 20 different standards have been included in the groups of 10, while 3 of these 20 have been included in every group of 10. From 1932 to 1938 only standards of the smaller double-walled type [89] were included in the groups of 10. More recently, only standards of the larger double-walled type constructed by Thomas in 1933 have been included in the groups of 10.

The unit established in 1910 and maintained in this Bureau in the way described above has been referred to by different names. The name now being used is the NBS International Ohm.

Obviously, a unit of resistance maintained in this way changes with time to the extent of the proportional decrease (or increase) of the resistances of the standard resistors used in its maintenance. Therefore, it is not to be presumed that the NBS International Ohm has remained constant during the time since its establishment. To obtain information on this point various investigations have been made in this Bureau. Most of those since 1927 have been made by Thomas. In addition, groups of standard resistors have been sent abroad for measurements of their resistances in other national laboratories, and standards of other national laboratories have been measured in this Bureau. More recently these international intercomparisons have been made at the International Bureau of Weights and Measures. Comparisons of the values found for the resistances of the same standard resistors in different national laboratories have given the relations between the units of resistance of the different countries. From information as to the manner in which the units of the different countries were established and have been maintained, the results of the international intercomparisons and the investigations made in this Bureau, it may be concluded that the probable change in the NBS International Ohm during the 30 years since its establishment has been less than 30 parts in 1 million. Since the standard resistors now being used in this Bureau in the maintenance of the units are fully one order better than those formerly used for this purpose, it is reasonable to presume that the present rate of change of the NBS International Ohm is less than 1 part in 1 million per year.

The relation between the NBS International Ohm, the mean international ohm, and the units of resistance of other countries is given in table 1, which contains data taken from the 1933, 1935, and 1937 reports of the International Committee of Weights and Measures (Comité International des Poids et Mesures, Procés-Verbaux des Séances) and Comptes Rendus, page 24, volume 209, 1939.

TABLE 1.—*Units of resistance of various countries, as determined from comparison of standards at the International Bureau of Weights and Measures* [1]

[Expressed as departures in microhms from the Mean International Ohm]

| Unit of— | Third comparison, November 1933 | Fourth comparison, March 1935 | Fifth comparison, December 1936 | Sixth comparison, February 1939 |
|---|---|---|---|---|
| Germany | +10.6 | +9.8 | +6.6 | +9.1 |
| United States | −6.4 | −5.5 | −3.7 | −3.2 |
| France | +7.3 | +69.5 | | |
| | | 0.0 | +0.9 | +3.7 |
| Great Britain | −5.2 | −3.6 | −3.9 | −6.5 |
| Japan | −8.3 | −11.2 | −10.0 | −14.4 |
| Russia | +9.5 | +10.6 | | |
| | | 0.0 | −0.4 | +0.9 |

[1] In this table a plus sign signifies that the unit of the country was larger than the Mean International Ohm.

The Mean International Ohm was defined as the mean of the units of electrical resistance, in 1935, of Germany, United States, Great Britian, Japan, and Russia. After this mean was obtained, it was taken by France as the new French unit and by Russia as the new Russian unit. The data given in the table are based on the assumption that units of resistance are being maintained with equal fidelity in each of the six countries.

The unit of the United States is the unit of resistance referred to above as the NBS International Ohm. Therefore, in the early part of 1939,

1 NBS International Ohm = 0.999 997 Mean International Ohm.

From measurements made in this Bureau by Wenner, Thomas, Cooter, and Kotter, the results of which were reported to the International Bureau of Weights and Measures in December 1938,

1 NBS International Ohm = 1.000 485 absolute ohm.

From measurements made in this Bureau by Curtis, Moon, and Sparks, the results of which were reported to the International Bureau of Weights and Measures in December 1938,

1 NBS International Ohm = 1.000 484 absolute ohm. In each case the uncertainties in the measurements were estimated to be 2 parts in 100,000.

In 1937 the International Committee on Weights and Measures adopted provisionally the relation

1 Mean International Ohm = 1.000 48 absolute ohm.

From this relation and the data given in table 1, it follows that provisionally

1 NBS International Ohm = 1.000 477 absolute ohm

as of February 1939.

## APPENDIX 8. REFERENCES

This list of references is by no means comprehensive, but an effort has been made to include most of the little known publications containing contributions of importance at the time. In cases in which original publications are not readily available, references are made to later publications or to text and handbooks.

### (a) AUTHORS

The letters included in brackets at the end of each reference indicate the subjects discussed. The key to these letters is given under the heading "Subjects" p. 293.

[1] R. Appleyard, Proc. Phys. Soc. (London) **14**, 243 (1895–96) [P].
[2] W. E. Ayrton and T. Mather, Electrician **32**, 627 (1893) [G].
[3] A. C. Becquerel, Ann. Chim. **32**, 425 (1826) [D].
[4] A. Blondel, Ann. phys. [9] **10**, 219 (1918) [G].
[5] P. W. Bridgeman, Proc. Am. Acad. Arts Sci. **57**, 129 (1922) [I].
[6] H. B. Brooks, BS J. Research **4**, 297 (1930) RP150 [F].
[7] H. B. Brooks, BS J. Research **11**, 211 (1933) RP586 [O].
[8] H. B. Brooks and A. W Spinks, BS J. Research **9**, 781 (1932) RP506 [O].
[9] H. L. Callendar, Proc. Phys. Soc. (London) **22**, 220 (1909–10) [K].
[10] G. Carey-Foster, Teleg. Eng. J. **1**, 196 (1872–13) [S].
[11] C. D. Child and O. M. Stewart, Phys. Rev. **4**, 502 (1897) [K].

[12] S. H. Christie, Trans. Roy. Soc. (London) **123**, 95 (1833) [U]
[13] G. Chrystal, Brit. Assn. Adv. Sci. Rep. **46**, **38** (1876) [I].
[14] E. Dibbern, Z. Instrumentenk. 31, 105 (1911) [F].
[15] H. Diesselhorst, Z. Instrumentenk. 31, 250 (1911) [F].
[16] P. H. Dike, Rev. Sci. Instr. **2**, 379 (1931) [Q, T].
[17] I. R. Edler, Proc. Am. Acad. Arts Sci. **40**, 549 (1904–05) [S].
[18] H. Edwards, Electrotech. u. Maschinenbau **33**, 165 (1915) [G].
[19] W. Esmarch, Ann. Physik [4] 69, 255 (1922) [K, P, S].
[20] J. Fischer, Z. Instrumentenk. 54, (137) 1934) [K].
[21] J. A. Fleming, Handbook for the Electrical Laboratory and Testing Room (Electrician, London, 1901) [K, P].
[22] R. T. Glazebrook, Phil. Mag. [5] **11**, 291 (1881) [O].
[23] R. T. Glazebrook, Brit. Assn. Adv. Sci. Rep. **63**, 136 (1893) [K].
[24] R. T. Glazebrook, Phil. Mag. [5] **50**, 410 (1900) [L].
[25] A. Gray, Absolute Measurements in Electricity and Magnetism, vol. I (Macmillan, London, 1888) [E, K, P, S].
[26] T. Gray, Phil. Mag. [5] 12, 283 (1881) [K].
[27] E. H. Griffiths, Trans. Roy. Soc. (London) 184, 301 (1893) [O].
[28] C. E. Guye, Arch. Sci. phys. nat. 27, 26 (1892) [F, K].
[29] B. Hague, Alternating Current Bridge Methods (Pitman & Sons, London, 1930) [C, K, O, T, U].
[30] T. R. Harrison and P. D. Foote, Trans. Am. Inst. Elec. Eng. **39**, 390 (1920) [C].
[31] H. Hausrath, Ann. Physik. [4] **16**, 134 (1905) [D].
[32] O. Heaviside, Phil. Mag. [4] **45**, 114 (1873) [K].
[33] O. Heaviside, Phil. Mag. [4] **45**, 245 (1873) [D].
[34] O. Heaviside, Phil. Mag. [4] **46**, 469 (1873) [D].
[35] H. Helmholtz, Pogg. Ann. **89**, 211 (1853) [F].
[36] R. Hieck, Electrotech. u. Maschinenbau **40**, 158 (1922) [P].
[37] A. V. Hill, J. Sci. Instr. **11**, 309 (1934) [F].
[38] W. Jaeger, Z. Instrumentenk. **23**, 261 and 353 (1903) [F].
[39] W. Jaeger, Z. Instrumentenk. **24**, 288 (1904) [D, K].
[40] W. Jaeger, Z. Instrumentenk. **26**, 69 (1906) [D, F, J, K].
[41] W. Jaeger, Ann. Physik [4] **21**, 64 (1906) [F].
[42] W. Jaeger, Z. Instrumentenk. **26**, 360 (1906) [K].
[43] W. Jaeger, Z. Instrumentenk. **28**, 206 (1908) [F].
[44] W. Jaeger, Elektrischemesstechnik (3d ed.) (Johann Ambrosius Barth, Leipzig, (1928) [F, G, H].
[45] W. Jaeger and H. Diesselhorst, Wiss. Abhandl. physik-tech. Reichsanstalt **4**, 119 (1904) [B, P].
[46] W. Jaeger, St. Lindeck and H. Diesselhorst. Z. Instrumentenk. **23**, **33** (1908) [H, P].
[47] G. Jones, J. Am. Chem. Soc. **50**, 1049 (1928) [Q, T, U].
[48] A. E. Kennelly, Elec. World **34**, 413 (1899) [N].
[49] G. Kirchhoff, Pogg. Ann. **72**, 497 (1847) [C].
[50] G. Kirchhoff, Phil. Mag. [5] **11**, 81 (1881) [D].
[51] F. Kohlrausch, Pogg. Ann. **72**, 497 (1847) [C].
[52] F. Kohlrausch, Ann. Physik **20**, 76 (1883) [D].
[53] K. Kuepfmueller, Arch. Electrotech. **12**, 160 (1923) [N].
[54] F. A. Laws, Electrical Measurements (McGraw-Hill Book Co., New York, N. Y., 1917) [D, F, P, S].
[55] M. G. Lloyd, Proc. Eng. Soc. West. Pa. (Oct. 1903) [H, M, P].
[56] T. Mather, Phil. Mag. [5] **29**, 434 (1890) [F].
[57] J. C. Maxwell, Brit. Assn. Adv. Sci. Rep. **46**, 36 (1876) [I].
[58] J. C. Maxwell, Electricity and Magnetism, 3d ed. (Clarendon Press, Oxford, 1892) [E, K, S].
[59] Michel, Compt. rend. **189**, 1257 (1929) [F].
[60] M. F. Molikov, Mémoirs de la Chambre Centrale des Poids et Mesures de l'U. R. S. S. Présentés au Comité. Consultatif d'Électricité et Photométrie. p. 5 (1932) [J].
[61] W. J. H. Moll, Proc. Phys. Soc. (London) **35**, 253 (1923) [F].
[62] W. J. H. Moll and H. C. Burger, Phil. Mag. [6] **50**, 626 (1925) [F].
[63] E. F. Mueller and F. Wenner, J. Research NBS **15**, 477 (1935) RP842 [A].
[64] F. H. Nalder, Brit. Assn. Adv. Sci. Rep. **63**, 702 (1893) [S].
[65] E. F. Northrup, Methods of Measuring Electrical Resistance. (McGraw-Hill Book Co., New York, N. Y., 1912) [D, F, G, K, P, S].
[66] G. S. Ohm, Schweiggers J. **46**, 137 (1826) [I].
[67] G. S. Ohm, Gesammelte Abhandlungen (Johann Ambrosius Barth, Leipzig, 1892) **17**, 25 [I].

242883—40——11

[68] A. Paalzow and H. Rubens, Ann. phys. [3] **37**, 529 (1889) [K].
[69] H. A. Perkins, Phys. Rev. **18**, 131 (1921) [I].
[70] H. A. Perkins, J. phys. [6] **4**, 246 (1923) [I].
[71] Lord Rayleigh, Proc. Roy. Soc. (London) **49**, 203 (1891) [C, K].
[72] Lord Rayleigh, Trans. Roy. Soc. (London) **173**, 697 (1882) [L].
[73] E. H. Rayner, Proc. Phys. Soc. (London) **27**, 384 (1914–15) [M].
[74] J. H. Reeves, Proc. Phys. Soc. (London) **14**, 166 (1895) [B, P].
[75] H. Rohmann, Physik. Z. **14**, 203 (1913) [F].
[76] E. B. Rosa, Bul. BS **5**, 413 (1908) S107 [R].
[77] A. Rosen, J. Inst. Elec. Eng. **62**, 916 (1924) [N].
[78] A. Schuster, Phil. Mag. [5] **39**, 175 (1895) [K].
[79] L. Schwendler, Phil. Mag. [4] **31**, 364 (1866) [K].
[80] G. F. C. Searle, Electrician **66**, 999 (1911) [E].
[81] G. F. C. Searle, Electrician, **67**, 12 (1911) [M, P].
[82] F. E. Smith, Electrician, **57**, 976 (1906) [B, D, H, J, K, L, S, U].
[83] F. E. Smith, Electrician, **57**, 1009 (1906) [K, L, P, S].
[84] F. E. Smith, Dictionary of Applied Physics, R. Glazebrook edition (Macmillan, London, 1922) **2**, 718 (1922) [K, L, M, P, S].
[85] S. G. Starling, Electricity and Magnetism (Longmans, London, 1912) [K, S].
[86] H. von Steinwehr, Handbuch der Physik (Springer, Berlin, 1927) **16**, 445 [D, F, J, K, U].
[87] F. Streintz, Ann. Physik [4] **3**, 1 (1900) [S].
[88] L. Thevenin, Comptes rendus **97**, 159 (1883) [C].
[89] J. L. Thomas, BS, J. Research **5**, 295 (1930) RP201 [R].
[90] J. L. Thomas, BS., J. Research **12**, 313 (1934) RP657 [R].
[91] J. L. Thomas, J. Research NBS **13**, 681 (1934) RP737 [R].
[92] J. L. Thomas, J. Research NBS **14**, 589 (1935) RP789 [R].
[93] J. L. Thomas, J. Research NBS 16, 149 (1936) RP863 [R].
[94] W. Thomson, Phil. Mag. [4] **24**, 156 (1862) [K, P].
[95] W. Volkman, Ann. Physik [4] **10**, 217 (1903) [G].
[96] K. W. Wagner, Elektrotech. Z. **32**, 1001 (1911) [T].
[97] K. W. Wagner, Elektrotech. Z. **33**, 635 (1912) [T].
[98] C. W. Waidner, H. C. Dickinson, E. F. Mueller, and D. R. Harper, 3d. Bul. BS **11**, 575 (1915) RP241 [RU].
[99] F. Wenner, Phys. Rev. **17**, 384 (1903) [S].
[100] F. Wenner, Bul. BS **6**, 347 (1909) RP134 [C, F, K].
[101] F. Wenner, Bul. BS **8**, 559 (1912) RP181 [B, E, F, K, P, S]
[102] F. Wenner, J. Wash. Acad. Sci. **2**, 114 (1912) [R].
[103] F. Wenner, BS Sci. Pap. **12**, 469 (1915) S258 [S].
[104] F. Wenner, Bul. BS **13**, 211 (1916) S273 [F].
[105] F. Wenner, Phys. Rev. [2] **15**, 531 (1920) [I].
[106] F. Wenner, Phys. Rev. [2] **23**, 780 (1924) [I].
[107] F. Wenner, J. Opt. Soc. Am. **11**, 495 (1925) [G].
[108] F. Wenner, BS Sci. Pap. **21**, 191 (1926) S531; Proc. Phys. Soc. **39**, 124 (1927) [C].
[109] F. Wenner, N. L. Forman, and A. R. Lindberg, Phys. Rev. [2] **20**, 589 (1922) [I].
[110] F. Wenner and J. L. Thomas, BS J. Research **12**, 147 (1934) RP639 [R].
[111] F. Wenner and A. Smith, BS Sci. Pap. **19**, 297 (1924) S481 [H].
[112] F. Wenner, E. H. Smith and F. M. Soule, BS J. Research **5**, 711 (1930) RP223 [U].
[113] F. Wenner and F. M. Soule, Phys. Rev. [2] **23**, 780 (1924) [I].
[114] F. Wenner and E. Weibel. Bul. BS **11**, 1 (1914) S 223 [S].
[115] F. Wenner and E. Weibel, Bul. BS **11**, 65 (1914) S225 [B, S].
[116] F. Wenner, E. Weibel, and F. C. Weaver, Phys. Rev. [2] **3**, 497 (1914) [F].
[117] C. Wheatstone, Trans, Roy. Soc. (London) **133**, 303 (1843) [U].
[118] C. Wheatstone, Pogg. Ann. **62**, 535 (1844) [U].
[119] W. P. White, Phys. Rev. **19**, 305 (1904) [F].
[120] W. P. White, Z. Instrumentenk. **34**, 71 (1914) [A, G, O].
[121] F. A. Wolff, M. P. Shoemaker, and C. A. Briggs, Bul. BS **12**, 449 (1915–16) S256 [B, H, P].
[122] A. Zeleny, Phys. Rev. **32**, 297 (1911) [F].
[123] F. Zernike, Koninkl. Akad. Wetenschappen Amsterdam **24**, 239 (1922) [F].
[124] L. A. Zurcher and A. C. Seletzky, Trans. Am. Inst. Elec. Engrs., 1939 [K].

### (b) SUBJECTS

The numbers included in brackets at the end of each subject refer to publications in which the subject is discussed. The key to these numbers is given under the heading "Authors."

[A] Adjustable resistance elements. [63, 120].
[B] Balancing of resistances of connectors. [45, 74, 82, 101, 115, 121].
[C] Current distribution in a linear network. [29, 30, 49, 100, 108].
[D] Differential galvanometer method. [3, 31, 33, 34, 39, 40, 50, 51, 52, 54, 65, 82, 86].
[E] Four-terminal conductors. [25, 35, 58, 80, 101].
[F] Galvanometers. [6, 14, 15, 28, 37, 38, 40, 41, 43, 44, 54, 56, 59, 61, 62, 65, 75, 86, 100, 101, 104, 116, 119, 122, 123].
[G] Galvanometer shunts. [2, 4, 17, 44, 65, 95, 107, 120].
[H] Measurement of resistances of connectors. [44, 46, 55, 82, 111, 121].
[I] Ohm's law. [5, 13, 57, 66, 67, 69, 70, 105, 106, 109, 113].
[J] Potentiometer method. [40, 60, 82, 86].
[K] Sensitivity of bridges. [9, 11, 19, 20, 21, 23, 25, 26, 28, 29, 32, 39, 40, 42, 51, 58, 65, 68, 71, 78, 79, 82, 83, 84, 85, 86, 94, 100, 101, 125].
[L] Series-parallel build up. [24, 72, 82, 83, 84].
[M] Shunting of standard resistors. [55, 73, 81, 84].
[N] Star-polygon transformations. [48, 53, 77].
[O] Thermoelectromotive forces. [7, 8, 22, 27, 29, 120].
[P] Thomson bridge. [1, 19, 21, 25, 36, 45, 46, 54, 55, 65, 74, 81, 83, 84, 94, 101, 121].
[Q] Shielding. [16, 47].
[R] Standard resistors. [76, 89, 90, 91, 92, 93, 98, 102, 110].
[S] Various methods and their modifications. [10, 18, 19, 25, 54, 58, 64, 65, 82, 83, 84, 85, 87, 99, 101, 103, 114, 115].
[T] Wagner branch. [16, 29, 47, 96, 97].
[U] Wheatstone bridge. [12, 29, 47, 82, 86, 98, 112, 117, 118].