DISSIPATION FACTORS OF FUSED-SILICA CAPACITORS IN THE AUDIO FREQUENCY RANGE

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Abstract
We describe dissipation factor measurements of 10 pF fused-silica capacitance standards from 50 Hz to 20 kHz, using a toroidal cross capacitor and a 10 pF nitrogen-filled capacitor as the references. The relative combined standard uncertainties are 0.56×10^{-6}, 0.16×10^{-6}, and 0.26×10^{-6} at 100 Hz, 1 kHz, and 10 kHz, respectively.

Introduction
We have recently reported progress on determining the frequency dependence of capacitance standards in the audio frequency range [1]. This effort is in part in response to industrial needs. Recently, ultra-precision multi-frequency (from 50 Hz to 20 kHz) capacitance bridges have become commercially available and secondary calibration laboratories have started using this new type of bridge for their impedance calibrations. Another closely related calibration need is the determination of dissipation factors of capacitance standards, needed not only for calibrations of capacitance bridges, but also for LCR meters and network analyzers. New measurement capabilities of dissipation factors will also enable improved traceability of energy and power measurements and better characterization of dielectric materials.

Dissipation factors of capacitors have previously been studied at NIST for various applications. Astin [2] studied loss mechanisms of air capacitors at 60 Hz, 200 Hz, and 1000 Hz, achieving uncertainties as low as 0.5×10^{-6}.

Shields [3] established a dissipation factor standard using a 0.5 pF toroidal cross capacitor, C_{0.5}, with an estimated uncertainty of 0.02×10^{-6} at 1592 Hz. So and Shields [4] used a variable parallel-plate guard-ring capacitor as the reference for dissipation factor measurements, achieving uncertainties as low as 0.01×10^{-6} at 1592 Hz. However, these previous studies have yet to be extended to other frequencies in the audio frequency range.

Kronig–Kramers Relations
For a simple parallel-plate capacitor, the dominant loss mechanisms include the dielectric loss between the electrodes and the resistance of the electrodes and their leads. Conductivity of a dielectric can be written as two components: \sigma = \sigma_0 + \omega \varepsilon'' (\omega), where the first term results from the dc conductivity and the second term is due to dielectric relaxation with \varepsilon'' being the imaginary part of the dielectric constant. When the dc conductivity is negligible as is the case for fused-silica capacitors, the dissipation factor
\begin{equation}
\tan \delta = \frac{G}{\omega C} = \frac{\varepsilon''}{\varepsilon'},
\end{equation}
where C is the capacitance and G is the conductance of the dielectric and \varepsilon' is the real part of the dielectric constant. \varepsilon'' and \varepsilon' are related via the Kronig-Kramers relations:
\begin{equation}
\varepsilon' (\omega) - \varepsilon'' = \frac{2}{\pi} \int_0^\infty \frac{\varepsilon'' (u)}{\omega^2 - u^2} d \ln u.
\end{equation}
Approximating the integral factor \frac{u^2}{(\omega^2 - u^2)} by the unit-step function [5], we obtain
\begin{equation}
\varepsilon' (\omega) - \varepsilon'' = \frac{2}{\pi} \int_0^\infty \varepsilon'' (u) d \ln u.
\end{equation}
This derivation effectively assumes that the distribution of the dielectric relaxation times is very broad, leading to relatively flat curves of \varepsilon''(\omega) and \varepsilon'(\omega). Differentiating Eq. (3), we have
\begin{equation}
\tan \delta = -\frac{\pi \frac{d \varepsilon'}{d \omega}}{2 \varepsilon' \ln \omega}.
\end{equation}
Eq. (4) is useful for estimating the dissipation factor of a capacitance standard from its frequency dependence of capacitance.

Experiment
The ultimate reference for dissipation factor measurements at NIST is the toroidal cross capacitor, C_{0.5}, which is made of stainless steel and sealed in a vacuum housing [3]. The toroidal arrangement also contains another cylindrical 40 pF capacitance, C_{10}, between two of the four active electrodes. The electrode separation of C_{10} is about 3 mm. Since all electrodes were made from the same material and were finished and cleaned in the same manner, Shields was able to determine that C_{10} has a dissipation factor less than 0.02×10^{-6} at 1592 Hz with comparable uncertainty. The dissipation factor due to the dielectric films is inversely proportional to the electrode separation and is less than 0.15×10^{-6} in the audio frequency range when the electrodes are well cleaned and their separation is 1 mm or more [3, 6]. Comparing C_{10} with an identically made capacitor shows that the difference of their dissipation factors is within the detection limit in the audio frequency range. Comparing with another 10 pF nitrogen-filled cylindrical capacitor whose frequency dependence of capacitance had been determined earlier with respect to a 1 pF cross capacitor shows that the frequency dependence of C_{10} is no more than 0.2×10^{-6} per decade change in frequency. Using this estimate of frequency dependence in Eq. (4), we es-
timate that the dissipation factor of $C_{10}$ is less than $0.14 \times 10^{-6}$ in the audio frequency range. Simple substitution techniques are employed to measure dissipation factors of 10 pF fused-silica capacitors with respect to $C_{10}$, using ac bridges which have been described previously [1].

Results and Uncertainty Analysis

Shown in Fig. 1 is the measured dissipation factor of a 140 pF fused-silica transfer standard, $C_{112}$, as a function of frequency from 50 Hz to 20 kHz. The main sources of uncertainties for the measurements are listed on Table 1 for four representative frequencies. The Type A uncertainty, which is directly linked to the signal-to-noise ratio of the ac bridge systems and the stabilities of the standards, dominates at low frequencies. The reference standard $C_{10}$ is a four-terminal pair capacitor, and its loss due to the leads and contacts are negligible in the frequency range. However, $C_{112}$ is a three terminal capacitor and its loss due to the leads resistance is the dominant loss mechanism at high frequencies. In the frequency range from 300 Hz to 6 kHz, the uncertainties of the reference standard dominates. The relative combined standard uncertainties are shown in Fig. 1 together with the dissipation factor data.

Also shown in Fig. 1 is the estimated dissipation factor below 1592 using Eq. (4) and the frequency dependence of capacitance of $C_{112}$ measured earlier [1]. The comparison is restricted to the low frequency region where the dominant source of frequency dependence results from dielectric relaxation. The leads effect becomes significant above 1592 Hz, and we have not attempted to separate the contributions from the two sources.

![Fig. 1](image_url). Measured dissipation factor of $C_{112}$ as a function of frequency (open circles), with $1\sigma$ uncertainty bars, and calculated dissipation factor (solid triangles).

Table 1. Contribution of component uncertainties to the total uncertainty at four representative frequencies for $C_{112}$.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>100 Hz</th>
<th>400 Hz</th>
<th>1 kHz</th>
<th>10 kHz</th>
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<tr>
<td>Type A</td>
<td>0.53</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
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<tr>
<td>Reference capacitor</td>
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<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
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<tr>
<td>Contact resistance</td>
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<td>0.01</td>
<td>0.02</td>
<td>0.2</td>
</tr>
<tr>
<td>Bridge linearity errors</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Relative combined standard uncertainty</td>
<td>0.56</td>
<td>0.16</td>
<td>0.16</td>
<td>0.26</td>
</tr>
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</table>

References